EURODYN 2020
XI International Conference on Structural Dynamics
M. Papadrakakis, M. Fragiadakis, C. Papadimitriou (eds.)
Athens, Greece, 23–25 November 2020

EFFECT OF NOISE IN THE TIME-FREQUENCY ESTIMATE OF THE PERIDYNAMIC *BOND ELASTIC CONSTANT* PARAMETER

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Keywords: Structural health monitoring, Masonry structures, Peridynamics, Structural features, Gaussian noise.

Abstract. *The Peridynamic (PD) theory is a modern nonlocal (nonlinear, elastic /inelastic, without/with memory) theory able to deal with long-range forces and discontinuity in materials. For this reason the theory is suitable for the monitoring of masonry structures. Starting from a special case of PD formulation, named Bond-Based Peridynamic (BBPD), a feature obtained by the idealization of real systems with BBPD is used for SHM purposes: the bond elastic constant parameter. To characterize the damage (i.e. permanent deterioration of material and/or geometric properties of the systems) occurring in systems idealized with PD models, a joint time-frequency direct estimate of the parameter values is performed using a Short Time Fourier Transform (STFF) of the systems response and the input acceleration at the base of the systems. The method is applied numerically and the effect of noise in the time-frequency evaluation of the parameter values is analyzed. The study concludes that PD can provides simply and strong information on the health of simulated systems, allowing at the same time an easy and scalable parametrization of civil, especially masonry, structures, while the bond elastic constant parameter can be used for the damage characterization, i.e. to detect, quantify and localize the damage in a generic system.*

1 INTRODUCTION

A modern nonlocal theory of continuum established in early 2000 by Stewart Silling [1], [2] and Silling et al. [3], named *Peridynamic* (PD), is recently attracting attention in the field of computational mechanics. The theory replaces the partial differential equations of the classical continuum theory with integral, spatial, equations. This allows to easily overcome problems related to the representation of discontinuities in the matter, which can rise during damage (e.g. cracks), where the differential formulations are not defined. A comprehensive literature review of Peridynamic applications and uses can be found in [4]. Basically, PD theory found its principle in the forces-interaction (*bonds*) of points. The matter is divided in infinitesimal portions characterized by their mass and volume. Each point *k* is then supposed to interact with its neighbours inside a region. The collection of all the points inside this region is named *family* of *k*. The family of *k* is in turn characterized by the *horizon*, i.e. the maximum distance for which the interaction between *k* and the other points of a body occurs. The horizon define the locality of the behaviour of a system; the smaller the horizon the more local the behaviour will be. For further insight on PD theory and its fundamental literature one can refer to the book of Madenci and Oterkus [5].

Despite the huge amount of literature on PD theory and its applications, and the emerging literature on damage simulation in PD, studies on the use of PD theory as a paradigm of SHM are still missing. In particular an interesting field of research in this perspective is the identification of PD model parameters for the damage detection of structural systems, in support to the SHM of the built environment. In this direction, an interesting reference work is the study of Ibrahim [6] on fracture mechanics, which assessed the recent advances of Structural Life Assessment (SLA) and explained the differences between SLA and SHM in PD [4].

On this perspective, the equations of motion written with a special case of PD formulation, the Bond-Based PD (BBPD) [1], reveal a new feature that can be used as control parameter in the monitoring of civil structures. The feature can be extracted thanks to the PD integral equations of motion, which directly relate the internal material state (e.g. internal force field, etc.), with the structural response (e.g. displacements, etc.). This feature is called *bond elastic constant*, and its value should decrease with damage.

In the paper, the method for a direct time-frequency estimate [7], [8], [9] of the values of the bond elastic constant over the joint Time-Frequency (TF) domain is proposed [10] for a single bond, using time-history variables (e.g. displacements, accelerations, etc.) (see Section 2). The method is then demonstrated in Section 3 with numerical simulations on a case study: a single bond is here used to monitor the health of an idealized Single Degree of Freedom (SDOF) system connected to the ground. Conclusions are finally drawn in Section 4.

2 DIRECT TIME-FREQUENCY ESTIMATE OF BOND CONSTANT

The linearized micro-linear viscoelastic BBPD equation of motion can be written in standard form as (bold is herein used for vectors, bold capital for matrices and italic for scalars):

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{z}(t)$$

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_k & \mathbf{O} & \cdots \\ \mathbf{O} & \ddots & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix};\ \mathbf{C} = \begin{bmatrix} \sum_{\substack{j=1 \\ j\neq k}}^{K} \mathbf{C}_{kj} & -\mathbf{C}_{kj} & \cdots \\ -\mathbf{C}_{kj} & \ddots & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix};\ \mathbf{K} = \begin{bmatrix} \sum_{\substack{j=1 \\ j\neq k}}^{K} \mathbf{K}_{kj} & -\mathbf{K}_{kj} & \cdots \\ -\mathbf{K}_{kj} & \ddots & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \quad (1)$$

where:

$$\begin{aligned} \mathbf{M}_k &= m_k \mathbf{I}_3 \\ \mathbf{C}_{kj} &= \nu_{kj} \mathbf{K}_{kj} \\ \mathbf{K}_{kj} &= c_{kj} V_j V_k \boldsymbol{\Xi}_{kj} \\ \boldsymbol{\Xi}_{kj} &= \frac{\boldsymbol{\xi}_{kj}\left(\boldsymbol{\xi}_{kj}{}^{\mathrm{T}}\right)}{\left\|\boldsymbol{\xi}_{kj}\right\|^3} \\ c_{kj} &= \begin{cases} c_{kj} \text{ if } \left\|\boldsymbol{\xi}_{kj}\right\| \leq h \\ 0 \text{ if } \left\|\boldsymbol{\xi}_{kj}\right\| > h \end{cases} \\ \boldsymbol{\xi}_{kj} &= \mathbf{x}_j - \mathbf{x}_k \end{aligned} \tag{2}$$

In equations (1) and (2) $\mathbf{x}_k = (x_{k,X}, x_{k,Y}, x_{k,Z})^{\mathrm{T}}$ are the coordinates *X*, *Y*, *Z* of point *k*, c_{kj} is called bond elastic constant, and *h* is the horizon. V_j and V_k are the volumes associated to the points *j* and *k* respectively while ν_{kj} is a bond damping constant; m_k is the mass associated to the point *k*, $\mathbf{I}_3$ is a 3x3 identity matrix and **O** is a 3x3 zero matrix. *K* is the number of points used to discretize the system. Finally, $\mathbf{u}(t)$ and $\mathbf{z}(t)$ denote the displacement vector and the external force vector respectively, being *t* the time variable.

The linearized equations of motions can be written in expanded form for each point *k* as:

$$\begin{aligned} \sum_{\substack{j=1 \\ j\neq k}}^{K} \left[c_{kj} \left(\frac{V_j V_k}{m_k} \frac{\left(\boldsymbol{\xi}_{kj}\left(\boldsymbol{\xi}_{kj}{}^{\mathrm{T}}\right)\right)}{\left\|\boldsymbol{\xi}_{kj}\right\|^3} \boldsymbol{\eta}_{kj} + \nu_{kj} \frac{V_j V_k}{m_k} \frac{\left(\boldsymbol{\xi}_{kj}\left(\boldsymbol{\xi}_{kj}{}^{\mathrm{T}}\right)\right)}{\left\|\boldsymbol{\xi}_{kj}\right\|^3} \dot{\boldsymbol{\eta}}_{kj} \right) \right] &= \mathbf{n}_{k,e}(t) \\ \mathbf{n}_{k,e}(t) &= -\mathbf{a}_k(t) + \ddot{\mathbf{u}}_k(t) \\ \boldsymbol{\eta}_{kj} &= \mathbf{u}_j(t) - \mathbf{u}_k(t) \end{aligned} \tag{3}$$

where $\mathbf{u}_j(t)$ and $\mathbf{u}_k(t)$ are the coordinates *X*, *Y*, *Z* of displacement at point *j* and *k*, while $\mathbf{a}_k(t)$ are the coordinates *X*, *Y*, *Z* of the external acceleration at point *k*. If we write the equations (3) for two points (the first representing the ground and the second representing a SDOF) relativized with respect the response of the first point, we have:

$$c_{21} \left(\frac{V_1 V_2}{m_2} \frac{\left(\boldsymbol{\xi}_{21}\left(\boldsymbol{\xi}_{21}{}^{\mathrm{T}}\right)\right)}{\|\boldsymbol{\xi}_{21}\|^3} \boldsymbol{\eta}_{21} + \nu_{21} \frac{V_1 V_2}{m_2} \frac{\left(\boldsymbol{\xi}_{21}\left(\boldsymbol{\xi}_{21}{}^{\mathrm{T}}\right)\right)}{\|\boldsymbol{\xi}_{21}\|^3} \dot{\boldsymbol{\eta}}_{21} \right) = -\ddot{\mathbf{u}}_1(t) + \ddot{\mathbf{u}}_2(t) \tag{4}$$

In the assumption of all the geometrical quantities known and supposing to know the mass and damping term, we can estimate the time-frequency values of the bond elastic constant as:

$$\begin{aligned} c_{21,d}(f,t) &= \frac{\mathrm{T}_{Re}\left(n_{21,d}(t)\right)\mathrm{T}_{Re}\left(n_{2,e,d}(t)\right) + \mathrm{T}_{Im}\left(n_{21,d}(t)\right)\mathrm{T}_{Im}\left(n_{2,e,d}(t)\right)}{\mathrm{T}_{Re}\left(n_{21,d}(t)\right)^2 + \mathrm{T}_{Im}\left(n_{21,d}(t)\right)^2} \\ \mathbf{n}_{21}(t) &= \frac{V_1 V_2}{m_2} \frac{\left(\boldsymbol{\xi}_{21}\left(\boldsymbol{\xi}_{21}{}^{\mathrm{T}}\right)\right)}{\|\boldsymbol{\xi}_{21}\|^3} \boldsymbol{\eta}_{21} + \nu_{21} \frac{V_1 V_2}{m_2} \frac{\left(\boldsymbol{\xi}_{21}\left(\boldsymbol{\xi}_{21}{}^{\mathrm{T}}\right)\right)}{\|\boldsymbol{\xi}_{21}\|^3} \dot{\boldsymbol{\eta}}_{21} \\ \mathbf{n}_{2,e}(t) &= -\ddot{\mathbf{u}}_1(t) + \ddot{\mathbf{u}}_2(t) \end{aligned} \tag{5}$$

where $c_{21,d}(f,t)$ is the time-frequency estimate of the bond elastic constant obtained with the observations along the direction *d* (i.e. *X*, *Y*, or *Z*) and the real and imaginary parts of the assumed linear Time-frequency Distribution (TFD), $T_{Re}()$ and $T_{Im}()$, respectively.

3 NUMERICAL APPLICATION

In order to study the effect of noise on the identification of the bond elastic constant, this section demonstrates the identification procedure reported in Section 2 applied on signals corrupted by adding to the original signals, the 7% of uncorrelated random time-histories extracted from Gaussian distributions having the same variance of the uncorrupted signals. The east-west record of the Loma-Prieta earthquake of the October 17, 1989 in the Santa Cruz Mountains (https://it.mathworks.com/help/matlab/matlab_prog/loma-prieta-earthquake.html, sampled at 200 Hz, has been used as input acceleration applied to the assumed SDOF, which was simulated numerically in this section. For the simulation, the following values were assumed:

- $\mathbf{x}_1=(0,0,0)^T$ m;
- $\mathbf{x}_2=(1,0,0)^T$ m;
- $V_1=V_2=1$ m^3;
- $m_2=1$ kg;
- $\nu_{21}=0.0036$ s.

Then, in order to simulate the occurrence of a low level of damage, potentially mistaken for noise in signals, the bond elastic constant has been reduced of a 0.002% each time instant, starting from 16.23 s up to 17.23 s, bringing to a total reduction of about 9.52% in 1 second (corresponding to a reduction in the value of the natural frequency of 4.88%). Figure 1 depicts the imposed acceleration at the base of the SDOF and the acceleration response of the SDOF, while Table 1 reports the reference values of bond elastic constant before and after the application of damage. The explicit forward-backward Euler time integration algorithm has been used to solve the nonlinear dynamic analysis, as suggested by [5].

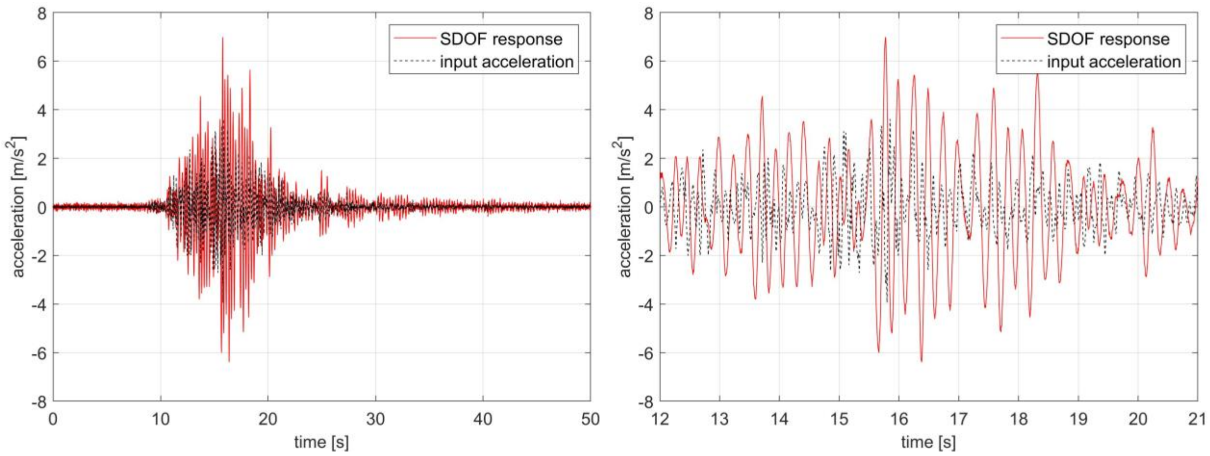

Figure 1: Accelerations obtained from the simulation (left) and zoom of the signals between 12 and 21 s (right).

c_{21} [N/m^6] before damage	c_{21} [N/m^6] after damage
636	575.462

Table 1: Reference values of bond elastic constant before and after the application of damage.

After having simulated the system, the identification procedure described in Section 2 has been applied. The TFD were calculated with a STFT using a periodic Hanning window over 128 points. The following page focuses on the discussion of the results of the procedure.

Figure 2 depicts the estimated bond elastic constant in the case of signals corrupted by 7% of uncorrelated Gaussian noise functions. From the figure is possible to conclude as the parameter estimate is mostly corrupted for joint time-frequency values that are not excited enough. This results is quite interesting because shows as the time-frequency estimate may be used as a tool to identify a time-frequency subdomain that brings the higher quantity of information in the reproduction of a specific signal. In particular, the high frequency values bring to overestimate the true values of the parameters, while the low frequencies bring to underestimate the true c_{21}. Then, it is worth noting that because very high and very low values of the bond elastic constant are associated to low values of probability, the definition of the time-frequency subdomain containing the higher quantity of information is also possible in case of lack of information on the real values of c_{kj}. From Figure 2 it is also possible to note as the estimated values of c_{21} in the area of high information content are almost close to the reference values reported in Table 1, also in case of corrupted signals.

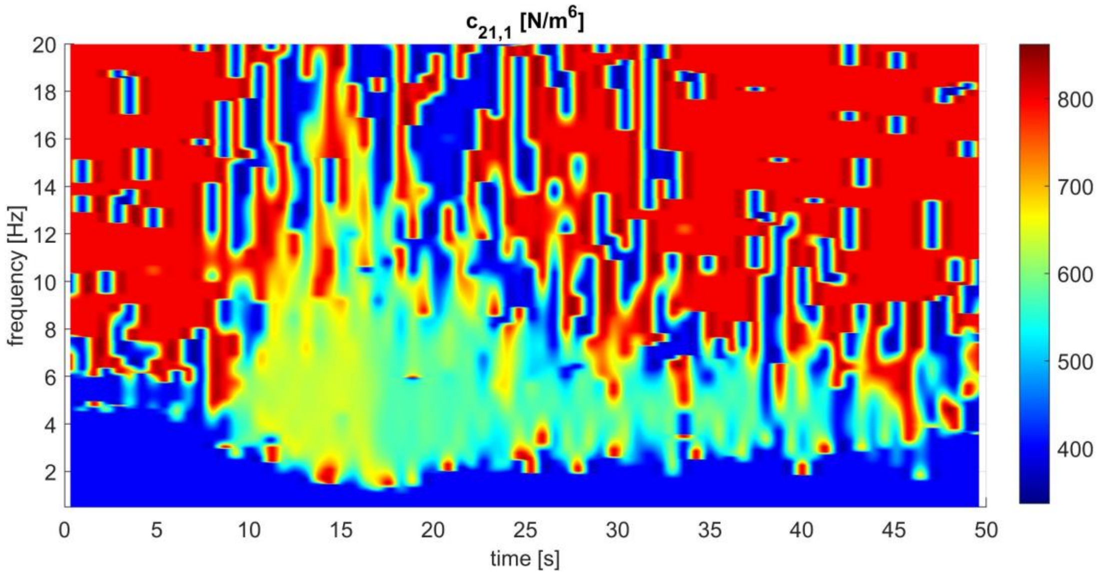

Figure 2: Time-frequency estimate of the bond elastic constant value.

4 CONCLUSIONS

Peridynamic can provides simply and strong information on the health of simulated systems. In the paper, a method to identify the *bond elastic constant* parameter of PD, over the time-frequency domain, has been proposed. The method has been demonstrated numerically on a nonlinear SDOF, whose simulated records has been corrupted by noise. The main results of the analysis are reported hereinafter:

- The method was able to estimate a correct parameter value also in presence of corrupted signals;
- The time-frequency estimate of the parameter reveals the definition of a time-frequency subdomain where the higher quantity of information for a system is located, making its use a potential instrument to define, for example, time-frequency filters specific for a given system.

Finally, it is worth mentioning that given the original aim of PD theory (emulating discontinuities and long-range forces in materials), masonry structures could greatly benefit from this PD idealization, as they are incline to crack under strong external actions, such as those ones due to earthquake.

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