Indonesian Journal of Mathematics Education
Vol. 5, No. 1, April 2022, pp: 1~18
p-ISSN: 2654-3907, e-ISSN: 2654-346X DOI: 10.31002/ijome.v5i1.5363
e-mail: ijome@untidar.ac.id, website: jurnal.untidar.ac.id/index.php/ijome

Beliefs about Mathematics and Mathematics Assessment in Post-Secondary Education

Milica Videnovic
Simon Fraser University, Burnaby, British Columbia, Canada
e-mail: mvidenov@sfu.ca

Abstract

This paper studied the beliefs about mathematics, mathematics assessment, and written and oral mathematics assessment in post-secondary education from the mathematics professors' perspectives. Seven mathematics professors and instructors were interviewed and asked to explain how they perceive mathematics and mathematics assessment and how they compare the oral exam to the written exam. Four out of seven mathematics professors and instructors were educated in Poland, Romania, Bosnia, and Ukraine, and they are currently teaching mathematics at a university in Canada. The other three professors were educated in Canada, Germany, and the United States, and they are currently teaching at a university in Germany. Five participants had previously experienced an oral examination in mathematics, while the other two had never been exposed to an oral examination in mathematics throughout their schooling. The results showed that similar beliefs about mathematics and mathematics assessment result in different beliefs about written and oral mathematics assessment.
Keywords: oral assessment, oral examination, mathematics, beliefs, culture

INTRODUCTION

A large amount of research on teachers' beliefs focuses on beliefs about mathematics, mathematics teaching, and mathematics learning (Beswick, 2007; Cross, 2009; Ernest, 1989; Handal, 2003; Liljedahl, 2009; Maasz & Schlöglmann, 2009; Philipp, 2007; Raymond, 1997; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1992; Žalská, 2012). However, in the most recent review of assessment in mathematics education, there has been almost no research on students' and teachers' beliefs about assessment in mathematics (Suurtamm et al., 2016), especially about oral assessment in mathematics.

Many countries, especially in Europe, have been implementing the interrogatory type of oral examination as an important part of assessment practice in their schooling system. For instance, this is not the case in Canada. The Canadian educational system is dominated by closed-book, written examinations. Moreover, the oral form of assessment in mathematics courses at the university level in Canada is not present at all, even though there are a number of research studies that indicate that oral assessment has a positive impact on students' learning of mathematics (Boedigheimer, Ghrist, Peterson & Kallemyn, 2015; Iannone & Simpson, 2012, 2015; Lianghuo & Mei, 2007; Nelson, 2010; Nor & Shahrill, 2014; Odafe, 2006).

This paper begins by presenting the research on the purpose of classroom assessment, as well as a brief description of types of oral assessment, the shift from oral to written assessment, and the disadvantages and advantages of oral assessment. Next, it presents the literature on teachers' beliefs about mathematics, along with the theoretical framework, research questions, and methodology of this study. The last part of the paper provides the results, a discussion of the results, and some recommendations for possible future research.

The Purpose of Assessment

According to Brown (2008), all teachers' beliefs about the purposes of assessment fall

into one of these four categories: assessment as improvement of teaching and learning (improvement); assessment as making schools and teachers accountable for their effectiveness (school accountability); assessment as making students accountable for their learning (student accountability); and assessment as irrelevant to the life and work of teachers and students (irrelevant). In assessment as improvement of teaching and learning, the purpose of assessing students' knowledge or understanding is to gather the information that would lead to changes in teaching and learning practices, so that improvement in students' achievement can be facilitated. When it comes to assessment as making schools and teachers accountable for their effectiveness, it focuses on demonstrating the quality of teaching and learning. For some, accountability has come to mean the responsibility of a school to produce high achievement test scores (Smith & Fey, 2000). Student accountability assessment has the goal of holding students individually accountable for their learning, grades, and whether they have met various curriculum objectives. Contrary to the first three categories of assessment, in the fourth category, there are a number of reasons for teachers to consider an assessment irrelevant to student growth and achievement. Moreover, the assessment may even unfairly impact certain students, create negative attitudes toward testing, or be so inaccurate that it is unreliable. All of these possibilities contribute to the notion that assessment could be irrelevant.

Types of Oral Assessment

There are two main types of assessment: oral and written. Joughin (1998) defines *oral assessment* as "assessment in which a student's response to the assessment task is verbal, in the sense of being 'expressed or conveyed by speech instead of writing' (*Oxford English Dictionary)*".

According to Joughin (2010), they can be categorized into three forms: presentation on a prepared topic (individual or in groups); interrogation (covering everything from short-form question-and-answer to a doctoral oral exam); and application (where candidates apply their knowledge live in a simulated situation, e.g., having trainee doctors undertake live diagnoses with an actor-patient).

This paper focuses on *oral examination*, with elements of both presentation and dialogue that come from the interrogation form of oral assessment, and combines oral medium with writing on paper and board. The term *assessment* is used to represent a broader range of evaluation activities, but also, as most assessments in undergraduate mathematics are exams, in this paper, the terms *assessment* and *exam* are used interchangeably.

The Shift from Oral to Written Assessment

The oral examination has a very long history in higher education. Prior to the beginning of the 20th century, oral examinations were a standard practice in the UK, which later failed because of accusations of bias and the apparent efficiency of written exams. According to Stray (2001), four factors are identified as being crucial in causing the shift from oral to written examinations: the move from group socio-moral to individual cognitive assessment in the later 18th century; the differential difficulty of oral testing in different subjects; the impact of increased student numbers; and the internal politics of Oxford and Cambridge.

Despite this shift, the transition from using oral to written only has not happened everywhere. There are still many countries that maintain oral assessment in most academic subjects as an important part of their assessment practice (Brown & Knight, 1994; De Vita & Case, 2003; Forrest, 1985; Hubbard, 1971). Some of these countries are Hungary, Italy, Germany, and the Czech Republic.

Looking at the history of written examinations, Stray (2001) notes that the written examination became the norm in the UK starting at the beginning of the 20th century. Since then, the primary method of assessment in the mathematics classroom has become strictly based on closed-book written

examinations. The USA, in particular, appears to be dominated by closed-book written examinations (Crannel, 1999; Nelson, 2010). Also, the majority of mathematics students in the UK are to be assessed predominately using high-stakes, closed-book examinations at the end of almost every module (Iannone & Simpson, 2011). Written assessment, most commonly in the form of examinations, can also take the form of quizzes, projects, assignments, papers, essays, journals, dissertations, and portfolios.

Disadvantages and Advantages of Oral Assessment

Although oral assessment is used in many areas, there is very little literature examining the use of oral assessment. The UK's comprehensive review of the literature on innovative assessment shows that less than 2% of the papers address oral assessment. Of 317 papers considered, only 31 dealt with 'non-written assessment', which includes: oral examination, group and individual oral presentation (oral group presentations were by far the most commonly cited non-written assessment, at 50% of the total sample), debate, the artifact (such as a display stand or non-written poster), audio or video recording, and role-play. Within this category of non-written assessment, only 4 papers addressed the use of oral examinations (Hounsell, Falchikov, Hounsell, Klampfleitner, Huxham, Thompson & Blair, 2007). All of these non-written assessments addressed undergraduate assessment, with the exception of a single article considering the use of oral exams for Ph.D. students. Also, most of the research on oral assessment focuses mainly on liberal arts subjects, indicating an almost complete absence of research that studies oral assessment in mathematics classrooms.

The main topic that has been discussed in the oral assessment literature is related to the disadvantages and advantages of oral in comparison to written assessment, specifically focusing on understanding assessment from the learner's and the teacher's perspectives. In terms of the disadvantages of oral assessment in comparison to written ones, two things came up: fairness and anxiety (Henderson, Lloyd, & Scott, 2002; Hounsell et al., 2007; Huxham, Campbell & Westwood, 2012; Joughin, 2007). Videnovic (2017b) reports from her study that the mathematics professors interviewed believe that it is not quite clear which type of an exam, oral or written, can be considered to be more or less fair in comparison to the other, and which one of these two can cause more or less anxiety among students.

Romagnano (2001) believes that all assessments of students' mathematical understanding are subjective and that objectivity does not exist. Also, he thinks that a conclusion about a student's knowledge would require the teacher's judgment and, therefore, "no objective" assessment occurs; subjective—that is, human—knowledge, beliefs, judgments, and decisions are unavoidable parts of any assessment scheme" (p. 36). Human judgment about mental constructs is introduced when test designers decide "what items to include on the test, the wording and content of the items, the determination of the 'correct' answer,... how the test is administered, and the uses of the results" (FairTest: The National Center for Fair and Open Testing).

When it comes to oral assessment and anxiety, there is a perception that oral assessment may make students more anxious than other forms of assessment for two reasons: oral assessment anxiety may be primarily related to its unfamiliarity, and oral assessment anxiety is associated with the conception that an oral task requires a deeper understanding and the need to explain to others. Hounsell et al. (2007) note that "It is not clear whether oral assessments are scarier or just more novel" (p. 34). Also, Huxham et al. (2012) note that oral assessment anxiety may be primarily related to its unfamiliarity. In his phenomenographic study of student experiences with oral presentations, Joughin (2007) notes that greater anxiety about oral compared to written assessment is associated with a richer

conception of the oral task as requiring deeper understanding and the need to explain to others.

Despite all of these disadvantages, at the same time, an oral assessment is not without its advantages. Videnovic (2017a) reports that the mathematics professors interviewed believe that written exams can mostly assess procedural knowledge and instrumental understanding, while oral exams can better assess conceptual knowledge and relational understanding in mathematics. Moreover, the research studies on the advantages of the oral assessment show that oral assessment in mathematics and in other subjects: 1) provides immediate feedback and immediate grade (Boedigheimer et al., 2015; Iannone & Simpson, 2012; Odafe, 2006; Roecker, 2007); 2) do not allow plagiarism (Huxham et al., 2012; Joughin, 1998; Nor & Shahrill, 2014); 3) helps develop better oral communication skills (Badger, 2010; Huxham et al., 2012); 4) promotes deep comprehension of the learned material (Iannone & Simpson, 2012, 2015; Joughin, 2007; Lianghuo & Mei, 2007; Nelson, 2010; Nor & Shahrill, 2014; Odafe, 2006; Roecker, 2007); 5) encourages students to deeply/actively engage with the course material (Boedigheimer et al., 2015; Iannone & Simpson, 2012; Nor & Shahrill, 2014; Odafe, 2006); 6) helps students gain ownership of the learned material (Boedigheimer et al., 2015); 7) is more personal/ provides individualized contact between teacher and student (Joughin, 2007); 8) helps students learn to express technical material clearly and concisely (Boedigheimer et al., 2015); 9) allows for probing knowledge through dialogue (Badger, 2010; Joughin, 1998; Odafe, 2006); 10) provides long-lasting mathematical knowledge (Iannone & Simpson, 2012); 11) is authentic/helps prepare students for their professional careers (ex. career interviews) (Boedigheimer et al., 2015; Henderson et al., 2002; Huxham et al., 2012; Iannone & Simpson, 2015; Joughin, 1998); 12) helps develop better presentation skills (Boedigheimer et al., 2015); 13) helps students build the confidence (Boedigheimer et al., 2015); 14) is reactive to students' needs (Iannone & Simpson, 2015); 15) provides the opportunity for assessing students' mental math skills (ex. mental calculation) and the use of sketches and free drawing in geometry (Fyhn, 2015); 16) provides the opportunity to better understand students' attitude, beliefs, motivation and creativity (Fyhn, 2015); 17) encourages students to put more effort/time in preparing for it (Iannone & Simpson, 2012; Joughin, 2007; Odafe, 2006); 18) prevents one small gap in knowledge completely stalling a solution (Joughin, 1998); 19) can determine students' critical thinking abilities (Badger, 2010).

LITERATURE ON TEACHERS' BELIEFS ABOUT MATHEMATICS

Beliefs about mathematics can be classified into three groups: *the traditional perspective, the formalist perspective*, and *the constructivist perspective* (Dionne, 1984). Törner and Grigutsch (1994) refer to these three groups as a *toolbox, systems*, and *processes*. In the *traditional perspective 'toolbox'*, mathematics is seen as a set of skills, which involves doing mathematical calculations and using rules, procedures, and formulas. In the *formalist perspective 'system'*, mathematics is seen as logic and rigor, whereby doing mathematics is considered writing rigorous proofs and exact definitions. In the *constructivist perspective 'process',* mathematics is seen as a constructive process, that involves learning mathematics through the process of problem-solving, building rules and formulas in order for students to be able to experience the actual doing of mathematics and find relations between different notions. In this constructive process, two types of understanding and knowledge of mathematics could be considered. *Relational understanding* – as knowing both what to do and why, and *instrumental understanding* – as the ability to execute mathematical rules and procedures (Skemp,1976). Similarly, *conceptual knowledge* – a knowledge-rich in relationships, which can be thought of as a connected web of knowledge, a network in which the linking

relationships are as prominent as the discrete pieces of information, and *procedural knowledge* – a knowledge that consists of rules or procedures for solving mathematical problems (Hiebert & Lefevre, 1986).

In terms of teachers' views of the nature of mathematics, like their belief systems on the nature of mathematics as a whole, they form the basis of the philosophy of mathematics, although some teachers' views may not have been elaborated into fully articulated philosophies. Therefore, "teachers' conceptions of the nature of mathematics by no means have to be consciously held views; rather they may be implicitly held philosophies" (Ernest, 1989, p. 249). Based on their observed occurrence in the teaching of mathematics, Ernest (1989) describes three philosophies of mathematics: instrumentalist, Platonist, and problem-solving. In the instrumentalist view of mathematics, mathematics is an accumulation of facts, rules, and skills to be used in the pursuance of some external end. Thus, mathematics is a set of unrelated but utilitarian rules and facts. In the Platonist view of mathematics, mathematics is a static but unified body of certain knowledge. Mathematics is discovered, not created. In the problem-solving view of mathematics, mathematics is a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of inquiry and coming to know, not a finished product, for its results remain open to revision.

According to Ernest (1989), these three philosophies of mathematics, as systems of beliefs, can be assumed to form a hierarchy. In this hierarchy, instrumentalism is at the lowest level, involving knowledge of mathematical facts, rules, and methods as separate entities. The Platonist view would be at the next level, involving a global understanding of mathematics as a consistent, connected, and objective structure. Finally, at the highest level, the problem-solving view perceives mathematics as a dynamically organized structure located in a social and cultural context.

THEORETICAL FRAMEWORK

Green (1971) introduced three dimensions of belief systems: *quasi-logical relationships*, *psychological strength*, and *isolated clusters*. In a *quasi-logical relationship*, beliefs can be either *primary* or *derivative* (a belief that is derived from a primary belief). For instance, if a student believes that learning mathematics is useful for his/her life, this would be considered a primary belief. If a student thinks that it would be important to work hard in mathematics class and tries to relate problem-solving exercises to everyday life, these would be considered derivative beliefs. In the *psychological strength* dimension, beliefs can be either *central* or *peripheral*. Central beliefs are held most strongly, whereas peripheral beliefs are held less strongly and can be changed more easily. For instance, an experienced teacher holds more central, deep-rooted beliefs, whereas a newly hired teacher holds more peripheral, changeable beliefs. In the *isolated clusters* dimension, beliefs are held in *clusters*, where "nobody holds a belief in total independence of all other beliefs. Beliefs always occur in sets or groups" (p. 41). An example of this would be when we talk about mathematics. We could broadly classify beliefs about mathematics in relation to the nature of mathematics, teaching and learning of mathematics, the nature of mathematical knowledge and understanding, etc.

Research Questions

Although the goal of this paper is to study beliefs about mathematics and mathematics assessment of participants who teach mathematics at the university level, the vast majority of research addresses the beliefs and practices of mathematics school teachers. Therefore, the lack of research studies on mathematics professors' beliefs about mathematics and mathematics assessment, led me to study the relationships between the mathematics professors' beliefs about mathematics, mathematics assessment and written and oral mathematics assessment. So,

the following questions have been investigated in this paper: 1) *What are the mathematics professors' beliefs about mathematics? 2) What are the mathematics professors' beliefs about mathematics assessment? 3) What are the mathematics professors' beliefs about written mathematics assessment?* 4) *What are the mathematics professors' beliefs about oral mathematics assessment?*

METHOD

The research design for this study is qualitative. Seven participants were interviewed using open-ended questions to gather information about their beliefs about mathematics and mathematics assessment, and their personal experiences and perspectives on using written and oral assessments in the mathematics classroom. These participants were selected based on the following criteria: each participant had been exposed to oral assessment either as a student and/or professor. In terms of recruitment, this study used a methodology of snowball sampling, a technique for finding research subjects in which one subject gives the researcher the name of another subject, who in turn provides the name of a third, and so on (Vogt, 1999).

Seven mathematics professors and instructors were selected for interviews: Melissa, Elisabeth, Van, Nora, Dave, James, and Jane. These names are pseudonyms. Melissa, Elisabeth, Van, and Nora were born and educated in Poland, Romania, Bosnia, and Ukraine, respectively, and are currently teaching at a Canadian university, while Dave, James, and Jane were born and educated in Canada, Germany, and the United States, respectively, and are currently teaching at a university in Germany. With respect to familiarity with oral assessment, Van, Melissa, Nora, and Elisabeth had been previously exposed to oral examination in mathematics prior to moving to Canada, while Dave and Jane, who were educated in Canada and the United States, had never been exposed to oral examination in mathematics prior to moving to Germany. James was born and educated in Germany, and thus, he has had a lot of exposure to oral assessment in mathematics.

This study used the qualitative research method for data collection and analysis because it focuses more on an in-depth understanding of mathematics professors' beliefs about mathematics assessment and mathematics assessment practice from various schooling and teaching cultures. The audio recordings of interviews were transcribed, and after they were transcribed, the possible patterns were looked for. By looking for patterns throughout the data, the goal was to identify some shared views among participants, which helped in defining the themes for discussing and organizing the results of the data. These themes are discussed in what follows.

RESULTS AND DISCUSSION

This section takes a look at the participants' beliefs about mathematics, mathematics assessment, and written and oral mathematics assessment. It also discusses the relationships between the participants' beliefs about mathematics, mathematics assessment, and written and oral mathematics assessment.

Beliefs about Mathematics and Mathematics Assessment

The participants' beliefs about mathematics and mathematics assessment are presented in Table 1 and Table 2. The words highlighted in italics in Table 1 and Table 2 are the common words identified in the participants' shared responses about their beliefs about mathematics and mathematics assessment. These common words are: 'understanding,' 'knowing,' 'application,' 'thinking,' 'reasoning,' 'concept,' and 'procedure.'

Table 1. Beliefs about Mathematics

	Definition	Participants
Mathematics is	*knowing* theory and *application*	Nora; Melissa; Elisabeth
	a way of *thinking* about things	Dave; Van
	art of working with abstract *concepts* towards their *understanding*	James
	pattern recognition	Jane; Nora

Table 2. Beliefs about Mathematics Assessment

	Purpose	Participants
Mathematics assessment should	assess *reasoning*, logical *thinking*, and *understanding* of the *concept* and *procedure*	Melissa; Van; Jane; Nora
	promote discussion	Elisabeth
	sort out future mathematics teachers who are good and not good in mathematics	Dave
	assess recipe mathematics to non-mathematics major students and *conceptual* mathematics to mathematics major students	James

When it came to the participants' beliefs about mathematics, it seemed that most of the participants' beliefs about mathematics were in relation to knowing and understanding the mathematical concepts and their applications. Similarly, when it came to the participants' beliefs about mathematics assessment, what most of the participants valued about mathematics assessment was the ability to assess students' reasoning and understanding of the concepts and procedures.

An interesting comment came from Dave when he talked about his beliefs on what would be the main purpose of mathematics assessment in his mathematics course for future school teachers that he teaches in Germany:

> If you like, the primary function of my assessment isn't to tell anybody how good they are at math. It is to sort out people who are bad at it. And in the best of all possible worlds, I would like to say, "Well you're only bad at it because you had ten years of bad math teaching in school," and so we'll just try and fix that. But my experience is that it takes more than ten weeks to fix ten years of bad teaching.

The participants' beliefs about mathematics assessment are based on their prior schooling and teaching experience. Oral examinations in mathematics were part of the educational system in some of the participants' prior schooling and teaching experience. Therefore, the oral exams were considered to be an essential and natural part of the examination process, from primary to higher education. Melissa, James, and Jane exemplified this:

> We were used to it: It was natural. It was not something that was that different in high school; it was a continuation of high school. *(Melissa)*

> Mathematics I think very much lives from discussions. So, for me, the oral examination is much more natural, and the written

> examination is just out of necessity. *(James)*
>
> I have reasons that I feel are good reasons why I prefer written exams, but, you know, maybe I wouldn't think those things if I had gone through a system with oral exams. *(Jane)*

On the other hand, oral exams can cause discomfort to those who have never been exposed to them as being something that is not completely natural or familiar. Then, Dave exemplified his view on the possibility of using the oral exams:

> It is primarily, I guess, if you like, a cultural issue […] I think there is going to be a difference between me doing an oral exam and somebody who has grown up with oral exams doing an oral exam […]. I'm doing something that is not part of my cultural background, so I don't have any intuitions about it, even if I have knowledge about it.

An interesting finding came from the participants' responses in relation to the questions that they were asked, specifically putting the emphasis on the word 'assessment.' Therefore, when the participants were asked 'What kind of assessment are they currently using in their mathematics courses?', Van, Melissa, Elisabeth, and Nora all used the word 'assessment' in their responses, while Dave, Jane, and James all responded by using the words 'exam' and/or 'examination' instead. On the other hand, when the participants were asked 'What kind of assessment were they exposed to in their mathematics courses before coming to Canada and Germany?,' Van, Melissa, Elisabeth, Nora, and James all responded by using the words' exam 'and/or' examination ', while Dave and Jane both used the word 'assessment' in their responses. Despite the fact that both questions emphasized the 'assessment' that participants are currently or have previously used, the words 'assessment,' 'exam,' and 'examination' were used interchangeably depending on the participants' current or previous experiences with mathematics assessment. It seems that the participants who responded with 'assessment', their current or past experiences consisted of using multiple forms of written assessment in mathematics, such as mid-term exams, final exams, quizzes, homework assignments, online assignments, participation, essays, reflections, etc. On the other hand, the participants who responded with 'exam/examination, said that their current or past experiences with mathematics assessment consisted of using only written and oral exams as the only two forms of assessment that count for students' final grades.

Beliefs about Written Assessment in Mathematics

The participants' beliefs about written assessment in mathematics can be divided between the positive aspects and the negative aspects of written assessment in mathematics. Based on the positive aspects, written assessment in mathematics: 1) allows the relationship only between the student and the subject that is being assessed; 2) provides an opportunity to answer questions in order of the student's preference; and 3) provides a written record of the student's performance.

Allows the relation only between the student and the subject that is being assessed

When asked to describe some of the positive aspects of written assessment, Melissa explained:

> I probably had a slight preference for writing because this was only between me and the subject.

Provides an opportunity to answer questions in order of student's preference

Jane exemplified this based on her experience with using the written assessment in mathematics courses during her previous education:

> When you have a written exam, you have a choice of answering questions in different orders. Then you really have time.

Provides a written record of student's performance

Jane also mentioned that one of the positive aspects of a written assessment is that during the written exams there is a written record or a proof of students' work. She explained:

> When you have a written exam, there is this record of like a complete detailed record of what happened on the exam, so the student has some sort of form of recourse if they feel they weren't graded correctly. And it's there, it's written.

On the other hand, based on the negative aspects, written assessment in mathematics: 1) does not prevent plagiarism; 2) does not provide an opportunity to redeem; and 3) limits an examiner to assess a wide range of students' knowledge and understanding of the subject.

Does not prevent plagiarism

When the participants were asked if it was possible for students to cheat during the written exams, Melissa responded:

> It was possible because, in a way, in a large room, possibly somebody would bring a piece of paper with something

Does not provide an opportunity to redeem

According to Elisabeth and Nora, when there is only a written exam, then there is no opportunity for students to have another chance to redeem themselves if they do not do well on the written exam. They explained:

> Sometimes, when there is only a written exam, some people may claim, "Oh, I knew it, but I got stuck," or "I had a bad day, and everything was on that exam." *(Elisabeth)*

> You have no chance to redeem yourself if, by accident, you got the questions exactly in the areas where you are not the best. *(Nora)*

Limits an examiner to assess a wide range of student's knowledge and understanding of the subject

Nora explained this when she was asked to describe some of the negative characteristics of written assessment:

> First of all, a written exam can test some parts, but not everything of what you have studied.

Beliefs about Oral Assessment in Mathematics

When it comes to the participants' beliefs about oral assessment in mathematics, they can also be divided between the positive aspects and the negative aspects of oral assessment in mathematics. Based on the positive aspects, oral assessment in mathematics: 1) is reactive to students' needs in terms of providing an opportunity for discussion, follow-up questions, and instant feedback; 2) reaffirms or improves students' grades; 3) prevents plagiarism; 4) provides an opportunity for students to assess themselves by listening to their classmates; 5) can assess students' thinking; 6) provides an opportunity to redeem; 7) allows differentiated assessment; 8) provides an opportunity to adapt the level of questions to each student's level of response.

Is reactive to student's needs in terms of providing an opportunity for discussion, follow-up questions, and instant feedback

Melissa and Elisabeth exemplified this when they were asked to describe some of the positive characteristics of oral assessment:

> It was this instant feedback and the possibility of follow-up questions or discussion, and sometimes it was also discussion, because in cases when the instructor felt that you were doing a good job, you had an opportunity for some sort of exchange. Sometimes it felt like

> a more rewarding experience than just a written one. *(Melissa)*
> So, you would submit an assignment, and maybe you get it back one week or maybe later than one week after that, and maybe you don't care anymore, and maybe you have read the solutions that are possible and then you don't care about comparing with your work and fixing your mistakes or something like that. But when you have an oral examination, maybe you will work on that instantaneously. This idea of learning from your mistakes, I think is more valuable in the oral examination or in the oral conversation. *(Elisabeth)*

Reaffirms/improves students' grades

Melissa commented that the oral assessment not only provides an opportunity for students to confirm their knowledge and understanding of the material but also usually provides an opportunity for students to improve their course grades:

> I felt that it was generally an opportunity to reaffirm or correct your written examination. And, I felt that in most cases it was, in practically all cases, that it was reaffirming or improving the grade.

Prevents plagiarism

In terms of cheating during the oral exams, Melissa, Elisabeth, and Van all expressed their own opinions based on their experiences with using the oral exams in mathematics courses during their previous teaching and schooling:

> Well, you did not have access to any of your notes or anything. In theory, you could have somebody else substitute for you, but I've never heard of anything like this. I don't remember whether there were any ID checks when entering. This, I don't remember. I don't see how you can cheat during the oral. *(Melissa)*
> I don't think they can cheat. When you step in front of an examiner, if the student has the topic on the notes, so sometimes, yeah, some people could cheat in that time given before stepping into the oral examination, so they could get access to notes and write something on that paper. But, once you are in front of the examiner, once you are in the out, what is on the paper through the process of discussion of question and answer, you can see if the student got access to that special formula. *(Elisabeth)*
> You cannot cheat on the oral exam. That was my always experience, and for good or bad, I mean on the oral exam, you are on your own. *(Van)*

Provides an opportunity for students to assess themselves by listening to their classmates

Elisabeth explained this when she talked about her experience during the oral examination as a student:

> That was a very good way to know where each and everyone stands because in college, and in elementary and high schools, everybody used to be there, and the examination is in front of the entire class.

Can assess students' thinking

Elisabeth also mentioned that one of the positive aspects of an oral assessment is that it provides an opportunity for students to show their thinking process about the material that is being assessed. She commented:

> That line of thinking is something that we can assess through the oral examination.

Provides an opportunity to redeem

If there is an oral exam, then there is an opportunity for students to have another chance to redeem themselves in case they do not do well on the written exam. Nora explained:

> What was fantastic about it, what I really loved, is that you had a chance to redeem yourself, because some students get really scared when they get a question. I have seen it. They get frozen. They stop writing. Some are so jittery. I have seen it all, here. If it happens to you, even if you come with a blank sheet of paper to the professor, they can start talking to you, and within 5 minutes you calm down normally. They would ask you, "Okay, so what do you say on this? What is the theorem? Okay, what do you recall? You forgot the proof. Okay. So, can you tell me the actual theorem? Okay. Where do you start in this theorem?"

Allows differentiated assessment

With the oral assessment, students also have the option to show and present their knowledge and understanding of the material orally, not just through writing. Dave explained:

> The advantage that I can see is that the written exam format is one format of assessment, and not everybody is equally good at dealing with that format. So, in general, in assessment, it is understood that multiple forms of assessment are better than a single form of assessment. Because then the format of the assessment is having less of a weight on the outcome. So, then it would make sense for everybody to have a mixture of oral, written performance, different kinds of assessment in every course, but that doesn't happen. And in courses with hundreds of people, I can imagine it would get quite complicated.

Provides an opportunity to adapt the level of questions to each student's level of response

During the oral examination, an examiner is able to adjust the exam questions according to the student's knowledge and the ability to answer the questions. James and Elisabeth exemplified this:

> In the oral exams, I get to know the person better, how he or she thinks, and I can very much adapt the level of questions to the level of response. This allows me for instance, from the homework, if I have the impression this is a really good person, I would start out asking difficult questions to allow him or her to get an A. And, for other people who I don't really know, I would also start high level, but not very high and then see whether I can go towards an A question or go towards a question that just gives a D or something. So, this flexibility is what I very much value in oral exams, but at the same time it involves subjective decisions. *(James)*
>
> So usually starting from one question, one problem or one exercise and then if that was going nicely, correctly, fluently, maybe all was good. But if the student will stumble and not be able to do it, then giving them easier and easier questions to solve because if somebody is stuck at some topic, maybe if they cannot do the derivative, you have to see if they can do limits, and if they cannot do

> limits, you have to get lower and so. It takes time. *(Elisabeth)*

On the other hand, based on the negative aspects, oral assessment in mathematics: can make the students feel intimidated or discriminated by an examiner.

Can make the students feel intimidated or discriminated by an examiner

As students, Van, Elisabeth, Nora, and Melissa shared their personal experiences with oral exams. Van and Elisabeth commented that during the oral examination, some professors would even mock a student if he/she was not well prepared for the exam. They explained:

> One thing that I didn't like about oral exams was that there were people that were misusing their powers over students. And, some kind of verbal abuse in front of your peers that we cannot even imagine here in Canada. Yeah, making fun of somebody that he/she feels stupid in front of twenty, thirty people was a regular, and, also, I mean there were extreme cases that sadly I'm aware of that some instructors were just corrupted. *(Van)*

> Sometimes in Romania, they could make fun of you even in university. Professors, yeah when writing something really wrong. If there was something wrong with some of the basics and you got it incorrect or something like that. Yeah, they would make fun of you. *(Elisabeth)*

Nora talked about how she felt being discriminated in mathematics class as a female student:

> Discrimination of some sort was going. I was a victim of discrimination myself because I was in the math department, but that was a subsection, which was more towards the engineering mathematics. And some of the professors were very old school, old gentleman, like 70, I would say plus. In their opinion, a girl could never get an A. So, I challenged that with a professor. I asked him, "Okay, if you think that I'm not getting the level, ask me as much as you want. Four hours." After that, I had the reputation 'don't touch her' because if I know that I know, they cannot do anything to me.

Melissa also had a similar experience during her schooling:

> There were a couple of those which I felt I've been too intimidated, but overall it was a positive experience. It was because of the feeling that the examiner is sort of this inaccessible person, which is just examining me and not […] yes, so it is not somebody I would be free to ask questions. So, intimidation was often related to the personality of the instructor.

Clusters of Beliefs and Their Relationships

The goal of this paper was to identify relationships between the participants' beliefs about mathematics, mathematics assessment, and written and oral mathematics assessment. Based on Green's (1971) concepts of *primary* and *derivative* beliefs, this study looked into beliefs that have a direct relation to mathematical beliefs. Moreover, with the addition of beliefs about written and oral mathematics assessment, a third belief system is proposed: *sub-derivative* belief — a belief that is derived from a *derivative* belief (a belief that is derived from a *primary* belief). Based on the participants' prior schooling and teaching experience in oral and written assessment cultures, two clusters of beliefs were identified. These clusters of beliefs are presented in Figure

1 and 2. This paper will be referring to the oral assessment cultures restricted to countries that are involved in this study, and these are: Bosnia, Germany, Poland, Romania, and Ukraine. It will also be referring to the written assessment cultures of Canada and the United States. Written assessment culture is defined as a culture in which oral assessment in mathematics is not part of the system of education, while oral assessment culture is one where oral assessment is an important part of assessment practice in mathematics.

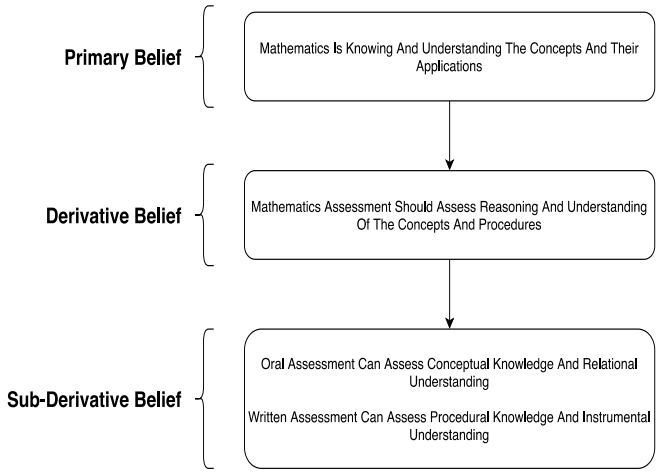

Figure 1. The Cluster of Beliefs Based on the Participants' Schooling and Teaching Experience in Oral Assessment Cultures

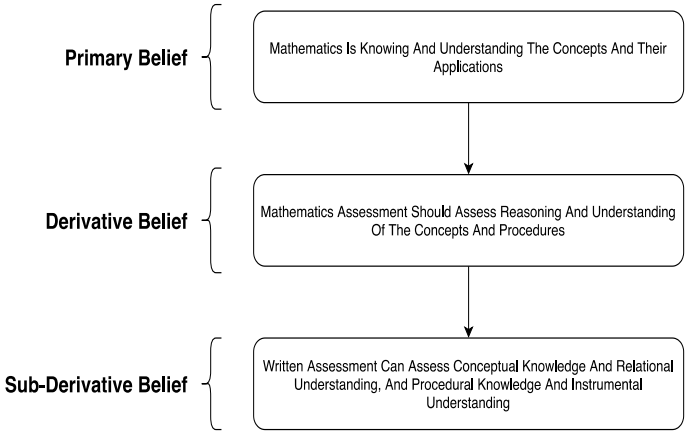

Figure 2. The Cluster of Beliefs Based on the Participants' Schooling and Teaching Experience in Written Assessment Cultures

Figure 1 and 2 represent both clusters with a primary belief 'mathematics is knowing and understanding the concepts and their applications' and a derivative belief 'mathematics assessment should assess reasoning and understanding of the concepts and procedures'. These primary and derivative beliefs are explained in Table 1 and 2. On the other hand, both sub-derivative beliefs, 'oral assessment can assess conceptual knowledge and relational understanding, and written assessment can assess procedural knowledge and instrumental understanding' (Figure 1), and 'written assessment can assess conceptual knowledge and relational understanding, and procedural knowledge and instrumental understanding' (Figure 2), are derived from a derivative belief. These sub-derivative beliefs are explained in my previous study (Videnovic, 2017a), in which these same mathematics professors and instructors were asked to share their personal experiences of using written and oral assessments in the mathematics classroom. Their sub-derivative beliefs are based on their prior (or lack of) exposure to oral assessment in mathematics.

CONCLUSION

Initially, the main purpose of this research was to find out how mathematics professors experience and view oral and written assessments in mathematics. In this paper, the results showed that similar beliefs about mathematics and mathematics assessment result in different beliefs about written and oral mathematics assessment.

The primary belief about mathematics ('mathematics is knowing and understanding the concepts and their applications') was also included in the participants' beliefs about mathematics assessment. In other words, the participants' belief about the purpose of mathematics assessment was derived from their primary belief about mathematics—'mathematics assessment should assess reasoning and understanding of the concepts and procedures'. This belief about the purpose of mathematics assessment is closely related to

one of the three major purposes of assessment according to Brown (2008), which is that the purpose of assessment is to improve teaching and learning. In this 'assessment as improvement of teaching and learning', the purpose of assessing students' knowledge and understanding is to gather the information that would lead to changes in teaching and learning practices, so that improvement in students' achievement can be facilitated.

The participants' beliefs about written and oral mathematics assessment were derived from their belief about the purpose of mathematics assessment ('mathematics assessment should assess reasoning and understanding of the concepts and procedures'), and they were strictly based on their prior exposure (or not) to oral assessment in mathematics. For the participants who had been exposed to oral assessment in mathematics, their sub-derivative belief about written and oral mathematics assessment was that 'oral assessment can assess conceptual knowledge and relational understanding, and written assessment can assess procedural knowledge and instrumental understanding'. On the other hand, among the participants who had not been exposed to oral assessment in mathematics, their sub-derivative belief was that 'written assessment can assess conceptual and procedural knowledge, and relational understanding and instrumental understanding'. These aspects of oral and written assessment in mathematics have not been specifically discussed in any of the literature on mathematics assessment. The closest research to these results on what can be assessed in oral assessment is that oral assessment promotes deep comprehension of the learned material (Iannone & Simpson, 2012, 2015; Joughin, 2007; Lianghuo & Mei, 2007; Nelson, 2010; Nor & Shahrill, 2014; Odafe, 2006; Roecker, 2007), provides long-lasting mathematical knowledge (Iannone & Simpson, 2012), and can determine students' critical thinking abilities (Badger, 2010).

The participants' beliefs about the positive aspects of oral assessment in mathematics are consistent with the research which showed that the oral assessment: is reactive to students' needs (Iannone & Simpson, 2015); allows for probing knowledge through dialogue (Badger, 2010; Joughin, 1998; Odafe, 2006); provides immediate feedback (Boedigheimer et al., 2015; Iannone & Simpson, 2012; Odafe, 2006; Roecker, 2007); prevents plagiarism (Huxham et al., 2012; Joughin, 1998; Nor & Shahrill, 2014); encourages students to deeply engage with the course material (Boedigheimer et al., 2015; Iannone & Simpson, 2012; Nor & Shahrill, 2014; Odafe, 2006); and prevents one small gap in knowledge completely stalling a solution (Joughin, 1998), or, in other words, provides an opportunity to adapt the level of questions to each student's level of response. The participants also mentioned that the oral assessment provides an opportunity for students to assess themselves by listening to their classmates during the oral examination. This aspect of oral assessment had not been introduced in any of the literature on oral assessment.

When it comes to the limitations of the findings, the study was conducted in a context that included participants who were very successful as students in learning mathematics. Therefore, this is an important factor in participants' perceptions of mathematics and mathematics assessment. If interviewees of this study were people who were not successful in mathematics subjects as students, they may have had different experiences and perspectives on mathematics and mathematics assessment. Other sources of data and other research methods can be used to study other aspects of the oral assessment context and experience. Thus, some of the following research topics are recommendations for possible future research studies: 1) to perform a quantitative study in order to understand whether these beliefs about mathematics and mathematics assessment, specifically written and oral mathematics assessment, from this study, expand to a larger population of mathematics professors from different schooling and teaching cultures; 2) to

compare mathematics professor-student interaction during the teaching in oral and written assessment cultures; 3) to compare students' oral and written responses on oral and written mathematics exams in order to see which response meets the standard of what is considered a satisfying mathematical answer (for instance, by video recording an oral examination in a mathematics classroom).

REFERENCES

Badger, J. (2010). Assessing reflective thinking: pre-service teachers' and professors' perceptions of an oral examination. *Assessment in Education: Principles, Policy & Practice*, *17*(1), 77-89. https://doi.org/10.1080/09695940903076022

Beswick, K. (2007). Teachers' beliefs that matter in secondary mathematics classrooms. *Educational Studies in Mathematics*, *65*(1), 95–120. https://doi.org/10.1007/s10649-006-9035-3

Boedigheimer, R., Ghrist, M., Peterson, D., & Kallemyn, B. (2015). Individual Oral Exams in Mathematics Courses: 10 Years of Experience at the Air Force Academy. *Primus (Problems, Resources, and Issues in Mathematics Undergraduate Studies)*, *25*(2), 99-120. https://doi.org/10.1080/10511970.2014.906008

Brown, S., & Knight, P. (1994). *Assessing learners in higher education.* London: Kogan Page.

Brown, G. (2008). *Conceptions of assessment: Understanding what assessment means to teachers and students*. New York, NY: Nova Science Publishers.

Crannel, A. (1999). Collaborative oral take-home exams. In: Gold, B., Keith, S. Z., & Marion, W. A. (Eds.) *Assessment practices in undergraduate mathematics*, pp. 143–145. Washington DC: Mathematical Association of America.

Cross, D. I. (2009). Alignment, cohesion, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education*, *12*(5), 325-346. https://doi.org/10.1007/s10857-009-9120-5

De Vita, G., & Case, P. (2003). Rethinking the internationalisation agenda in UK higher education. *Journal of further and higher education*, *27*(4), 383-398. https://doi.org/10.1080/03098770320000128082

Dionne, J. (1984). The perception of mathematics among elementary school teachers. In J. M. Moser (Ed.), *Proceedings of 6th Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 223–228). Madison (WI): University of Wisconsin: PME-NA.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. *Mathematics teaching: The state of the art*, 249-254. http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/impact.htm

FairTest: The National Center for Fair and Open Testing. (2012, May 22). *"What's Wrong with Standardized Tests."* www.fairtest.org/facts/whatwron.htm

Forrest, G. M. (1985). Oral examinations. In: T. Husen, & T. N. Postlethwaite (Eds.) *The International Encyclopedia of Education*, pp. 3688-3688). Oxford, England: Pergamon Press.

Fyhn, A. B. (2015). How a local oral examination considers affective aspects of knowing mathematics. In *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education*, pp. 1174-1180.

Green, T. F. (1971). *The activities of teaching.* New York, NY: Mcgraw-Hill.

Handal, B. (2003). Teachers' Mathematical

Beliefs: A Review. *The Mathematics Educator*, *13*(2), 47–57. https://openjournals.libs.uga.edu/tme/article/view/1863

Henderson, J., Lloyd, P., & Scott, H. (2002). 'In the real world we're all put on the spot at some time or other, so you need to be prepared for it': An exploratory study of an oral method of assessing knowledge of mental health law. *Social work education*, *21*(1), 91-103. https://doi.org/10.1080/02615470120107040

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In: Hiebert, J. (Ed.) *Conceptual and procedural knowledge: The case of mathematics*, pp. 1-27. New York, NY: Roudledge. https://doi.org/10.4324/9780203063538

Hounsell, D., Falchikov, N., Hounsell, J., Klampfleitner, M., Huxham, M., Thompson, K., & Blair, S. (2007). *Innovative assessment across the disciplines: An analytical review of the literature*. New York, NY: Higher Education Academy.

Hubbard, M. (1971). *Measuring medical education*. Philadelphia: Lea and Febiger.

Huxham, M., Campbell, F., & Westwood, J. (2012). Oral versus written assessments: a test of student performance and attitudes. *Assessment & Evaluation in Higher Education*, *37*(1), 125-136. https://doi.org/10.1080/02602938.2010.515012

Iannone, P., & Simpson, A. (2011). The summative assessment diet: how we assess in mathematics degrees. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *30*(4), 186-196. https://doi.org/10.1093/teamat/hrr017

Iannone, P., & Simpson, A. (2012). Oral assessment in mathematics: implementation and outcomes. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, *31*(4), 179-190. https://doi.org/10.1093/teamat/hrs012

Iannone, P., & Simpson, A. (2015). Students' views of oral performance assessment in mathematics: Straddling the 'assessment of' and 'assessment for' learning divide. *Assessment & Evaluation in Higher Education*, *40*(7), 971-987. https://doi.org/10.1080/02602938.2014.961124

Joughin, G. (1998). Dimensions of oral assessment. *Assessment & Evaluation in Higher Education*, *23*(4), 367-378. https://doi.org/10.1080/0260293980230404

Joughin, G. (2007). Student conceptions of oral presentations. *Studies in Higher Education*, *32*(3), 323-336. https://doi.org/10.1080/03075070701346873

Joughin, G. (2010). *A short guide to oral assessment*. Leeds, Britania Raya: Leeds Metropolitan University. https://eprints.leedsbeckett.ac.uk/id/eprint/2804/1/100317_36668_ShortGuideOralAssess1_WEB.pdf

Lianghuo, F., & Mei, Y. S. (2007). Integrating oral presentation into mathematics teaching and learning: An exploratory study with Singapore secondary students. *The Montana Mathematics Enthusiast, Monograph*, *3*, 81-98. https://eprints.soton.ac.uk/174317/

Liljedahl, P. (2009). Teachers' insights into the relationship between beliefs and practice. In J. Maasz & W. Schlöglmann (Eds.) *Beliefs and attitudes in mathematics education: New research results* (pp. 44–54). Rotterdam, The Netherlands: Sense Publishers.

Maasz, J. & Schlöglmann, W. (Eds.). (2009).

Beliefs and attitudes in mathematics education: New research results. Rotterdam, The Netherlands: Sense Publishers.

Nelson, M. A. (2010). Oral assessments: Improving retention, grades, and understanding. *PRIMUS*, *21*(1), 47-61. https://doi.org/10.1080/10511970902869176

Nor, H. N. H. M., & Shahrill, M. (2014, June). Incorporating the use of poster and oral presentations as an alternative assessment in the teaching of secondary mathematics. In *Proceedings of the 2nd International Conference on Social Sciences Research* (pp. 369-378). ICSSR 2014, WorldConferences. net Kota Kinabalu, Sabah, Malaysia. https://doi.org/10.13140/2.1.3291.3603

Odafe, V. U. (2006). Oral examinations in college mathematics: An alternative assessment technique. *PRIMUS*, *16*(3), 243-256. https://doi.org/10.1080/10511970608984149

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In: F. Lester (Ed.) *Handbook of research on mathematics teaching and learning*, pp. 257–315. Charlotte, NC: Information Age Publishing.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, *28*(5), 550–576. https://doi.org/10.2307/749691

Roecker, L. (2007). Using oral examination as a technique to assess student understanding and teaching effectiveness. *Journal of Chemical Education*, *84*(10), 1663-1666. https://doi.org/10.1021/ed084p1663

Romagnano, L. (2001). Implementing the assessment standards: The myth of objectivity in mathematics assessment. *Mathematics Teacher*, *94*(1), 31-37. https://doi.org/10.5951/MT.94.1.0031

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching*, *12*(2), 88-95. https://doi.org/10.5951/MTMS.12.2.0088

Smith, M. L., & Fey, P. (2000). Validity and accountability in high-stakes testing. *Journal of Teacher Education*, *51*(5), 334-344. https://doi.org/10.1177/002248710005100500 2

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*(2), 213–226. https://doi.org/10.1016/S0742-051X(00)00052-4

Stray, C. (2001). The shift from oral to written examination: Cambridge and Oxford 1700–1900. *Assessment in Education: Principles, Policy & Practice*, *8*(1), 33-50. https://doi.org/10.1080/09695940120033243

Suurtamm, C., Thompson, D. R., Kim, R. Y., Moreno, L. D., Sayac, N., Schukajlow, S., … Vos, P. (2016). *Assessment in Mathematics Education*. Switzerland, AG: Springer. http://doi.org/10.1007/978-3-319-32394-7

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In: D. A. Grouws (Ed.) *Handbook of research on mathematics teaching and learning*, pp. 127–146. New York, NY: Macmillan.

Törner, G., & Grigutsch, S. (1994). Mathematische Weltbilder bei Studienanfängern – eine Erhebung. *Journal für Mathematikdidaktik, 15*(3/4), 211–252. https://doi.org/10.1007/BF03338808

Videnovic, M. (2017a). Oral vs. written exams: what are we assessing in mathematics? *Open Mathematical Education Notes*, *7*(1), 1-7. https://www.potsdam.edu/sites/default/files/omen,%207(1)(2017,1-7)_0.pdf

Videnovic, M. (2017b). Evidential vs. non-evidential beliefs in case of oral assessments: A sequel. *Open Mathematical Education Notes*, *7*(2), 43-47. http://www.imvibl.org/omen/7_2_2017/omen_7_2_2017_43_47.pdf

Vogt, W. P. (1999). *Dictionary of statistics and methodology: A nontechnical guide for the social sciences*. London: Sage.

Žalská, J. (2012). Mathematics teachers' mathematical beliefs: A comprehensive review of international research. *Scientia in Educatione*, *3*(1), 45–65.