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Primary School Pupils' Performance on the Addition of Fractions: Conceptual and Procedural Knowledge

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Abstract

Fractions remain predominantly one of the most challenging topics to teach and learn. Brunei Darussalam is no exception, where a few local researches reported that students performed poorly in fraction topics. To understand this situation, this study focuses on pupils' understanding in solving problems on fractions. Specifically, this study investigated Year 5 pupils' conceptual and procedural performance on the addition of fractions. This study employed a quantitative research approach involving 572 primary school children. A test consisting of six carefully planned questions on fractions was used as the research instrument. The questions were divided into two sections to test pupils' conceptual and procedural understanding laterally. The findings from this study revealed that children performed better in the procedural than in the conceptual questions. It can be concluded that most Year 5 pupils can correctly attempt the addition of fractions via procedural approach without understanding the essential concepts involved. Recommendation for future research was also discussed.

Keywords: Fractions, Quantitative Approach, Conceptual and Procedural Knowledge, Primary School Mathematics

Abstrak

Pecahan tetap menjadi salah satu topik yang paling menantang untuk diajarkan dan dipelajari. Tidak terkecuali Brunei Darussalam, di mana beberapa penelitian lokal melaporkan bahwa siswa kurang berprestasi dalam topik pecahan. Untuk memahami situasi ini, penelitian ini berfokus pada pemahaman siswa dalam menyelesaikan masalah pada pecahan. Secara khusus, penelitian ini menyelidiki kinerja konseptual dan prosedural siswa kelas 5 pada penjumlahan pecahan. Penelitian ini menggunakan pendekatan penelitian kuantitatif dengan melibatkan 572 anak sekolah dasar. Sebuah tes yang terdiri dari enam soal yang direncanakan dengan cermat pada pecahan digunakan sebagai instrumen penelitian. Soal-soal tersebut dibagi menjadi dua bagian untuk menguji pemahaman konseptual dan prosedural siswa secara lateral. Temuan dari penelitian ini mengungkapkan bahwa siswa tampil lebih baik dalam prosedural daripada pertanyaan konseptual. Kesimpulannya kebanyakan siswa sekolah kelas 5 dapat mencoba penjumlahan pecahan dengan tepat melalui pedekatan prosedural tanpa memahami konsep yang terlibat. Rekomendasi untuk penelitian lebih lanjut juga dibahas.

Kata kunci: Pecahan, Pendekatan Kuantitatif, Pengetahuan Konseptual dan Prosedural, Matematika Sekolah Dasar

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INTRODUCTION

Pupils seem to be able to perform effectively in solving a mathematical problem with working shown as evidence. However, how well they have grasped the underlying concept of the problem cannot be easily seen or measured. In the teaching and learning of Mathematics, it is pertinent for children to grasp conceptual and procedural understanding of any mathematical concept learnt, as they are mutually related (Davis, 2000), regardless of how the understanding develops and in which order. Failure to gain

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such understanding can impact subsequent mastery of concepts like fractions and topics relevant to them. Fractions are a crucial topic to be learnt as it is a prerequisite requirement for advanced mathematics and the advancement of technology (Siegler et al., 2013; Torbeyns et al., 2015). This topic is related to other topics in mathematics that may not necessarily be covered at the primary school level. Fractions that are covered at the primary level include fractions basic skills up to fractions operations limited to fraction with the same notation. Pupils' foundation on fractions will determine their performance in learning subsequent topics relevant to fractions (Karika & Csíkos, 2022).

More than two decades ago, Suffolk and Clement (2003) reported that students at the secondary level in Brunei Darussalam found questions on primary level fractions difficult. Findings about the difficulties in learning fractions taught at primary and secondary schools were subsequently reported by only a handful of studies conducted in the country (Abbas et al., 2020; Abdullah & Leung, 2019; Finti et al., 2016; Gani et al., 2019; Harun, 2003, 2011; Japar et al., 2022; Laidin & Tengah, 2021; Low et al., 2020; Lubis et al., 2017; Simpol et al., 2017; Yusof, 2003; Yusof & Langkan, 2016; Yusof & Malone, 2003). The above-mentioned studies look into different aspects of pupils' performances such as misconceptions, errors, and practices to support pupils' learning on fractions and fractions operations however, one that delves into children's procedural and conceptual performances on addition of fractions is still scarce in Brunei. Therefore, this study signifies its importance in informing the procedural and conceptual competence involving fractions addition among Bruneian pupils. It can thus provide a significant platform for future investigations on problems involving fractions or perhaps act as a basis for making necessary changes in the teaching of fractions in order to realise Brunei's 21st Century National Education System (*Sistem Pendidikan Negara Abad ke-21* or SPN21), which is to increase students' achievement in Mathematics.

Hiebert and Lefevre (1986) distinguished procedural knowledge as "algorithms, or rules for completing mathematical tasks" (p. 6), while conceptual knowledge is "characterised most clearly as knowledge that is rich in relationships" (p. 3). These two types of knowledge focus on two different aspects of skills when it comes to solving mathematics problems. However, when children can solve mathematics problems, this does not necessarily mean they understand the underlying concepts of the mathematical processes (Kerslake, 1986; Khairunnisak et al., 2012; Nasution et al., 2018). It may arise from the teaching approaches that children are accustomed to (Ginting et al., 2018; Julie et al., 2013). In the local context, Yusof and Malone (2003) discovered that Year 5 pupils have difficulties with basic facts on fractions, possibly due to their learning of fractions that emphasise the algorithm tasks rather than tasks focusing on the relationships of fraction concepts (Idris & Narayanan, 2011).

In the quest to understand the children's performance of fraction procedural and conceptual competence, Li (2014) has conducted a comparative study of 561 British and 648 Taiwanese (aged 12 and 13) students' conceptual and procedural knowledge of fractions. Subsequently, in 2017, Li and her colleague proposed a framework towards developing fraction proficiency based on five dimensions: the five constructs of fractions, equivalent fractions, fraction procedural operations and conceptual understanding, the relationship with other relevant topics, and the transition between various representations involving fractions (Tsai & Li, 2017). In this present study, we adapted Li's (2014) instrument for exploring Year 5 pupils' conceptual and procedural knowledge of fractions in Brunei. Hence, this study aims to explore Year 5 pupils' performances on the addition of fractions. The guiding research question for this study, in the context of Brunei specifically, is, What are the Year 5 pupils' conceptual and procedural performances on the addition of fractions?

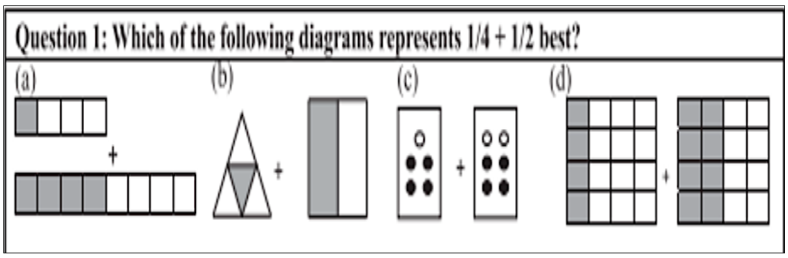
METHODS

This study employed a quantitative research method approach. The instrument used was a test adapted from Li's study (2014). The test was administered to 572 Year 5 pupils (aged 10 to 11) randomly selected from 17 government primary schools throughout the four districts in Brunei. The test instrument had six questions in total, and the average duration of the test was 20 minutes. The pupils individually completed the test without any assistance. The researchers developed the codes for the answers where '0' for a wrong answer and '1' for a correct answer to ease the keying in of data as well as the process of analysing. The quantitative analysis was computed using the IBM SPSS Version 21 software. The basic analysis of the survey data involved frequencies, mean and cross-tabulations. The Spearman correlation test was used to analyse the relationship between procedural and conceptual questions.

Test Questions

The pen and paper test consisted of two parts: Sections A and B. Section A consists of three conceptual questions (denoted as C1, C2 and C3), while Section B consists of three procedural questions (denoted as P1, P2 and P3). Although there were six questions asked altogether, questions in Section A are actually mirroring questions in Section B, as seen in Tables 1, 2 and 3 below.

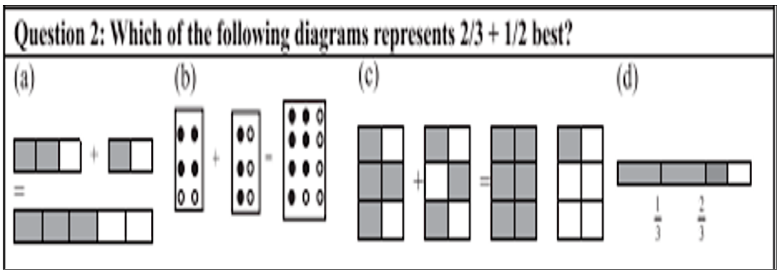
Table 1. Test question C1 and P1

Section A – Conceptual (C1)	Section B – Procedural (P1)
	$\frac{1}{4}+\frac{1}{2}$

The correct answer for C1 in Table 1 above is diagram (d), and the answer for P1 is $\frac{3}{4}$. For C1, the question seeks the Year 5 pupils' knowledge on which is the best pictorial representation of $\frac{1}{4}+\frac{1}{2}$. The multiple choices of pictorial representation or fraction models of $\frac{1}{4}+\frac{1}{2}$ shown required pupils to carefully select that indicate the concept of $\frac{1}{4}+\frac{1}{2}$. For choice (a) the fraction models showed two fraction strips with shaded and unshaded parts. It shows $\frac{1}{4}$ and $\frac{4}{8}$. This is not the correct answer because the fraction models are not from the same sizes. For the multiple-choice answer or diagram (b) in Table 1, although the fraction models represent $\frac{1}{4}$ and $\frac{1}{2}$ correctly, it is not the correct answer because the shapes of the fraction models are different. Meanwhile, for the multiple-choice answer (c) the fraction models are denoted like a domino model where it represents $\frac{1}{5}$ and $\frac{2}{6}$, unless it refers to another sub-construct of fraction related to ratio. The multiple-choice answer (d) has the same shapes of fraction models, same equal parts, and the shading denotes $\frac{4}{16}$ and $\frac{8}{16}$ The first diagram that shows $\frac{4}{16}$ is equivalent to $\frac{1}{4}$ whereas $\frac{8}{16}$ is equivalent to $\frac{1}{2}$. Hence, this is the correct answer.

To arrive at the answer in P1, a series of steps needed to be done, normally in sequential order. To add fractions of different denominators, the pupils have to convert the fractions so that the denominators are the same. To do this, they see denominators 4 and 2 by looking for the same common factor, which is 4. Thus, $\frac{1}{2}$ is converted to $\frac{2}{4}$ by multiplying $\frac{1}{2}$ with $\frac{2}{2}$. Then the pupils can write $\frac{1}{4}+\frac{1}{2}$ and add the numerators together arriving to $\frac{3}{4}$ as the final answer.

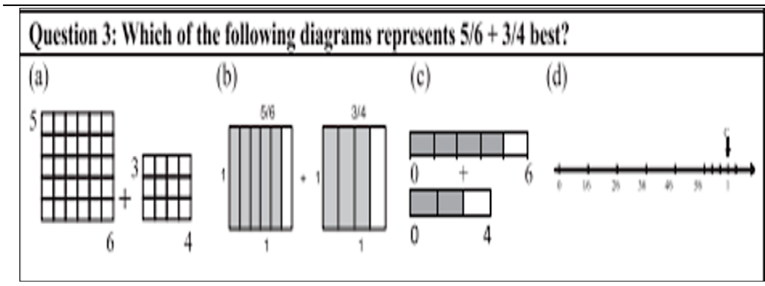
Table 2. Test question C2 and P2

Section A – Conceptual (C2)	Section B – Procedural (P2)
	$\frac{2}{3}+\frac{1}{2}$

Referring to Table 2, the correct answer for C2 is diagram (c) and the answer for P2 is $1\frac{1}{6}$. For C2, the question seeks the Year 5 pupils' knowledge on which is the best pictorial representation of $\frac{2}{3}+\frac{1}{2}$. The multiple choices of pictorial representation or fraction models of $\frac{2}{3}+\frac{1}{2}$ shown required the pupils to carefully select that indicate the concept of $\frac{2}{3}+\frac{1}{2}$. For choice (a) the fraction models showed two fraction strips with shaded and unshaded parts. It shows $\frac{1}{4}$ and $\frac{4}{8}$. This is not the correct answer because the fraction models are not from the same sizes. For the multiple-choice answer (b), although the fraction models represent $\frac{4}{6}$ and $\frac{3}{6}$, which is the equivalent fractions of $\frac{2}{3}+\frac{1}{2}$, it is not the correct answer because diagram that showed the answer does not represent the correct final answer as it can be read as $\frac{7}{12}$. There is an indication of whole number bias after the first few steps, i.e. converting the question into equivalent fractions. Both the numerators and denominators were added like in the whole number. For the multiple-choice answer (d), instead of showing diagrams of the fractions added, this multiple-choice showed the answer directly of a combined diagram from different sizes. For the multiple-choice answer (c) the fraction models represent $\frac{4}{6}$ and $\frac{3}{6}$ which are the equivalent of $\frac{2}{3}+\frac{1}{2}$ respectively. The answer showed is the correct representation where when these two fractions are added, it produced an improper fraction of $1\frac{1}{6}$. This can be seen from the diagram with all shaded parts that represented a whole. Hence, this is the correct answer.

To arrive at the answer in P2, a series of steps needed to be done, normally in sequential order. To add fractions of different denominators, the pupils have to covert the fractions so that the denominators are the same. To do this, they see denominators 3 and 2 by looking for the same common factor which is 6. Thus, $\frac{2}{3}$ is converted to $\frac{4}{6}$ by multiplying it with $\frac{2}{2}$, whereas $\frac{1}{2}$ into $\frac{3}{6}$ by multiplying it with $\frac{3}{3}$. Then the pupils can write $\frac{4}{6}+\frac{3}{6}$ and add the numerators together arriving to $\frac{7}{6}$, simplified as $1\frac{1}{6}$ for the final answer.

Table 3. Test questions C3 and P3

Section A – Conceptual (C3)	Section B – Procedural (P3)
	$\frac{5}{6}+\frac{3}{4}$

The correct answer for C3 in Table 3 is diagram (b) and the answer for P3 is $1\frac{7}{12}$. For C3, the question seeks pupils' knowledge on which is the best pictorial representation of $\frac{5}{6}+\frac{3}{4}$. The multiple choices of pictorial representation or fraction models of $\frac{5}{6}+\frac{3}{4}$ shown required the pupils to carefully select that indicate the concept of $\frac{5}{6}+\frac{3}{4}$. For choice (a) both fraction models were unshaded. This is not the correct answer because the fraction models are not from the same sizes and the information given in the diagrams does not relate to $\frac{5}{6}$ and $\frac{3}{4}$. For the multiple-choice answer (c) it is not the correct answer because not just it is of different sizes but the shaded parts do not represent $\frac{5}{6} + \frac{3}{4}$. The multiple-choice answer (d), what were represented in the number line is combined fractions from different sizes. For the multiple-choice answer (b) the fraction models represent $\frac{5}{6}$ and $\frac{3}{4}$ based on the shaded parts. The answer showed is the correct representation although no answer of the fraction additions was given. Hence, this is the correct answer.

To arrive at the answer in P3, a series of steps needed to be done, normally in sequential order. To add fractions of different denominators, the pupils have to covert the fractions so that the denominators are the same. To do this, they see denominators 6 and 4 by looking for the same common factor which is 24. Thus, $\frac{5}{6}$ is converted to $\frac{20}{24}$ by multiplying it with $\frac{4}{4}$, whereas $\frac{3}{4}$ into $\frac{18}{24}$ by multiplying it with $\frac{6}{6}$. Then the pupils can write $\frac{20}{24} + \frac{18}{24}$ and add the numerators together arriving to $\frac{38}{24}$, which is then simplified to $1\frac{7}{12}$ as the final answer. Another way is by using LCM of 12, where the pupils can get an equivalent fraction of $\frac{10}{12} + \frac{9}{12}$ and get $\frac{19}{12}$, simplified as $1\frac{7}{12}$. From the above series of steps, they can find the answers without understanding why they are doing so.

RESULTS AND DISCUSSION

When analysed, the overall mean score of the whole group was 2.7540, indicating a failed result out of 6 of the overall pupils (see Table 4). Similarly, the overall mean for the conceptual format of the questions (C1, C2, and C3) resulted in an also failed mark of 0.9719 for the group of pupils. However, for the procedural format of the questions (P1, P2, and P3) produced a slightly better pass mean result of 1.7885. This indicated that more pupils are familiar with and correctly attempt the addition of fractions via a procedural approach, and most pupils were unable to attempt the questions in the conceptual format correctly.

Table 4. Mean score of students for all questions: Procedural and conceptual questions

	Overall [Out of 6]	Format of Question	
		Procedural [Out of 3]	Conceptual [Out of 3]
Mean	2.7540	1.7885	0.9719

When the item-by-item analysis was conducted (refer to Table 5), mixed results were produced with mean ranging from low fail (0.08 and 0.13 respectively for C1 and C2), to borderline pass (0.51 for P3), and to reasonable pass (0.66, 0.62 and 0.76 for P1, P2 and C3 respectively).

For the procedural format of questions, almost half of the pupils correctly attempted P3, while slightly higher correct attempts for P1 and P2 (in Table 5). For the conceptual questions, C3 produced the highest number of correct attempts, also producing the highest mean among the six questions. While

C1 and C2 had the most incorrect attempts, not only for the conceptual form of the questions but among all six questions (Table 5). This again supports the initial claim that, in general, most pupils can attempt questions procedurally without understanding the underlying concept of the question (Questions 1 and 2 for conceptual and procedural versions).

Table 5. Descriptive statistics for procedural questions (P1, P2 and P3) and conceptual questions (C1, C2 and C3) (n=573)

	Nature of Question	Mean	Attempt (%)		
			Correct	Incorrect	None
P1	Procedural	0.66	381 (*66.5%*)	192 (*33.5%*)	0 (*0.0%*)
P2	Procedural	0.62	354 (*61.8%*)	219 (*38.2%*)	0 (*0.0%*)
P3	Procedural	0.51	290 (*50.7%*)	282 (*49.3%*)	1 (*0.2%*)
C1	Conceptual	0.08	48 (*8.4%*)	524 (*91.4%*)	1 (*0.2%*)
C2	Conceptual	0.13	72 (*12.6%*)	498 (*86.9%*)	3 (*0.5%*)
C3	Conceptual	0.76	434 (*75.7%*)	136 (*23.7%*)	3 (*0.5%*)

Referring to Table 6, when Spearman's correlation analysis was done to the corresponding procedural versus conceptual questions, there is a positive correlation between C3 and P3 $r(568) = .11$, $p<0.07$. This evidence supports there is a correlation between the correct response of the pupils in the procedural format to incorrect answer to the conceptual format of the question or vice versa. As we previously claimed, in general, some pupils were able to correctly attempt the procedure of addition of fractions without understanding the concept underlying the question.

Table 6. Spearman's correlation between correctly attempted procedural versus correctly attempted conceptual fractions addition questions

	Spearman's correlation	
	Coefficient	Sig. (2-tailed)
Procedural versus Conceptual	0.174	.000

When Spearman's correct-correct correlation analysis was done to corresponding procedural to conceptual formats for Questions 1 and 2, both P1 vs C1 and P2 vs C2 did not show significant correct-correct correlation (displayed in Table 7), with only 34 pupils obtaining correct-correct combination for P1 vs C1 and 72 pupils for P2 vs C2. The high combination frequencies for questions with the correct procedural format versus the incorrect conceptual format (346 and 302 for Questions 1 and 2, respectively) give evidence and further support that although some pupils are able to attempt the addition of fractions via the procedural approach correctly, most pupils may not necessarily understand the underlying concepts of these questions.

Table 7. Spearman's correlation between correctly attempted procedural versus correctly attempted conceptual fractions addition questions and their breakdown combination frequencies

	Spearman's correlation		Combination frequencies			
	Coefficient	Sig. (2-tailed)	Correct-correct	Correct-incorrect	Incorrect-correct	Incorrect-incorrect
P1 vs. C1	.028	.501.	34	346	14	178
P2 vs. C2	.051	.227	72	302	23	196
P3 vs. C3	.112**	.007	232	55	201	81

However, when a similar Spearman's correlation analysis was done to Question 3, a significant correct-correct correlation ($p<0.05$) was achieved for P3 vs C3, with 232 pupils obtaining a correct-correct combination (refer to Table 7). In addition, the incorrect-correct combination of Question 3 has also garnered a high number of responses with 201 pupils. This indicated that pupils who understood the concept skill of adding fractions still did poorly on the procedural question. This is opposite to the study done by Kerslake (1989).

Upon closer inspection of Question 3 in the conceptual format C3 (refer to Table 3 earlier), the multiple-choice options may provide some indicators of a high number of correct attempts. The choices (a) and (d) might be unfamiliar to pupils when solving fraction-related questions, where (a) has no shading and (d) involves a number-line. Some pupils, by elimination, might choose the answer between (b) and (c). However, when looking at option (c), if pupils are familiar with the diagrammatic shading representation of fractions (while ignoring the number on the side of the diagrams), the pupils will immediately realise that the fractions represented by option (c) are $\frac{4}{5}$ and $\frac{2}{3}$. Could this be the reason why a majority of pupils did not choose option (c) and, by default, choose (b) instead? This could explain why there were more correct responses in C3, to begin with (high mean value). This could also indicate an accidental finding that most pupils possibly understand the concept of fractions and how to represent individual fractions in diagrammatic form.

The findings revealed that the pupils in this present study performed better on procedural questions (P1, P2) on the addition of fractions than on conceptual questions (C1 and C2). Perhaps, our initial observation is that they memorised the procedures without understanding the rationale for doing so. Such an act is similar to learning the techniques by rote learning, which has long been known not to have a lasting effect on one's learning. Thus, this indicates that apart from procedural knowledge, the pupils would also need assistance developing their conceptual knowledge of the addition of fractions. In addition, this information also revealed that those who had obtained all correct answers to the procedural questions might not necessarily have answered the conceptual questions accurately. This concurred with Kerslake's (1989) study and, more particularly, with Li (2014), who found that Taiwanese and British children had also showed patterns similar to those recorded in this study.

Another possible explanation is that the conceptual multiple-choice questions were represented in pictorial form, which should be the basic representations of fractions before doing the algorithm representation. The findings conclude that the pupils' performance on the conceptual questions (pictorial presentations of fractions) is still weak and underdeveloped. Furthermore, the results of this study have raised the awareness that arriving at a procedurally correct answer does not necessarily reflect on the pupils correct conceptual knowledge of the problem. Still, their knowledge of one skill may influence the other or vice versa (Hecht & Vagi, 2010).

CONCLUSION

The findings of this study have shed some insights on Year 5 pupils' conceptual and procedural performances on the addition of fractions in Brunei primary schools. There is evidence indicating that pupils can correctly attempt the addition of fractions via a procedural approach without understanding the essential concepts involved. One accidental finding in this study is that although most pupils may not necessarily understand the concept of addition of fractions, there is evidence that they do know the concept of individual proper fractions and how to represent them diagrammatically. The limitation of this study is that we only explored pupils' conceptual and procedural skills quantitatively without delving into the reasoning for their mathematical performances, especially in the conceptual section. Hence, further in-depth item-by-item analysis or perhaps extending the study to be done qualitatively is recommended in future studies.

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