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# Mode I fatigue limit of V- and U-notches

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**Abstract**

The fatigue limit of structures containing V- and U-notches is assessed through the coupled stress-energy criterion of Finite Fracture Mechanics (FFM). The analysis is limited to mode I loading conditions. The FFM criterion is a critical-distance-based approach whose implementation requires the knowledge of two material properties, namely the plain material fatigue limit and the threshold value of the stress intensity factor (SIF) range for the fatigue crack growth of long cracks. Differently from other criteria based on a critical distance, the FFM crack advance results a structural parameter, being a function also of the notch geometry. The approach is validated by a comparison with experimental notch fatigue results taken from the literature and referred to a variety of materials and geometrical configurations.



*Keywords:* Finite Fracture Mechanics; fatigue limit; mode I; U-notch; V-notch

## 1. Introduction

The coupled Finite Fracture Mechanics (FFM) criterion was introduced in the framework of static fracture, and applied to brittle elements containing cracks (Cornetti et al. 2014), notches (Leguillon 2002, Carpinteri et al. 2010, Sapora et al. 2015, Doitrand et al. 2019) and holes (Torabi et al. 2017 , Sapora and Cornetti 2018), under different loading conditions. The approach has been recently generalized in the fatigue regime to predict the fatigue limit (Sapora et al. 2020). The analysis was limited to mechanical components subjected to tensile loading conditions and

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weakened by a central sharp crack or a circular hole, thus focusing on the crack/notch sensitivity (Atzori and Lazzarin 2001). The FFM fatigue limit condition can be expressed by a system of two equations with two unknowns: the critical crack advance $l_c$, which results a structural parameter (function of both the material and the geometry of the structure), and the fatigue strength $\Delta\sigma_f$. In formulae:

$$\begin{cases} \dfrac{1}{l_c}\displaystyle\int_0^{l_c} \Delta\sigma_y(x)\,dx = \Delta\sigma_0 \\ \dfrac{1}{l_c}\displaystyle\int_0^{l_c} \Delta K_I^2(c)\,dc = \Delta K_{th}^2 \end{cases} \tag{1}$$

The goal of the present contribution is twofold: i) to estimate the mode I fatigue limit of structures weakened by sharp V-notches or U-notches by applying the coupled FFM criterion; ii) to validate the approach against experimental notch fatigue results taken from the literature and referred to a variety of materials and geometrical configurations.

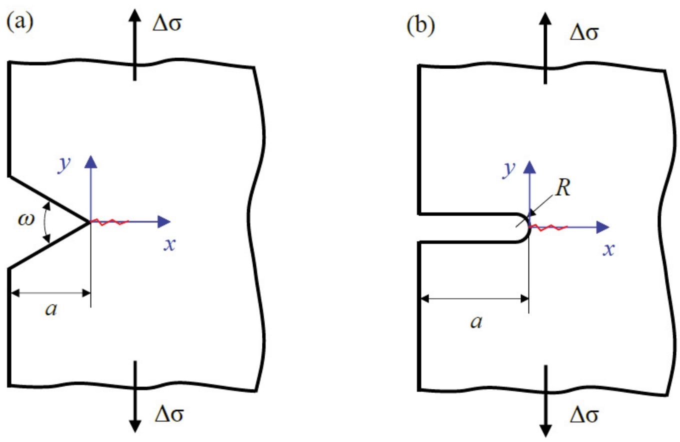


Fig. 1. Semi-infinite V-notched plate (a) and U-notched plate (b) under tensile load.

## 2. V-notched structures

The notch stress intensity factor (NSIF) $K_I^V$ represents the coefficient of the dominant term of the stress field at the notch tip. Within brittle structural behaviour, $K_I^V$ can be reasonably assumed as the governing failure parameter. The fatigue limit condition under mode I loading conditions can be thus expressed as:

$$\Delta K_I^V = \Delta K_{I,th}^V \tag{2}$$

$K_{I,th}^V$ being the threshold range of the NSIF.

In case of a semi-infinite V-notched slab ($a$ being the notch depth) under uniaxial remote tension $\Delta\sigma$ (Fig. 1a), we have that

$$\Delta K_I^V(a) = \beta(\omega)a^{1-\lambda}\Delta\sigma \tag{3}$$

$\lambda$ representing the well-known William's eigenvalue. The shape function $\beta$ related to the present geometry was evaluated by (Dunn et al. 1997): it varies from $1.12\sqrt{\pi}$ for the crack case ($\omega$ = 0°) to 1 for the plane geometry ($\omega$ = 180°).
In order to implement the FFM criterion expressed by Eq. (1), the stress field and the SIF functions are needed. The former can be approximated by the asymptotic relationship

$$\Delta\sigma_y(x) = \frac{\Delta K_I^V}{(2\pi x)^{1-\lambda}} \tag{4}$$

whereas the latter through the expression proposed by (Hasebe and Iida 1978):

$$\Delta K_I(c) = \mu(\omega)\Delta K_I^V c^{\lambda-0.5} \tag{5}$$

The parameter $\mu$ increases from unity, when $\omega$ = 0°, up to 1.12 $\sqrt{\pi}$, when Eq. (5) coincides with the formula for the SIF of an edge crack. Accurate values can be found in tabulated form in (Livieri and Tovo 2009).
The substitution of Eqs. (4) and (5) into system (1) yields :

$$\Delta K_{I,th}^V = \xi(\omega) l_{th}^{2(1-\lambda)} \Delta\sigma_0 \tag{6}$$

where

$$\xi(\omega) = \lambda^{\lambda}\left[\frac{(2\pi)^{2\lambda-1}}{\mu^2/2}\right]^{1-\lambda} \tag{7}$$

and

$$l_{th} = \left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \tag{8}$$

The crack advance $l_c$ takes the following form:

$$l_c = \frac{2}{\lambda\mu^2(2\pi)^{2(1-\lambda)}} l_{th} \tag{9}$$

Indeed, for approaches based on a critical distance it has been proved (Lazzarin and Zambardi 2001; Atzori et al. 2005; Carpinteri et al. 2010) that Eq. (6) still keeps true, but for a different definition of the function $\xi$ (Eq. (7)), which depends on the adopted criterion.

Substituting Eqs. (3) and (6) into the fatigue limit condition (2) yields:

$$\frac{\Delta\sigma_f}{\Delta\sigma_0} = \frac{\xi}{\beta\,\bar{a}^{1-\lambda}} \tag{10}$$

where $\bar{a} = a/l_{th}$ is the dimensionless notch depth. Predictions according to Eqs. (10) are reported in Fig. 2a for increasing notch amplitudes $\omega$, leading to the so-called generalized Kitagawa diagram for V-notches.

For finite geometries, the equations presented above are still valid, but for the shape function $\beta$ (see Eq. (3)), which refers to the particular structure under investigation and must be evaluated through a finite element analysis (FEA). By considering the shape functions estimated in (Atzori et al. 2005), we can apply the FFM approach to the experimental data presented in (Kihara and Yoshii 1991) on HT60 class high strength steel (full marks) and SS41 class mild steel (empty marks), respectively. The comparison is presented in Fig. 2b revealing a good agreement.

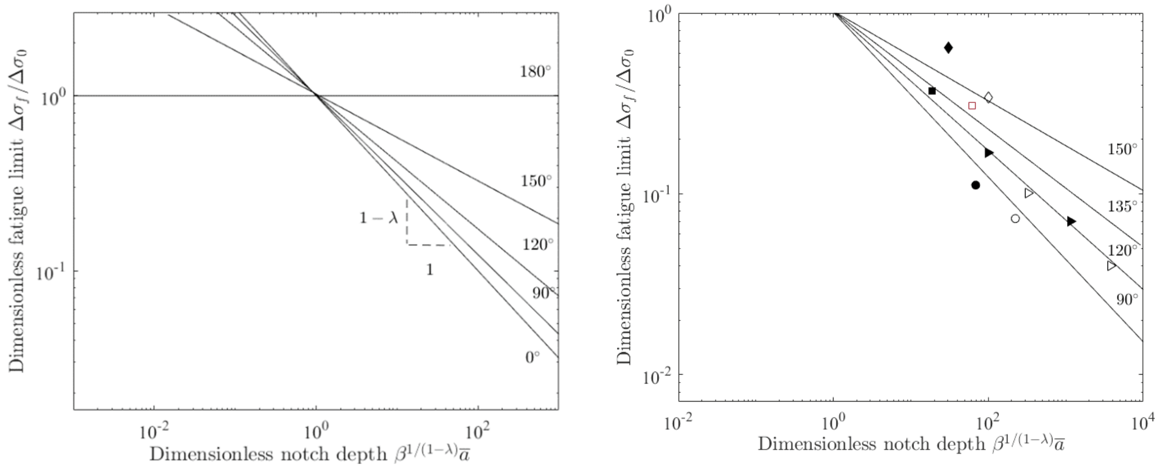


Fig.2. V-notches: FFM generalized Kitagawa diagram for different notch amplitudes $\omega$ (a); comparison with experimental data from (Kihara and Yoshii 1991) (b).

## 3. U-notched structures

Let us now consider a U-notched geometry (Figure 1b), which has already been treated by (Sapora et al. 2015) in the FFM framework. For sufficiently slender notches, the stress field can be approximated by means of (Creager and Paris 1967) relationship:

$$\Delta\sigma_y(x) = \frac{2\Delta K_I^U}{\sqrt{\pi}} \frac{x+R}{(2x+R)^{3/2}} \tag{11}$$

which provides error less than 4% as $x<R/2$.

On the contrary, the SIF can be expressed through the expression proposed by (Sapora et al. 2015):

$$\Delta K_I(c) = \left\{1+\left[\frac{R}{5.02c}\right]^n\right\}^{-\frac{1}{2n}} \Delta K_I^U \tag{12}$$

provided that the length $c$ is much smaller than the notch depth $a$. The fitting parameter $n$ was estimated equal to 1.82 by means of a FEA and the maximum percentage error is below 1%.

In Eqs. (11) and (12) $K_I^U$ represents the apparent SIF, which can be expressed as a function of the applied stress (Glinka 1985):

$$\Delta K_I^U = Y\Delta\sigma\sqrt{\pi a} \tag{13}$$

where the shape factor $Y$ depends on the considered geometry.

By introducing the notch acuity $\zeta = a / R$ and keeping it fixed, substituting Eqs. (11) and (12) into the FFM system (1) yields

$$\frac{l_c}{2a} + \frac{1}{4\zeta} = \frac{l_c l_{th}}{\pi a^2} \left( \int_0^{l_c/a} \frac{1}{\left[1 + \left(\frac{1}{5.02\zeta t}\right)^{1.82}\right]^{\frac{1}{1.82}}} dt \right)^{-1} \tag{14}$$

which is an implicit equation providing the crack advance $l_c$, and

$$\frac{\Delta\sigma_f}{\Delta\sigma_0} = \sqrt{\frac{l_c}{2Y^2 a} + \frac{1}{4Y^2\zeta}} \tag{15}$$

which provides the fatigue strength, once $l_c$ is derived from Eq. (14).

The dimensionless crack advance $\overline{l_c} = l_c / l_{th}$ is plotted in Fig. 3a, for different $\zeta$ values. For very large notch sizes $a$, the notch tip radius $R$ is large too ($\zeta$ being constant for each curve). The fatigue limit $\Delta\sigma_f$ can be estimated by the range of the peak stress at the notch tip: $\Delta\sigma_f = \Delta\sigma_0 / K_{tg}$, $K_{tg} = 2\sqrt{\zeta}$ being the stress concentration factor related to the gross section. The FFM solution is thus stress-governed, and the energy condition defines the crack advance. On the other hand, as the size $a$ decreases (and so the radius $R$), the notch is equivalent to a long crack of the same size and, therefore, the fatigue limit is dictated by LEFM: $\Delta K_I = \Delta K_{th}$. The fatigue limit according to FFM is energy driven, and the stress condition defines the crack advance. Fatigue limit predictions are presented in Fig. 3b, together with experimental results summarized in (Atzori et al. 2005) from (Harkegard 1981; Nisitani and Endo 1988; Lazzarin et al. 1997). The matching reveals again satisfactory.

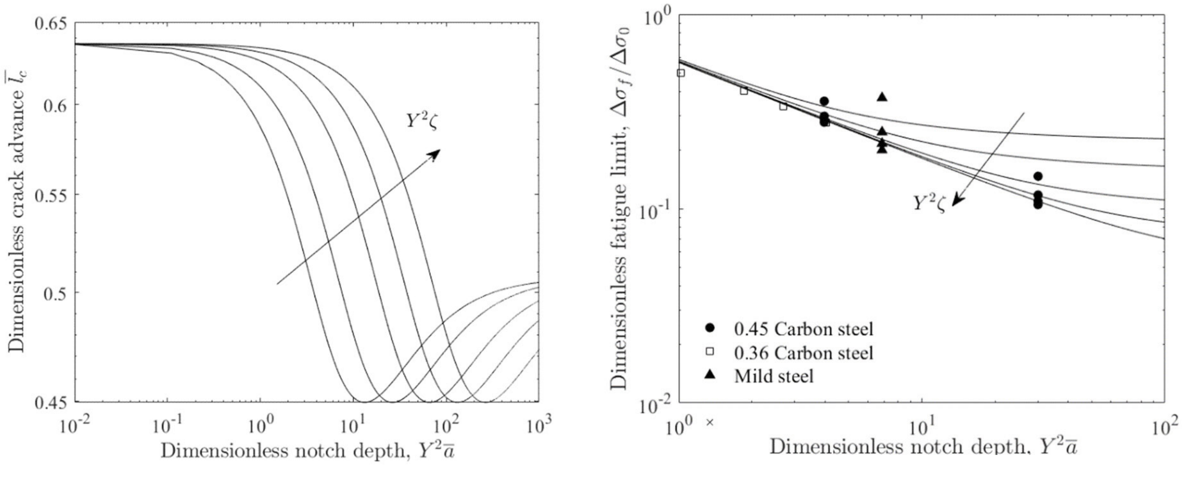


Fig. 3. U-notches: FFM crack advance (a) and fatigue limit (b) referring to different values $Y^2\zeta$ = 5, 10, 25, 50 and 100. The comparison with experimental data is also depicted.

## 5. Conclusions

The FFM criterion was extended to predict the fatigue limit of structures weakened by sharp V-notches or U-notches under mode I loading. The FFM is a critical-distance-based criterion (Taylor 2007) which involves the simultaneous fulfilment of a stress-based requirement and an energy-based condition, resulting in a system of two equations in two unknowns: the critical distance and the fatigue strength (Liu et al. 2020, Sapora et al. 2020). The FFM approach was validated against experimental results taken from the literature and involving different materials and geometries. A good agreement between theoretical estimations and experimental results was observed.

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