Hardware tests for knee implants

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WEAR TESTING OF KNEE IMPLANTS

Simulation and optimization

Hardware wear tests for knee implants, necessary for market authorization, are expensive and time-consuming. Faster than realtime wear simulations have the potential to speed up the design process for implants by providing frequent and early feedback. Computational support for virtual wear testing requires the detailed simulation of the mechanics of load cycles as well as the integration of resulting wear over many cycles. Moreover, sensitivity information helps to improve the implant‵s shape and to set up the hardware test correctly. Uncertainty quantification helps to judge reliability and variance of virtual and real test outcomes. The numerical simulation of implant wear, however, is computationally challenging and requires new algorithmic developments in order to be applicable in practice.

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Industrial challenge and motivation

The articular surfaces of total knee implants consist of a metallic femoral (upper) part and a tibial (lower) part usually made of high-density polyethylene. Due to daily motions, wear particles in particular from the tibial part accumulate in the joint and lead to inflammations, which is one of the limiting factors for the implant's life time. For marketing authorization, implant manufacturers have to demonstrate sufficiently low wear of their products by standardized wear tests.

Wear testing of total knee replacements is formalized by the ISO 14243 standard (Fig. 2). It requires to track the wear over a course of five million specified load cycles, and may require repeated manual intervention. A complete test takes about three months, at a cost of approximately 30 000 €. Both, time and cost, put a significant burden in particular on the design process. Faster than real time computational simulation of the wear rate can support the design process and reduce the time to market as well as the testing expenses.

Figure 1: Hardware wear testing of knee implants. The femoral (upper) components undergo a prescribed motion for one to five million load cycles, after which the total wear is determined by the loss of mass in the tibial (lower) part.

The requirements formulated by the industry partners T. Batsch from aap Implantate AG, an implant manufacturer, and C. Abicht from Questmed GmbH, a testing laboratory, were, first, a quantitatively reliable and fast computation of total wear, and, second, the qualitatively correct reproduction of wear patterns on the articulating surfaces. A complete computational shape design was not required due a multitude of competing criteria in the design process. Instead, optimization of the freedom in the test setup left by the standard procedure as well as optimization-based hints for design improvements were desired.

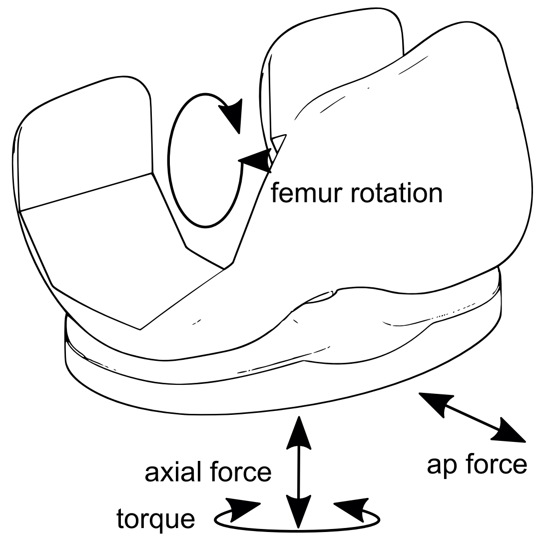

Figure 2: Load directions of the ISO 14243 standard.

Mathematical research

The computational support for wear testing has four aspects: (i) simulating the mechanics of single load cycles, (ii) computing the resulting wear over many cycles, (iii) determining the wear sensitivities with respect to the implant's shape and test setup, and (iv) the quantification of uncertainties. The problem size, in terms of required spatial resolution, test duration, and parametric variance, implies a huge computational effort that makes an industry-scale application of simulation challenging. New algorithmic developments led to viable simulation methods.

(i) Mechanical simulation. The simulation of a single load cycle requires the accurate solution of a sequence of mechanical contact problems. Since inertia is negligible compared to the applied forces, a quasi-static approximation is sufficient, and allows for the parallel computation of the displacement at all time steps. For such contact problems, robust and efficient non-smooth multigrid methods have been developed [2, 5]. From the resulting contact pressure and relative velocity of the sliding articular surfaces, the local wear rate can be computed by Archard's wear law [1]. Comparison with experimental data shows a very good agreement of the total wear rate, but some deviations in the wear pattern (Fig. 3).

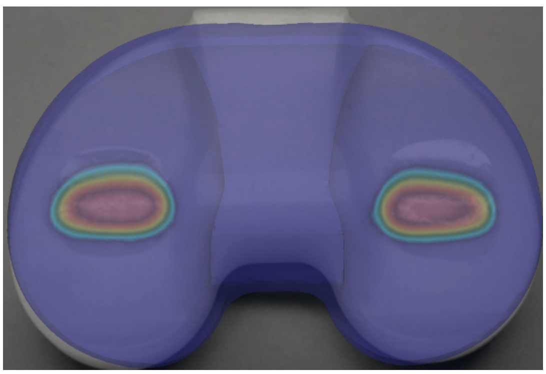

Figure 3: Overlay of simulated and experimental wear after 5 million gait cycles.

(ii) Long-time integration. The wear affects the implant's shape $\partial\Omega$ and thus the wear rate, but the high number of load cycles required by the ISO test prevents the simulation of every single cycle. Cycle jumping extrapolates the wear from a single cycle to a larger time interval, and permits to select a compromise between accuracy and simulation time (Fig. 4).

Essentially, it is equivalent to an explicit first order Euler time stepping for integration of the wear dynamics averaged over one load cycle of duration τ:

$$\dot{\overline{\partial\Omega}}(t) \approx \frac{1}{\tau}\int_{s=t}^{t+\tau} w(u(\overline{\partial\Omega}(t), s)\, ds$$

Higher-order time stepping schemes can, however, be much more efficient, in particular, if they exploit the accuracy-effort trade-off available in the mechanical simulation. Fast inexact spectral deferred correction (SDC) methods [4, 9] with theoretically optimal selection of cycle simulation accuracy have been developed [10], and applied to implant wear, demonstrating a superior efficiency compared to simple cycle jumping, see Fig. 5. Both higher order and inexact evaluation of the right hand sides with adaptive accuracy increase the simulation efficiency significantly.

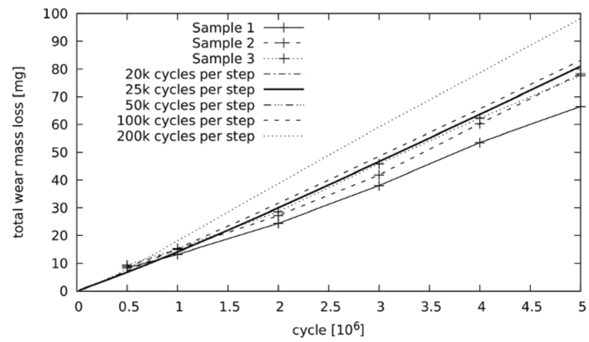

Figure 4: Total wear mass loss as a function of time for different cycle jumping step sizes.

(iii) Optimization of time-dependent contact problems. As a step towards the optimization of implant shapes, numerical methods for the optimal control of time-dependent contact problems were developed. The main difficulty is the non-smoothness of the problem, introduced by the contact constraint. This requires non-standard techniques, concerning analysis and optimization [3]. Time discretization techniques for dynamic contact problems were extended to the case of optimal control problems, optimality conditions and sensitivity results were derived, and numerical solution algorithms were implemented and tested [7, 8].

(iv) Uncertainty quantification. ISO wear tests are subject to subtle differences in environmental and initial conditions, such that their results have a certain distribution. Quantifying this uncertainty by sampling for mean increases the computational effort substantially. Therefore, adaptive hierarchical Monte Carlo methods have been developed [6], which reduce the sampling overhead significantly by exploiting again the accuracy-effort trade-off available in the mechanical simulation.

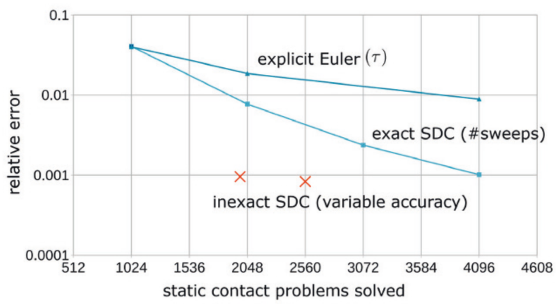

Figure 5: Efficiency of different time integrators for long-time wear simulation on $200\ 000$ load cycles. The computational effort is measured by the number of static contact problems to be solved.

Implementation

In several mutual visits per year, both industry partners formulated their requirements and provided data in form of implant geometries and wear curves for validation as well as engineering experience and insight. The latter proved extremely helpful in formulating and addressing the mathematical problem in a practically relevant way. During the project, a prototype wear simulation software was implemented and validated in close collaboration with Questmed GmbH. Simulating wear patterns correctly turned out to be much more challenging than computing wear curves. This is mostly due to the particular stiffness structure of the wear dynamics, for which a preliminary treatment has been designed in the form of spatial wear rate smoothing approximating the action of an implicit integrator.

Industrial relevance and summary

The project consortium has successfully produced an efficient, massively parallel prototype code for the over-night simulation of wear during an ISO 14243 test. Simulated total mass loss and wear patterns were compared with experimental data provided by Questmed GmbH, with very good agreement. Moreover, aspects of shape and test optimization and treatment of uncertainties have been investigated as well. The approach is currently implemented by the Algo4U Sagl, Lugano (https://algo4u.ch), for offering a commercial wear simulation service.

Acknowledgements

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