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Observable Measure of Bipartite Quantum Correlations

Davide Girolami[*] and Gerardo Adesso[†]
School of Mathematical Sciences, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom

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We introduce a measure Q of bipartite quantum correlations for arbitrary two-qubit states, expressed as a state-independent function of the density matrix elements. The amount of quantum correlations can be quantified experimentally by measuring the expectation value of a small set of observables on up to four copies of the state, without the need for a full tomography. We extend the measure to $2 \times d$ systems, providing its explicit form in terms of observables and applying it to the relevant class of multiqubit states employed in the deterministic quantum computation with one quantum bit model. The number of required measurements to determine Q in our scheme does not increase with d. Our results provide an experimentally friendly framework to estimate quantitatively the degree of general quantum correlations in composite systems.

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Quantum entanglement is one of the most fundamental consequences of the superposition principle and undoubtedly plays a key role in designing faster-than-classical algorithms, teleportation protocols, and superdense coding [1]. However, it may not be the ultimate resource behind the power of quantum computation [2]. It has been recently found that, even with no entanglement, some mixed-state based schemes [such as the so-called deterministic quantum computation with one quantum bit (DQC1) [3]] allow an improvement of performance in computing tasks [4], and, more generally, separable states possess genuinely quantum correlations (QCs) [5], captured, e.g., by the quantum discord [6,7], which cannot be described within a classical scenario [6–11]. In general, QCs in a state ρ_{AB} can be defined as the minimum amount of total correlations (measured, e.g., by the mutual information) between Alice and Bob that are destroyed by a local measurement on one or both subsystems [6–8,10,12–15]. For pure states, QCs coincide with entanglement [6]. For mixed states, even if some operational interpretations have been proposed [11,15–18], basic technical issues still prevent us from reaching a full comprehension of their nature. Indeed, theoretical evaluation and experimental detection of QCs both represent hard challenges: Any attempt to determine the QCs in a given state ρ_{AB} is hindered by the difficulty of solving an optimization to determine the least disturbing measurement for that state, thus requiring the full knowledge of it. Recently, some nontomographic detection schemes for witnessing nonvanishing QCs by measuring just one observable have been proposed and implemented [14,19,20]. However, by noting that *all* states possess nonzero QCs but a null-measure set [21], the most worthwhile question becomes that of evaluating, by a proper measure, the actual *amount* of QCs encoded in a state. Only then can quantitative connections be drawn between the QC content and the performance of some quantum protocol using them as a resource [4,22,23].

In this Letter, we show that QCs in a general two-qubit state ρ can be reliably quantified without any explicit optimization and with no need to know the full shape of the state. We define a QC measure Q which is a state-independent function of the density matrix elements. In particular, Q can be expressed in terms of the expectation values of a set of nine observables $\{O_i\}$. Consequently, such a function could be evaluated by designing simple quantum circuits simulating the measurements of $\{\langle O_i \rangle\}$ [24–31]. However, following the alternative approach of Refs. [32–34], we further show that the quantity Q can be even less demandingly measured by performing seven local projections on up to four copies of the state ρ. Then, we extend our measure to capture bipartite QCs in states of $2 \times d$ dimensional systems, finding that seven projective measurements are always sufficient to experimentally determine Q; i.e., the number of measurements required is independent of d. Specifically, we use this construction to obtain a quantitative estimate of QCs in a recent experimental implementation [35] of the DQC1 model [3] with four qubits.

A number of conceptually different measures of general QCs have been recently proposed [5–8,10,12–15]. In the following, we consider a two-qubit state $\rho \equiv \rho_{AB}$ and adopt a geometric perspective, quantifying the QCs in terms of the minimum distance of ρ from the set Ω of classical-quantum states. The states $\chi \in \Omega$ filling such a set are left unperturbed by at least one choice of projective measurement on Alice and take the form [11] $\chi = \sum_i p_i |i\rangle\langle i| \otimes \rho_{iB}$, where p_i are probabilities, $\{|i\rangle\}$ is an orthonormal vector set, and ρ_{iB} is the marginal density matrix of Bob. Adopting the Hilbert-Schmidt norm $\|M\|_2 = \sqrt{\mathrm{Tr}(MM^\dagger)}$, one obtains a QC measure known as "geometric discord," introduced in Ref. [14], operationally interpreted in Refs. [17,23], and defined as

$$D_G(\rho) = 2\min_{\chi\in\Omega}\|\rho - \chi\|_2^2, \qquad (1)$$

where we add a normalization factor 2. The geometric discord enjoys a closed expression for two-qubit states. First, one needs to express the state in the Bloch basis: $\rho = \frac{1}{4}\sum_{i,j=0}^{3} R_{ij}\sigma_i \otimes \sigma_j = \frac{1}{4}(\mathbb{I}_4 + \sum_{i=1}^{3} x_i\sigma_i \otimes \mathbb{I}_2 + \sum_{j=1}^{3} y_j \mathbb{I}_2 \otimes \sigma_j + \sum_{i,j=1}^{3} t_{ij}\sigma_i \otimes \sigma_j)$, where $R_{ij} = \mathrm{Tr}[\rho(\sigma_i \otimes \sigma_j)]$, $\sigma_0 = \mathbb{I}_2$, σ_i $(i = 1, 2, 3)$ are the Pauli matrices, $\vec{x} = \{x_i\}$, $\vec{y} = \{y_i\}$ represent the three-dimensional Bloch column vectors associated to A and B, and t_{ij} are the elements of the correlation matrix t. Then, following Ref. [14], we have $D_G(\rho) = \frac{1}{2}(\|\vec{x}\|^2 + \|t\|_2^2 - 4k_{\max}) = 2\,\mathrm{Tr}[S] - 2k_{\max}$, with $k_{\max}$ being the largest eigenvalue of the matrix $S = \frac{1}{4}(\vec{x}\vec{x}^{\mathsf{T}} + tt^{\mathsf{T}})$. We now provide an explicit expression for $k_{\max}$. The characteristic equation of the matrix S is cubic and can be solved analytically [36]. Being constrained to real solutions only, we write the eigenvalues $\{k_i\}$ of S as

$$k_i = \frac{\mathrm{Tr}[S]}{3} + \frac{\sqrt{6\,\mathrm{Tr}[S^2] - 2\,\mathrm{Tr}[S]^2}}{3}\cos\left(\frac{\theta + \alpha_i}{3}\right), \qquad (2)$$

where $\{\alpha_i\} = \{0, 2\pi, 4\pi\}$ and $\theta = \arccos\{(2\,\mathrm{Tr}[S]^3 - 9\,\mathrm{Tr}[S]\mathrm{Tr}[S^2] + 9\,\mathrm{Tr}[S^3])\sqrt{2/(3\,\mathrm{Tr}[S^2] - \mathrm{Tr}[S]^2)^3}\}$. Since θ is an arccosine, we have $0 \le \theta/3 \le \pi/3$ and the maximum of $\cos\frac{\theta+\alpha_i}{3}$ is reached for $\alpha_i \equiv \alpha_1 = 0$. Hence, $k_{\max} \equiv \max\{k_i\} = k_1$, and the geometric discord for an arbitrary two-qubit state ρ assumes the form of a state-independent function of its entries (ρ_{ij}), that is,

$$D_G(\rho) = 2(\mathrm{Tr}[S] - k_1). \qquad (3)$$

However, we aim to define a simpler, and more accessible experimentally, QC quantifier. By replacing θ with 0 in Eq. (2), we obtain a meaningful and remarkably tight lower bound $Q \le D_G$ (see Fig. 1) to the geometric discord, given by

$$Q(\rho) = \tfrac{2}{3}(2\,\mathrm{Tr}[S] - \sqrt{6\,\mathrm{Tr}[S^2] - 2\,\mathrm{Tr}[S]^2}). \qquad (4)$$

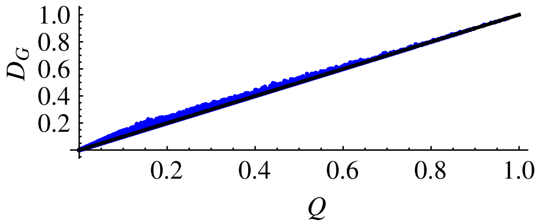

FIG. 1 (color online). Geometric discord D_G versus its tight lower bound Q for 3×10^4 random two-qubit states. The plotted quantities are dimensionless.

The quantity Q is still a state-independent expression of the entries of ρ, but a rather easier one to manage than D_G, and can be regarded as a *bona fide* measure of QCs in its own right. Indeed, it is non-negative by definition; it is faithful (i.e., vanishes only on classical-quantum states χ) and coincides with D_G for pure states. The latter two properties can be proven as follows. Faithfulness is equivalent to showing that $Q = 0 \Leftrightarrow D_G = 0$; the condition for vanishing Q is $\mathrm{Tr}[S]^2 = \mathrm{Tr}[S^2]$, and by the Cayley-Hamilton theorem this implies $\mathrm{Tr}[S]^3 = \mathrm{Tr}[S^3]$, i.e., $D_G = 0$. The equality between D_G and Q for pure states follows from the fact that, writing a bipartite pure state (where subsystem A is a qubit) in the Schmidt decomposition, we have $\theta = 0$. We also find that Q provides a nontrivial upper bound on an entanglement measure, specifically the squared negativity $\mathcal{N}^2$ [1]. In fact, the chain $D_G \ge Q \ge \mathcal{N}^2$ holds for arbitrary two-qubit states, with all inequalities saturated on pure states [37]. Finally, let us mention that a simple upper bound on D_G can be obtained as well from Eq. (2): $D_G \le 4\,\mathrm{Tr}[S]/3$.

From now on, we adopt Q as a rightful QC quantifier for two qubits, endowed with the advantage of requiring neither theoretical optimizations nor experimental state tomography for its evaluation. Specifically, the task of providing a recipe for measuring Q reduces to writing $\mathrm{Tr}[S]$ and $\mathrm{Tr}[S^2]$ as functions of suitable observables and is accomplishable as follows. Defining the matrices $X = \vec{x}\vec{x}^{\mathsf{T}}$ and $T = tt^{\mathsf{T}}$, we have $\mathrm{Tr}[S] = (\mathrm{Tr}[X] + \mathrm{Tr}[T])/4 = \mathrm{Tr}[\rho^2] - \mathrm{Tr}[\rho_B^2]/2$ and $\mathrm{Tr}[S^2] = \frac{1}{16}(\mathrm{Tr}[X^2] + \mathrm{Tr}[T^2] + 2\mathrm{Tr}[XT])$. After some algebra we obtain

$$\begin{aligned}\mathrm{Tr}[S^2] = \tfrac{1}{4}(&-2 - 8\,\mathrm{Tr}[\rho^4] + 8\,\mathrm{Tr}[\rho^3] + 6\,\mathrm{Tr}[\rho^2]^2 - 2\,\mathrm{Tr}[\rho^2](5 + \mathrm{Tr}[\rho_B^2]) - 2\,\mathrm{Tr}[\rho_A^2]^2 + 10\,\mathrm{Tr}[\rho_A^2] - \mathrm{Tr}[\rho_B^2]^2 \\ &+ 12\,\mathrm{Tr}[\rho_B^2] - 6\,\mathrm{Tr}[\rho_A^2]\mathrm{Tr}[\rho_B^2] + 4\,\mathrm{Tr}[\rho(\mathbb{I}_2 \otimes \rho_B)\rho(\mathbb{I}_2 \otimes \rho_B)] - 24\,\mathrm{Tr}[\rho(\rho_A \otimes \rho_B)] \\ &+ 8\,\mathrm{Tr}[\rho(\rho_A \otimes \mathbb{I}_2)\rho(\rho_A \otimes \mathbb{I}_2)] + 8\,\mathrm{Tr}[\rho^2(\rho_A \otimes \rho_B)]).\end{aligned} \qquad (5)$$

By substituting in Eq. (4), Q takes the form of a function of fourth-order polynomials of (ρ_{ij}); in particular, it is written in terms of traces of matrices powers. Now, given a general density matrix ρ, it holds [24–27,30,31] that $\mathrm{Tr}[\rho^k] = \mathrm{Tr}[V^k \rho^{\otimes k}]$, where V^k is the shift operator, $V^k|\psi_1\psi_2\ldots\psi_k\rangle = |\psi_k\psi_1\ldots\psi_{k-1}\rangle$. Also, for two unknown states ρ_1 and ρ_2, it has been proven [27] that $\mathrm{Tr}[V^2\rho_1 \otimes \rho_2] = \mathrm{Tr}[\rho_1\rho_2]$. More generally, we have [38] $\mathrm{Tr}[\rho_1\rho_2\ldots\rho_k] = \mathrm{Tr}[V^k\rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_k]$.

We can exploit these results and follow the approach proposed in Ref. [27] for estimating the expectation values of the appropriate unitary operators $\{O_i\}_{i=1}^{9}$ to associate with each of the nine independent factors in Eq. (5) (which include those appearing in $\mathrm{Tr}[S]$ as well). They can all be expressed as shift operators $O_i = V^k$ on a number k ($k \leq 4$) of copies of the global and/or marginal density matrices and their overlaps, depending on each particular term in Eq. (5). The circuit to be implemented, which includes an ancillary meter qubit, is depicted in Fig. 2(a). For each O_i, we build an interferometer modified by inserting a controlled-O_i gate: Defining the visibility v, we obtain in general $\mathrm{Tr}[O_i \rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_k] = v$. Hence, QCs can be measured quantitatively from the expectation values of the nine operators $\{O_i\}$ only—as opposed to 15 observables required for complete state tomography—obtained in the laboratory via readouts on the ancillary qubit.

We wish now to provide an alternative scheme for the exact measurement of Q that further reduces the required resources for its implementation and appears even more experimentally friendly. This is done by rephrasing the detection scheme in terms of local (with respect to the Alice-Bob split) projectors on multiple (up to 4) copies $\rho^{\otimes n}$ of the *same* state ρ [28,29,32–34]. We observe that

$$\mathrm{Tr}\,[S] = \mathrm{Tr}[\rho^2] - \mathrm{Tr}[\rho_B^2]/2. \tag{6}$$

It is known [29] that $\mathrm{Tr}[\rho^2] = \mathrm{Tr}[V^2 \rho^{\otimes 2}] = \mathrm{Tr}[(P^+ - P^-)\rho^{\otimes 2}] = 1 - 2\,\mathrm{Tr}[P^- \rho^{\otimes 2}]$, where V^2 is the swap operator and $P^{\pm}$ are the projectors on the symmetric or antisymmetric subspaces. By naming A_i (B_j) the subsystems controlled by Alice (Bob) in the ith (jth) copy of the bipartite state ρ, we have then

$$\mathrm{Tr}[S] = \tfrac{1}{2} - 2\,\mathrm{Tr}[P^-_{(A_1B_1)(A_2B_2)} \rho^{\otimes 2}] + \mathrm{Tr}[P^-_{B_1B_2} \rho_B^{\otimes 2}], \tag{7}$$

where for two qubits $P^-_{B_1B_2} = |\psi^-_{B_1B_2}\rangle\langle\psi^-_{B_1B_2}|$, $|\psi^-_{B_1B_2}\rangle = (1/\sqrt{2})(|01\rangle - |10\rangle)$, while $P^-_{(A_1B_1)(A_2B_2)} = \frac{1}{8}(3\mathbb{I}_{16} - \sum_i \sigma_i^{(4)} \otimes \sigma_i^{(4)})$, where with $\sigma^{(d)}$ we indicate the generalized (and normalized) Gell-Mann matrices for dimension d. Alternatively, we can exploit the very recent results of Ref. [34] and write

$$\begin{aligned} \mathrm{Tr}[S] &= 4c_1 - 2c_2 - c_3 + \tfrac{1}{2}, \\ c_1 &= \mathrm{Tr}[(P^-_{A_1A_2} \otimes P^-_{B_1B_2})\rho^{\otimes 2}]; \\ c_2 &= \mathrm{Tr}[(P^-_{A_1A_2} \otimes \mathbb{I}_{B_1B_2})\rho^{\otimes 2}]; \\ c_3 &= \mathrm{Tr}[(\mathbb{I}_{A_1A_2} \otimes P^-_{B_1B_2})\rho^{\otimes 2}]. \end{aligned} \tag{8}$$

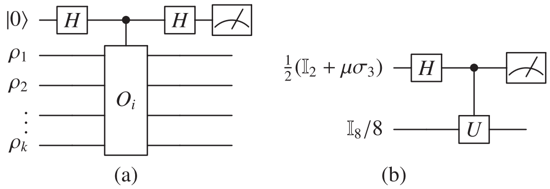

FIG. 2. (a) Quantum circuit estimating $\mathrm{Tr}[O_i \rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_k] = v$. Two Hadamard gates H are applied to an ancilla, followed by a measurement in the computational basis. The interferometer is modified by inserting a controlled-O_i gate acting on the overlap of states. (b) DQC1 model with a register of three maximally mixed qubits and an ancillary qubit in a state of polarization μ. Expectation values of σ_1 and σ_2 on the ancilla return the real and imaginary parts of $\mathrm{Tr}[U]/8$.

In Ref. [29], a method to measure the purity of a quantum state, which is all that we need, is presented and demonstrated by means of the implementation of an all-optical setup. Reference [39] presents a more comprehensive detection scheme for projective measurements; see also [40] for a very recent alternative method. To sum up, in this framework we need three measurements of two-qubit projectors [Eq. (8)]—or two measurements, one on two qubits and the other (nonlocal with respect to the Alice-Bob split) on four qubits [Eq. (7)]—and two copies of the state, to measure $\mathrm{Tr}[S]$.

The detection of $\mathrm{Tr}[S^2]$ can also be recast in terms of local projections. Following Ref. [34], we obtain

$$\begin{aligned} \mathrm{Tr}[S^2] &= 16c_4 + 8(c_7 - c_5 - 2c_6) + c_3^2 \\ &\quad + 4c_2^2 - c_3 - 2c_2 + \tfrac{1}{4}, \\ c_4 &= \mathrm{Tr}[(P^-_{A_1A_4} \otimes P^-_{A_2A_3} \otimes P^-_{B_1B_2} \otimes P^-_{B_3B_4})\rho^{\otimes 4}]; \\ c_5 &= \mathrm{Tr}[(P^-_{A_1A_4} \otimes \mathbb{I}_{A_2A_3} \otimes P^-_{B_1B_2} \otimes P^-_{B_3B_4})\rho^{\otimes 4}]; \\ c_6 &= \mathrm{Tr}[(P^-_{A_1A_4} \otimes P^-_{A_2A_3} \otimes P^-_{B_1B_2} \otimes \mathbb{I}_{B_3B_4})\rho^{\otimes 4}]; \\ c_7 &= \mathrm{Tr}[(\mathbb{I}_{A_1A_4} \otimes P^-_{A_2A_3} \otimes P^-_{B_1B_2} \otimes \mathbb{I}_{B_3B_4})\rho^{\otimes 4}]. \end{aligned} \tag{9}$$

Compared to the measurement of $\mathrm{Tr}[S]$, here we have again projectors on pairs of qubits; however, they need to be implemented on four copies of the state ρ. As Eq. (9) shows, we can evaluate the value of $\mathrm{Tr}[S^2]$ by measuring four such independent projectors in the laboratory. Therefore, in the most economical scheme devised here, the full quantitative detection of bipartite QCs in an arbitrary two-qubit state ρ as measured by Q demands six or seven projective measurements on (up to) four copies of the state ρ. Notice for comparison that, to measure the geometric discord D_G exactly [Eq. (3)], one would need 11 projective measurements on up to six copies of the state [34]. On the other hand, at a qualitative level, a single observable witness suffices to reveal whether Q (or the discord) is zero or not [14,19,20,35].

We now extend our measure to higher-dimensional systems, in particular, to $2 \times d$ systems, which include the practically relevant case of one qubit (A) versus a register (B) of n qubits. The geometric discord for an arbitrary state ρ of a $2 \times d$ system has been derived in Ref. [41] and has the same expression as Eq. (3), just amended with the following generalizations: $\mathrm{Tr}[S] = \frac{1}{2d}(\mathrm{Tr}[X] + \mathrm{Tr}[T])$, $x_i = \mathrm{Tr}[\rho(\sigma_i \otimes \mathbb{I}_d)]$, and $t_{ij} = \mathrm{Tr}[\rho(\sigma_i \otimes \tau_j)]$, where, for example, we can assume $\{\tau_j\} \equiv \{\sigma_j^{(d)}\}$ as the d-dimensional basis for Bob's subsystem. We can repeat the steps done for 2×2 systems and obtain a state-independent form for D_G and for the lower bound Q as well, since the subsystem A is still a qubit and therefore the characteristic equation of S remains a cubic. Thus, for $2 \times d$ states, the task we face is again to express $\mathrm{Tr}[S]$ and $\mathrm{Tr}[S^2]$ in terms of observables.

The most practical way to proceed is to consider the scheme in terms of local projectors. In this respect, it is straightforward to verify that Eqs. (7)–(9) still hold for $2 \times d$ systems: Their expression can be written in exactly the same form as for the two-qubit case, provided we generalize the swap and the projectors P^- to arbitrary dimension d as follows: $V^2 = \frac{1}{d}(\mathbb{1}_{d^2} + \sum_i \tau_i \otimes \tau_i)$ and $P^-_{S_j S_k} = \frac{1}{2d}[(d-1)\mathbb{1}_{d^2} - \sum_i \tau_i \otimes \tau_i]$, where S_j and S_k denote two d-dimensional systems and the τ_i's reduce to Pauli matrices in dimension $d = 2$ (e.g., when we want to calculate $\mathrm{Tr}[\rho_B^2]$ in the 2×2 case). This observation, combined with the previous analysis, allows us to conclude that, even for arbitrary states ρ of $2 \times d$ dimensional systems, we just need six or seven projective measurements on up to four copies of the state ρ to quantify bipartite QCs between the qubit and the remaining qudit system. The number of measurement settings thus does not increase with d, which demonstrates the efficiency and scalability of our scheme. Clearly, the optical implementation of projectors of the type $P_{B_i B_j}$, i.e., multiqubit projectors, is more complicated than the two-qubit case; see, e.g., [42]. However, the method demonstrated in Ref. [39] can be extended to arbitrary dimensions without dramatically increasing the complexity of the experimental setting (as claimed by the authors in the last section of Ref. [39]). More precisely, the number of optical elements required to implement each projector (basically interferometers) should increase polynomially—namely, linearly—with d [39,43], in stark contrast with a complete quantum state tomography for which the required resources scale exponentially [44]. Note also that our scheme for $2 \times d$ systems is completely general and no prior knowledge of the form of the state is required; it relies only on the implicit assumption that the subsystem A has dimension 2—i.e., it is indeed a qubit. This assumption can be verified in the laboratory *a priori*, e.g., by measuring suitable Hilbert space dimension witnesses [45] or, possibly, with tomography on the marginal state of subsystem A, which consumes only a fixed, small amount of extra resources.

We consider as an example the four-qubit implementation of the DQC1 model for quantum computation [3]. This algorithm estimates the trace of a normalized unitary matrix U. We consider the instance recently implemented experimentally in Ref. [35] (where only a discord witness rather than a quantitative estimate was measured), where $U = (a, a, b, 1, a, b, 1, 1)$, with $a = -(e^{-i3\pi/5})^4$ and $b = (e^{-i3\pi/5})^8$. Such a specific gate is used for the approximation of Jones polynomials [46]. The first qubit (the ancilla) is initially in a state of polarization μ, while the remaining qubits are maximally mixed. Referring to the scheme of Fig. 2(b), the final state of the system before readout is

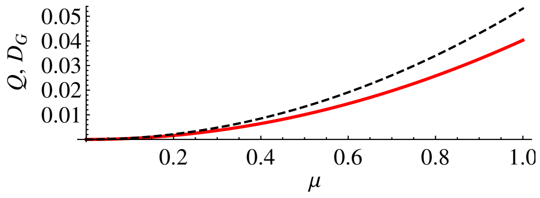

FIG. 3 (color online). Bipartite quantum correlations as measured by Q (red continuous line) and D_G (black dashed line) for the output state of the DQC1 model as implemented in Ref. [35] with four qubits, plotted as functions of the initial polarization μ of the ancilla qubit. All the plotted quantities are dimensionless.

$$\rho_{\mathrm{out}} = \frac{1}{16}\begin{pmatrix} \mathbb{1}_8 & \mu U^\dagger \\ \mu U & \mathbb{1}_8 \end{pmatrix}. \tag{10}$$

We calculate the bipartite QCs $Q(\rho_{\mathrm{out}})$, measurable in the laboratory according to the scheme detailed above, between the ancilla and the residual three-qubit system, and we compare them with the geometric discord D_G, while entanglement is always zero across this bipartition. The plots in Fig. 3 reveal that Q is in good agreement with D_G, being a monotonic function of the polarization μ of the ancilla, hence showcasing its reliability as a QC quantifier [4].

In conclusion, we presented a scheme to quantify theoretically and experimentally general bipartite QCs for arbitrary two-qubit and qubit-qudit states. We introduced a measure Q that is a state-independent function of polynomials of the density matrix elements and can be measured by implementing a restricted number of quantum circuits or, alternatively, a restricted number of local projections, on up to four copies of the state, which appears in reach of current technology [33,39,43]. We used our measure to evaluate quantitatively the degree of QCs created in a recent experimental implementation [35] of the DQC1 model with four qubits [3].

Providing experimentally friendly recipes for the measure of QCs in n-partite realizations of quantum information protocols is key to clarifying their usefulness for the performance of such practical tasks [2]. In this respect, much attention is being devoted to the QC dynamics in open quantum systems [47–50] and, independently, to characterizing the transition from Markovian to non-Markovian regimes [51–55]. Non-Markovianity can be witnessed by monitoring entanglement between one subsystem, coupled to the environment, and another clean subsystem [54]. One might imagine that more general QCs could be somehow more sensitive to the properties of dynamical maps. However, it is known that even local Markovian channels (as well as Markovian common environments [56]) can induce an increase of discordlike QCs in a composite system [57]. Therefore, the question needs to be formulated properly and with care, demanding a dedicated analysis which is beyond the scope of this work [58].

We hope our Letter may contribute to render the general quantumness of correlations a more accessible (theoretically and experimentally) concept in the study of complex quantum systems.

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*pmxdg1@nottingham.ac.uk
†gerardo.adesso@nottingham.ac.uk

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