# Multistage Reliability-Based Expansion Planning of AC Distribution Networks Using a Mixed-Integer Linear Programming Model

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## Abstract

A new mathematical model for the multistage distribution network expansion planning problem considering reliability is proposed in this paper. Decisions related to substation and branch expansion are driven by the minimization of the total cost, which comprises investment and operating costs including the impact of reliability. The proposed model features two main novelties. First, a set of novel algebraic expressions is devised for a standard reliability index, namely the expected energy not supplied. As a result, the dependence of reliability on network topology is explicitly and effectively cast in the mathematical formulation of the planning problem at hand. In addition, the effect of the network is characterized by a computationally efficient piecewise linear representation of the ac power flow model that takes into account both real and reactive power. The resulting optimization problem is formulated as an instance of mixed-integer linear programming, which provides a suitable framework for the attainment of high-quality solutions with acceptable computational effort using efficient off-the-shelf software with well-known convergence properties. The effectiveness of the proposed planning methodology is empirically demonstrated by providing cheaper expansion plans that enhance system reliability and by achieving better computational results as compared with state-of-the-art models.

*Keywords:* AC network model, distribution network expansion planning, mixed-integer linear programming, multistage, reliability.

## 1. Introduction

Due to the capital intensive nature of the transmission and generation systems and the catastrophic social and environmental consequences of their inadequacy, the assessment of reliability has usually focused on such infrastructures. Nonetheless, the analysis of customer interruption occurrences indicates that service unavailability is mostly related to faults at the distribution system level [1, 2]. System-wide reliability can thus be greatly enhanced by adequately planning distribution network investment decisions. To that end, reliability-based distribution planning models should minimize the duration and frequency of customer interruptions through the improvement of standard reliability assessment indices [2–5]. This paper addresses the incorporation of reliability into multistage distribution network expansion planning.

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Existing techniques for reliability assessment can be categorized in two groups, namely simulation-based methods and non-simulation-based approaches. In the former, reliability indices are obtained via examining network operation for every component outage, Monte Carlo simulation being typically adopted [6]. To that end, a power flow is run to determine the impact of every interruption, which is computationally demanding. Moreover, simulation-based methods cannot be directly integrated into the optimization models used for distribution operation and planning, thereby requiring the application of inexact solution techniques wherein reliability assessment is decoupled from the optimization process. Relevant examples are [7–11].

Non-simulation-based methods usually derive the impact range of interruptions through topological analysis [4], or by formulating an interruption incidence matrix based on topological information from distribution networks [12]. Both approaches involve the characterization of the network topology, which, within the context of the optimization problems related to operation and planning, is *a priori* unknown, hence being represented by decision variables. As a result of this topological dependence, the use of non-simulation-based methods for the evaluation of reliability indices within such optimization problems drastically increases their mathematical complexity.

The first non-simulation-based formulation explicitly incorporating reliability assessment into an optimization problem related to distribution networks is presented in [13], which addresses the network reconfiguration problem. Using a fictitious power flow to describe the load nodal affiliation, the effects of failures occurring in the shortest upstream path between each load node and the corresponding substation are quantified, but the effects of fault isolation are disregarded. Hence, following the terminology coined in [14], the non-simulation-based formulation in [13] solely considers repair-and-switching interruptions in the reliability assessment, i.e., switching-only interruptions are neglected. This practical issue is addressed in the linear-programming-based formulation proposed in [14], which, similarly to [13], relies on fictitious power flows. Although this formulation can calculate all load-node and system-level reliability indices, its reliance on the enumeration of all possible component outages can considerably increase the number of decision variables and constraints of the resulting optimization model, which may lead to prohibitive computational effort. In an attempt to reduce the dimension of the formulation presented in [14], an alternative albeit equivalent approach is proposed in [15], whereby reliability indices are represented by a set of recursive algebraic expressions. The previous works [14, 15] calculate the standard system-level reliability indices by modeling the impact of every interruption on load nodes. In [16], a linear-programming-based approach is proposed to obtain such system reliability indices without explicitly formulating the impact of each interruption on node-level reliability indices. The reliability assessment approaches in [14–16] disregard post-fault reconfiguration mechanisms based on the operation of tie switches. This gap is addressed in [17, 18] by the consideration of fault scenarios, thereby requiring the explicit and, hence, computationally expensive characterization of system operation under every component outage, similar to the enumeration-based approach relying on fictitious flows used in [13, 14]. In [17], the reliability assessment is performed taking into account the actions of circuit breakers and switches through a set of linear constraints that determine the post-fault reconfiguration strategies and lead to a reduction in the impact and duration of interruptions. Furthermore, in [18], the model described in [17] is integrated in the circuit breaker and switch allocation problem to account for reliability.

The above findings on non-simulation-based reliability assessment have triggered the development of new approaches for reliability-constrained distribution network expansion planning. The first use of a non-simulation-based approach for the incorporation of reliability into the multistage distribution network expansion planning problem is reported in [19]. In that work, the topology-dependent linear programming formulation for reliability assessment described in [14] is explicitly accommodated in terms of the decision variables of the optimization problem. This approach features a major shortcoming, namely replacing the conventional use of simulation with the enumeration-based expressions of [19] may lead to intractability due to the large dimension of the resulting planning problem. Attempts to reduce the increase in the size of the planning problem while considering non-simulation-based reliability assessment are presented in [20] and its extended version [21], which also accounts for reliability incentive schemes and distributed generation. However, the problem size reduction is gained at the expense of disregarding switching-only interruptions, thereby potentially leading to an imprecise reliability assessment [16] that may hinder the compliance with standards and regulations. Moreover, previous works [19–21] rely on an approximate network model based on a constant power factor across the system, which can give rise to optimistic planning solutions or even impractical investment decisions eventually requiring load shedding. Both modeling simplifications are recently addressed in [22, 23] by adopting the enumerative scenario-based reliability assessment formulation of [17] and an approximate ac network model. Unfortunately, in [22], a static approach is adopted, which may lead to oversized expansion plans, as shown in [24], whereas practical network-related aspects such as losses and current limits are neglected. The enumeration-based model presented in [22] is extended to a multistage setting in [23]. However, such an extended modeling capability yields prohibitively large optimization problems, thereby requiring the adoption of practical simplifications related to network topology, transfer nodes, and demand.

Motivated by the issues of existing works, this paper presents a new non-simulation-based approach for the precise incorporation of reliability into multistage distribution network expansion planning. Within the context of reliability-constrained distribution network expansion planning [7–11, 19–23], the proposed approach features two main novelties:

1. The modeling of reliability using both a set of algebraic expressions built on the formulation described in [15] and a set of radiality constraints, which have been adapted from [25] for the planning problem at hand.
2. The characterization of the effect of the distribution network using the accurate linearized ac load flow presented in [24].

Compared with existing works on non-simulation-based reliability assessment [13–18], the novelty of the proposed approach lies not only in the incorporation of investment decisions and system operation, which are disregarded therein, but also in the reliability model itself. Major distinctive aspects include 1) the consideration of switching-only interruptions, which are neglected in [13], 2) the characterization of the dependence of reliability on the *a priori* unknown network topology, which is not featured in [13–15], 3) the reliance on simple recursive algebraic nodal expressions, unlike [13, 14, 16–18], and 4) in contrast to [13, 14, 17, 18], the use of a non-enumerative formulation.

It should be noted that the explicit representation of reliability as part of the problem formulation renders the proposed model suitable for non-heuristic techniques, which is a relevant salient feature over previous approximate approaches for distribution network expansion planning relying on simulation-based reliability assessment [7–11]. In comparison with recent non-simulation-based reliability-constrained planning formulations [19–23], the benefits associated with the novelties of the proposed approach are related to 1) the modeling of reliability, 2) the characterization of the distribution network, and 3) the decision-making framework.

Regarding reliability, unlike [20, 21], switching-only interruptions are precisely considered while 1) overcoming the potential intractability of [19, 22, 23] stemming from the explicit representation of system operation for all possible faults at every stage, and 2) accounting for the practical features neglected in [22, 23], namely radial operation under the normal state, transfer nodes, and the chronological aspect of demand for network operation.

As for the effect of the distribution network, the proposed approach significantly differs from [8, 11, 19–23] as reactive power is explicitly modeled while considering network losses and current

Table 1: Proposed Approach versus the Related Literature

| Feature | [7] | [8] | [9] | [10] | [11] | [19] | [20] | [21] | [22] | [23] | Proposed approach |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Mathematical-programming-based | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| AC power flow | ✓ | ✗ | ✓ | ✓ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ | ✓ |
| Non-enumerative reliability assessment | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ | ✗ | ✗ | ✓ |
| Switching-only interruptions | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | ✗ | ✓ | ✓ | ✓ |
| Multistage | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | ✓ | ✓ |
| Radiality | ✓ | ✓ | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | ✗ | ✓ |
| Load levels | ✓ | ✓ | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ | ✗ | ✗ | ✓ |
| Active power losses | ✓ | ✓ | ✗ | ✓ | ✓ | ✓ | ✗ | ✗ | ✗ | ✗ | ✓ |
| Current limits | ✓ | ✓ | ✗ | ✓ | ✓ | ✓ | ✗ | ✓ | ✗ | ✗ | ✓ |
| Transfer nodes | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ | ✓ | ✓ | ✗ | ✗ | ✓ |

limits, thereby providing a more accurate and practical characterization. Moreover, in contrast to the static planning framework adopted in [22], the timing of investment decisions is an outcome of the optimization process.

Table 1 summarizes the main differences between this work and the state of the art of reliability-constrained distribution network expansion planning [7–11, 19–23]. In this table, symbols "✓" and "✗" respectively indicate whether a particular aspect is considered or not. As can be observed, the proposed approach substantially departs from the relevant body of literature [7–11, 19–23] in both modeling and methodological features.

The main contributions of this work are twofold:

1. From a modeling perspective, an explicit non-enumeration-based formulation is presented, for the first time in the literature, for multistage reliability-constrained distribution network expansion planning with an ac power flow model and radiality constraints. Unlike previously reported works, the novel ac-based planning model benefits from the availability of effective mathematical programming techniques and the more precise representation of distribution network operation including radiality, losses, current limits, and different loading conditions. Moreover, the proposed planning model features a novel mathematical characterization of reliability assessment that overcomes the potential intractability of the state-of-the-art enumeration-based formulations while accounting for the effect of transfer nodes and switching-only interruptions.
2. From a methodological point of view, the multistage distribution network expansion planning problem is addressed by an accurate approach based on mixed-integer linear programming considering technical, economic, and reliability aspects. The superiority of the proposed method is backed by its computationally effective ability to optimally solve cases for which the state-of-the-art formulations require substantially longer computing times to identify lower-quality and even infeasible solutions.

To the best of our knowledge, there is no current literature contribution on multistage reliability-constrained distribution network expansion planning using a mixed-integer linear programming framework to jointly and effectively consider 1) a precise power flow model including reactive power, losses, and current limits, 2) radial operation under the normal state, 3) the impact of transfer nodes and switching-only interruptions on reliability, and 4) the chronological characterization of demand for network operation. Thus, both contributions constitute an original and effective solution to the major issues of the state of the art [7–11, 19–23], which may lead to intractability, suboptimality, and even impractical solutions.

The remainder of this paper is organized as follows. Section 2 is devoted to distribution network reliability assessment. Section 3 presents the problem formulation. In Section 4, numerical results are reported and analyzed. Relevant conclusions are drawn in Section 5. Finally, the nomenclature and the linearization schemes used in Section 3 are described in Appendices A and B, respectively.

## 2. Distribution Network Reliability Assessment

The proposed approach relies on the analytical predictive reliability assessment described in [1, 4, 5] for standard reliability metrics such as expected energy not supplied (EENS), system average interruption frequency index (SAIFI), and system average interruption duration index (SAIDI), among others [3]. Thus, the effect of component outages is characterized using two pieces of information, namely failure rates and interruption durations.

As is done in [14, 15], it is considered that 1) the resulting meshed network operates radially, 2) every branch has a switch, 3) at the output of each substation, the feeder is equipped with a circuit breaker without a recloser, 4) interruptions are caused by the sustained outage of individual branches, and 5) the healthy part of the system can be re-energized after fault isolation.

In the occurrence of a fault, the circuit breaker in the feeder of the branch involved is opened, affecting all the nodes belonging to the feeder. Then, the system is reconfigured in order to minimize the energy not supplied. More specifically, the loads located upstream of the fault are re-energized by opening the first switch upstream of the fault followed by the closing of the feeder circuit breaker. After the cause of the fault is eliminated, the corresponding switch is closed and service is fully restored.

As a consequence, load nodes are affected by switching-only interruptions and repair-and-switching interruptions [14, 15]. Switching-only interruptions are associated with reconfiguring the network to isolate a damaged component. Repair-and-switching interruptions correspond to those for which the supply is not restored until the damage is repaired.

Both types of nodal interruptions are characterized by the corresponding expected rates and durations. Such magnitudes depend solely on the information related to branches, i.e., lengths, failure rates, and durations of repair-and-switching and switching-only interruptions [14, 15]. Therefore, using the expected nodal interruption rates and durations, the standard reliability metrics can be calculated as described in [14, 15].

It is worth mentioning that the above reliability-related modeling aspects have been commonly considered in distribution system operation and planning [26]. Within such a context and according to industry practice [27], the reliability worth is estimated based on the costs associated with standard reliability metrics. Here, a widely used reliability index, namely EENS, is adopted to quantify reliability and, hence, investment decisions are driven by the costs due to the unserved energy during contingencies, i.e., the costs of EENS. Note, however, that other practical reliability metrics can be considered.

## 3. Problem Formulation

Using the notation described in Appendix A, this section presents the mathematical model proposed for multistage reliability-constrained distribution network expansion planning, which is a challenging instance of mixed-integer nonlinear programming [28]. The application of the linearization schemes provided in Appendix B eventually yields a mixed-integer linear programming model. As a result, finite convergence to the global optimum is guaranteed, while a measure of the distance to optimality is readily available [29].

As done in the closely related literature [7–11, 19–23], the model assumes that future nodal peak demands are obtained from appropriate forecasting procedures that are beyond the scope of this paper. For further details, the interested reader is referred to the recent survey provided in [30].

### 3.1. Objective Function

As is customary in industry practice [27] and the relevant literature on multistage distribution network expansion planning [7–9, 11, 19–21, 23], the objective of the proposed model is to minimize the present value of the total cost, which comprises investment and operating costs. The operating cost includes maintenance, energy production, load shedding, and reliability costs, the latter being modeled here by the costs of EENS, as explained in Section 2. Mathematically, the economic goal of the proposed optimization is formulated as:

$$\text{Minimize}\, c^{PV} = \sum_{y \in \mathcal{Y}} \frac{(1+I_r)^{-y}}{I_r} c_y^I + \sum_{y \in \mathcal{Y}} \left[ (1+I_r)^{-y} \left( c_y^M + c_y^E + c_y^{Sh} + c_y^{ENS} \right) \right] + \frac{(1+I_r)^{-|\mathcal{Y}|}}{I_r} \left( c_{|\mathcal{Y}|}^M + c_{|\mathcal{Y}|}^E + c_{|\mathcal{Y}|}^{Sh} + c_{|\mathcal{Y}|}^{ENS} \right). \tag{1}$$

As done in [8, 11, 19], the present value of the total cost $c^{PV}$ is modeled in (1) under the hypothesis of a perpetual or infinite planning horizon [31], i.e., the investment is amortized annually throughout the lifetime of the installed equipment and after its lifetime expiration there is a reinvestment in the same type of equipment. Thus, according to [31], the present value of the total cost minimized in (1) is equal to the sum of the present value of the total investment cost, represented by the first summation term in (1), and the present value of the total operating cost, which is characterized by the remaining terms in (1). Note that the last term in (1) solely involving operating costs at the last planning stage represents the perpetual portion of such costs. Individual cost terms in (1) are cast as follows:

$$c_y^I = RR^S \sum_{i \in \mathcal{SN}} \left( C_i^{I,S} x_{i,y}^S + \sum_{t \in \mathcal{T}} C_t^{I,T} x_{i,t,y}^T \right) + RR^B \left( \sum_{ij \in \mathcal{AB}} \sum_{c \in \mathcal{C}} C_{ij,c}^{I,AB} x_{ij,c,y}^{AB} + \sum_{ij \in \mathcal{RB}} \sum_{\substack{c \in \mathcal{C} | \\ c \neq \beta^{RB}}} C_{ij,c}^{I,RB} x_{ij,c,y}^{RB} \right); \forall y \in \mathcal{Y} \tag{2}$$

$$c_y^M = \sum_{i \in \mathcal{SN}} C_i^{M,S} w_{i,y}^{S,ex} + \sum_{i \in \mathcal{SN}} \sum_{t \in \mathcal{T}} C_t^{M,T} w_{i,t,y}^T + \sum_{ij \in \mathcal{B}} \sum_{c \in \mathcal{C}} C_{ij,c}^{M,B} w_{ij,c,y}^B; \forall y \in \mathcal{Y} \tag{3}$$

$$c_y^E = \sum_{l \in \mathcal{L}} \Delta_l \left( \sum_{i \in \mathcal{SN}} C_{i,l}^E P_{i,l,y}^S \right); \forall y \in \mathcal{Y} \tag{4}$$

$$c_y^{Sh} = \sum_{l \in \mathcal{L}} \Delta_l \left( \sum_{i \in \mathcal{SN}} C^{Sh} P_{i,l,y}^{Sh} \right); \forall y \in \mathcal{Y} \tag{5}$$

$$c_y^{ENS} = C^{ENS} EENS_y; \forall y \in \mathcal{Y}. \tag{6}$$

In (2), the amortized cost of the investment at each stage is formulated as the sum of the costs associated with the reinforcement and construction of substations, and the replacement and addition of branches. The capital recovery rates for substations and circuits are computed as $RR^S = \frac{I_r(1+I_r)^{\eta^S}}{(1+I_r)^{\eta^S}-1}$ and $RR^B = \frac{I_r(1+I_r)^{\eta^B}}{(1+I_r)^{\eta^B}-1}$, respectively. Expressions (3) model the maintenance costs of existing substations, newly added transformers, and branches at each stage. The cost of energy production at each stage is modeled in (4). Using a sufficiently large penalty cost coefficient, $C^{Sh}$, load shedding costs are formulated in (5). Finally, reliability costs, i.e., the costs of expected energy not supplied along the planning horizon, are cast in (6). Based on [7, 9, 11, 19, 20, 23], for each stage $y$, the reliability cost is equal to the cost coefficient for EENS, $C^{ENS}$, times the expected energy not supplied at that stage, $EENS_y$. According to [1], the calculation of EENS involves products of two terms, namely 1) the expected duration of the interruptions experienced by load nodes, and 2) the corresponding average load curtailment. As described in Section 2, branch outages give rise to the curtailment of nodal demands with two different durations depending on whether switching or repair-and-switching actions are implemented. Thus, the levels of expected energy not supplied along the planning horizon are formulated as follows [14–19, 23]:

$$EENS_y = \sum_{i \in \mathcal{N}} \left(\Gamma^{RS}_{i,y} + \Gamma^{SO}_{i,y}\right) \sum_{l \in \mathcal{L}} \frac{\Delta_l F^D_l P^D_{i,y}}{8760}; \forall y \in \mathcal{Y} \tag{7}$$

where the term within parentheses represents expected nodal interruption durations whereas the other term is related to nodal load curtailment. Note that the effect of branch outages on both interruption durations is precisely formulated in Section 3.6. As for load curtailment, the seasonal or chronological aspect of demand is modeled by the discretization of the annual system load-duration curve into a set of blocks each characterized by its load level and duration. Furthermore, for the sake of simplicity, such a system-wide demand characterization is applied on a nodal basis.

### *3.2. Investment and Operational Constraints*

Based on [8, 11, 19, 32], investment and operational variables are constrained by (8)–(30):

$$\sum_{y \in \mathcal{Y}} \sum_{c \in \mathcal{C}} x^{AB}_{ij,c,y} \leq 1; \forall ij \in \mathcal{AB} \tag{8}$$

$$\sum_{y \in \mathcal{Y}} \sum_{c \in \mathcal{C} | c \neq \beta^{RB}} x^{RB}_{ij,c,y} \leq 1; \forall ij \in \mathcal{RB} \tag{9}$$

$$w^B_{ij,c,y} \leq 1; \forall ij \in \mathcal{FB} | \varphi_{ij} = 1, \forall c \in \mathcal{C}, \forall y \in \mathcal{Y} \tag{10}$$

$$w^B_{ij,c,y} \leq \sum_{p=1}^{y} x^{RB}_{ij,c,p}; \forall ij \in \mathcal{RB} | \varphi_{ij} = 1, \forall c \in \mathcal{C} | c \neq \beta^{RB}, \forall y \in \mathcal{Y} \tag{11}$$

$$w^B_{ij,c,y} \leq 1 - \sum_{p=1}^{y} \sum_{e \in \mathcal{C} | e \neq \beta^{RB}} x^{RB}_{ij,e,p}; \forall ij \in \mathcal{RB} | \varphi_{ij} = 1, \forall c \in \mathcal{C} | c = \beta^{RB}, \forall y \in \mathcal{Y} \tag{12}$$

$$w^B_{ij,c,y} \leq \sum_{p=1}^{y} x^{AB}_{ij,c,p}; \forall ij \in \mathcal{AB} | \varphi_{ij} = 1, \forall c \in \mathcal{C}, \forall y \in \mathcal{Y} \tag{13}$$

$$w^B_{ij,c,y} = 1; \forall ij \in \mathcal{FB} | \varphi_{ij} = 0, \forall c \in \mathcal{C}, \forall y \in \mathcal{Y} \tag{14}$$

$$w^B_{ij,c,y} = \sum_{p=1}^{y} x^{RB}_{ij,c,p}; \forall ij \in \mathcal{RB} | \varphi_{ij} = 0, \forall c \in \mathcal{C} | c \neq \beta^{RB}, \forall y \in \mathcal{Y} \tag{15}$$

$$w^B_{ij,c,y} = 1 - \sum_{p=1}^{y} \sum_{e \in \mathcal{C} | e \neq \beta^{RB}} x^{RB}_{ij,e,p}; \forall ij \in \mathcal{RB} | \varphi_{ij} = 0, \forall c \in \mathcal{C} | c = \beta^{RB}, \forall y \in \mathcal{Y} \tag{16}$$

$$w_{ij,c,y}^{B} = \sum_{p=1}^{y} x_{ij,c,p}^{AB}; \forall ij \in \mathcal{AB} | \varphi_{ij} = 0, \forall c \in \mathcal{C}, \forall y \in \mathcal{Y} \tag{17}$$

$$w_{ij,y}^{B+} + w_{ij,y}^{B-} = \sum_{c \in \mathcal{C}} w_{ij,c,y}^{B}; \forall ij \in \mathcal{B}, \forall y \in \mathcal{Y} \tag{18}$$

$$w_{ij,c,y}^{B} = 0; \forall ij \in \mathcal{FB}, \forall c \in \mathcal{C} | c \neq \beta^{FB}, \forall y \in \mathcal{Y} \tag{19}$$

$$\sum_{y \in \mathcal{Y}} x_{i,y}^{S} \leq 1; \forall i \in \mathcal{SN} \tag{20}$$

$$\sum_{y \in \mathcal{Y}} \sum_{t \in \mathcal{T}} x_{i,t,y}^{T} \leq 1; \forall i \in \mathcal{SN} \tag{21}$$

$$x_{i,t,y}^{T} \leq \sum_{p=1}^{y} x_{i,p}^{S}; \forall i \in \mathcal{SN}, \forall t \in \mathcal{T}, \forall y \in \mathcal{Y} \tag{22}$$

$$w_{i,t,y}^{T} \leq \sum_{p=1}^{y} x_{i,t,p}^{T}; \forall i \in \mathcal{SN}, \forall t \in \mathcal{T}, \forall y \in \mathcal{Y} \tag{23}$$

$$\sum_{t \in \mathcal{T}} w_{i,t,y}^{T} \leq 1; \forall i \in \mathcal{SN}, \forall y \in \mathcal{Y} \tag{24}$$

$$x_{i,y}^{S}, w_{i,y}^{S,ex} \in \{0,1\}; \forall i \in \mathcal{SN}, \forall y \in \mathcal{Y} \tag{25}$$

$$x_{i,t,y}^{T}, w_{i,t,y}^{T} \in \{0,1\}; \forall i \in \mathcal{SN}, \forall t \in \mathcal{T}, \forall y \in \mathcal{Y} \tag{26}$$

$$x_{ij,c,y}^{RB} \in \{0,1\}; \forall ij \in \mathcal{RB}, \forall c \in \mathcal{C}, \forall y \in \mathcal{Y} \tag{27}$$

$$x_{ij,c,y}^{AB} \in \{0,1\}; \forall ij \in \mathcal{AB}, \forall c \in \mathcal{C}, \forall y \in \mathcal{Y} \tag{28}$$

$$w_{ij,c,y}^{B} \in \{0,1\}; \forall ij \in \mathcal{B}, \forall c \in \mathcal{C}, \forall y \in \mathcal{Y} \tag{29}$$

$$w_{ij,y}^{B-}, w_{ij,y}^{B+} \in \{0,1\}; \forall ij \in \mathcal{B}, \forall y \in \mathcal{Y}. \tag{30}$$

Expressions (8) and (9) ensure that a maximum of one installation or reinforcement is performed for each branch throughout the planning horizon. In (10)–(19), branch operation is modeled by binary variables different from those used to represent investment decisions, which allows handling radially operated meshed distribution networks. Expressions (10)–(17) guarantee that a branch with a specific conductor type can be used once its corresponding investment has already been made. Expressions (10)–(13) are associated with the set of branches that can be switched under normal operation, for which the corresponding binary parameters $\varphi_{ij}$ are equal to 1. Analogously, expressions (14)–(17) are related to non-switchable branches under normal operation, which are characterized by $\varphi_{ij}$ equal to 0.

The operating state of a given branch $ij$ is represented by two binary variables in (18), as proposed in [33]. If $w_{ij,y}^{B+}$ or $w_{ij,y}^{B-}$ is equal to 1, then branch $ij$ is in operation at stage $y$, whereas if both are equal to 0, then branch $ij$ is out of operation at that stage. Furthermore, the direction of the flow across a particular branch $ij$ at a given stage $y$ is also modeled by the values of variables $w_{ij,y}^{B+}$ and $w_{ij,y}^{B-}$. Thus, the combination $w_{ij,y}^{B+} = 1$ and $w_{ij,y}^{B-} = 0$ is used to identify that node $i$ is upstream of node $j$ and, hence, the flow is from $i$ to $j$. Conversely, the combination $w_{ij,y}^{B+} = 0$ and $w_{ij,y}^{B-} = 1$ is used to identify that node $i$ is downstream of node $j$ and, hence, the flow is from $j$ to $i$. Expressions (19) ensure that fixed branches solely use the original conductor type.

According to (20), the planner can only invest once at most in each substation. Similarly, expressions (21) impose a maximum of one new transformer installation per substation throughout the planning horizon. As per (22), the installation of new transformers happens after the corresponding substation investment. Expressions (23) guarantee that new transformers operate only if the related investment has already been made. Moreover, constraints (24) ensure the use of one new transformer type at most for each substation at each stage of the planning horizon. Lastly, the binary nature of investment and operational variables is set in (25)–(30).

### 3.3. Effect of the Distribution Network

Expressions (31)–(41) represent the ac power flow model for a radially operated distribution network based on the set of recursive equations proposed in [34]:

$$\sum_{ki\in\mathcal{B}}\sum_{c\in\mathcal{C}}P_{ki,c,l,y}-\sum_{ij\in\mathcal{B}}\sum_{c\in\mathcal{C}}\left(P_{ij,c,l,y}+R_c\ell_{ij}I^{sqr}_{ij,c,l,y}\right)+P^{Sh}_{i,l,y}+P^{S}_{i,l,y}=F^D_lP^D_{i,y};$$
$$\forall i\in\mathcal{N},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{31}$$

$$\sum_{ki\in\mathcal{B}}\sum_{c\in\mathcal{C}}Q_{ki,c,l,y}-\sum_{ij\in\mathcal{B}}\sum_{c\in\mathcal{C}}\left(Q_{ij,c,l,y}+X_c\ell_{ij}I^{sqr}_{ij,c,l,y}\right)+Q^{Sh}_{i,l,y}+Q^{S}_{i,l,y}+Q^{CB}_{i,l,y}=F^D_lQ^D_{i,y};$$
$$\forall i\in\mathcal{N},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{32}$$

$$V^{sqr}_{i,l,y}=V^{sqr}_{j,l,y}+\sum_{c\in\mathcal{C}}[2\left(R_cP_{ij,c,l,y}+X_cQ_{ij,c,l,y}\right)\ell_{ij}-Z^2_c\ell^2_{ij}I^{sqr}_{ij,c,l,y}]+\Delta^V_{ij,l,y};$$
$$\forall ij\in\mathcal{B},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{33}$$

$$-\overline{\Delta}^V\left[1-\left(w^{B+}_{ij,y}+w^{B-}_{ij,y}\right)\right]\le\Delta^V_{ij,l,y}\le\overline{\Delta}^V\left[1-\left(w^{B+}_{ij,y}+w^{B-}_{ij,y}\right)\right];$$
$$\forall ij\in\mathcal{B},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{34}$$

$$V^{sqr}_{j,l,y}\hat{I}^{sqr}_{ij,l,y}=\hat{P}^2_{ij,l,y}+\hat{Q}^2_{ij,l,y};\forall ij\in\mathcal{B},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{35}$$

$$\hat{I}^{sqr}_{ij,l,y}=\sum_{c\in\mathcal{C}}I^{sqr}_{ij,c,l,y};\forall ij\in\mathcal{B},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{36}$$

$$\hat{P}_{ij,l,y}=\sum_{c\in\mathcal{C}}P_{ij,c,l,y};\forall ij\in\mathcal{B},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{37}$$

$$\hat{Q}_{ij,l,y}=\sum_{c\in\mathcal{C}}Q_{ij,c,l,y};\forall ij\in\mathcal{B},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{38}$$

$$P^{Sh}_{i,l,y}\le F^D_lP^D_{i,y};\forall i\in\mathcal{N},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{39}$$

$$Q^{Sh}_{i,l,y}=\tan(\cos^{-1}(pf_i))P^{Sh}_{i,l,y};\forall i\in\mathcal{N},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{40}$$

$$S^{S,sqr}_{i,l,y}\ge\left(P^S_{i,l,y}\right)^2+\left(Q^S_{i,l,y}\right)^2;\forall i\in\mathcal{SN},\forall l\in\mathcal{L},\forall y\in\mathcal{Y}\tag{41}$$

where $V^{sqr}_{i,l,y}$ and $I^{sqr}_{ij,c,l,y}$ represent $V^2_{i,l,y}$ and $I^2_{ij,c,l,y}$, respectively.

Expressions (31) and (32) respectively ensure the active and reactive power balances at each node, i.e., Kirchhoff's first law, while accounting for network losses. Expressions (33) and (34) correspond to Kirchhoff's second law. Expressions (33) model branch voltage drops through auxiliary variables $\Delta^V_{ij,l,y}$, which are bounded in (34). As per (34), $\Delta^V_{ij,l,y}$ is equal to 0 for those branches $ij$ in operation at stage $y$, for which $w^{B+}_{ij,y}+w^{B-}_{ij,y}$ is equal to 1. Thus, $\Delta^V_{ij,l,y}$ has no effect on (33), as desired. Conversely, for unused branches $ij$ at stage $y$, for which $w^{B+}_{ij,y}+w^{B-}_{ij,y}$ is equal to 0, $\Delta^V_{ij,l,y}$ lies in the interval $[-\overline{\Delta}^V,\overline{\Delta}^V]$. Hence, a sufficiently large value for $\overline{\Delta}^V$ ensures the deactivation of (33), as desired. Expressions (35) establish the relationship between active and reactive power flows $\hat{P}_{ij,l,y}$ and $\hat{Q}_{ij,l,y}$, squared current magnitudes $\hat{I}^{sqr}_{ij,l,y}$, and squared voltage magnitudes $V^{sqr}_{j,l,y}$. $\hat{I}^{sqr}_{ij,l,y}$, $\hat{P}_{ij,l,y}$, and $\hat{Q}_{ij,l,y}$ are respectively defined in (36)–(38), which rely on the fact that each branch uses a single conductor type at each planning stage. Additionally, nodal load shedding is limited as per

(39) and (40), whereas the active, reactive, and apparent power injections at substation nodes are related by (41).

Note that (35) and (41) contain nonlinear terms that can be linearized as described in [24]. Based on the limited range within which nodal voltage magnitudes lie in practice, the product $V^{sqr}_{j,l,y}\hat{I}^{sqr}_{ij,l,y}$ in (35) can be linearized as follows:

$$V^{sqr}_{j,l,y}\hat{I}^{sqr}_{ij,l,y} \approx V^{est}_{j,l,y}\hat{I}^{sqr}_{ij,l,y}; \forall ij \in \mathcal{B}, \forall l \in \mathcal{L}, \forall y \in \mathcal{Y} \tag{42}$$

where parameters $V^{est}_{j,l,y}$ are estimated squared voltage magnitudes.

Analogously, the quadratic terms in the right-hand sides of (35) and (41) can be linearized using a piecewise approximation, as described in Appendix B.

Expressions (31)–(34), (36)–(40), and the linearized versions of (35) and (41) are essential pillars of the modeling contribution featured by this paper. The significant departure from closely related non-simulation-based models [19–23] comprises the capability to consider different power factors across the system, unlike [19–21], while overcoming the limitation of [22, 23], namely the absence of network losses and current limits.

### *3.4. Operational Limits*

Expressions (43)–(48) set the acceptable ranges of the operational variables taking into account the investment decisions and operational statuses of branches and substations:

$$\underline{V}^2 \leq V^{sqr}_{i,l,y} \leq \overline{V}^2; \forall i \in \mathcal{N}, \forall l \in \mathcal{L}, \forall y \in \mathcal{Y} \tag{43}$$

$$0 \leq I^{sqr}_{ij,c,l,y} \leq \overline{I}^2_c w^B_{ij,c,y}; \forall ij \in \mathcal{B}, \forall c \in \mathcal{C}, \forall l \in \mathcal{L}, \forall y \in \mathcal{Y} \tag{44}$$

$$-\overline{S}^B_{ij,c} w^B_{ij,c,y} \leq P_{ij,c,l,y} \leq \overline{S}^B_{ij,c} w^B_{ij,c,y}; \forall ij \in \mathcal{B}, \forall c \in \mathcal{C}, \forall l \in \mathcal{L}, \forall y \in \mathcal{Y} \tag{45}$$

$$-\overline{S}^B_{ij,c} w^B_{ij,c,y} \leq Q_{ij,c,l,y} \leq \overline{S}^B_{ij,c} w^B_{ij,c,y}; \forall ij \in \mathcal{B}, \forall c \in \mathcal{C}, \forall l \in \mathcal{L}, \forall y \in \mathcal{Y} \tag{46}$$

$$0 \leq Q^{CB}_{i,l,y} \leq \overline{Q}^{CB}_i; \forall i \in \mathcal{N}, \forall l \in \mathcal{L}, \forall y \in \mathcal{Y} \tag{47}$$

$$S^{S,sqr}_{i,l,y} \leq (\overline{S}^{S,ex}_i w^{S,ex}_{i,y} + \sum_{t \in \mathcal{T}} \overline{S}^T_t w^T_{i,t,y})^2; \forall i \in \mathcal{SN}, \forall l \in \mathcal{L}, \forall y \in \mathcal{Y}. \tag{48}$$

The above expressions represent the limits for voltages (43), currents (44), active power flows (45), reactive power flows (46), reactive power injections of capacitor banks (47), and levels of apparent power injected by substations (48).

Nonlinear expressions (48) can be cast in a linear way as follows. First, the right-hand-side term is expanded:

$$\begin{aligned} S^{S,sqr}_{i,l,y} \leq & \left(\overline{S}^{S,ex}_i w^{S,ex}_{i,y}\right)^2 + \left(\overline{S}^T_1 w^T_{i,1,y}\right)^2 + ... + \left(\overline{S}^T_{|\mathcal{T}|} w^T_{i,|\mathcal{T}|,y}\right)^2 + 2\left(\overline{S}^{S,ex}_i w^{S,ex}_{i,y}\right)\left(\overline{S}^T_1 w^T_{i,1,y}\right) + ...+ \\ & 2\left(\overline{S}^{S,ex}_i w^{S,ex}_{i,y}\right)\left(\overline{S}^T_{|\mathcal{T}|} w^T_{i,|\mathcal{T}|,y}\right) + 2\left(\overline{S}^T_1 w^T_{i,1,y}\right)\left(\overline{S}^T_2 w^T_{i,2,y}\right) + ... + 2\left(\overline{S}^T_1 w^T_{i,1,y}\right)\left(\overline{S}^T_{|\mathcal{T}|} w^T_{i,|\mathcal{T}|,y}\right) + ...+ \\ & 2\left(\overline{S}^T_{|\mathcal{T}|-1} w^T_{i,|\mathcal{T}|-1,y}\right)\left(\overline{S}^T_{|\mathcal{T}|} w^T_{i,|\mathcal{T}|,y}\right); \forall i \in \mathcal{SN}, \forall l \in \mathcal{L}, \forall y \in \mathcal{Y}. \end{aligned} \tag{49}$$

The terms in the right-hand side of (49) involve products of binary variables $w^{S,ex}_{i,y}$ and $w^T_{i,t,y}$, which are subsequently linearized as described in Appendix B.

### *3.5. Radiality Constraints*

In [25], a set of constraints was presented to guarantee the radial operation for the reconfiguration problem of distribution systems. Expressions (50)–(52) together with (18) extend that model for the radially operated meshed distribution networks considered in the planning problem at hand:

$$\sum_{ji\in\mathcal{B}} w_{ji,y}^{B+} + \sum_{ij\in\mathcal{B}} w_{ij,y}^{B-} = 0; \forall i \in \mathcal{SN}, \forall y \in \mathcal{Y} \tag{50}$$

$$\sum_{ji\in\mathcal{B}} w_{ji,y}^{B+} + \sum_{ij\in\mathcal{B}} w_{ij,y}^{B-} = 1; \forall i \in \mathcal{N}\backslash\mathcal{SN}, \forall y \in \mathcal{Y} | P_{i,y}^{D} > 0 \tag{51}$$

$$\sum_{ji\in\mathcal{B}} w_{ji,y}^{B+} + \sum_{ij\in\mathcal{B}} w_{ij,y}^{B-} \leq 1; \forall i \in \mathcal{N}\backslash\mathcal{SN}, \forall y \in \mathcal{Y} | P_{i,y}^{D} = 0. \tag{52}$$

Expressions (50) ensure that, for those branches connecting load nodes to substations, the substation node is the sending node. Expressions (51) guarantee that each load node is the receiving node of a single branch. Finally, transfer nodes, i.e., nodes without demand that are used to connect two load nodes, are modeled in (52).

It is worth emphasizing that the proposed radiality constraints (50)–(52), which are suitable for an unknown network topology, extend those presented in [25] for a given grid layout. Such a nontrivial extension constitutes a relevant part of the first contribution listed in Section 1 as it represents a salient modeling feature over the state-of-the-art radiality constraints used in distribution system planning [35]. Note that, unlike [35], neither auxiliary variables nor fictitious flows are required, thereby leading to a more efficient equivalent model from a computational perspective. Moreover, the proposed radiality model substantially differs from those used in closely related formulations for multistage reliability-constrained distribution network expansion planning [19–21], which rely on a single binary variable for branch operation, and [23], where radial operation under the normal state and transfer nodes are both disregarded.

### *3.6. Reliability Constraints*

This section is devoted to modeling the effect of branch outages on EENS. To that end, the expected durations of both repair-and-switching and switching-only nodal interruptions, $\Gamma_{i,y}^{RS}$ and $\Gamma_{i,y}^{SO}$, which are required to determine $EENS_y$ as per (7), are formulated in terms of branch failure rates, $\lambda_{ij}$, and branch interruption durations, $\tau_{ij}^{RS}$ and $\tau_{ij}^{SO}$.

In [15], linear algebra was proposed to model standard reliability indices for a radial system. As a major computational advantage, such a reliability assessment algebraic model overcomes the dimensionality issue of the non-simulation-based approaches reported in [14] and [17, 18] and applied in [19] and [22, 23], respectively. Here, we propose leveraging the findings of [15] to explicitly incorporate reliability in the mathematical formulation of multistage reliability-constrained distribution network expansion planning. It is worth mentioning that the proposed reliability model features a major distintive aspect over the formulation of [15], namely the capability of handling the lack of knowledge of the network topology along the planning horizon, which is an outcome of the optimization process. The proposed reliability constraints are formulated as follows:

$$\Gamma_{i,y}^{RS} = \Gamma_{j,y}^{RS} + \tau_{ij}^{RS}\lambda_{ij}\ell_{ij}w_{ij,y}^{B-} - \tau_{ij}^{RS}\lambda_{ij}\ell_{ij}w_{ij,y}^{B+} + \Delta_{ij,y}^{RS}; \forall ij \in \mathcal{B}, \forall y \in \mathcal{Y} \tag{53}$$

$$-\overline{\Delta}^{RS}\left[1 - \left(w_{ij,y}^{B+} + w_{ij,y}^{B-}\right)\right] \leq \Delta_{ij,y}^{RS} \leq \overline{\Delta}^{RS}\left[1 - \left(w_{ij,y}^{B+} + w_{ij,y}^{B-}\right)\right]; \forall ij \in \mathcal{B}, \forall y \in \mathcal{Y} \tag{54}$$

$$\Gamma_{i,y}^{SO} = \Gamma_{j,y}^{SO} - \tau_{ij}^{SO}\lambda_{ij}\ell_{ij}w_{ij,y}^{B-} + \tau_{ij}^{SO}\lambda_{ij}\ell_{ij}w_{ij,y}^{B+} + \Delta_{ij,y}^{SO}; \forall ij \in \mathcal{B} | i, j \notin \mathcal{SN}, \forall y \in \mathcal{Y} \tag{55}$$

$$-\overline{\Delta}^{SO}\left[1 - \left(w_{ij,y}^{B+} + w_{ij,y}^{B-}\right)\right] \leq \Delta_{ij,y}^{SO} \leq \overline{\Delta}^{SO}\left[1 - \left(w_{ij,y}^{B+} + w_{ij,y}^{B-}\right)\right];$$
$$\forall ij \in \mathcal{B} | i, j \notin \mathcal{SN}, \forall y \in \mathcal{Y} \tag{56}$$

$$\sum_{\substack{ji\in\mathcal{B}|\\ j\notin\mathcal{SN}}} f_{ji,y}^{SO} - \sum_{\substack{ij\in\mathcal{B}|\\ j\notin\mathcal{SN}}} f_{ij,y}^{SO} + \Psi_{i,y}^{SO} = \sum_{\substack{ji\in\mathcal{B}|\\ j\notin\mathcal{SN}}} \tau_{ji}^{SO}\lambda_{ji}\ell_{ji}w_{ji,y}^{B+} + \sum_{\substack{ij\in\mathcal{B}|\\ j\notin\mathcal{SN}}} \tau_{ij}^{SO}\lambda_{ij}\ell_{ij}w_{ij,y}^{B-};$$
$$\forall i \in \mathcal{RN}, \forall y \in \mathcal{Y} \tag{57}$$

$$\sum_{\substack{ji\in\mathcal{B}| \\ j\notin\mathcal{SN}}} f^{SO}_{ji,y} - \sum_{\substack{ij\in\mathcal{B}| \\ j\notin\mathcal{SN}}} f^{SO}_{ij,y} = \sum_{\substack{ji\in\mathcal{B}| \\ j\notin\mathcal{SN}}} \tau^{SO}_{ji}\lambda_{ji}\ell_{ji}w^{B+}_{ji,y} + \sum_{\substack{ij\in\mathcal{B}| \\ j\notin\mathcal{SN}}} \tau^{SO}_{ij}\lambda_{ij}\ell_{ij}w^{B-}_{ij,y};$$

$$\forall i \in \mathcal{N}\backslash\mathcal{RN}, \forall y \in \mathcal{Y} \tag{58}$$

$$-\overline{\Delta}^{SO}\left(w^{B+}_{ij,y} + w^{B-}_{ij,y}\right) \leq f^{SO}_{ij,y} \leq \overline{\Delta}^{SO}\left(w^{B+}_{ij,y} + w^{B-}_{ij,y}\right); \forall ij \in \mathcal{B}|i, j \notin \mathcal{SN}, \forall y \in \mathcal{Y} \tag{59}$$

$$0 \leq \Psi^{SO}_{i,y} \leq \overline{\Delta}^{SO}\left(w^{B+}_{ij,y} + w^{B-}_{ij,y}\right); \forall ij \in \mathcal{B}|j \in \mathcal{SN}, \forall y \in \mathcal{Y} \tag{60}$$

$$0 \leq \Psi^{SO}_{j,y} \leq \overline{\Delta}^{SO}\left(w^{B+}_{ij,y} + w^{B-}_{ij,y}\right); \forall ij \in \mathcal{B}|i \in \mathcal{SN}, \forall y \in \mathcal{Y} \tag{61}$$

$$\Gamma^{SO}_{i,y} \geq \Psi^{SO}_{i,y}; \forall i \in \mathcal{RN}, \forall y \in \mathcal{Y} \tag{62}$$

$$\Gamma^{RS}_{i,y} = 0; \forall i \in \mathcal{SN}, \forall y \in \mathcal{Y} \tag{63}$$

$$\Gamma^{SO}_{i,y} = 0; \forall i \in \mathcal{SN}, \forall y \in \mathcal{Y} \tag{64}$$

$$\Gamma^{SO}_{i,y} \geq 0; \forall i \in \mathcal{N}, \forall y \in \mathcal{Y} \tag{65}$$

$$\Gamma^{RS}_{i,y} \geq 0; \forall i \in \mathcal{N}, \forall y \in \mathcal{Y}. \tag{66}$$

The calculation of nodal reliability indices in distribution networks requires modeling the direction of branch flows. Here, this topological information is directly provided by variables $w^{B+}_{ij,y}$ and $w^{B-}_{ij,y}$ as described in Section 3.2. Note that, unlike [19], the use of variables $w^{B+}_{ij,y}$ and $w^{B-}_{ij,y}$ precludes the need for additional topology-dependent variables.

Expressions (53) and (54) model the expected durations of repair-and-switching interruptions affecting load nodes. If the flow at stage $y$ is from $i$ to $j$, i.e., $w^{B+}_{ij,y} = 1$ and $w^{B-}_{ij,y} = 0$, then expressions (53) become $\Gamma^{RS}_{i,y} = \Gamma^{RS}_{j,y} - \tau^{RS}_{ij}\lambda_{ij}\ell_{ij}$, as desired. Note that $\Delta^{RS}_{ij,y}$ is equal to 0 as per (54). On the other hand, if the flow at stage $y$ is from $j$ to $i$, i.e., $w^{B+}_{ij,y} = 0$ and $w^{B-}_{ij,y} = 1$, then expressions (53) become $\Gamma^{RS}_{i,y} = \Gamma^{RS}_{j,y} + \tau^{RS}_{ij}\lambda_{ij}\ell_{ij}$. This desired result stems from the fact that $\Delta^{RS}_{ij,y}$ is equal to 0 according to (54). When branch $ij$ is not in operation at stage $y$, i.e., $w^{B+}_{ij,y} = w^{B-}_{ij,y} = 0$, expressions (53) are relaxed as $\Delta^{RS}_{ij,y}$ may vary freely within a sufficiently large range as modeled in (54) for a suitable value of parameter $\overline{\Delta}^{RS}$.

Expressions (55) and (56) characterize the expected durations of switching-only interruptions affecting load nodes. Similar to (53)–(54) for repair-and-switching interruptions, variables $w^{B+}_{ij,y}$ and $w^{B-}_{ij,y}$ and the bounding parameter $\overline{\Delta}^{SO}$ are used to compute the expected nodal durations of switching-only interruptions. Moreover, note that expressions (53)–(54) and (55)–(56) are respectively similar to the formulation for the branch voltage drops (33)–(34).

Expressions (57)–(62) allow modeling the expected duration of switching-only interruptions of every root node, which is equal to the sum of the durations of switching-only interruptions of all branches downstream [15]. Based on the findings of [15] for a given radial topology, expressions (57)–(62) represent the operation, i.e., the load flow, under a special loading condition of a particular fictitious lossless system with an *a priori* unknown topology. According to [15], such a load flow allows characterizing via optimization variables the topological information required to compute the standard reliability index considered in this paper, namely the expected energy not supplied.

Fig. 1 is useful to gain insight into how expressions (57)–(62) work. Fig. 1(a) shows the system used for illustration purposes. The original system comprises eight load nodes represented by circles, one existing substation depicted as a solid square, one candidate substation plotted as a dashed square, five existing branches indicated by solid lines, and seven candidate branches represented by dashed lines. The fictitious system is obtained from the original system by removing all substation nodes and the branches (existing and candidate) that connect them to the root nodes. Consequently,

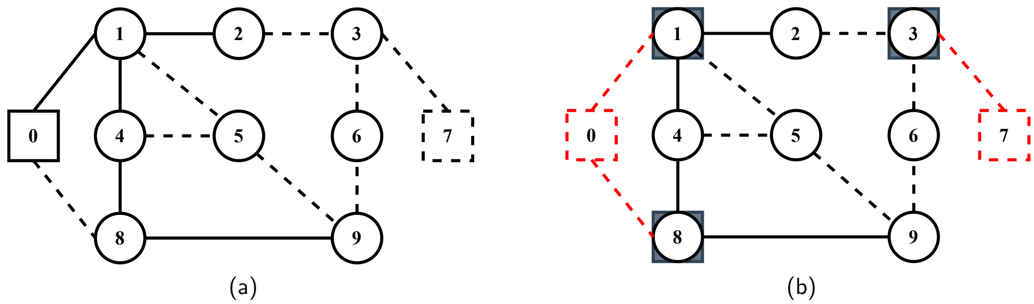


Figure 1: Illustrative example. (a) Original system. (b) Fictitious system.

the root nodes in the original system are potential source nodes in the fictitious system. To play such a role in the fictitious system, the root nodes in the original system must be connected to a substation by an operating branch, which is modeled by $w_{ij,y}^{B+} = 1$ with $i$ indexing a substation node, or by $w_{ij,y}^{B-} = 1$ with $j$ indexing a substation node. In addition, all branches connecting the nodes in the fictitious system are respectively identical to the corresponding branches (existing and candidate) in the original system. The fictitious system associated with the original system represented in Fig. 1(a) is shown in Fig. 1(b), where the removed assets are highlighted using dotted red lines and the original root nodes are inside a square to indicate their role as potential source nodes in the fictitious system.

The following nodal demands characterize the special loading condition for the fictitious system. Demands at fictitious source nodes are all zero. Fictitious demand nodes comprise original load nodes together with the root nodes without connection to any substation. The demand at each of these fictitious nodes is equal to the product of the failure rate and the switching-only interruption duration of the only operating branch that connects this node to the upstream node.

Bearing in mind that 1) the fictitious system is lossless, topologically similar to the original network, and radially operated, 2) the sources of this fictitious system are located at the root nodes of the original system, and 3) the non-zero-valued demands of the special loading condition are equal to the products of failure rates and switching-only interruption durations, the expected durations of switching-only interruptions for the root nodes becoming sources of the fictitious system are equal to the corresponding generation levels resulting from the load flow solution. Further details on the underlying rationale for (57)–(62) can be found in [15].

In (57)–(62), branch power flows in the fictitious system are modeled by variables $f_{ij,y}^{SO}$, which depend on $w_{ij,y}^{B+}$ and $w_{ij,y}^{B-}$ (59). Expressions (57) model the power balance for the root nodes. If root node $i$ becomes a source node in the fictitious system at stage $y$, i.e., there is at least one branch $ij$ in operation such that $j$ indexes a substation node, expressions (60)–(61) allow fictitious generation to be injected at node $i$, which is modeled by variables $\Psi_{i,y}^{SO}$. Consequently, as per (51), the other branches feeding root node $i$ are not operating at stage $y$, i.e., $\sum\limits_{ji \in \mathcal{B} | j \notin \mathcal{SN}} w_{ji,y}^{B+} + \sum\limits_{ij \in \mathcal{B} | j \notin \mathcal{SN}} w_{ij,y}^{B+} = 0$, and, thus, the right-hand side of (57) is equal to 0. Conversely, if root node $i$ is not connected to any substation of the original system by a branch in operation at stage $y$, i.e., $w_{ij,y}^{B+} + w_{ij,y}^{B-} = 0, \forall ij \in \mathcal{B} | j \in \mathcal{SN}$, then $\Psi_{i,y}^{SO} = 0$ as set in (60)–(61). Hence, according to (51), another branch is operating at stage $y$ to feed root node $i$, i.e., $\sum\limits_{ji \in \mathcal{B} | j \notin \mathcal{SN}} w_{ji,y}^{B+} + \sum\limits_{ij \in \mathcal{B} | j \notin \mathcal{SN}} w_{ij,y}^{B+} = 1$. Moreover, the right-hand side of (57) determines the fictitious demand for such a root node, which is equal to the product of the failure rate and the switching-only interruption duration of that branch in operation.

Expressions (58) represent the power balance for the load nodes of the original system excluding the root nodes. For a given load node, the right-hand side of (58) sets the demand equal to the product of the failure rate and the switching-only interruption duration of the only branch in operation connecting this node to the upstream node.

Expressions (62) set the relationship between $\Gamma_{i,y}^{SO}$ and $\Psi_{i,y}^{SO}$ for the root nodes. Note that, for the root nodes that become source nodes of the fictitious system, the minimization of the objective function leads to $\Gamma_{i,y}^{SO} = \Psi_{i,y}^{SO}$. On the other hand, for the root nodes that are load nodes of the fictitious system, $\Gamma_{i,y}^{SO} \geq 0$. The standard values of the expected interruption durations at substation nodes are established in (63)–(64). Finally, the non-negativity of expected interruption durations is set in (65)–(66).

Overall, expressions (53)–(66) play a key role in terms of the modeling contribution of this paper as reliability is effectively incorporated into multistage distribution network expansion planning without requiring the explicit characterization of system operation for every outage and while considering switching-only interruptions, which are the main limitations of state-of-the-art works [19, 22, 23] and [20, 21], respectively.

### 3.7. Mixed-Integer Linear Formulation

The proposed planning model is a mixed-integer nonlinear program that can be recast as an instance of mixed-integer linear programming. To that end, nonlinear expressions (35), (41), and (49) are replaced with linear terms using (42) and the linearization schemes described in Appendix B. For the sake of completeness, the resulting mixed-integer linear program is formulated as:

$$\begin{aligned} \text{Minimize}\, c^{PV} = & \sum_{y \in \mathcal{Y}} \frac{(1+I_r)^{-y}}{I_r} c_y^I + \sum_{y \in \mathcal{Y}} \left[ (1+I_r)^{-y} \left( c_y^M + c_y^E + c_y^{Sh} + c_y^{ENS} \right) \right] + \\ & \frac{(1+I_r)^{-|\mathcal{Y}|}}{I_r} \left( c_{|\mathcal{Y}|}^M + c_{|\mathcal{Y}|}^E + c_{|\mathcal{Y}|}^{Sh} + c_{|\mathcal{Y}|}^{ENS} \right) \end{aligned} \tag{67}$$

subject to:

$$\text{Expressions (2)–(34), (36)–(40), (43)–(47), and (50)–(66)} \tag{68}$$

$$\text{Linearized versions of (35), (41), and (49).} \tag{69}$$

Problem (67)–(69) relies on the selection of appropriate values for bounding parameters $\overline{\Delta}^{RS}$, $\overline{\Delta}^{SO}$, and $\overline{\Delta}^{V}$. It should be noted that the tuning of such parameters benefits from two relevant aspects:

1. No matter the system size and the number of stages, only three bounding parameters require adjustment.
2. Suitable practical values are readily available for the three bounding parameters. Thus, $\overline{\Delta}^{RS}$ can be set equal to the sum over all branches in the network of the products of the corresponding repair-and-switching interruption duration and failure rate. Similarly, $\overline{\Delta}^{SO}$ can be set equal to the sum over all branches in the network of the products of the corresponding switching-only interruption duration and failure rate. Finally, $\overline{\Delta}^{V}$ can be set equal to the difference between the upper and lower voltage limits of the network under planning.

Moreover, as tighter values for $\overline{\Delta}^{RS}$, $\overline{\Delta}^{SO}$, and $\overline{\Delta}^{V}$ may be advantageous from a computational perspective, using as initial values those mentioned in the above item 2, we have implemented a trial-and-error selection procedure relying on the stabilization of the resulting investment plan whereby the values of such parameters are iteratively tightened.

The use of mixed-integer linear programming in problem (67)–(69) to effectively address multistage reliability-constrained distribution network expansion planning represents the methodological contribution of this paper as finite convergence to optimality is guaranteed and a measure of the distance to optimality is provided. Moreover, problem (67)–(69) is suitable for off-the-shelf software based on the state-of-the-art branch-and-cut algorithm [29], which is beneficial for practical implementation purposes.

## 4. Numerical Results

The proposed model has been applied to the 54-node test system addressed in [19] considering a 10-year planning horizon, a \$2-million investment budget, and a 15-block piecewise linear approximation for the ac power flow. Based on [19, 27], the cost coefficient for EENS, $C^{ENS}$, is \$11200/MWh. Due to space limitations, a complete description of the case study, including all system parameters, is available in [36], thereby enabling full reproducibility and a comprehensive analysis of results.

For assessment purposes, we have implemented the formulation presented in [19] and a modified version of the model described in [23]. Note that both references represent the state of the art for the exact incorporation of reliability into the multistage distribution network expansion planning problem. In the first benchmark, 1) an alternative albeit significantly larger reliability constraint set is considered, and 2) reactive power is not explicitly modeled as a constant power factor is assumed. As for the second benchmark, major modifications to the formulation presented in [23] include 1) the replacement of the objective function with that considered in our formulation, 2) the extension of the model to consider different load levels and radial operation under normal conditions, as, unlike in our approach, both aspects are disregarded in [23], and 3) the removal of the constraints associated with infrastructure aging, as such a modeling aspect is neglected in our formulation. Simulations have been run on a Dell PowerEdge R920X64 with four Intel® Xeon® E7-4820 processors at 2.00 GHz and 768 GB of RAM using GAMS 24.7 and CPLEX 12.6 [37]. For all models, the optimality tolerance of CPLEX was set at 0%.

Table 2 reports the investment plans attained for the three models, the corresponding investment topologies being available in [36]. It should be noted that the simulations of both the proposed approach and the benchmark relying on [19] were successfully run to optimality. Unfortunately, for the model based on [23], the solver was unable to reduce the optimality gap under 99.07% after one week. This result reveals the computational impact of considering both a yearly discretization of the planning horizon and the practical aspects disregarded in [23] such as radial operation under the normal state and the chronological aspect of demand for network operation.

In Table 2, single figures designate nodes, pairs of figures connected by a hyphen indicate branches, whereas A$k$, R$k$, and T$k$ respectively denote the installation of alternative $k$ for a branch subject to addition, for a branch subject to replacement, and for a candidate transformer. As can be observed, the expansion plans for the proposed model and the benchmark based on [19] mainly differ in 9 branches, namely 3-4, 3-51, 8-25, 16-40, 33-39, and 42-47, which are installed according to the proposed model, and 24-25, 38-39, and 46-47, which are built as per the benchmark. Additional significant differences are featured in 1) the alternatives installed in the substations at nodes 51, 52, and 54, and in branch 35-36, and 2) the installation times of branches 1-9, 11-52, 12-45, 33-34, 34-35, 35-36, 38-45, and 44-45. Analogously, Table 2 also shows that the solutions identified for the proposed approach and the benchmark based on [23] are significantly different regarding 1) the installation of new substations and branches, 2) the use of investment and replacement alternatives, and 3) the timing of expansion decisions. Such substantial disparities motivate the need for the proposed approach.

Table 2: Investment Plans

| Stage | Proposed model | | Benchmark [19] | | Benchmark [23] | |
|---|---|---|---|---|---|---|
| 1 | 9-17(A1) | 30-43(A1) | 9-17(A1) | 30-43(A1) | 17-18(A2) | |
| | 10-31(A1) | 30-54(A2) | 10-31(A1) | 30-54(A2) | 18-19(A1) | |
| | 18-19(A1) | 31-37(A1) | 18-19(A1) | 31-37(A1) | 19-20(A1) | |
| | 18-21(A1) | 37-43(A1) | 18-21(A1) | 37-43(A1) | | |
| | 21-54(A1) | 54(T2) | 21-54(A1) | 54(T1) | | |
| 2 | 19-20(A1) | | 19-20(A1) | | 9-17(A1) | 18-21(A2) |
| | 22-54(A1) | | 22-54(A1) | | 9-22(A1) | |
| 3 | 1-51(R2) | 23-24(A1) | 1-9(R1) | 24-25(A1) | 8-25(A1) | |
| | 3-51(R1) | 51(T2) | 1-51(R2) | 51(T1) | 23-24(A1) | |
| | 8-25(A1) | | 23-24(A1) | | | |
| 4 | 26-27(A1) | 28-53(A1) | 26-27(A1) | 28-53(A1) | 8-27(A2) | 28-53(A2) |
| | 27-28(A1) | 53(T2) | 27-28(A1) | 53(T2) | 26-27(A1) | |
| 5 | 29-30(A1) | 34-35(A1) | 12-45(A1) | 38-44(A1) | 10-31(A2) | |
| | 32-39(A1) | 35-36(A2) | 29-30(A1) | 44-45(A1) | 29-30(A1) | |
| | 33-34(A1) | 36-53(A2) | 32-39(A1) | | 30-54(A2) | |
| | 33-39(A1) | | 38-39(A1) | | 32-39(A1) | |
| 6 | | | 33-34(A1) | 35-36(A1) | 8-33(A1) | 34-35(A2) |
| | | | 34-35(A1) | 36-53(A2) | 33-34(A2) | 35-36(A2) |
| 7 | 12-45(A1) | 44-45(A1) | | | 31-37(A2) | 37-43(A2) |
| | 38-44(A1) | | | | 33-39(A1) | 38-39(A2) |
| 8 | 40-41(A1) | 41-53(A1) | 11-52(R1) | 41-42(A1) | 13-43(A2) | 41-53(A1) |
| | 41-42(A1) | | 40-41(A1) | 41-53(A1) | 40-41(A1) | 42-48(A1) |
| 9 | 3-4(R1) | 16-40(A1) | 14-46(A1) | | 12-45(A2) | 44-45(A1) |
| | 11-52(R1) | 42-47(A1) | 14-52(R2) | | 14-46(A1) | |
| | 14-46(A1) | 52(T1) | 46-47(A1) | | 16-40(A2) | |
| | 14-52(R2) | | 52(T2) | | 42-47(A1) | |
| 10 | 1-9(R1) | 48-49(A1) | 14-50(A1) | 49-50(A1) | 14-50(A1) | 49-50(A1) |
| | 14-50(A1) | 49-50(A1) | 48-49(A1) | | 46-47(A1) | |

Table 3: Present Values of Costs ($10^3$ \$)

| | | Benchmark [19] | | Benchmark [23] | |
|---|---|---|---|---|---|
| | Proposed model | Approximate results | Actual results | Approximate results | Actual results |
| Investment | 4,268.40 | 3,811.50 | 3,811.50 | 557.43 | – |
| Production | 134,026.01 | 135,044.60 | 134,032.57 | 68,307.72 | – |
| Maintenance | 281.51 | 279.23 | 279.23 | 190.46 | – |
| Load shedding | 0.00 | 0.00 | 26,797.48 | 12,063,895.10 | – |
| Reliability | 33,805.13 | 33,195.86 | 33,275.00 | 367,042.50 | – |
| Total | 172,381.05 | 172,331.19 | 198,195.78 | 12,499,993.21 | – |

Table 3 presents the economic results associated with the investment plans shown in Table 2. The columns labeled as "Approximate results" list the costs provided by the models based on [19] and [23], respectively. It is worth emphasizing that the operational costs, and, hence, the total cost, reported in such columns are optimistic due to the use of either an approximate network model that does not properly consider reactive power or a potentially infeasible reliability model neglecting the effect of transfer nodes. Thus, for the sake of a fair comparison, we have computed the costs resulting from solving the proposed model with expansion decisions fixed to those featured by the investment plans identified by the benchmarks. Such actual costs are displayed in the columns of Table 3 labeled as "Actual results".

As can be seen in Table 3, the approximate operational costs characterizing the optimal solution to the benchmark based on [19] are very close to those associated with the optimal solution to the proposed model. Moreover, according to the approximate network representation of the benchmark, the resulting investment plan apparently complies with nodal power balance as the cost of load shedding is null. However, the fourth column of Table 3 reveals that the operation of the optimal investment plan resulting from the benchmark is far more expensive in reality and, hence, a substantial 13.02% reduction in the total cost is attained by the proposed model. This cost reduction is a consequence of the use of a more accurate characterization of the effect of the distribution network. Note that the optimal investment plan for the benchmark, albeit giving rise to a lower investment cost, actually fails to meet nodal power balance due to its reliance on an approximate network model based on a constant power factor across the system. As a result, load shedding is featured. By contrast, the proposed model yields an investment plan that does not require load shedding, which offsets the investment cost increase.

Regarding the solution to the benchmark based on [23], the substantially larger values for load shedding and reliability costs and the significantly lower investment cost reported in Table 3 indicate that the resulting investment plan is far from optimal. More importantly, as can be seen in the last column of this table, no actual results were reported for this poor-quality investment plan, which led to infeasibility for the proposed model.

The suitability of the proposed linearized ac network model for multistage reliability-constrained distribution network expansion planning has been further verified empirically. First, we have implemented a modified version of the proposed planning model wherein the effect of the distribution network is characterized by the formulation presented in [38], which is based on second-order cone programming. Unfortunately, for this more accurate planning model, CPLEX failed to find a single feasible solution after one week. In addition, we have used a full load flow model to quantify

Table 4: Reliability-Unconstrained Investment Plan

| Stage | Investment decisions | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1-9(R2) | 3-51(R2) | 11-52(R2) | 18-19(A1) | 54(T2) | |
| | 1-51(R2) | 9-17(A1) | 14-15(R2) | 18-21(A1) | | |
| | 3-4(R2) | 11-12(R2) | 14-52(R2) | 21-54(A1) | | |
| 2 | 19-20(A1) | 22-54(A1) | | | | |
| 3 | 8-25(A1) | 23-24(A1) | 51(T2) | | | |
| 4 | 26-27(A1) | 27-28(A1) | 28-53(A2) | 53(T2) | | |
| 5 | 29-30(A1) | 30-54(A2) | 32-39(A1) | 33-39(A1) | 35-36(A2) | 37-43(A1) |
| | 30-43(A1) | 31-37(A1) | 33-34(A1) | 34-35(A1) | 36-53(A2) | |
| 6 | | | | | | |
| 7 | 12-45(A1) | 38-44(A1) | 44-45(A1) | | | |
| 8 | 40-41(A1) | 41-42(A1) | 41-53(A1) | | | |
| 9 | 52(T1) | 14-46(A1) | 16-40(A1) | 46-47(A1) | | |
| 10 | 14-50(A1) | 48-49(A1) | 49-50(A1) | | | |

the operational accuracy of the linearized network model. It is important to note that the load flow determines the system operation for all stages in a single run. The maximum absolute errors obtained for branch current flows, power injections at substations, and nodal voltage magnitudes were 0.39 A, 0.05 MVA, and 1.35 kV, respectively. The corresponding mean absolute errors were 0.04 A, 0.01 MVA, and 0.52 kV, with standard deviations equal to 0.06 A, 0.01 MVA, and 0.04 kV, respectively. Finally, the relative errors considering the total values of branch current flows, power injections at substations, and nodal voltage magnitudes amounted to 1.03%, 0.01%, and 0.31%, respectively. These results, which are consistent with those reported in [24], corroborate the relevant trade-off between tractability and modeling accuracy featured by the use of the proposed ac network formulation for planning purposes.

This case study is also useful to illustrate the computational benefits of the proposed algebraic formulation for reliability. To that end, we have run two modified versions of the proposed planning model wherein reliability constraints are replaced with those recently presented in [19] and [23], respectively. The proposed approach attained optimality in 14.53 h, whereas the modified model relying on [19] required 48.87 h, thereby increasing the computational burden by 3.36 times. As for the model based on the reliability formulation described in [23], the simulation failed again to identify a solution with acceptable quality. These results substantiate the computational superiority of the proposed model.

Finally, the impact of reliability has been examined by solving a reliability-unconstrained version of the proposed model wherein all reliability-related terms are dropped. The comparison of the resulting investment plan provided in Table 4 with that shown in Table 2 for the proposed model reveals that neglecting reliability yields the postponement of some branch investments, as is the case of branches 30-43, 30-54, 31-37, and 37-43, and the earlier implementation of several branch replacements, as is experienced by branches 3-4, 11-52, and 14-52.

The economic effect of the above changes in the investment plan are summarized in Table 5. In this table, column 2 lists the actual costs for the resulting expansion decisions should reliability be modeled, whereas $\varepsilon$ in column 3 denotes the percent cost differences with respect to the results reported in Table 3 for the proposed approach. For this particular case study, disregarding reliability

Table 5: Economic Results for the Reliability-Unconstrained Investment Plan

| Cost term | Present value ($10^3$ \$) | $\varepsilon$ (%) |
|---|---:|---:|
| Investment | 4,258.14 | -0.24 |
| Production | 133,494.76 | -0.40 |
| Maintenance | 300.34 | 6.69 |
| Load shedding | 0.00 | – |
| Reliability | 35,484.73 | 4.97 |
| Total | 173,537.97 | 0.67 |

yields slight reductions in the investment and production costs while moderately rising maintenance and reliability costs. Overall, the optimal expansion plan resulting from the reliability-unconstrained model is more expensive by \$1.16 million, which represents a 0.67% increase in the total cost.

## 5. Conclusion

This paper has presented a novel model for the multistage distribution network expansion planning problem wherein reliability and radiality are explicitly formulated. The proposed approach features two relevant novelties. First, a novel and computationally efficient set of algebraic expressions is developed to explicitly incorporate a standard topology-dependent reliability metric, namely the expected energy not supplied, in the formulation of the planning problem. Second, both active and reactive power are precisely accounted for through an effective piecewise linear approximation of the ac power flow. As empirically evidenced, the resulting mixed-integer linear program outperforms the state-of-the-art models in terms of both solution quality and computational performance.

## A. Nomenclature

The nomenclature used throughout this paper is provided below for quick reference.

*Sets and Indices*

| | |
|---|---|
| $\mathcal{B}$ | Set of indices $ij$, $ji$, $ki$ of branch types. $\mathcal{B} = \mathcal{AB} \cup \mathcal{FB} \cup \mathcal{RB}$, where $\mathcal{AB}$, $\mathcal{FB}$, and $\mathcal{RB}$ denote added branch, existing fixed branch, and existing replaceable branch, respectively. |
| $\mathcal{C}$ | Set of indices $c$, $e$ of conductor types. |
| $\mathcal{L}$ | Set of indices $l$ of load levels. |
| $\mathcal{N}$ | Set of node indices $i$, $j$. $\mathcal{RN} \subseteq \mathcal{N}$, $\mathcal{SN} \subseteq \mathcal{N}$, where $\mathcal{RN}$ and $\mathcal{SN}$ are related to load nodes that may be directly connected to a substation (root nodes) and substation nodes, respectively. |
| $\mathcal{T}$ | Set of indices $t$ of transformer alternatives. |
| $\mathcal{Y}$ | Set of indices $p$, $y$ of yearly time stages. |

*Parameters*

| | |
|---|---|
| $\beta^{FB}, \beta^{RB}$ | Initial conductor alternatives for existing fixed and replaceable branches. |
| $\Delta_l$ | Duration of load level $l$. |
| $\overline{\Delta}^{RS}$, $\overline{\Delta}^{SO}$, $\overline{\Delta}^{V}$ | Upper bounds for the absolute value of $\Delta^{RS}_{ij,y}$, $\Delta^{SO}_{ij,y}$, and $\Delta^{V}_{ij,l,y}$. |
| $\eta^B, \eta^S$ | Lifetimes of branches and substations. |
| $\lambda_{ij}$ | Failure rate per unit length of branch $ij$. |
| $\tau^{RS}_{ij}, \tau^{SO}_{ij}$ | Durations of the repair-and-switching and switching-only interruptions associated with the failure of branch $ij$. |
| $\varphi_{ij}$ | Binary parameter that is equal to 1 if branch $ij$ is switchable under normal operation, being 0 otherwise. |
| $C^E_{i,l}$ | Cost coefficient for the energy supplied by the substation at node $i$ and load level $l$. |
| $C^{ENS}$ | Cost coefficient for the expected energy not supplied under branch outages. |
| $C^{I,AB}_{ij,c}, C^{I,RB}_{ij,c}$ | Cost coefficients for the installation of conductor type $c$ in added and replaceable branch $ij$. |
| $C^{I,S}_i, C^{I,T}_t$ | Investment cost coefficients for substation installation at node $i$ and for transformer alternative $t$. |
| $C^{M,B}_{ij,c}$, $C^{M,S}_i$, $C^{M,T}_t$ | Maintenance cost coefficients for conductor type c for branch $ij$, for the substation at node $i$, and for transformer alternative $t$. |
| $C^{Sh}$ | Cost coefficient for load shedding under normal operation. |
| $F^D_l$ | Demand factor of load level $l$. |
| $\overline{I}_c$ | Maximum current magnitude of conductor type $c$. |
| $I_r$ | Interest rate. |
| $\ell_{ij}$ | Length of branch $ij$. |
| $P^D_{i,y}, Q^D_{i,y}$ | Active and reactive power peak demands at node $i$ and stage $y$. |
| $pf_i$ | Power factor at node $i$. |
| $\overline{Q}^{CB}_i$ | Reactive power capacity of the capacitor bank at node $i$. |
| $R_c, X_c, Z_c$ | Resistance, reactance, and impedance per unit length for conductor type $c$. |
| $RR^B, RR^S$ | Capital recovery rates for investments in branches and substations. |
| $\overline{S}^B_{ij,c}$ | Apparent power capacity of branch $ij$ for conductor type $c$. |
| $\overline{S}^{S,ex}_i$ | Original apparent power capacity of the existing substation at node $i$. |
| $\overline{S}^T_t$ | Apparent power capacity of transformer alternative $t$. |
| $V^{est}_{i,l,y}$ | Estimated squared voltage magnitude at node $i$, load level $l$, and stage $y$. |

*Continuous Variables*

| | |
|---|---|
| $\Delta^{RS}_{ij,y}, \Delta^{SO}_{ij,y}$ | Auxiliary variables used to compute $\Gamma^{RS}_{i,y}$ and $\Gamma^{SO}_{i,y}$. |
| $\Delta^{V}_{ij,l,y}$ | Squared voltage drop magnitude of branch $ij$ for load level $l$ and stage $y$. |

| | |
|---|---|
| $\Gamma^{RS}_{i,y}, \Gamma^{SO}_{i,y}$ | Expected durations of repair-and-switching and switching-only interruptions affecting node $i$ at stage $y$. |
| $\Psi^{SO}_{i,y}$ | Generation supplied at node $i$ of the fictitious system used at stage $y$ to compute $\Gamma^{SO}_{i,y}$. |
| $c^{E}_{y}$, $c^{ENS}_{y}$, $c^{I}_{y}$, $c^{M}_{y}$, $c^{Sh}_{y}$ | Costs of production, reliability, investment, maintenance, and load shedding at stage $y$. |
| $c^{PV}$ | Present value of the total cost. |
| $EENS_y$ | Expected energy not supplied at stage $y$. |
| $f^{SO}_{ij,y}$ | Flow across branch $ij$ of the fictitious system used at stage $y$ to compute $\Gamma^{SO}_{i,y}$. |
| $I_{ij,c,l,y}$ | Magnitude of the current across branch $ij$ for conductor type $c$, load level $l$, and stage $y$. |
| $I^{sqr}_{ij,c,l,y}$ | Squared magnitude of the current across branch $ij$ for conductor type $c$, load level $l$, and stage $y$. |
| $\hat{I}^{sqr}_{ij,l,y}$ | Squared magnitude of the current across branch $ij$ for load level $l$ and stage $y$. |
| $P_{ij,c,l,y}$, $Q_{ij,c,l,y}$ | Active and reactive power flows across branch $ij$ for conductor type $c$, load level $l$, and stage $y$. |
| $\hat{P}_{ij,l,y}, \hat{Q}_{ij,l,y}$ | Active and reactive power flows across branch $ij$ for load level $l$ and stage $y$. |
| $P^{S}_{i,l,y}, Q^{S}_{i,l,y}$ | Active and reactive power injections at substation node $i$, load level $l$, and stage $y$. |
| $P^{Sh}_{i,l,y}, Q^{Sh}_{i,l,y}$ | Active and reactive load shedding at node $i$, load level $l$, and stage $y$. |
| $Q^{CB}_{i,l,y}$ | Reactive power injection of the capacitor bank at node $i$, load level $l$, and stage $y$. |
| $S^{S,sqr}_{i,l,y}$ | Squared value of the apparent power injected at substation node $i$, load level $l$, and stage $y$. |
| $V_{i,l,y}$ | Voltage magnitude at node $i$, load level $l$, and stage $y$. |
| $V^{sqr}_{i,l,y}$ | Squared voltage magnitude at node $i$, load level $l$, and stage $y$. |

*Binary Variables*

| | |
|---|---|
| $x^{AB}_{ij,c,y}, x^{RB}_{ij,c,y}$ | Investment variables for conductor type $c$ for candidate and replaceable branch $ij$ at stage $y$. |
| $x^{S}_{i,y}$ | Investment variable for the substation at node $i$ and stage $y$. |
| $x^{T}_{i,t,y}, w^{T}_{i,t,y}$ | Investment and operational variables for alternative $t$ for new transformers at substation node $i$ and stage $y$. |
| $w^{B}_{ij,c,y}$ | Operational variable for conductor type $c$ for branch $ij$ at stage $y$. |
| $w^{B+}_{ij,y}, w^{B-}_{ij,y}$ | Forward and backward directions of the flow across branch $ij$ at stage $y$. |
| $w^{S,ex}_{i,y}$ | Operational variable for the existing substation at node $i$ and stage $y$. |

## B. Linearization Schemes

The linearization schemes used in Section 3 are described next.

### *B.1. Linearization of the Quadratic Terms in (35) and (41)*

For expository purposes, the quadratic terms in (35) and (41) are represented by the generic form $z^2$, where $z$ is a continuous variable. If $z$ denotes a variable that can take both positive and negative values, the linearization is reduced to the positive orthant, i.e., $|z|^2$ is linearized rather than $z^2$. According to [39], $|z|$ can be equivalently represented by the sum of two auxiliary non-negative variables $z^+$ and $z^-$ complying with the following set of linear constraints:

$$z^+ - z^- = z \tag{B.1}$$

$$0 \le z^+ \le \overline{z} \tag{B.2}$$

$$0 \le z^- \le \overline{z} \tag{B.3}$$

where $\overline{z}$ is the upper bound for $|z|$.

The piecewise linearization thus comprises two steps:

1. The replacement of $z^2$ in (35) and (41) with $\sum\limits_{\kappa=1}^{K} \sigma_{z,\kappa}\Delta_{z,\kappa}$, where $K$ is the number of blocks into which $|z|$ is discretized, $\sigma_{z,\kappa}$ is the slope of $|z|^2$ in the $\kappa th$ block, and $\Delta_{z,\kappa}$ is a continuous variable representing the contribution of the $\kappa th$ block to the value of $|z|$. The slopes are defined as $\sigma_{z,\kappa} = \dfrac{1}{\bar{\Delta}_{z,\kappa}}\left[\left(\sum\limits_{\nu=1}^{\kappa}\bar{\Delta}_{z,\nu}\right)^2 - \left(\sum\limits_{\nu=1}^{\kappa-1}\bar{\Delta}_{z,\nu}\right)^2\right]$, $\kappa = 1,\ldots,K$, where $\bar{\Delta}_{z,\kappa}$ is the width of the $\kappa th$ block.
2. The incorporation of (B.1)–(B.3) and the following constraints:

$$z^+ + z^- = \sum_{\kappa=1}^{K}\Delta_{z,\kappa} \tag{B.4}$$

$$0 \le \Delta_{z,\kappa} \le \bar{\Delta}_{z,\kappa};\ \kappa = 1,\ldots,K. \tag{B.5}$$

The relationship between $z^+$, $z^-$, and $\Delta_{z,\kappa}$ is modeled in (B.4) whereas the upper and lower bounds for variables $\Delta_{z,\kappa}$ are set in (B.5).
Note that if $z$ denotes a non-negative variable, auxiliary variables $z^+$ and $z^-$ and constraints (B.1)–(B.3) are no longer needed and the left-hand side of (B.4) can be replaced with $z$.

### *B.2. Linearization of the Products of Two Binary Variables in (49)*

A linear equivalent for the product of two binary variables $x \in \{0,1\}$ and $y \in \{0,1\}$ is obtained as follows [40]:

1. Replace the product $xy$ with a new binary variable $z$.
2. Introduce the following new expressions:

$$z \in \{0,1\} \tag{B.6}$$

$$z \le x \tag{B.7}$$

$$z \le y \tag{B.8}$$

$$z \ge x + y - 1. \tag{B.9}$$

Expression (B.6) imposes the integrality of the newly added binary variable $z$. Expressions (B.6)–(B.8) make sure that if either $x$ or $y$ is equal to 0, the new binary variable $z$ is also equal to 0. Analogously, expressions (B.7)–(B.9) ensure that $z$ is equal to 1 if both binary variables, $x$ and $y$, are equal to 1.

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