



UNIVERSITETET I AGDER

**AN INVESTIGATION OF STUDENTS' LEARNING OF  
INTEGRAL CALCULUS WITH MAPLE SOFTWARE AND  
PAPER-PENCIL STRATEGIES IN THE WESTERN REGION  
OF GHANA**

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## PREFACE

I owe the sincerest gratitude to the Almighty God for granting me knowledge and strength in conducting and writing this thesis to fulfil my master's degree in Mathematics Education at the University of Agder, Kristiansand in Norway. My Ten years' experience of teaching Mathematics at Senior High School in Ghana Education Service as well as an Examiner in Mathematics for West African Examination Council (WAEC), which organise a standard test called West African Senior School Certificate Examination (WASSCE) for all students living in the five English – speaking West Africa countries including Ghana, Nigeria, Sierra Leone, Gambia and Liberia motivated me to research how Maple software can be used to enhance students understanding of Integral Calculus at Senior High School in Ghana. Most students fail a section of the WASSCE that examines Integral calculus because of a lack of understanding of concepts in integration. Students fear that Integral calculus is the most challenging branch of mathematics.

My grateful appreciation extends to my supervisor Professor John David Monaghan at the Department of Mathematical Sciences at the University of Agder, for his time and effort in commenting, guiding, and making recommendations when writing this thesis. Another immense appreciation to all the lecturers at the Department of Mathematics at the University of Agder who encouraged me during my entire studies.

I cannot forget my best friend called, Joshua Kofi Sogli, who proofread the thesis for me.

My heartfelt thanks go to my Late mother, Miss Comfort Dufie, and Grandmother called Mad Yaa Asamoah, who supported and encouraged me to learn that determination and hard work leads to success.

Kumasi- Ghana, May 15, 2022.

## ABSTRACT

The goal of the research was to investigate the impact of Maple software instruction on senior high school students' understanding of integral calculus. The study adopted a mixed-method design comprising qualitative and quantitative research designs. The researcher used both purposive and simple random sampling techniques to select one hundred (100) participants: fifty (50) participants for the control group and fifty (50) participants for the experimental group. The data collection instruments used in the study were an interview, pre-test and post-test. The analysis of the data was carried out with the assistance of descriptive statistics and a t-test for independent samples. The study found that 7(7%) participants found it difficult to execute correct substitution of the lower and upper limits of definite integral questions.

Moreover, most of the participants, 35(35%), omitted the constant of integration after responding to the indefinite integral test item of the pre-test. It was noted that 18(18%) of the participants could not correctly integrate the polynomial or quadratic function administered to them. The independent samples t-test analysis of the post-test scores for the experimental and control groups revealed a statistically significant difference between the experimental group ( $M = 24.80$ ;  $SD = 9.48$ ) and the control group ( $M = 20.65$ ;  $SD = 7.67$ ). The estimated t-statistic was ( $t = 2.986$ ;  $p = 0.005$ ). This shows that Maple Software's experimental group outperformed the control group using the paper and pencil strategy. The analysis of the interview data indicated that Maple Software has contributed to the success of students' achievement in the integral calculus by arousing and sustaining the student's interest. The Maple Software also made it easier for students to follow the calculus instruction. The findings recommended that technology and mathematical software should be used in the teaching and learning of integration at schools.

## SAMMENDRAG

Studien ble designet for å undersøke effekten av Maple-programvareinstruksjon på videregående elevers forståelse av integralregning. Studien tok i bruk et blandet metodedesign bestående av kvalitative og kvantitative forskningsdesign. Forskeren brukte både målrettede og enkle tilfeldige prøvetakingsteknikker for å velge hundre (100) deltakere: femti (50) deltakere for kontrollgruppen og femti (50) deltakere for eksperimentell gruppen. Datainnsamlingsinstrumentene som ble brukt i studien var intervju, pre-test og post-test. Analyse av data ble utført ved hjelp av beskrivende statistikk, og en uavhengige Samples t-test. Studien fant at 7 (7 %) av deltakerne fant det vanskelig å utføre korrekt substitusjon av de nedre og øvre grensene for bestemte integrerte spørsmål. Dessuten, de fleste av deltakerne, 35 (35%) utelot integrasjonskonstanten etter å ha svart på det ubestemte integraltelementet i pre-testen. Det ble bemerket at 18 (18%) av deltakerne ikke var i stand til å integrere polynomfunksjonen eller kvadratisk funksjon som ble administrert til dem korrekt. Den uavhengige t-testanalysen av post-testskårene for eksperimentelle og kontrollgruppene viste at det var en statistisk signifikant forskjell mellom den eksperimentelle gruppen ( $M = 24,80$ ;  $SD = 9,48$ ) og kontrollgruppen ( $M = 20,65$ ;  $SD = 7,67$ ). Den estimerte t-statistikken var ( $t = 2,986$ ;  $p = 0,005$ ). Dette viser at den eksperimentelle gruppen som brukte Maple Software overgikk kontrollgruppen ved å bruke papir- og blyantstrategien. Analysen av intervjudataene indikerte at Maple Software har bidratt til suksessen til elevenes prestasjoner i integralregningen ved å vekke og opprettholde studentens interesse. Maple-programvaren gjorde det også enklere for elevene å følge kalkulusinstruksjonen. Basert på funnene ble det anbefalt at teknologi og matematisk programvare skulle brukes i undervisning og læring av integrasjon.

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## CHAPTER ONE

### INTRODUCTION

#### 1.1 Background to the Study

The Education system worldwide has begun to change drastically with the emergence of Information and Communication Technology (ICT). Most countries have started integrating ICT into their education system, and Ghana is no exception. In the view of Chen and Wu (2020) and Guillén-Gámez and Mayorga-Fernández (2020), the incorporation of ICT in instruction has changed the approach to teaching and learning, especially for mathematics instruction. According to Takyi and Obeng (2013), education is one of the essential tools for national development and a means for achieving developmental goals such as economic growth and poverty reduction. Moreover, mathematics is the bedrock for any progressive nation's economic, scientific, and technological advancement. For this reason, Obodo (2004) and Carpenter et al. (2003) state that governments seeking growth in their countries have placed a premium on educational policies that enhance the study of mathematics.

Fletcher (2005) and Gyasi-Agyei et al. (2014) (2014) indicate that mathematics is considered a foundation subject and identification tool for students' entry into high education and other professions. Also, this can be seen in how mathematics is applied in our daily lives and how it forms a powerful binding force among the various branches of science. In this vein, Adu-Agyem and Osei-Poku (2012) and Gyasi-Agyei et al. (2014) agree that mathematics should be made easy and more accessible for students at the pre-tertiary level through the teaching and learning process to meet the educational needs of learners. Meanwhile, Blazar and Kraft (2017) claim that the instructional materials used by teachers, classroom management, teacher content awareness and personality, relating topics to real-life circumstances and teaching methods are all factors that affect students' attitudes toward mathematics. According to Ningsih and Paradesa (2018), mathematics can be considered abstract and challenging. Understanding the

theories and formulas that explain a concept is essential for learning mathematics. The task for students in a traditional mathematics classroom is to investigate complex problems; however, learning challenges are now being resolved through multimedia applications and accomplishments. The difficulty is even more significant in mathematics teaching and learning. Teachers must evaluate mental and learning situations and interactive methods for teaching and learning abstract mathematical concepts difficult for students to grasp naturally.

Furthermore, Abukari et al. (2015) view the creation of public operating systems (Maple Soft, Maxima, GeoGebra, Scilab, Geometer's Sketchpad, Mathematica, and MathLab) has helped many Ghanaians to solve issues that were related to disbursement. Mahmudi (2010) states that one of the open-source software in the mathematics community is Maple Software. According to Salleh and Zakaria (2016), Maple Software is an application program for studying mathematics, including functions, diagrams, calculus, and matrices. To improve the study of calculus, maple operating software has surrounded itself with enhanced illustration and robust quantitative algorithms for a better understanding of tools in its internal interface. Moreover, Milovanović et al. (2016) claim that Maple software creates an educational environment with student-centred teaching strategies than traditional instruction. Therefore, the Maple application used in mathematical studies serves as a backbone for students in the classrooms. This helps in explaining some problematic concepts of integral calculus in facilitating mathematical notation, as demonstrated in the following studies (Bali et al., 2016; Kumar et al., 2019; Salleh & Zakaria, 2016; Samková, 2012; Vieira, 2015) and contribute to the understanding of science and the theories of mathematics.

Anderson et al. (2014) stated that Mathematical problems could be solved more effectively with the aid of visualisations. Solares and Kieran (2013) and Anderson et al. (2014) argue that graphical representations in mathematics (number lines, strips, graphs) are found in the maple software translate pictorial and graphic statistical data, which in turn grants students the needed

aid in resolving and reaching higher standards of accuracy.

Scholarly articles have given solid proof of maple software's ability to be used as an aid in teaching and learning. Nonetheless, Buneci (2014) and Pyke et al. (2015) claim Maple Software offers less information on visualization in integral calculus at the senior high school level.

In their Study, Jahanshahi et al. (2015), comprehensive applications of trapezoidal rule and maple answer mathematically Abel integral equations of the first kind. Also, Yurttas et al. (2012) asserted that to solve the polynomial of  $2\cos(\pi/n)$  of a group of rational numbers, Maple is the most structured tool. Students can as it is taking advantage of readily available technologies like Maple, GeoGebra, geometer's sketchpad, Mathematica, and Matlab. According to Tang and Austin (2009), technology is relevant since it's only a tool that enhances educational themes and objectives. The question arises about how one can effectively use this software as an instructional tool in a senior high school mathematics classroom when the concept of focus is integral calculus. According to Nurzannah et al. (2021), the usage of many different mathematical software's in the learning and teaching of the subject has been proven both suitable and successful. However, the results of these studies are somewhat mixed. For instance, some studies demonstrate significant student achievements with instructional use of mathematical software (LeBeau et al., 2012; Montijo, 2017; Unodiaku, 2013; e.g., Gyedu et al., 2020). Others indicate otherwise (e.g., Atuahene & Russell, 2016; Lim & Chapman, 2015; Montijo, 2017).

Moreover, most of these studies focused on using either paid mathematical software or handheld technology. With the increasing infiltration of open-source software in the mathematics classroom in Ghana, there is the need to research their potential impact on students' understanding of historically problematic concepts such as integral calculus. Gyedu et al. (2020) asserted that in Ghana, one of the required subjects to complete senior high school regarding STEM subjects is mathematics, particularly calculus. However, Mynbaev et al.

(2008) assert that the prevailing obstacle that inhibits students from continuing with STEM courses at the university level is poor performance in mathematics. Salleh and Zakaria (2016) opine that a potential explanation for students' poor performance in mathematics as a subject could be that they are not well imbued with the fundamental concepts in mathematics. The study of integral calculus is an essential concept for STEM courses at the university level in mathematics.

Nonetheless, students entering the courses at the university lack basic concepts on this topic. Henderson and Broadbridge (2007) and Lavicza (2010) agree that this under-preparedness calls for refresher instructions to bridge the gap in students' comprehension. The institution in this research established refresher tuition to deal with students having challenges with students with integral calculus, but all efforts proved to be futile. Technological advancements drive the government's economic progress. When engineers and scientists collaborate, commercial and industrial activity expand enormously. To achieve this idea a reality, all sector workforces must be maintained, as stated by Salleh and Zakaria (2016).

According to Osborne (2007), any teaching and learning situation comprises curriculum, pedagogy, and assessment. The result is the development and implementation of a completely new method of instruction to teach integral calculus. Nonetheless, Bryant et al. (2011) and Haripersad (2011) agree that integral calculus is one of the topics that learners studying science, technology, engineering, and maths (STEM) need to have the upper hand over. However, many students have difficulties comprehending this topic, as revealed in the following studies (Bryant et al., 2011; Grove, 2012; Haripersad, 2011; Kashefi et al., 2012). It was realised that students at the school where this research was conducted seem to be more willing to rely on others for help in mathematics than to investigate mathematics concepts on their thought. Meanwhile, tutees often expect an excellent academic score defining rewards as marks for every well-executed work done. They are, however, completely unaware that theoretical comprehension

is a reward. This mindset is why they are inadequately prepared for the coursework, where former knowledge acquisition has been made through routine repetition instead of understanding concepts (Mac and Bhaird et al., 2017).

In addition, Salleh and Zakaria (2016) providing students with solutions to the problems they face in mathematics is a way to strengthen confidence in learning mathematics. In this regard, most learners in the field are mainly geared towards getting the final answers to questions instead of the step-by-step analysis of any mathematical solutions. Moreover, Özkan and Ünal (2009) and Salleh and Zakaria (2016) emphasize that the application of calculus is one major problem students encounter in their studies. Unpreparedness in educational calculus affects the knowledgeability of first year undergrads. Thus, this study has put a careful strategy and carried out. The approach was designed to help students better understand the subject. For the West African Senior School Certificate Examination this is essential if we are to get as many students as possible to pass the test (WASSCE).

In addition, learning mathematics using technology positively impacts student understanding (Genlott & Grönlund, 2016; Zainuddin et al., 2020; e.g., Ayub et al., 2010). Furthermore, not many research works have elaborated the benefits of technology on theoretical, systematic, and metacognitive know-how in studying integral calculus. This systematic investigation engulfs modern teaching technique, which is enhanced by integrating technological advancement. In this study, technology is used to enhance the existing teaching method. This pedagogy is designed to help students improve their knowledge of integral calculus by using a numerical operating system called Maple Software.

The teaching methods formed hammers on the elements of theoretical knowledge function as an integral module. The significance of pedagogical competence, on the other hand, has not been overlooked. This component was given similar relevance in the strategy's design. Competency in the procedure could need remembering key processes. Nevertheless, Tai and



Wei (2020) assert that the combination of rote learning and understanding is preferable in a constant process instead of two independent forms. As a result, the improved teaching technique seeks to assist senior high school students grasp integral calculus better by studying the strand employing maple-integrated methods.

## **1.2 Statement of the Problem**

Considering the abstractness of calculus, it is necessary to understand the responsibility of Maple application in learning integral calculus. The studies conducted by Covington-Ward (2017), Buelin et al. (2016) and Salleh and Zakaria (2016) indicate that students have difficulties in understanding the concepts of integrals calculus. Moreover, Kossivi (2020), Opach et al. (2014) and Persson (2014) assert that the literature on the use of Maple software in teaching integral calculus at the senior high school level is scarce, despite the application of Maple software in other fields. The issue is contemporary and significant in the field of mathematics education, as demonstrated by this research. Integral calculus is a difficult subject that necessitates additional instruction if students are to be adequately prepared for college-level STEM courses and professions, as asserted by Kossivi, (2020).

Furthermore, given the recommendations from mathematics educators and researchers about the importance of calculus in mathematics, the mathematical concepts must be taken seriously in senior high schools. However, in Ghana, most Second Cycle students encounter calculus related problems and, for that matter, integral calculus. Hence their performance on the topic continues to decline, as cited in the West African Examination Council (WAEC) Chief Examiner's report (WAEC, 2017; 2018; 2019).

The traditional teaching method makes the teacher dominate the classroom and turns students into mere listeners, according to Mereku (2010) and Emaikwu (2012). According to Chimuka (2017), there are many possible reasons for this traditional talk-and-chalk teacher-centred type of teaching, which assumes that students are passive recipients of knowledge and is not an

exception. Many research studies have support, such as stated by Akgül, (2014), that Education may benefit significantly from using information and communication technologies (ICTs).

The researcher believes that the integration of Maple software into the teaching and learning of integral calculus in the senior high school mathematics in Ghana can act as scaffold to improve students' performance as pointed by Chimuka (2017). The study focused on investigating students' learning of integral calculus with maple software and paper-pencil strategies in the Western Region of Ghana. The research was inspired by the West Africa Examinations Council (WAEC) chief examiners' annual reports of 2017, 2018, and 2019 which discovered students' performance in integral calculus in the West Africa Secondary School Certificate Examination (WASSCE) was abysmal as compared to other areas of mathematics.

### **1.3 Purpose of the Study**

The research aimed to investigate whether teaching integral calculus using the Maple software was more beneficial to senior high school students' grasp of the topic than teaching using the conventional paper-and-pencil method (traditional approach).

### **1.4 Research Objectives**

The study explored the following objectives:

1. To investigate difficulties senior high school students face in solving integral calculus questions.
2. To find out students' views on the use of Maple Software in learning integral calculus.
3. To investigate the effect of teaching integral calculus using Maple Software on senior high school students' mathematics achievement in the study.

### **1.5 Research Questions**

The following research questions underpinned the study.

1. What difficulties do students face in solving integral calculus questions?
2. What are students' views on using Maple Software in learning integral calculus?

3. What effect does teaching integral calculus using Maple Software have on senior high school students' mathematics achievement in the study?

### **1.6 Research Hypotheses**

The study explored the following research hypothesis in the study:

Null Hypothesis, H<sub>0</sub>: There is no significant difference between students' calculus achievement using Maple and those taught by traditional teaching methods.

Alternative Hypothesis H<sub>1</sub>: there is significant difference between students' calculus achievement using Maple and those taught by traditional teaching methods.

### **1.7 Significance of the Study**

This research aims to provide information on Maple software as a pedagogical tool in teaching mathematics (integral calculus) in Ghana's senior high school. The study findings will serve as a framework for introducing a new practical teaching approach. The study's result would be invaluable to practising mathematics educators, school administrators, curriculum planners, and mathematics trainers at universities to place interventions that will result in high-quality teaching and learning experiences. The results would also be helpful to policymakers, those interested in education research and policy development, and other partners in the field of education. Finally, these results will affect parents heavily burdened with educational resources when choosing what support services their children should have, access and use daily.

### **1.8 Delimitation of the Study**

*"Delimitations describe the scope of the study or establish parameters or limits for the study"* (Baron, 2008, p. 6).

Although integral calculus covers a wide range of topics, this investigation focused solely on polynomial integration, sketching curves, and finding the area under a curve. Again, a lot of software can be used for teaching mathematics, but the research was only done on Maple software. Finally, the Tarkwa-Nsuaem Municipality in Ghana's Western Region is home to

many senior high schools. However, the participants in the study were drawn from the learners of Tarkwa Senior High School.

### **1.9 Limitation of the Study**

The study's time constraints prevented it from proceeding outside its stated framework. Hence, the findings, inferences, and deductions from this could not be generalized to other senior high schools in districts or municipalities other than the study area. In addition, the lack of good literature on the use of Maple software in teaching integral calculus in Ghana was a problem. Financial resources also limited the study in terms of data collection.

### **1.10 Organization of the Study**

There are six chapters in this study. Chapter One's introduction includes the background of the research, statement of the problem, research hypothesis, research questions, research objectives, organization of the study, the significance of the study, the purpose of the study, delimitation of the study, and limitation to the study. In addition, a review of relevant literature is presented in Chapter two. Chapter three focuses on the methodology, which consists of an overview of the chapter, the population, sample size and sampling procedures, data instrumentation, data collection and analysis procedures, and ethical considerations. The study's findings and conclusions are detailed in Chapter four. The results are presented in the fifth chapter of the study. The findings, conclusions, and recommendations of the study are summarized in chapter six of the thesis

### **1.11 Definition of Constructs or Terms as used in the Study**

Active learning: - is an instructional approach in which students play an active role in learning by solving problems.

Algorithm: - refers to a procedure or set of rules to be followed in calculations or other problem-solving processes.

Attitude: - Attitudes can be categorized as positive or negative. A favourable attitude toward the study of integral calculus using Maple Software corresponds to a cheerful emotional disposition, or the students tend to behave or feel toward the topic. On the other hand, the negative attitude toward integral calculus instruction with Maple Software corresponds to a negative emotional disposition toward the topic.

Classroom management: - encompasses a wide range of skills and tactics that teachers employ to keep students organized and attentive in the classroom.

Complex problems: - Complex problems need the capacity to approach them from many perspectives and may have multiple feasible answers.

Confidence: Confidence is the belief in one's capacity to perform at a high level.

Content knowledge or awareness: - refers to the facts, theories, concepts, and ideas that teachers must master to be effective in the classroom.

Creativity: - is the combination of the physical and social learning environment, teacher and student attitudes and characteristics, and a clear problem-solving process that results in a learning outcome.

Environment: - it refers to the various physical locations and situations in which students learn.

Graphical representation: - it refers to the use of charts and graphs to analyze, clarify, and comprehend functions and other information visually how.

Information Communication Technology: - it refers to communication, information, and technological tools that are used to improve learning, teaching, and assessment.

Instruction: - it is the preparation and implementation of well-thought-out strategies for guiding learners' acquisition of knowledge and comprehension, as well as their development of skills, attitudes, and values.

Instructional materials: - refers to lectures, readings, textbooks, multimedia components or tools, and other resources.

Integral calculus: - is a discipline of mathematics that deals with determining, describing, and applying integrals.

Intuitive interface: - it is an interface that performs as expected by the user.

Maple Software: - is a mathematics program that combines the world's most sophisticated arithmetic engine with an automatic interface or medium that makes it simple to analyze, explore, visualize, and manipulate data.

Mathematical abstraction: - it is the process of thinking about and manipulating procedures, rules, methods, and concepts that are not tied to real-world events or circumstances.

Mathematical theory: - is a model based on a set of axioms that describe a branch of mathematics.

Metacognitive awareness: - Being aware of how you think is known as metacognitive awareness. Metacognition is the ability to recognize one's thoughts and strategies. It helps students to be more about what they are doing and why they are doing it, and how the abilities they are acquiring might be applied differently in different settings.

Open-source software: - is software that has the source code available for anyone to examine, use, edit and improve.

Pre-tertiary education: - refers to Basic Education which runs from Kindergarten to Senior High School.

Scaffolding: - It is the process of breaking down learning into manageable parts and assigning a tool or structure to each one.

Self-efficacy: - is the belief in one's own ability to accomplish specific goals (Bandura, 1997)

Sensory channel: - it refers to a group of sensory receptors processing information in the spinal cord and the brain.

STEM education: - it is a deliberate integration of science, technology, engineering, and mathematics to create a student-centred learning environment in which students discover and engineer answers to issues.

Symbolic computation: - is a type of computation that uses symbols, and computers are used to handle such mathematical equations and statements in symbolic form rather than numerical numbers.

Visualization: - is the ability to form mental images based on the concept of integral calculus.

Visualization in this study means the ability to create pictures by sketching graphs of function, lines, and areas under the curve.

## CHAPTER TWO

### LITERATURE REVIEW

#### 2.1 Introduction

This chapter reviews related literature. The study comprises the theory of multiple representations, visualisations as a practical learning tool in calculus, mathematics education with technology, teachers' perceptions and attitudes towards technology use in the classroom, and maple software as an effective mathematics teaching tool. It also includes maple software as a tool for computing, understanding calculus conceptually and procedurally, building a functional understanding, concept of function in calculus, function, polynomial function, analytic geometry of polynomial and its graph, and integral calculus instruction. It then concludes with the fundamental theorem of calculus, teaching the fundamental theorem of calculus, indefinite integral of polynomial, definite integral of polynomial, errors in working with indefinite integral of functions and the conceptual framework of the study.

#### 2.2 The Theory of Multiple Representations

In the realm of education, Jerome Bruner offered many theories. Bruner's works in education were centred on cognitive development and psychology in education. Bruner's (1966) learning strategy was based on logico-scientific processes. As Bruner (1966) found in his research cited by Aydos (2015) and Ainsworth (2006), students' comprehension of abstract concepts would improve if a learning strategy tailored to each student's individual learning needs were devised and implemented. According to Bruner's (1966) theory, which stressed representational difference, they maintained that each mode of cognition has three stages. These stages were enactive, iconic and symbolic. However, learning occurs through movement or activities in the enactive stage. The enactive stage includes manipulating a solid object and learning about its qualities. This step is demonstrated or represented by manipulating graphs using the maple software in the study. Through vivid representations, the iconic stage is aided in developing



mental processes. Here, learning takes place using visuals and iconography. For instance, investigating the qualities of a solid structure with photographs from textbooks is an iconic stage. This stage is perceived as watching a teacher demonstrate graphs or tables in a virtual setting. Storage metaphors define the symbolic stage, where information is encoded as codes or representations, as pointed out by Aydos (2015). For instance, using mathematical symbols to calculate a solid's surface area or volume is an example of the symbolic state of learning, according to Bruner (1966).

Moreover, Bruner's (1966) work on representations in mathematics education has been viewed as multiple representations theory. Dreher et al. (2016) believed that multiple representation theory would explain how students learn abstract mathematical concepts through various mathematical representations. This view was shared by several other reformist mathematics educators, including Brantlinger (2014), as well as mathematics education organisations such as the National Council of Teachers of Mathematics (NCTM) (2000). Furthermore, Kang and Liu (2018) emphasised that multiple representation theory can help students' cognitive processes in authentic, real-world issues and learning contexts. Moreover, some studies suggested that the best way to build learning settings that enhance conceptual understanding through multiple representation theory is to leverage technology (Ott et al., 2018; Samsuddin & Retnawati, 2018)

According to Alacacı and McDonald (2012), technology provides various possibilities for learners to master abstract concepts in methods that are tailored to their particular learning styles and interests. As cited by Aydos (2015) and other studies, for example, Fowler (2020); Demetriou et al. (2019); Abdurrahman et al. (2019) advocated for the use of multiple representation theory to bridge the gap between technology and mathematics education. Aydos (2015) confirmed that present-day math educators have established that multiple representation theory is an essential part of reformist math education and that technology helps achieve the

reforms' goals.

### **2.3 Visualizations as an Effective Learning Tool in Calculus**

Visualisations allow students to observe and evaluate concepts before visually recalling and interpreting knowledge, which leads to comprehension and understanding. According to Quintero et al. (2015), visual learning is a potent sort of learning because it requires five skills: observation, recognition, interpretation, and perception. Also, Ziatdinov and Valles (2022) state that students use visuals to analyse and generate hypotheses and communicate ideas to others by sketching or drawing images.

Attaining the aim of student participation in the classroom is aided by the utilisation of visualisation approaches, which is a challenging undertaking, especially in mathematics. More so, visuals, in addition to these abilities, aid in grabbing students' attention and engaging them in their learning situations. Because mathematics educators or teachers want students to remember material provided to them, images are essential because students only remember half of what they see and hear (Cumino et al., 2021).

More so, the use of visuals assists students to understand mathematical topics or concepts better when they are presented in a visual format. For instance, in this study, students were asked to find the area obtained between the curve of the given function  $f(x) = x^2 - 4x$ , and  $0 \leq x \leq 4$ . According to Kossivi (2020), the mathematics instructor represented it using Maple operating system and visualization techniques resources to guide students to comprehend. Students drew a graph using their foreknowledge and understanding of integral concepts to answer the task.

Furthermore, Nobre, De Resenda Da Costa, and Da Rocha (2016), believe that students can better understand mathematical concepts by visualizing them in their minds. Students can make the necessary transition from the known to the unknown by constructing visual imagery Murphy (2009). Visuals aid in the decomposition of abstract arithmetic concepts, resulting in

improved knowledge and comprehension and advanced mathematical skills Nobre et al. (2016). Additionally, the use of graphics can benefit students of all learning styles by providing content that is easier for them to understand (Murphy 2009 cited by Vasquez 2015).

#### **2.4 Mathematics Education with Technology**

Technology is becoming increasingly important for promoting conceptual comprehension in mathematical education, according to the National Council of Teachers of Mathematics (NCTM) (2000). In other words, it is becoming an indispensable tool in teaching and studying mathematics at all levels since it increases students' decision making, reasoning, and problem-solving abilities. Similarly, Bray and Tangney (2016) and Young et al. (2018) believe that teaching mathematics in technologically rich surroundings is more effective than using traditional teaching techniques, focusing on paper and pencil.

Moreover, researchers have encouraged teachers to integrate technology into mathematics lessons to help consolidate mathematical concepts. In support of this viewpoint, the Ministry of Education (MOE) (2015) stressed the intelligent application of technology in all levels of education in Ghana. Furthermore, the Ministry of Education (2015) urged mathematics teachers to teach students the knowledge needed to effectively use digital resources (ICT) in mathematics. In addition, the National Council of Teachers of Mathematics (NCTM) (2014: 2015) says it's critical that educators have regular access to technologies that help students and teachers make sense of mathematics and solve problems. According to NCTM (2015), effective teachers make the most of technology to deepen students' knowledge, pique their attention, and boost their mathematical skills. Teachers can increase access to mathematics for all students by strategically using technology. Students' learning can be supported by content-specific and digital resources in the classroom. Therefore, it is crucial to determine which tools are appropriate for a particular classroom learning situation. According to NCTM (2015), the content-specific technological tools used in mathematics education include computer algebra

systems, dynamic geometry settings, interactive applets, handheld mathematics devices, data gathering, analytical devices, and computer-based applications. The tools let students discover and explore mathematical concepts and their relevant relationships. Moreover, communication and collaboration tools and digital media are examples of open-source technologies available to make teaching mathematics enjoyable to learners. These technologies give students more access to information, ideas, and relationships that can help them make better sense of concepts (NCTM, 2015).

Meanwhile, Ampadu and Danso (2018) state that the ability to arouse and sustain students' interest and increase their mathematics performance is a laborious task now confronting mathematics education in Ghana. Most Ghanaians agree that mathematics is essential and beneficial. It has a variety of applications, including social, artistic, utility, and communication. According to Ampadu and Danso (2018), mathematics serves as a fulcrum for studying other disciplines such as Physics, ICT, Geography, and Economics. Mathematics is a valuable fundamental skill in learning other topics without which learners may face difficulties.

Moreover, almost all mathematics syllabuses indicate unequivocally that a mathematics course is intended to help students develop attitudes and knowledge that will be useful in their lives after school. It also tries to promote a positive attitude toward understanding mathematics' utility and relevance in our current life. The utilisation of mathematics in real-world application and effective techniques to teaching and studying the subject are highly emphasised, as seen by mathematics textbooks in the senior high schools in the country.

Mathematics has been seen as a subject that necessitates practice to meet the objectives of teaching the subject (Ampadu & Danso, 2018). Mathematics tests what a student knows and how quickly and precisely they can communicate it without making mistakes. One of the essential aims of mathematics teaching is ensuring that all students grasp the material being

taught. In the same vein, mathematics is regarded as one of the most challenging and demanding topics in Ghanaian senior high schools (Ampadu & Danso, 2018).

Nonetheless, it is one of the essential disciplines of science, given the importance of mathematical abilities and knowledge in everyday life and the numerous mathematical applications in other subjects and sciences. As a result, mathematics is a subject that needs to be studied seriously. Nonetheless, Ampadu and Danso (2018) opine that teacher should improve students' knowledge of mathematical concepts and provide a high-quality educational environment. This reasoning is based on mathematical concepts being complex for many students to grasp. Similarly, Wu and Rau (2019) opine that student must be actively engaged with abstract or concrete concepts for learning to occur in a mathematics classroom. Students' interests and accomplishment levels increased when teachers effectively integrated technology into the learning process (Wu & Rau, 2019).

When technology is an essential active element of the mathematical education process, it is employed appropriately and judiciously to achieve learning outcomes. In today's school environment, it is thought that using digital resources (ICT) in the classroom has a good effect on students' success and attitudes towards mathematics education. The judicious use of technology in a mathematics classroom improves mathematics instruction (Eyyam & Yaratan, 2014; Wu & Rau, 2019). The judicious use of technological tools can support both the learning of mathematical methods and skills as well as the development of desired mathematical proficiencies such as problem-solving, reasoning, and justifying, according to the findings of several studies (Hill & Uribe-Florez, 2020; Leung, 2017; Stein et al., 2020; Önal, 2017). Furthermore, Hill and Uribe-Florez (2020) believe that incorporating ICT into our daily teaching and learning of mathematics can open new ways to promote classroom instruction and mathematical skills. Teachers and students must have consistent access to technologies that encourage and enhance mathematical reasoning, problem-solving, and communication, as

asserted by the National Council of Teachers of Mathematics (NCTM, 2011)

According to Stein et al. (2020), technology in mathematics instruction has expanded dramatically. It has earned widespread recognition and approval as an instructional tool in primary, senior high schools and tertiary levels of mathematics education. The Government of Ghana has made information and communication technology (ICT) integration a priority to increase educational standards and promote teachers' and students' access to new technology and their skills and knowledge. In the opinion of Leung (2017), one of the main problems for educators worldwide is the use of ICT in education. The kind of technologies utilised in mathematics education has also changed dramatically due to the growth in utilisation. Overhead projection technology has evolved from the slide and overhead projectors (which are still in use) to data projectors, video gadgets, and digital video disc players. Students and teachers can also access online learning materials over the internet. Technology has been a vital aspect of mathematics instruction for decades, although not in every school. However, it was initially just for a small number of well-resourced top institutions.

Eyyam and Yaratan (2014) state that teachers and students now have more access to computers, interactive whiteboards, the internet, and more instructional software packages. Technology is here to stay, and mathematicians must make efficient use of it in their classrooms. Technology is now extensively used in most developed and developing countries' classrooms.

The incorporation of technology into educational settings has been the subject of many studies, all of which have found positive results. Students can be motivated to develop their critical thinking and problem-solving skills by using technology. In addition, technology can be used to restructure and rebuild the classroom to create an atmosphere that encourages students to acquire higher-order thinking skills (Nobre et al., 2016).

More so, collaboration among students is aided by technology. Collaboration is a potent tool for learning. Students collaborate to produce projects or learn from one another by reading their

classmates' work (Önal, 2017). Another issue of utilising technology in the mathematics classroom is discussed in MacBride and Luehmann's (2016) paper. For the Pre-Calculus lesson, a school mathematics teacher integrated an online blog. However, a student wrote a blog entry on the class's activities each day. It was expected of students to respond to each other's posts. The teacher's only entries were online resources for remediation and enrichment because students largely maintained the website. The blog acted as a reflection tool for the class. Students were pleased to see how other students replied to their blog articles (MacBride & Luehmann, 2016).

Shirley et al. (2011) conducted a study on connected classroom technology in the classroom. The task of the student can be displayed on a classroom monitor using the technology, which allows them to use their calculators. During the school year, the researchers observed seven (7) teachers. The teachers were given the tools to start using the Connected Classroom Technology (CCT) in their classes. Throughout the school year, the researchers conducted telephone interviews with the teachers. They also planned observation in the classrooms. The teachers indicated that they could monitor students' work, provide feedback in a shorter time, and ensure that each student was on task (Shirley et al., 2011). They found an improvement in student participation and achievement due to this technology benefit. Because of the usage of CCT in the classroom, teachers could immediately detect if a student misunderstood a concept. Remediation and re-teaching took place right away To get students back on track in the session (Shirley et al., 2011). As soon as students make a mistake, they can see it and figure out how to rectify it. Because of the benefits of CCT, students in the classroom where it was utilised tended to score higher on given assessments.

Furthermore, pre-service teachers' experiences utilising technology in mathematics classrooms were investigated in a study conducted by Wu and Rau (2019). The findings suggest that the study positively impacts student mathematics learning. The pre-service instructors observed

that the internet supplied arithmetic tasks at many levels, allowing students to select the desired level they felt most comfortable with. Students were motivated throughout the mathematics session using technology, and they were able to discuss what they learned the next day, according to the findings. The teachers were surprised at the students' ability to recall the concepts learnt. Some of the students who participated in the lessons thought that the computer-assisted them in understanding what they studied. Wu and Rau (2019) concluded that technology could be used to create more interactive and relevant mathematics instruction.

Another study indicated that combining technology with peer-led literature conversations can boost student enthusiasm and engagement. Wikis, online literary circles, and online book clubs are examples of technology utilised in small group literature discussions. Using these tools, students could connect with readers from other schools, states, and even nations. This form of technology can assess and motivate students to learn about various concepts. In addition, these online literature conversations can generate beneficial social interaction and establish a sense of community belonging (Costley, 2014).

The effectiveness of how mathematics teachers present content to students determines how well students succeed in mathematics. According to Martinovic and Manizade (2018), technology should be employed in the calculus and geometry classroom as a partner. These researchers gave pre-service instructors technology-based calculus and geometry assignments intending to improve their mathematical reasoning to back up their assertion. They discovered that these teachers could engage in carefully designed exercises and use technology abilities to generate and evaluate geometric conjectures, yielding positive results. Technology was identified as a significant aspect of building pre-service teachers' professional integrity in the study.

Furthermore, many studies have also looked at how dynamic geometry systems (DGS) can be used or integrated into teaching specific mathematical concepts. Again, Štrausová and Hašek



(2013) used DGS to study dynamic visual proofs. They believe that pictures and diagrams help learn many mathematical concepts and that an appropriate picture or diagram can be utilised to prove a geometric property or theorem visually. Additionally, researchers assert that uncommunicative proofs appeal to learners far beyond conventional reasoning. They, however, showed secondary school mathematics teachers and students' examples of dynamic visual proofs made with dynamic geometry software. However, they recognise that dynamic visual proofs utilising DGS have a drawback in that they do not record the chain of thinking that led to the proof, instead of focusing just on the conclusion.

Karaibryamov et al. (2013) investigated the optimisation of geometry courses using the DGS known as 'Sam' (mathematical software). With the help of DGS, a new way of teaching synthetic geometry in schools and universities was applied in this study. Their goal was to make the instruction process more efficient. Their purpose has been to increase the efficiency of the learning process. The Researchers asserted that their unique method enhanced the merits of DGS in geometry classroom instruction, notably in simplifying classroom instruction by lowering hours spent sketching, generalizing large groups of issues, and motivating and supporting students in their studies.

According to Chimuka (2017), they used PCaRD mode to implement three games for a year in a second cycle school with treatment and control groups. Tests (Pre and Post) were used to measure progress and engagement. The PCaRD, they found, aided, and stimulated students to learn. An adaptable foundation for introducing games into classroom learning, they stated.

According to their findings, the PCaRD enhanced students' learning and inspired them to learn. They also said that PCaRD gave teachers a flexible framework for incorporating games into their instruction.

Another study that examined the effectiveness of mathematical software is Ertekin (2014). Chimuka (2017), asserted that his research examined how using Cabri 3D to teach analytical

geometry affected pre-service teachers' opportunity to write an equation for a particular plane, find the plane's norm of a vector, and construct its graphing. The application helped the trained instructors with their geometry and algebra skills. A case study by Swallows (2015) examined the year-two decline of a technology initiative. That technology initiative produces positive benefits in the first year but falls in future years. The findings of this case study demonstrated that enthusiasm for using technology fades over time, resulting in less positive results with ongoing use. To grasp more hard tasks, Perjési-Hámori (2015), as cited in Chimuka, (2017) proposed utilizing the algebraic computer system (CAS) Maple. These tasks involve multivariate interpolations and multivariate regression, partial differential equations, and utilizing the algebraic computer system (CAS) Maple. This research shows the relevance may assist learners in understanding arithmetic. This study is another example of how technology may be used to help students learn mathematics. McAndrew (2015) conducted a similar study in which the third-year pre-service teachers were taught and explored numerical methods using CAS calculators. Despite their modest power compared to traditional computer-based numerical systems, the study found that CAS calculators can address textbook problems and provide a relatively accessible learning environment.

Vajda (2015) introduced the conventional Chebyshev polynomials as extremal polynomials using computer algebra tools. The use of computer algebra in this research has made students' discovery of extremal polynomials pleasant and straightforward. Soon and Ang (2015) used simulations to introduce the queuing theory, another study that employed computer technology to introduce a concept. The researchers highlighted how simulations might be used in the classroom to provide students with real-world learning experiences. The mathematical rules that regulate queues are complex for students to grasp, especially initially. Actual data was obtained from automated machine (ATM) lines and movie ticket counter lines, and it was used to model queue-related actions. According to the study, without learning traditional queueing

theory, learners comprehended fundamental probabilistic and statistical concepts, including the Poisson and exponential distribution. Interactive whiteboards (Promethean, SMART, and Active Boards), electronic tablets, e-readers, iPods, Geogebra, Maple, Geometer's Sketch Pad, computer algebra systems, Desmos, electronic voting devices, screencasting tools, pen casts, podcasts, and other technologies can all be used in the classroom.

## **2.5 Perceptions and Attitudes of Teachers towards Technology Use in the Classroom**

When implementing technology in the classroom, one factor to consider is the teachers' level of experience and disposition toward it. Haciomeroglu et al. (2009) studied teachers who learnt how to utilise GeoGebra and construct dynamic worksheets and found that their knowledge was improved, and their attitudes were improved. The teachers realised how adding technology to teaching mathematics benefits students in learning how to use the software. The teachers learned that using GeoGebra allows for dynamic linkage, which is crucial since it helps students see and understand concepts. Teachers discovered that students might use multiple dynamic representations and mathematical modelling to investigate, solve, and express mathematical problems in various ways. According to the researchers, mathematical concepts can be studied, and learners can engage in collaborative links up algebraic, visual, and numerical depictions of these concepts, without spending a considerable amount of classroom time sketching figures, objects, or functions. Teachers' Technological pedagogical Content Knowledge grew due to their use of technology (TPCK). The study synthesised their content knowledge, pedagogical expertise, and technology knowledge as they generated dynamic worksheets for their lessons. Most teachers expressed satisfaction with GeoGebra's teaching and learning abilities.

Researchers Stols and Kriek (2011) looked at the impact of behaviourism, normativism, and control beliefs in grade 10 - 12 mathematics teachers. The researchers looked at how these three attitudes influenced instructors' intentions to use dynamic geometry software in their classes to develop concepts in transformations, functions, and geometry. The research was

conducted in South Africa with two instructors, one from a semi-urban and one from an urban school; data were collected from 22 teachers (twelve males and ten females). The teachers' actual use of the program was compared to their intention to utilise it. There were two main factors that influenced whether the program was used: the perception of the technology and teachers' general technological proficiency Vasquez (2015). The combination of PU and GTP determines behavioural intention. According to the researchers, teachers' actual behaviour was compared to the perceived usefulness of technology or simplifying their classroom lives. The dynamic geometry software was not used in the classroom if the teachers did not have a basic understanding of technology (Stols & Kriek, 2011).

One strategy to increase teachers' usage of software in the classroom is to ensure that they have a basic understanding of computers and give them hands-on experience (Stols & Kriek, 2011). Teachers who did not use the application software were of the opinion that an efficient way to teach the mathematics concepts was to be patient. Traditional methods of repeating, drilling, and repeating were supplemented with a more cognitivism by teachers using the software. Teachers that refused to utilise the software had a teaching method incompatible with its utilisation, so they did not use it. If we intend to expand teachers' use of technology, primarily uses that increase student learning, we must understand how teachers' current classroom practices are anchored in, and influenced by, existing pedagogical assumptions, according to the researcher (Stols & Kriek, 2011).

In addition, Ifenthaler and Schweinbenz's (2016) qualitative study examined teachers' perceptions of Tablet-PCs (TPC) in the classroom. In the German study, 18 instructors (nine female and nine male) from three different middle schools used iPads to introduce technology in their classrooms. Some teachers had a class set of iPads, whereas others did not or only had iPads for the teachers or kids with disabilities. The findings found that ten out of ten teachers had a good view of TPC, with two of them seeing themselves as open-minded about it.

According to one teacher, TPC made the students more enthusiastic about their studies, and the more variation in classroom instruction, the better the students' ability to concentrate. Another teacher stated that TPCs are beneficial since they expose students to current technology. It was hypothesised that the instructors' negative opinions and scepticism of TPC use stemmed from their lack of familiarity with iPad use (technology illiteracy) and the fact that they were not given any training. Few of them had a clear sense of how they would use the iPad in their classrooms; most assumed the iPads would be used solely for research. While students may be more interested in school subjects and content when TPC is used in education, it will not turn them into 'Little Einsteins', according to a teacher in the study. TPC can help students reintegrate by presenting themselves in a favourable light during regular sessions (Ifenthaler & Schweinbenz, 2016). TPC may be beneficial for students with poor self-efficacy but strong computing skills.

## **2.6 Maple Software as an Effective Mathematics Teaching Tool**

According to Salleh and Zakaria (2016), Maple Software is mathematical application software that can be used to teach mathematics concepts. It operates on any operating system, such as Windows or Linux. Maple Software's friendly interface provides active engagement in hands-on activities that can help students learn mathematics in a meaningful way. Unlike the CAS-based graphing calculators, Maple Software provides various instructional tools, including a powerful mathematical software package that includes graphics, computation, and programming. Maple Software is preset to a two-dimension animation for animated visualisation of graphs to emphasise abstract mathematical concepts. Moreover, Salleh and Zakaria (2016), pointed out that in addition, maple Software provides colour editing capabilities that can be used to arouse students' interest as far as visualisation is concerned. Studies have found that Maple Software can generate metacognitive cues or signals among students in the learning of integral calculus.

Furthermore, CAS graphing calculators are based on Maple Software. More so, Padilla-López et al. (2015) emphasise that Maple Software has several mathematics programs not available on the CAS graphing calculators. Also, Maple Software has more processing capacity thanks to active animation and visualisation. It could be used to build vision-based intelligent monitoring systems that can automatically extract relevant information from visual data to analyse actions. Using hypertext and hypermedia approaches in computer-based learning to increase geometric modelling, the program supplies linkages and nodes between materials and students' activities, representing interaction and its impacts on learning (Padilla-López et al., 2015).

More so, Sözcü et al. (2013) view Maple Software as an excellent tool for teaching and researching geometric modelling problems because it includes sophisticated symbolic manipulations, which are programming languages that allow users to build their algorithms. Sozcu et al. (2013) stated that CAS add-ons are used in programming languages for physics and bioinformatics and packages for physical computing and graphic production and editing, such as computer-generated imagery (CG) and sound synthesis (synthetic sound).

Maple Software can be used in a mathematics classroom to help students learn by imitating constructivist educational approaches.

## **2.7 Maple Software as a Tool for Computing**

The research on Maple Software's use as an instructional tool emphasised its computational aspects (Zamuda & Brest, 2013) while ignoring its animation and visualisation features. For instance, previous studies looked at the efficiency of Maple Software for the characterisation of parametric equations (Thompson, 2013), Differential Geometry (Anderson & Torre, 2012), and Abel equations (Jahanshahi et al., 2015). When solving equations, Anderson, and Torre (2012) used Differential Geometry to isolate the values of functions and parameters and solve complex calculus problems.

Furthermore, by applying the collocation method, fractional Riccati differential equations with delay terms can be solved, Öztürk et al. (2013) employed Maple Software to solve a non-linear algebraic equation system and obtain the coefficients of truncated Taylor sum in matrix form. Maple Software was discovered to be a powerful and popular computer algebraic system (CAS) with its graphing capabilities to provide substantial insight into providing theorems by Meikle and Fleuriot (2012), who included it in their work. They realised that Maple Software could replace chalkboard lectures and static PowerPoint slides as a presenting tool, allowing users to speed up proving and verifying interactively complicated theorems and complex algorithms (Meikle & Fleuriot, 2012).

Awang and Zakaria (2012) discovered that the experimental group outperformed the control group significantly from a pre-test to post-test in integral calculus on incorporating Maple Software into the teaching process for the integral calculus topic for first-year students in their quasi-experimental non-equivalent control group design 101 randomly selected participants. Vieira (2015) also utilised Maple to solve Euler's type of non-homogeneous fractional differential equations and found that Maple's visual representation allowed students to see the roots of polynomial functions in a complex variable.

In their study, Salleh and Zakaria (2016) studied the effectiveness of a learning technique for integral calculus using Maple Software, which used a quasi-experimental non-equivalent control group design. According to this study, there are considerable differences in learning integral calculus between those who use Maple Software and those who use the traditional technique. According to the findings of Kossivi (2020), first-year university students who took the integral calculus session using Maple Software performed better than the control group in terms of procedural and conceptual understanding.

The vast majority of published peer-reviewed articles were far more descriptive than analytical, and there was no statistical evidence to support the claim that Maple Software could pique and

maintain the interest of pre-tertiary students in integral calculus.

## **2.8 Understanding Calculus Conceptually and Procedurally**

According to Serhan (2015), a fundamental principle of comprehension is connecting conceptual and procedural information. Procedural skills are algorithms or steps related to mathematical problems, while conceptual understanding is information-rich in relationships. According to Yusri et al. (2020), conceptual knowledge is an associated knowledge that is networked with relationships. Therefore, grasping mathematical concepts, procedures, and relationships is a conceptual understanding of mathematics (Maciejewski & Star, 2016; Ocal, 2017). While procedural comprehension is the ability to flexibly, accurately, efficiently, and appropriately carry out procedures, strategic competencies are defined as designing, expressing, and solving mathematical problems (Ocal, 2017).

Students' procedural skills tend to increase when instruction focuses on conceptual knowledge. On the other hand, the contrary is not always true (Hodara & Xu, 2016). Reinholz et al. (2015) investigation of students' conceptual understanding of fundamental calculus concepts incorporated calculus tasks meticulously designed to evaluate learners' desire for a visual solution method. This method of problem solving involved graphical representations and analytical processing, which necessitated the use of algebraic representations and analyses of the challenges faced by students (Quarles & Davis, 2017)

According to Quarles and Davis (2017), mathematical competency is characterised by conceptual knowledge, procedural knowledge, strategic competence, the capability of adapting reasoning, and a productive disposition. They emphasized that conceptual comprehension included operations and relationships. In contrast, procedural understanding contained attitudes toward conducting compliant, competent, and appropriate procedures. These researchers revealed a richer mathematical grasp that stemmed from students' conceptual, procedural, and strategic understanding, which aids retrieval and increases retention.



According to the hypothesis put forth by Quarles and Davis (2017), procedural algebraic knowledge was not correlated with higher grades, whereas conceptual mathematics knowledge did. To conduct their research on learning in developmental mathematics, the researchers utilized a pre-test and post-test research design, descriptive statistics, linear regression, and logistic regression analyses. In terms of practice, their findings showed that (a) learning mathematics that emphasised procedural skills did not prepare students for college-level mathematics, and (b) students with procedural skills could forget things within a few months. In terms of research, the study contributed to the body of knowledge by highlighting the need for more research on student assessments. Moreover, Quarles and Davis's (2017) work were significant for this study on students' conceptual and procedural comprehension, despite certain limitations encountered in the study. Finally, both static and dynamic visualisation were available on the Maple platform, which was necessary for a conceptual and procedural knowledge of calculus.

## **2.9 Building a Functional Understanding**

According to Herawaty et al. (2020), to interpret the meanings of mathematical concepts like function, students rely on prior information or their function concept picture. If the concept image is incorrect, it is reasonable to predict that a student's ability to apply the definition will be limited. Teachers must spot gaps in students' comprehension and help them fill them (Afriyani & Sa'dijah, 2018). Unfortunately, many mathematics teachers assume that students come to class knowing everything they need about a topic. García-García and Dolores-Flores (2018) believe that students will grasp more topics in mathematics, such as functions when introduced at an opportune moment in their understanding of mathematical concepts and definitions. Making connections between mathematical concepts and previously formed notions helps students understand the concepts. According to researchers, mathematics teachers must allow students to employ multiple representations in the mathematics classroom.

Students should see that a function is more than just a rule; it may also be represented as a statement or a graph. Students could graph, verbally, algebraically, or in a table related values of variables (domain and co-domain) and observe the impacts of the application of functional notations to relate the values. Students may begin to construct a process perspective of functions in this manner. In their study, Panaoura et al. (2017) conclude by stating that addressing students not just how a function manipulates a number but also what causes this change will aid in the development of function comprehension.

In addition, Perbowo and Anjarwati (2017) also argue that making connections to real-life circumstances is critical for students' learning of functions. Proving students with a cause and a connection make a lesson more important to them and help them understand it better. Students' difficulties with functions can arise at any time during their studies. Accurso et al. (2017) investigated if students' ability to switch between object and process views was a factor in their difficulty working with algebraic formulas. Six students enrolled in a mathematics education class were interviewed who used questions that either used the process view or the object view. An algebraic expression is a process used to manipulate numbers, whereas the object view refers to treating algebraic expression as an object or quantity.

## **2.10 Function of Single Variable**

In mathematics, functions are indispensable, and they are everywhere, and they are crucial for articulating physical links in the sciences. In mathematics, a function of a single variable is an expression or rule that establishes a relationship between one variable, called the independent variable and the other variable, called the dependent variable. For example, suppose a variable  $y$  is so closely tied to a variable  $x$  that a rule determines a unique value of  $y$  whenever a numerical value is assigned to  $x$ . In that case,  $y$  is a function of the independent variable  $x$ . This relationship is often written as  $y = f(x)$ , which means 'f of x' and  $y$  and  $x$  are related so that there is a unique value of  $y$  for every  $x$ . For the same  $x$ ,  $f(x)$  cannot have more than one value.

In other words, a function connects one set's element  $x$  to another set's element  $f(x)$ . Furthermore, the domain of the functions is the set of values of  $x$ , and the range of the function refers to a set of values of  $f(x)$  generated by the values in the domain. In addition to  $f(x)$ , other symbols such as  $g(x)$  and  $p(x)$  are often written to represent functions of the independent variable  $x$ . On the other hand, when the nature of the function is unknown or unspecified,  $f(x)$  is used (Panaoura et al., 2017; Perbowo & Anjarwati, 2017; Toeplitz, 2018)

### **2.11 Concept of function in Calculus**

Carlson et al. (2010), argue that students learn how to evaluate functions and solve equations before beginning calculus. As a result, they consider functions to be static. On the other hand, students are trained mainly on functions at the action view level. Students use functions to manipulate the output of a number, even while they can view functions in the best way suited for mathematical problems. Accurso et al. (2017) argue that students will be better equipped for high school calculus instruction if they have established a solid understanding of functions. Functions are essential in the study of calculus. The slope of a function and the equation of tangent lines are two examples of issues students face in a typical calculus class. As a result, student success depends on developing the function notion in the previous mathematics courses.

Furthermore, any student who wishes to comprehend calculus must have the capacity to apply the concept of function. Students' comprehension of functions accounts for a large part of their calculus success. According to García-García and Dolores-Flores (2018), almost every component of a calculus course in senior high school is dependent on students' comprehension of functions. Finding function limits demands students to interpret a function's behaviour around a specific  $x$ -coordinate. Students must have a dynamic understanding of function, at the very least at the process level. Understanding the limits of functions requires visualising and connecting the mathematical form and the graphical form.

According to Norton et al. (2018), a critical change in calculus has been a greater emphasis on visualisation in recent years. Once students have a basic understanding of derivatives, they can go deeper into the concept of function. At the senior high school level, they calculate rates of change, and tangent line slopes, compare rates and use the derivative concept to express a function's graphical representation. Success with derivatives requires an understanding of functions and the ability to connect their many representations and analyse and apply specific aspects of functions. The ability to articulate relationships of change between variables, explain parameter changes, and read and analyse graphs depends on the student's understanding of function. The integral is the last topic in a regular calculus senior high school course. Students utilise integration to find solids generated by rotating a region around an axis and solve physical and mathematical problems.

Leikin et al. (2018) argue that many students find it challenging to connect the role functions play in each of these areas. Students who have a solid mathematics background and a good comprehension of function are more likely to grasp these concepts. Students who do not grasp function have difficulty understanding most of what transpires in calculus. Functions play a considerable part in senior high school calculus. Therefore, it is crucial to ensure that students attending calculus-level mathematics courses understand functions thoroughly. Students' comprehension of function is required in every part of senior high school mathematics. Functions have a role in every step of the process, whether looking at limits, determining derivatives, or using the integral to compute region areas.

## **2.12 Polynomial Function**

According to Britannica and Editors of Encyclopedia (2021)), a polynomial in  $x$  is an expression of the form  $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0$ , where the coefficients  $a_0, a_1, \dots, a_n$  are given,  $x$  can be any real number, and all the powers of  $x$  are integers 1, 2, 3, .... The result is an algebraic function when the exponents of  $x$  can be any real

number. Because of their applicability, polynomial functions have been studied since earlier civilizations; a polynomial function can closely approximate practically any relationship involving real numbers. The largest power of the independent variable defines polynomial functions. Linear, quadratic, cubic, quartic, and quintic are common names for powers of one to five Britannica and Editors of Encyclopedia (2021)

### **2.13 Analytic Geometry of Polynomial and Its Graph**

Analytic geometry can be used to show polynomial functions geometrically. The x-axis represents the independent variable  $x$ , while the y-axis is used to plot the dependent variable  $y$ . When a graph of a relationship between  $x$  and  $y$  is plotted in the  $xy$ -plane, if a vertical line goes through only one point of the graphed curve, the connection is a function. For each  $x$ , there would only be one point  $f(x)$ , as defined by the definition of a function. The coordinates of the points  $(x, y)$  where  $y = f$  make up the graph of the function  $(x)$  (Toeplitz (2018)).

### **2.14 The Fundamental Theorem of Calculus**

The Fundamental Theorem of Calculus (FTC) is an essential component of integral calculus since it helps connect definite and indefinite integrals and efficiently assess definite integrals with anti-derivatives Radmehr et al. (2017). It illustrates the relationship between the rate at which a quantity increases and decreases Speiser et.al (2001) . The fundamental Theorem of Calculus has been regarded as a significant intellect milestone in the development and refinement of this branch of mathematics Carlson et.al (2003).

The FTC has two parts. The first part stipulates, if  $f$  is continuous on  $[a, b]$  and  $F$  is any anti-derivative of  $f$  on  $[a, b]$ , then  $\int_a^b f(x)dx = F(b) - F(a)$  (Nedaei et al., 2021, p. 63). The second part states that if  $f$  is continuous on an interval, then  $f$  has an anti-derivative on that interval. In particular, if  $a$  is any point in the interval, then the function  $F$  defined  $F(x) = \int_a^x f(x)dt$  by is an anti-derivative of  $f$ ; that is,  $F'(x) = f(x)$  for each  $x$  in the interval, or

in an alternative notation  $\frac{d}{dx} \left[ \int_a^x f(t) dt \right] = f(x)$  ((Nedaei et al., 2021, p. 70). Encapsulating both differentiation and integration appears necessary for understanding the FTC (Radmehr & Drake, 2017).

Literature says that many students can use the FTC to find the definite integral, but they don't comprehend what it means Radmehr et al. (2017). Students' knowledge of function limits the rate of change, and the notational element of the accumulation function have been reported to be connected to their difficulties with FTC (Radmehr & Drake, 2017). Several studies have shown that students in high school have difficulty comprehending limits, which may make it difficult for them to understand FTC, as stated by Thompson et al. (2008) and cited by Radmehr and Drake (2020). In mathematical notation terms, the role of  $t$  in  $\int_a^x f(t) dt$  is confusing for several students (Radmehr & Drake, 2017). The concept of accumulation function in the FTC denoted by  $F(x) = \int_a^x f(t) dt$  involves several parts that make it difficult for some students to understand it (Thompson & Silverman, 2008). First, students need to understand  $f(t)$  is a number depending on the value of  $t$ . Secondly, Co-variational understanding is also needed by students (Nedaei et al., 2021; Radmehr & Drake, 2017) of the relationship between  $t$  and  $f$ , which means understanding that as the value of variable  $t$  changes from  $[a, x]$ , the value of  $f(t)$  changes in accordance with the change in the value of  $t$  Radmehr and Drake (2020). Thirdly, the students need to understand the bounded area accumulating, as  $t$  and  $f(t)$  vary in tandem (Nedaei et al., 2021).

### **2.15 Integral Calculus Instruction**

Many researchers have made recommendations for teaching integral calculus. For example, Awang and Zakaria (2012) stated that integral calculus instruction should establish the enclosed area as a sum limit rather than integration strategies for solving various forms of

integrals. Using as many diagrams and graphs as possible to help pupils grasp the definite integral and area relationship is also essential.

According to Hong and Thomas (2015), using technology to teach integral calculus could also help students focus on intellectual concepts. Because many students are confused regarding differentiation and integration strategies, Radmehr and Drake (2020) asked them to compare them. They also mentioned that many students make technical errors when handling integral problems, so he recommended remedial sessions and revision worksheets to help pupils prepare for this area. Finally, Radmehr and Drake (2020) claim that teaching limits are another way of teaching integral calculus, particularly in senior high school. Unfortunately, limit is not a focus of senior high school curriculum in several countries, although it is essential for understanding the definition of the definite integral. Therefore, it is recommended that more attention be paid to this topic to help students understand integral calculus better.

### **2.16 Teaching the Fundamental Theorem of Calculus**

The literature suggests that the emphasis on integral calculus instruction be shifted. This paper presents a 'call for increased emphasis on the FTC as explicating an inherent relationship between accumulation of quantities in bits and the rate at which an incremental bit accumulates' rather than the traditional emphasis on finding a number representing the area enclosed by curves over an interval (Bowers, 2019, p. 51). Furthermore, the concept of co-variation should be included in calculus classes (Bowers, 2019).

### **2.17 Indefinite Integral of Polynomial**

If  $\frac{dy}{dx} = x^n$  then  $dy = x^n dx$  now, integrating both sides of the equation, we have  $\int dy = \int x^n dx$  which is read as the integral of  $x^n dx$ ; the above result yields  $y = \int x^n dx$  where,  $\int$  is the symbol of integration or the integral sign and  $dx$  represents the variable with which the integration is being done (Toeplitz, 2018). Thus,  $y = \frac{x^{n+1}}{n+1} + c$ ,  $n \neq -1$ ; thus, the integration

process increases the result's exponent by 1, and  $c$  is an arbitrary constant of integration which is added to the final result. Therefore, if  $f(x)$  is a polynomial consisting of the sum or difference or both of a number of terms, then the integral of  $f(x)$  with respect to  $x$  is denoted by  $\int f(x)dx$ . For instance, if  $f(x) = 6x^2 - 4x + 2$ , then the integration of  $f(x)$  with respect to  $x$  is:  $\int(6x^2 - 4x + 2)dx = \frac{6x^{2+1}}{2+1} - \frac{4x^{1+1}}{1+1} + \frac{2x^{0+1}}{0+1} + c$ . Which simplifies to  $\int(6x^2 - 4x + 2)dx = 2x^3 - 2x^2 + 2x + c$ .

### **2.18 Definite Integral of Polynomial**

The notation  $\int_a^b f(x)dx$  denotes the value of the integral of  $f(x)$  with respect to  $x$  when  $x = b$  minus the value of the integral when  $x = a$ ; the symbols  $a$  and  $b$  are called lower and upper limits of the integration, respectively such integrals are called definite integrals. Thus, integrals with the lower and upper limits are called definite integrals because their exact values can be determined. Moreover, the definite integration process terminates the arbitrary constant of integration. The definite integration is necessary when the area under a curve (polynomial) is determined (Toeplitz, 2018).

### **2.19 Errors in working with indefinite integral of functions**

Sallah et al. (2020) claim that students in senior high schools find it difficult to distinguish between integral symbols and variables in indefinite integration problems. The integral sign  $\int$  and  $dx$ , which are critical in the presentation of the solution, are frequently overlooked by students. In their attempts to solve indefinite integration problems, senior high school students frequently make the symbol errors of integral sign  $\int$  and  $dx$  Sallah et al. (2020). The errors associated with symbols and variables in integral calculus may appear inconsequential to those students who have grasped the concept of integral calculus but may not be simple for students who are yet to grasp the concept of integral calculus Sallah (2021). Moreover, in their study, Sallah et al., (2020) indicated that most of the senior high school students who

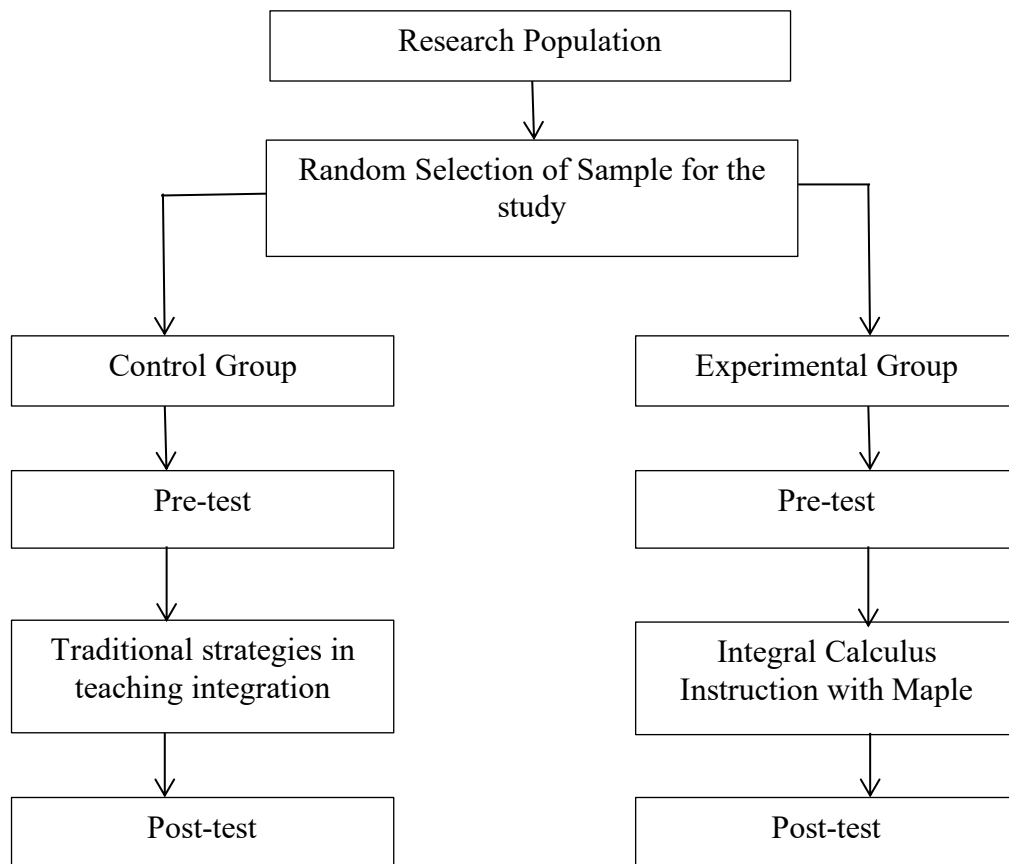


participated in their study omitted the arbitrary constant of integration.

## **2.20 Conceptual Framework of the Study**

In this research, respondents were chosen from the study population. The research sample is then randomly categorized as experimental or the treatment group and control group.

By administering a pretest in the early stages, the experimental and control group were compared. This activity was carried out to find the entry behaviour of the participants. The experimental group was then taken through integral calculus instruction with Maple Software as a teaching resource. At the same time, the control group was exposed to only conventional approaches to the learning of the integral calculus. However, after the intervention phase, the effectiveness of the learning strategies meted out to the two study groups (experimental and control) were determined by administering a post-test. Figure 2.1 below presents the conceptual framework of the study.



**Figure 2.1: Conceptual Framework of the Study**





## CHAPTER THREE

### METHODOLOGY

#### 3.1 Introduction

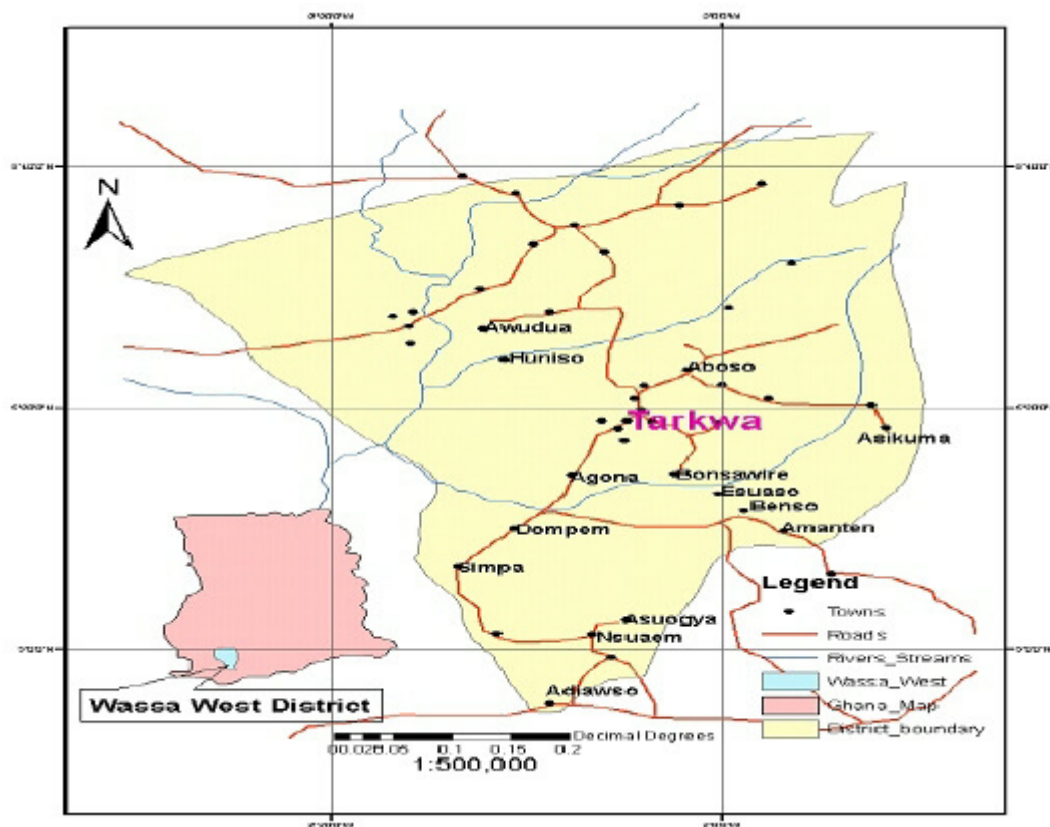
The study's goal was to explore the impact of Maple software instruction on students of second cycle school understanding of integral calculus. The research examines difficulties senior high school students face in solving integral calculus problems to find out students' views on the use of Maple Software in learning integral calculus and to investigate the effect of teaching integral calculus using Maple Software on senior high school students' mathematics achievement in the study. This chapter discusses the methods and techniques used in carrying out the study.

The chapter discussed the study area, research design, research population, sample and sampling procedure, research instruments, pilot study, validity, reliability, intervention, data collection procedure, data analysis procedure, and ethical considerations.

#### 3.2 Study Area

Tarkwa-Nsuaem Municipality was the study area for the research. It's a mining town in Ghana's Western Region. The administrative capital of Ghana's Wassa West District, Tarkwa-Nsuaem Municipality is situated 160 meters above sea level and roughly at longitude 20 59'45" West and latitude 5017'42" North in the southwest of Ghana Soni & Patel (2019). The town is located about 85 kilometres from Takoradi, the regional capital, 233 kilometres from Kumasi, and 317 kilometres from Accra, the country's capital (Ziggah & Youjian, 2013)

Figure 3.1 is the map of Wassa West District showing the location of Tarkwa-Nsuaem Municipality and its surrounding towns from which students come or commute daily to the two schools used in the study. The topography of the Tarkwa area is generally described as a remarkable series of ridges and valleys (Ziggah et al., 2012).



**Figure 3.2: Map of Wassa West District showing Tarkwa-Nsuaem Municipality**

(Source: Google Maps)

### 3.3 Research Design

According to Sileyew (2019), a research design is a set of guidelines and instructions used in conducting research. According to Turner et al. (2017), research design aims to achieve greater study control. The research was conducted using a mixed-method design, including qualitative and quantitative research approaches. Creswell (2014) indicates that since the problems addressed by social and health science are complex, using either qualitative or quantitative methods by themselves is inadequate to address the complex problems. Tobi and Kampen (2018) agree that mixed-method research provides better inferences and minimises method bias. The mixed method is the best research paradigm because it eschews metaphysical concepts that have caused endless discussion and debate and present an efficient and applied

research philosophy (Guetterman & Fetters, 2018; Lucero et al., 2018). The type of mixed method used was sequential explanatory. According to Turner et al. (2017), a sequential explanatory design first collects quantitative data and then collects qualitative data to help explain or elaborate on the quantitative results. This study aimed to investigate the effect of Maple software instruction on senior high school students' understanding of integral calculus compared to the paper and pencil approach (traditional approach). Moreover, to determine how efficient Maple software is as a pedagogical tool for student innovation, attitude, and achievement in senior high school integral calculus.

### **3.4 Research Population**

According to Majid (2018), population refers to all the elements that meet the criteria for inclusion in a study. Moreover, Asiamah et al. (2017) defined a population as the total number of individuals to whom the results of a study apply. Therefore, it is the group that the researcher wishes to explore. The study population comprised students in the Tarkwa Senior High School in the Tarkwa-Nsuaem Municipality in the Western Region of Ghana. Although there are other schools in the Municipality, the study focused on Tarkwa Senior High School.

### **3.5 Sample and Sampling Procedure**

According to Lohr (2021), a sample is a small group obtained from the accessible population. A sampling frame is a group of people drawn from a sample (Shannon-Baker, 2016). Moreover, Shaheen and Pradhan (2019) opine that purposive sampling is a sampling procedure where a researcher relies on his or her judgement to choose research subjects. A non-probability sampling method occurs when subjects selected for a sample are done by the researcher's judgment.

Moreover, according to Etikan et al. (2016), a simple random sampling technique is probability sampling, whereby the researcher ensures that all the population members have equal chances of being selected for a study. Singh and Masuku (2014) explained that in a simple random



sampling, each sample unit included has an equal chance of inclusion in the sample. Therefore, it provides an unbiased and better estimate of the parameters if the population is homogeneous. To choose the study sample, the researcher used purposive and simple random selection techniques.

The purposive sampling technique was used to select all Senior High School Form Two (SHS Form 2) classes. The school has two SHS Form 2 tracks, namely the Gold Track and Green Track, with five subject disciplines on each for the tracks (General Arts, Business, General Science, Visual Arts and Home Economics). After selecting all the SHS Form 2 classes in the school purposively, the simple random was used to select two in-tact classes for the study. Meanwhile, after selecting the two SHS Form 2 classes, the simple random was further used to determine the class assigned as the control group and the one assigned as the experimental group.

The study sampled one hundred participants since each SHS Form 2 class has 50 students. The purposive sampling technique was used to select all Senior high school Form 2 classes out of Senior high school Form 1 and Senior high school Form 3 classes because Senior high school Form 2 classes were expected to have already covered some prerequisites in Geometry from the Form 1 curriculum or syllabus. Further at this level, fundamental topics in calculus are covered hence integral calculus. Also, it was expected that the students had settled in the school as opposed to SHS Form 1 classes and had already acquired basic computer knowledge. The SHS Form 3 classes were not used in the study as they prepared for the West African Senior School Certificate Examination.

**Table 3.1 presents the sample distribution of the study.**

**Table 3.1: Sample Distribution of the Study**

Gender	Number of Participants	Percentage (%)
Male	48	48
Female	52	52
Total	100	100

From Table 3.1, 48 male students representing 48% and 52 female students representing 52% were sampled for the study. The information contained in the Table 3.1 revealed that the female students dominated the study.

### **3.6 Research Instruments**

The study used three (3) instruments. The instruments were pre-test, post-test and interview.

#### **3.6.1 Pre-test**

The pre-test was used to determine the difficulties students have in solving integral calculus questions involving polynomial functions. The researcher designed the pre-test items to establish the learners' entry behaviour and initial knowledge of the intended learning areas. Also, it aimed to ensure that students were of the same relative ability in performance in the integral calculus. The pre-test items comprised one question on functions, one question on indefinite integral, two questions on definite integral, one question on finding the area under a curve, and one question on the concept of the fundamental theorem of calculus. The questions were taken from previous West African Senior School Certificate Examination questions (WASSCE).

The concepts presented in the pre-test were on functions of single variable or polynomial, FTC, indefinite and definite integration as discussed in the literature. In designing the pre-test items, the age, mathematics fluency, and scope of the mathematics syllabus of senior high school students in Ghana were considered to ensure that all aspects of the integral calculus concepts captured in the syllabus were explored in the pre-test.

### **3.6.2 Post-test**

Students were given a post-test after the intervention or treatment had been given to them. Specific tests evaluating the work done on each topic were given at the end. These were graded and eventually complied with at the end of the study.

### **3.6.3 Interviews**

In Guetterman and Fetters (2018), interviews are one of the essential tools of qualitative research. Of all the qualitative methods of gathering information, interviews are the most effective because in-depth data is often obtained during interviews since most questions are open-ended. With the interview approach, the researcher hopes to grasp better the participant's thoughts on utilizing Maple Software to study integral calculus.

The researcher prepared a comprehensive interview guide containing the questions posed to the participants during the interview session of the study. Ten (10) students were randomly selected from the experimental group for the interview. Each participant is allotted a minimum of 20 minutes to respond to the interview questions.

### **3.7 Pilot Study**

According to In (2017), a pilot study is a conventional scientific instrument that allows researchers to do a preliminary examination before committing to a full-fledged investigation. Morin (2013) defines pilot research as a small-scale replica of the proposed study used to fine-tune the technique intended to be employed in the study.

Morin (2013) defines pilot research as a small-scale replica of the proposed study used to fine-tune the technique intended to be employed in the study. Sileyew (2019) claims that a pilot study allows the researcher to discover potential problems with the proposed study and change the method and instrument before the actual study is undertaken.

The test questions were first piloted to see if they generated the desired results to improve the study's validity. SHS Form 2 students from a different school participated in the pilot project, which was not intended for the actual study. According to Evbuomwan (2013), the pilot project is design to test if the plan questions to students could give the necessary data to address the research questions. Students in the pilot study were in the same class and age group as those in the final group study. Therefore, the researcher decided to assess the time it took them to complete the tasks and determine how well the students understood the questions and whether parts of the questions' content should be rewritten. After administering and collecting the pilot study data, the researcher noted that some questions were not attempted or glossed over. The researcher also noted that some of the questions demanded more time for the participants to respond appropriately to the questions. These findings enable the researcher to fine-tune the pre-test items for the actual study.

### **3.8 Validity**

The study instrument was submitted to a lecturer, who patiently went over it and made the required suggestions and revisions. Some lecturers from the Department of Mathematics Education of the University of Education of Winneba were consulted to validate the test items and determine the content and face validity of the items. According to Vakili and Jahangiri (2018), validity establishes if the study tools accurately measure what they are designed to measure.

### 3.9 Reliability

The Split-Half method examined students' pre-and post-test scores' reliability. First, the pre-test and post-test were divided into two halves using the odd-even items, and the scores were associated or correlated. Based on Pearson's Product Moment Correlation, this resulted in an internal consistency of 0.88. This result was then compared to the tabulated dependability or reliability coefficient, which was satisfactory at 0.8 by Vakili and Jahangiri (2018).

### 3.10 Intervention

During lessons on integral calculus, tutorial questions were practised in this study. The students in the experimental group of the study were taken through the principal procedure underlying the indefinite, definite integration and determination of the area under a curve of polynomial expressions as demonstrated below before they were exposed to Maple Software. Given that  $y = x^a$ , where  $a$  is an integer. The integration of  $y$  is denoted as:  $\int y dx$ . Thus:  $\int y dx = \int x^a dx = \frac{x^{a+1}}{a+1} + c$ . The participants were guided to understand that the  $c$  is an arbitrary constant called the constant of integration. Therefore, the integration of  $y = x^3$  is  $\int y dx = \int x^3 dx = \frac{x^{3+1}}{3+1} + c = \frac{x^4}{4} + c = \frac{1}{4}x^4 + c$ . The participants were taken through the integration of a constant. The researcher guides students to understand that  $y = k = kx^0$ , where  $k$  as a constant can be integrated as follows:

$\int y dx = \int kx^0 dx = k \int x^0 dx = \frac{x^{0+1}}{0+1} + c = \frac{x^1}{1} + c = x + c$ . Students were guided to discover that the procedure outlined above in integrating  $y = x^3$  and  $y = k = kx^0$  can be repeated on any number of terms of a polynomial to integrate it. For instance, the integration of the function,  $f(x) = x^5 + 3x^2 - 2x^{-3} + 2$  is:

$$\int f(x) dx = \int (x^5 + 3x^2 - 2x^{-3} + 2) dx = \frac{x^{5+1}}{5+1} + 3 \frac{x^{2+1}}{2+1} - 2 \frac{x^{-3+1}}{-3+1} + c$$

$$\int f(x) dx = \int (x^5 + 3x^2 - 2x^{-3} + 2) dx = \frac{x^6}{6} + \frac{3x^3}{3} - \frac{2x^{-2}}{-2} + c$$

$$\int f(x)dx = \int (x^5 + 3x^2 - 2x^{-3} + 2)dx = \frac{1}{6}x^6 + x^3 + x^{-2} + c$$

Therefore,  $\int (x^5 + 3x^2 - 2x^{-3} + 2)dx = \frac{1}{6}x^6 + x^3 + x^{-2} + c$ .

Participants were also taught how to produce a definite integration of a function. The researcher assisted students to understand that a definite integral must have lower and upper limits and are finally substituted into the integration result of the function. Students were guided to discover that indefinite integration produces a function while definite integration produces a value or number as far as their level is concerned. For instance, evaluate  $\int_1^3 (3x^2 + 2x + 4)dx$

$$\int_1^3 (3x^2 + 2x + 4)dx = \left[ 3 \frac{x^{2+1}}{2+1} + 2 \frac{x^{1+1}}{1+1} + 4x \right] \Bigg|_1^3$$

$$\int_1^3 (3x^2 + 2x + 4)dx = \left[ 3 \frac{x^3}{3} + 2 \frac{x^2}{2} + 4x \right] \Bigg|_1^3$$

$$\int_1^3 (3x^2 + 2x + 4)dx = [(x)^3 + (x)^2 + 4(x)] \Bigg|_1^3$$

$$\int_1^3 (3x^2 + 2x + 4)dx = \{[(3)^3 + (3)^2 + 4(3)] - [(1)^3 + (1)^2 + 4(1)]\}$$

$$\int_1^3 (3x^2 + 2x + 4)dx = (48) - (6)$$

Therefore;  $\int_1^3 (3x^2 + 2x + 4)dx = 42$ .

Students were guided to discover that to find the area under a curve, it is always ideal for sketching the region and then applying the concept of definite integration to the function of the curve and selecting appropriate limits. To sketch a polynomial, students were guided to follow the steps below:

1. Find the x-intercepts of the function (for the x-intercept,  $y = 0$ )
2. Find the y-intercepts of the function (for the y-intercept,  $x = 0$ )

3. Determine the turning points of the function or curve (for the turning point of the curve,

$$\left(\frac{dy}{dx} = 0\right)$$

4. Determine the maximum and minimum points of the curve. For the maximum point

$$\left(\frac{d^2y}{dx^2} < 0\right) \text{ and for the minimum point } \left(\frac{d^2y}{dx^2} > 0\right).$$

The information derived from step 1 to step 4 is then put together to sketch a polynomial on the Cartesian plane.

The topics (polynomial integration, curve sketching, and determining the area under a curve of a polynomial function) were also taught using Maple Software during the teaching part of the study. Participants were taken through the console or window of Maple Software. This enabled the participants to get familiarized with the various commands on the console. The students were given much time to explore independently, with very little supervision from the researcher. This strategy aroused and sustained participants' interest in the teaching process. After the students became used to the console, the researcher introduced the concepts of polynomial integration, curve sketching and finding area under a curve.

The concepts were treated one after the other at the learners' pace. Throughout the intervention phase, group discussion was encouraged. In this manner, the researcher was crucial in teaching the concepts to the students. However, to stimulate students' cognitive processes, students were encouraged to consistently share ideas among themselves in groups on the questions they provided. Throughout the tutorial sessions, students used Maple software as a learning resource to aid their learning in the experimental group. Throughout the intervention, the researcher always insisted that participants verified their solutions of indefinite and definite integrals generated by the Maple Software by using the principal procedures outlined for integrating polynomial functions.

Students were responsible for completing the tutorial questions independently during these sessions, with the researcher acting as a facilitator. As a result, students were encouraged to work in groups to complete their tasks within the allotted time. The activities during the tutorial sessions followed the instructional cycle of activities, class discussions, and class exercises. However, the activities in this cycle were designed with a strong emphasis on reflection. Reflective exercises helped students enhance their thinking skills hence their metacognitive awareness in learning. In addition, the researcher ensured that the quality of reflection activities was tailored to affect and stimulate students' achievement and their learning awareness inclined for them to learn better.

Students in the control group, on the other hand, received their integral calculus instruction as usual. This group, like the experimental group, rehearsed teaching and tutorial questions. The teaching strategy in this group was based on conventional approaches to teaching integral calculus topics, which does not involve Maple Software. The fluency of the mechanical portions, which entail mathematical procedures, was stressed. Students were exposed to a large number and sample of previous years' examination questions to recognise patterns of questions. To prepare them for the post-test, they were encouraged to memorise the patterns. In this group, the researcher taught integral calculus concepts without Maple and guided students to solve problems. Participants in the control group did their tutorial activities mechanically in the teaching sessions without using any mathematical application.

### **3.11 Data Collection Procedure**

Before data collection, the researcher obtained permission from the school's headmaster and the Head of the Department of Mathematics and followed the school's policy for authorisation to conduct research, including a confirmatory letter from the school. Furthermore, the researcher held a familiarisation meeting with the teachers in the mathematics department and



students to inform them of the study's aim and assure them that the data obtained would be used solely for the study's purposes.

### **3.11.1 The Pre-test and the Post-test**

The following technique was used to collect data: a pre-test examination was given to the sampled students to assess their knowledge of integral calculus with polynomial functions and their difficulties in answering the problems. These difficulties allowed the researcher to respond to the first research question and, by extension, the study's initial objective. Secondly, the sampled students were taught integral calculus utilising Maple Software as a teaching medium. It took two weeks for the intervention to take place. After the researcher's intervention, a post-test was given to measure the learnt concepts. The purpose of the pre-test was to determine students' challenges with an essential question involving polynomial functions and their entering behaviour and basic knowledge of the desired learning areas of the study.

### **3.11.2 Interviews**

The researcher believes that the interview technique will allow for more in-depth questioning, resulting in a deeper understanding of the participant's ideas and thought processes regarding using Maple Software in studying integral calculus with polynomial functions. The interview data were gathered from eight (8) randomly chosen students for the experimental group. The interview took place immediately after the school hours, for three days and two hours every day. Three primary questions and relevant follow-up questions were included in the interview guide. The primary goal of the interview was to find out what the participants thought about using Maple Software as an educational tool. A participant was allocated a maximum of 20 minutes for the interview. The researcher ensured that the atmosphere for the interview was friendly to enable the interviewees to relax in responding to the questions. Moreover,

interviewees were advised to ask for the questions to be repeated if they did not understand the questions. Since the data gathered from the interview session was spoken words, the researcher transcribed the spoken words into text, allowing the researcher to analyze the transcribed data qualitatively.

### **3.12 Data Analysis Procedure**

The study yielded both quantitative and qualitative data, including student scores on the pre-test and the post-test items. Inferential statistics were used to examine the achievement test results, which were used to answer research question three. The software known as the statistical package for social sciences (SPSS) was utilized in order to conduct analysis of the data. The t-test was performed to determine whether there was a statically significant difference between the participants' pre-test and post-test scores. This was accomplished mainly by comparing the pre-test and post-test mean scores. Descriptive statistics, such as percentages, means, and standard deviations, were used to examine the results. The students' interview responses were transcribed and examined to answer the second study question.

### **3.13 Ethical Considerations**

When carrying out this study, ethical procedures were followed. Permission to collect the data was sought from the school administration. The fact that the data is solely being used for research was stressed. Participants were also instructed not to use their real names on the pre-test or the post-test items but rather to use a study-specific identification number. This identification number was done to maintain the confidentiality and anonymity of the investigation. Furthermore, all information acquired during the survey was saved only for research purposes.



## **CHAPTER FOUR RESULTS**

### **4.1 Introduction**

The study aimed to find out how Maple software instruction affected senior high school students' grasp of integral calculus and the effectiveness of Maple software as a pedagogical resource for student inventiveness, attitude, and accomplishment in senior high school integral calculus. In furtherance, pre-test, post-test, and interview were deployed to gather information from the respondents. One hundred (100) sampled student-participants constitute the sample of the study. The statistical analyses were performed using qualitative and quantitative (Mixed method) research methodologies. This chapter focuses on the results that emerged from the data gathered.

A total of one hundred (100) scripts (50 scripts from the control group and 50 scripts from the experimental group) were collected on the pre-test items. The exact number was collected on the post-test items, indicating a 100 percent retrieval rate for the pre-test and post-test. Thus, the 100 respondents were used as the final sample, and their responses were used in the analysis. The demographic information on the sample and the results are presented in this chapter captured under sub-headings which reflect the research questions.

The following research questions underpinned the study:

1. What difficulties do students face in solving integral calculus questions?
2. What are students' views on using Maple Software in learning integral calculus?
3. What effect does teaching integral calculus using Maple Software have on senior high school students' mathematics achievement in the study?

### **4.2 Demographic Information**

The information on the biographic characteristics of the respondents was obtained from the research questionnaire and presented using frequency counts and percentages.

**Table 4.1: Demographic Characteristics of Respondents (N =100)**

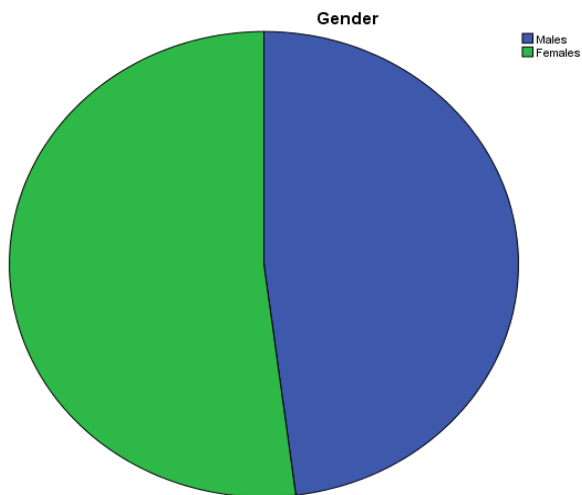
Variables	Categories	Number of Participants	Percentage (%)
Gender	Males	48	48.00
	Females	52	52.00
Total		100	100.0
Age	15 – 17	54	54.00
	18 – 20	32	32.00
	Above 21	14	14.00
Total		100	100.0

Table 4.1 reveals the demographic characteristics of students sampled for the study. The statistics show that out of 100 student-participants, 48 (48%) of the respondents were males, while 52 (52%) were females. The information contained in the Table 4.1 revealed that the female students dominated the study. Moreover, from Table 4.1, 54(54 %) of participants' age were from 15 to 17 years, and 32 (32%) corresponded to students aged from 18 to 20 years, while 14(14%) were above 21 years. This information indicates that most of the participants have their age falls in the 15-17 age category compared to the rest of the age category of the study.

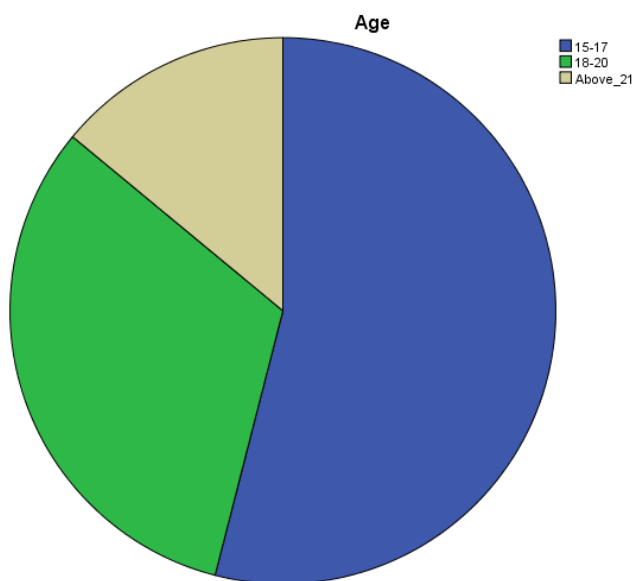
The Western Region of Ghana is home to the city of Tarkwa, which is primarily known for its mining industry. The school is a public institution (senior high school) in Ghana's Western Region. Because the institution is a mixed school, both boys and girls are admitted. The school's residential status for students is either boarding or day. General Science, Business, Home Economics, General Arts, and Visual Art are among the courses offered at the school. Since the school is in a mining region, many students help their parents with shallow mining, also known as Galamsey (meaning "to gather them and sell"). By the time the students start senior

high school, many of the student's age falls between 17 and 20.

Figure 4.1 and Figure 4.2 show a pictorial representation of the gender and the age categories, respectively.



**Figure 4.1: Pictorial Representation of the Distribution of Gender**



**Figure 4.2: Pictorial Representation of Age Distribution of Participants**

### **4.3 What difficulties do students face in solving integral calculus questions?**

The first research question of the study attempts to find out the difficulties the student-participants faced in solving integral calculus questions. To accomplish this goal, the researcher devised an initial test or pre-test to determine the learners' entry behaviour and prior understanding of the intended learning areas to discover the difficulties or challenges students face in solving integral calculus questions. The pre-test items were drawn from the Ghanaian senior high school mathematics syllabus. The pre-test questions on integral calculus were given to the participants in both the study's control and experimental groups. A critical examination of the participants' scripts revealed their difficulty with the pre-test items. After collecting the scripts for the pre-test, the researcher marked each of the pre-test items using a marking scheme prepared by the researcher (See Appendix C). At the same time, marking each script, the researcher paid attention to the students' systematic presentation of the solutions. The researcher also observed how properly the participants wrote the integral symbols  $\int$  and  $dx$  and the constant of integration in the case of indefinite integration and how participants concluded their final answers. Each of the questions was scored five marks. The total marks on the entire pre-test were 20 marks. The data generated by the pre-test items (see Appendix A) was used to answer this study question. Concerning the study, the operational definition of difficulty is the things that students find tough to do in solving integral calculus problems. The difficulties or challenges the student-participants faced in the pre-test have been tabulated and presented in Table 4.2.

**Table 4.2: Difficulties Students Encounter in Solving Integral Calculus Questions (N = 100)**

Difficulties	Number of Students	Percentage (%)
Wrong Substitution of limits	7	7.00
Omission of $dx$	22	22.00
Omission of constant of integration	35	35.00
Improper integration of polynomial	18	18.00
Inability to subtract lower limit values from upper limit values (definite integral problems)	11	11.00
Wrong choice of integration interval (definite integral problems)	5	5.00
Using differentiation instead of integration	2	2.00
Total	100	100

The information presented in Table 4.2 revealed that 7(7%) of the participants found it challenging to execute the correct Substitution of the lower and upper limits of definite integral questions. Also, the results indicated that 22(22%) of the participants omitted the  $dx$  in their attempt to set up the integration process for definite and indefinite integral questions. Moreover, most of the participants, 35(35%), omitted the constant of integration after responding to the indefinite integral test item of the pre-test. It was noted that 18(18%) of the participants could not correctly integrate the polynomial function or quadratic function that was administered to them. Furthermore, a critical examination of the scripts of the participants revealed that 11(11%) were not able to subtract lower limit values from the upper limit values to execute the complete evaluation of definite integral questions contained in the pre-test. This resulted in 11% of the participants not getting the maximum score allotted for such questions.



Meanwhile, 5(5%) participants could not correctly choose the integration interval for the word problem on the definite integral. However, it was noted that those who could not provide the desired results for the word problem question on the definite integral in the pre-test found it difficult to sketch the curve before responding to the question. However, on 2 participants corresponding 2% used differentiation instead of integration. The researcher randomly selected three (3) scripts of the pre-test on the omission of constant of integration, improper execution of the polynomial integration and the omission of  $dx$  in the construction of the integral and included them in the study. The following samples show participants' difficulty in the pre-test.

Question No. Q2  
Write on both sides of the paper

Find the integral of  
 $\int \frac{1}{2} (3x^2 - 2x) dx$

Soln:

$$\int \frac{1}{2} (3x^2 - 2x) dx = \int \left( \frac{3}{2}x^2 - x \right) dx$$

$$= \frac{3}{2} \frac{x^{2+1}}{2+1} - \frac{x^{1+1}}{1+1} + C$$

$$= \frac{3}{2} \frac{x^3}{3} - \frac{x^2}{2} + C$$

$$= \frac{1}{2} \frac{3x^3}{3} - \frac{x^2}{2} + C$$

$$= \frac{x^3}{2} - \frac{x^2}{2} + C$$

$\therefore \int \frac{1}{2} (3x^2 - 2x) dx = \frac{x^3}{2} - \frac{x^2}{2} + C$

omission of constant of integration.

**Figure 4.3: Omission of Constant of Integration**

On the script above, one would see that the participant integrated the given indefinite integral,  $\int \frac{1}{2}(3x^2 - 2x)dx$  correctly but failed to add the constant of integration (as indicated on the script as variable C with red ink), which is essential in carrying out indefinite integration successfully. As captured in the above script, the researcher used red ink to indicate where the constant of integration is supposed to have been placed accurately.

Write the question number in this column

Question No. Q3  
Write on both sides of the paper

Evaluate  $\int_1^3 (3x^2 + 2x + 4) dx$

Soln:

$$\int_1^3 (3x^2 + 2x + 4) dx = (3x^{2+1} + 2x^{1+1} + 4x) \Big|_1^3$$
$$= 3x^3 + 2x^2 + 4x \Big|_1^3 = [3(3)^3 + 2(3)^2 + 4(3)] - [3(1)^2 + 2(1) + 4(1)]$$
$$= (81 + 18 + 12) - (3 + 2 + 4)$$
$$= 111 - 9$$
$$= 102$$

$$\therefore \int_1^3 (3x^2 + 2x + 4) dx = 102$$

Improper execution of the integration of polynomial (quadratic function).

**Figure 4.4: Improper Execution of the Integration of Quadratic Function**

As shown above, this particular participant could not successfully execute the integration of the function (quadratic function) presented in the pre-test. The student realised that in the integration process, the indices or exponents of the terms in a polynomial function are increased

by one but failed to divide the terms in the polynomial with the same powers as shown in the script.

Question No... 94  
Write on both sides of the paper

Calculate to one decimal place the area of the finite region enclosed by the curve  $p(x) = x^3$ ,  $p(x) = x^3 - 7x^2 + 15x - 9$  and the lines  $y = 0$ ,  $x = 1$  and  $x = 3$

Solution

$$\text{Area} = \int_1^3 (x^3 - 7x^2 + 15x - 9) dx$$

$$= \frac{x^{3+1}}{3+1} - \frac{7x^{2+1}}{3} + \frac{15x^2}{2} - 9x \Big|_1^3$$

$$= \frac{x^4}{4} - \frac{7}{3}x^3 + \frac{15}{2}x^2 - 9x \Big|_1^3$$

$$= \left( \frac{(3)^4}{4} - \frac{7(3)^3}{3} + \frac{15(3)^2}{2} - 9(3) \right) - \left( \frac{1}{4} - \frac{7}{3} + \frac{15}{2} - 9 \right)$$

$$= -\frac{9}{4} - \left( -\frac{43}{12} \right)$$

$$\therefore \text{Area} = -\frac{9}{4} + \frac{43}{12} = \frac{-9}{4} + \frac{43}{12} = \frac{-27}{12} + \frac{43}{12} = \frac{16}{12} = \frac{4}{3} = 1.3$$

omission of dx

Figure 4.5: Omission of dx

In the above script, the participant accurately executed the integration of the polynomial function, but the student failed to add the symbol  $dx$  (indicating that the function is being integrated with respect to the variable  $x$ ) when forming the definite integral. This could be termed a procedural challenge. The symbols that identify an integration are the symbols  $\int$  and  $dx$ . Therefore, if one of the symbols is missing in the construction of integration, the resulting expression in the researcher's view cannot be termed an integral since the notation  $dx$ , which stands for differential of the variable  $x$ , denotes that the integration variable is  $x$ .

#### **4.4 What are students' views on using Maple Software in learning integral calculus?**

Research question two sought to find out students' views on the use of Maple Software in learning integral calculus. To address this study question, interview data were collected from eight (8) randomly chosen students for the experimental group. The interview took place immediately after regular school hours. The interview guide includes three main questions and related follow-up questions, which were asked to the experimental participants to get their feedback. The primary goal of the interview was to find out what the participants thought about using Maple Software as an educational tool. A participant was allocated a maximum of 20 minutes for the interview. The interview session afforded the researcher to elicit participants' views on how they were aroused and motivated by the Maple Software integration into the teaching and learning of integral calculus. Since the data gathered from the interview session was verbal communication, the researcher transcribed the interviews into text, allowing the researcher to qualitatively analyse the transcribed data. The following illustrations demonstrate participants' responses to the questions posed to them.

When asked the question, have you ever used any software in studying mathematics? If yes, which software have you ever used?

The interviewee remarked that:

*"No, I have not used any software in learning mathematics".*

When asked, to what extent did participation in using Maple Software in solving integral calculus questions aid or hinder your mathematics learning? Please explain

The student indicated that:

*"Using Maple software in solving integral calculus question did not hinder my ability to study mathematics; rather, it has aroused and sustained my interest in learning of calculus. The majority of the concepts in the study of calculus have been made easier and straightforward to understand. Using the Maple Software to study calculus has made it easier for me to remember the salient concepts compared to learning calculus without the Maple Software."*

The above response from the student suggests that Maple Software has contributed to the success of the student's achievement in the integral calculus by arousing and sustaining the student's interest. The Maple Software also made it easier for students to follow the calculus instruction. That view was supported by another student who remarked that:

*"Applying Maple Software to the teaching and learning of integral calculus has made the whole calculus session fun because most of the concepts we were exposed were practically based and so it was easy for me to understand than compared to learning calculus without Maple Software."*

When the student was asked, will you recommend using Maple software in learning mathematics? If yes, why? (What are the benefits of Maple Software?) Please elaborate.

The student indicated that:

*"Maple Software should be acquired by the school and made compulsory for teachers to use in teaching us abstract concepts that are difficult to understand so that we can appreciate mathematics and its application in other fields like engineering, banking and industries. When Maple Software is made to be part of the study of mathematics, the number of students who*

*will pass the mathematics of West African Senior School Certificate Examination will increase."*

When the interviewee was asked, is there anything more you would like to add? The student indicated that:

*"I was motivated to learn calculus all because of the Maple Software. I was fortunate to be in the group that used the Maple Software to study calculus. I think that I will be able to apply the concepts in future".*

The above remark suggests that the integration of Maple Software in calculus instruction has motivated the students in the experimental group to study the integral calculus efficiently and effectively.

When a student was asked, to what extent did participation in using Maple Software in solving integral calculus questions aid or hinder your mathematics learning? Please explain.

The student stated that:

*"The first time we studied integration, I did not understand, I became so sad because quizzes and tests were administered on the integration, and I performed poorly. But now, I have understood every concept taught us so far with the Maple software. I performed excellently in our second test because I have grasped the fundamental and salient concepts in integration. I can now tell the difference between indefinite and definite integrals. In our end of semester examination, when I see calculus questions, I will select and answer them with ease."*

The above revealed that the students' fears concerning the study of integral calculus had been dealt with by the integration of Maple Software in calculus instruction. This student would not have been confident to select and answer any integral calculus question during their semester examination if Maple Software had not been used to aid students'

understanding of integral calculus.

Furthermore, the students were encouraged to learn mathematics due to the applicability of the lessons that Maple Software integration enables, as evidenced by their comments. The interviews revealed that Maple Software's presentation of the integral calculus concept produced an environment that was favourable to practical activities and real-world connections. The post-test results confirmed that students who were taught with Maple Software made substantial improvements from pre-test to post-test.

In furtherance, when a student was asked this question: Is there anything more you would like to add?

The student remarked that:

*"In spite of the fact that the Maple Software is capable of displaying a step-by-step solution to an integral problem, one must understand the commands in the window of the Maple Software in order to learn the concepts without difficulty. If you don't know which button to press to integrate a given function, it will be difficult to display the solution."*

The above statement indicates that for students to use Maple Software efficiently and effectively, they must be taught every command on the console or window of the Maple Software. If students do not master the meaning of the commands in the window of the Maple Software, it will be challenging for them to integrate polynomial functions properly. Hence, they will not be able to appreciate the ability of Maple Software to undertake robust computations in the learning of integral calculus.

Another student also indicated that:

*"I have realised that even though Maple Software can solve integral calculus problems, it is also important to double check using the integration procedures to confirm the answers generated by the Maple Software. I have*



*also realised that when the function to be integrated has higher powers or exponents on the variable  $x$ , the Maple Software takes a little bit of time to bring out the results."*

This statement emphasised that even though Maple Software can perform integration for a given function, it is essential to verify the output of results from Maple Software using the polynomial integration strategy or approach. This is because all computer algebra systems (CAS) or mathematical applications are not one hundred percent efficient. Errors could be generated during the computation process, so verification of the output of Maple Software is essential. Moreover, the statement also revealed that complex functions tend to reduce the processing speed of Maple Software.

#### **4.5 What effect does teaching integral calculus using Maple Software have on senior high school students' mathematics achievement in the study?**

This research question sought to determine what effect teaching integral calculus using Maple Software has on senior high school students' mathematics achievement. The study tested the hypothesis that there is no significant difference in students' achievement in calculus using Maple and those taught by traditional teaching methods. The post-test (See Appendix B) items administered to the control group and the experimental group generated quantitative data to answer this question. Below are the descriptive statistics for the pre-test and post-test scores.

**Table 4.3: Descriptive Statistics of Pre-test for Experimental and Control**

Group	N	Mean	Stand Dev.	Maximum	Minimum
Experimental	50	17.72	7.24	12.00	7.00
Control Group	50	17.48	6.55	10.00	6.00

An independent sample t-test with a 95 percent confidence interval was used to see if the difference in mean scores was statistically significant. Table 4.3 shows that the experimental and control groups had mean scores of 17.72 and 17.48, respectively, with a mean difference

of 0.24. The experimental group's maximum and minimum scores were 12 and 7, respectively, while the control groups were 10 and 6.

**Table 4.4: Independent Samples T-test of Pre-test Scores**

Group	N	Mean	Std. Dev.	t-value	df	p-value
Experimental Group	50	17.72	7.24	0.388	98	0.887
Control Group	50	17.48	6.55			

The results of the independent samples t-test, as shown in Table 4.4, were that there was no statistically significant difference between the experimental group (M=17.72; SD = 7.24) and the control group (M = 17.48, SD = 6.55) on the pre-test scores of the two independent research groups. The estimated t-statistic ( $t = 0.388$ ;  $p = 0.887 > 0.05$ ) indicates that both the experimental and the control groups had similar conceptual knowledge of the concepts of integral calculus before implementing the treatment (Maple Software).

To answer research question three, the following hypothesis was developed. An independent samples t-test with a 95 percent confidence interval was conducted to see if there was a statistically significant difference in post-test scores between the experimental group using Maple Software and the control group using the control group's traditional teaching methods.

Null Hypothesis, H0: There is no significant difference in post-test scores between the experimental group using Maple Software and the control group using traditional methods.

Alternative Hypothesis H1: there is significant difference between students' calculus achievement using Maple and those taught by traditional teaching methods.

**Table 4.5: Independent Samples t-test of Post-test Scores (N = 50)**

Groups	N	Mean	Std. Dev.	t-value	df	p-value	Eta Squared
Experimental Group	50	24.80	9.48	2.98	98	0.005	0.096
Control Group	50	20.65	7.67				

Table 4.5 revealed a statistically significant difference between the experimental group (M = 24.80; SD = 9.48) and the control group (M = 20.65; SD = 7.67). The estimated t-statistic was (t = 2.986; p = 0.005). This shows that Maple Software's experimental group outperformed the control group using the traditional method. The eta squared value of 0.096 indicate that medium effect size (Cohen, 1988), implying that the teaching method (independent variable) accounted for 9.6% of the variance in the post-test scores (dependent variable). Consequently, the null hypothesis (H0) that there is no significant difference in post-test scores between the experimental group using Maple Software and the control group using traditional methods was rejected in favour of the alternate hypothesis (H1) that there is a significant difference in post-test scores between the experimental group using Maple Software and the control group using traditional methods. As a result, compared to the conventional approaches, the results demonstrated that Maple Software integration in calculus instruction was effective.



## **CHAPTER FIVE DISCUSSION OF RESULTS**

### **5.1 Introduction**

This chapter focuses on discussing findings that emerged from the analysis of the data gathered. The goal of the study was to investigate the effect of Maple software instruction on senior high school students' understanding of integral calculus to determine how efficient Maple software is as a pedagogical tool for student innovation, attitude, and achievement in senior high school integral calculus. The research findings are addressed under the following subheadings, which coincide to the study's goals.

### **5.2 Difficulties Senior High School Students Face in Solving Integral Calculus Questions**

The findings from the study revealed that 7(7%) of the participants found it challenging to execute correct Substitution of the lower and upper limits of definite integral questions. Also, the results indicated that 22(22%) of the participants omitted the  $dx$  in their attempt to set up the integration process for definite and indefinite integral questions. Moreover, most of the participants, 35(35%), omitted the constant of integration after responding to the indefinite integral test item of the pre-test. It was noted that 18(18%) of the participants could not correctly integrate the polynomial function or quadratic function that was administered to them. Furthermore, a critical examination of the scripts of the participants revealed that 11(11%) were not able to subtract lower limit values from the upper limit values to execute the complete evaluation of definite integral questions contained in the pre-test. This resulted in 11% of the participants not getting the maximum score allotted for such questions.

Meanwhile, 5(5%) participants could not correctly choose the integration interval for the word problem on the definite integral. However, it was noted that those who could not provide the desired results for the word problem question on the definite integral in the pre-test found it difficult to sketch the curve before responding to the question. However, on 2 participants

corresponding 2% used differentiation instead of integration. This finding agrees with the study conducted by Sallah et al. (2020) when they emphasised that students in senior high schools find it difficult to distinguish between integral symbols and variables in indefinite and definite integration problems. The errors associated with symbols and variables in integral calculus may appear inconsequential to those students who have grasped the concept of integral calculus but may not be simple for students who are yet to get the concept of integral calculus, Sallah (2021) and, therefore, mathematics instructors must adopt innovative strategies in assisting students to do away with such errors. Again, they indicated that the integral sign  $\int$  and  $dx$ , which are critical in the presentation of the solution, are frequently overlooked by students. In their attempt to solve indefinite integration problems, senior high school students often omit the constant of integration. The above assertion by Sallah et al. (2020) is corroborated by the works of Nedaei et al. (2021). They also mentioned that many students make technical errors when handling integral problems. Therefore, conscious remedial sessions and revision should be provided to assist students in preparing for this area.

### **5.3 Students' Views on the Use of Maple Software in Learning Integral Calculus**

Students' views on Maple Software integration in integral calculus instruction were investigated in the study. The results revealed that Maple Software has contributed to the success of students' achievement in the integral calculus by arousing and sustaining the student's interest. The Maple Software also made it easier for students to follow the calculus instruction.

According to the findings, incorporating Maple Software into the teaching and learning of integral calculus has motivated students to study mathematics because of its numerous representation possibilities and practical nature. This results of the study are consistent with the conclusions drawn by Ampadu and Danso (2018). They found that incorporating technology into the study of mathematics has the potential to pique and maintain students' interest while

also improving their mathematics performance. They indicated that the ability to arouse and sustain students' interest and increase their mathematics performance is an uphill task now confronting mathematics education in Ghana. Therefore, to get more students taking up STEM courses at the university or tertiary levels, mathematics teachers must make their mathematics instructions lively and relate concepts to real-life situations using computer software to explain abstract mathematical concepts. This will enable students to appreciate mathematics.

The findings also support the claims made by Onal (2017), Wu and Rau (2019), Hill and Uribe-Florez (2020), and Stein et al. (2020) that students must be actively engaged with abstract or concrete concepts for learning to occur in a mathematics classroom. Students' interests and accomplishment levels increase when teachers effectively integrate technology into the learning process. When technology is an essential active element of the mathematical education process, it must be employed appropriately and judiciously to achieve learning outcomes. They emphasised that using technology in the classroom positively impacts students' success and attitudes towards mathematics education. They stressed that judicious use of technological tools could support both the learning of mathematical methods and skills as well as the development of desired mathematical proficiencies such as problem-solving, reasoning, and justifying. The findings are also consistent with those of Leung (2017), who affirmed that incorporating Maple Software into mathematics instruction increases student motivation by virtue of its practical nature and visual representation of concepts. Therefore, students are able to retain abstract mathematical concepts, which are indications that desired learning outcomes have taken place.

#### **5.4 The Effect of Teaching Integral Calculus Using Maple Software on Senior High School Students' Mathematics Achievement.**

The findings indicated that the experimental and control groups had mean scores of 17.72 and 17.48, respectively, with a mean difference of 0.24. The experimental group's maximum and minimum scores were 12 and 7, respectively, while the control groups were 10 and 6. Moreover, the independent samples t-test analysis results showed that there was no statistically significant difference between the experimental group ( $M=17.72$ ;  $SD = 7.24$ ) and the control group ( $M = 17.48$ ,  $SD = 6.55$ ) on the pre-test scores of the two independent research groups. The estimated t-statistic ( $t = 0.388$ ;  $p = 0.887 > 0.05$ ) indicates that both the experimental and the control groups had similar conceptual knowledge of the concepts of integral calculus before implementing the treatment (Maple Software). Furthermore, an independent samples t-test analysis of the post-test scores for the experimental and control groups demonstrated a statistically significant difference between the experimental group ( $M = 24.80$ ;  $SD = 9.48$ ) and the control group ( $M = M = 20.65$ ;  $SD = 7.67$ ). The estimated t-statistic was ( $t = 2.986$ ;  $p = 0.005$ ). This shows that Maple Software's experimental group outperformed the control group using the traditional method. The eta squared value of 0.096 indicates a medium effect size (Cohen, 1988), implying that the conventional teaching method accounted for 9.6% of the variance in the post-test scores.

These findings strengthen Hill and Uribe-Florez's (2020) claim that daily use of technology in mathematics education can help students learn and develop mathematical knowledge and skills. The above findings resonate with the study conducted by Haciomeroglu et al. (2009), who found that using GeoGebra in a mathematics classroom improved the teacher-participants attitudes. The teacher-participants realised how adding technology to mathematics teaching benefits students learning. The teacher-participants learned that using GeoGebra allows for dynamic linkage, which is crucial since it helps students see and understand concepts. The



teachers-participants discovered that students might use multiple dynamic representations and mathematical modelling to investigate, solve, and express mathematical problems in various ways. Students can study mathematical concepts and interactively connect algebraic, pictorial and numeric representations of these concepts without spending a considerable amount of classroom time sketching figures, objects, or functions. The results are in-line with Perjesi-Hamori (2015) asserted that by utilising the Symbolic Algebra System (SAS), learners with inadequate achievement in mathematics were able to comprehend increasingly complicated problems.

Notwithstanding, the findings agree with Salleh and Zakaria (2016). They argued that Maple Software provides colour editing capabilities that can be used to arouse students' interest as far as visualisation is concerned. They emphasised that studies have found that Maple Software can generate metacognitive cues or signals among students to learn integral calculus. Unlike the CAS-based graphing calculators, Maple Software provides various instructional tools, including a powerful mathematical software package that includes graphics, computation, and programming.



## CHAPTER SIX SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

### 6.1 Introduction

The purpose of the study was to investigate the effect of Maple software instruction on senior high school students' understanding of integral calculus. The objectives of the study were to examine difficulties senior high school students face in solving integral calculus questions, to find out students' views on the use of Maple Software in learning integral calculus and to investigate the effect of teaching integral calculus using Maple Software on senior high school students' mathematics achievement or performance in the study. The findings of the study are summarised in this chapter, together with conclusions and recommendations for improving senior high school mathematics instruction in the study area.

### 6.2 Summary of the Study

The study found that 7(7%) of the participants found it challenging to execute correct Substitution of the lower and upper limits of definite integral questions. Also, the results indicated that 22(22%) of the participants omitted the  $dx$  in their attempt to set up the integration process for definite and indefinite integral questions. Moreover, most of the participants, 35(35%), omitted the constant of integration after responding to the indefinite integral test item of the pre-test. It was noted that 18(18%) of the participants could not correctly integrate the polynomial function or quadratic function that was administered to them. Furthermore, a critical examination of the scripts of the participants revealed that 11(11%) were not able to subtract lower limit values from the upper limit values to execute the complete evaluation of definite integral questions contained in the pre-test. This resulted in 11% of the participants not getting the maximum score allotted for such questions.

Meanwhile, 5(5%) participants could not correctly choose the integration interval for the word problem on the definite integral. However, it was noted that those who could not provide the

desired results for the word problem question on the definite integral in the pre-test found it difficult to sketch the curve before responding to the question. However, on 2 participants corresponding 2% used differentiation instead of integration. These findings are in consonance with the study conducted by Sallah et al. (2020), when they emphasised that students in senior high schools find it difficult to distinguish between integral symbols and variables in indefinite and definite integration problems. The integral sign  $\int$  and  $dx$ , which are critical in the presentation of the solution, are frequently overlooked by students. In their attempts to solve indefinite integration problems, senior high school students often omit the constant of integration. The above assertion by Sallah et al. (2020) is corroborated by the works of Nedaei, Radmehr, and Drake (2021) and Radmehr and Drake (2017), who also mentioned that many students make technical errors when handling integral problems, therefore, conscious remedial sessions and revision should be provided to assist students in preparing for this area.

Notwithstanding, the study findings suggest that incorporating Maple Software into the teaching and learning of integral calculus has motivated students to study integral calculus by arousing and sustaining their interests in the lesson. These findings are in-line with the results of Ampadu and Danso (2018). They found that incorporating technology into the study of mathematics has the potential to pique and maintain students' interest while also improving their mathematics performance.

In addition, the findings indicated that the experimental and control groups had mean scores of 17.72 and 17.48, respectively, with a mean difference of 0.24. The experimental group's minimum and maximum scores were 12 and 7, respectively, while the control groups were 10 and 6. Moreover, the independent samples t-test analysis results showed that there was no statistically significant difference between the experimental group ( $M=17.72$ ;  $SD = 7.24$ ) and the control group ( $M = 17.48$ ,  $SD = 6.55$ ) on the pre-test scores of the two independent research groups. The estimated t-statistic ( $t = 0.388$ ;  $p = 0.887 > 0.05$ ) indicates that both the

experimental and the control groups had similar conceptual knowledge of the concepts of integral calculus prior to implementing the treatment (Maple Software). Furthermore, an independent samples t-test analysis of the post-test scores for the experimental and control groups demonstrated a statistically significant difference between the experimental group ( $M = 24.80$ ;  $SD = 9.48$ ) and the control group ( $M = M = 20.65$ ;  $SD = 7.67$ ). The estimated t-statistic was ( $t = 2.986$ ;  $p = 0.005$ ). This shows that Maple Software's experimental group outperformed the control group using the traditional method. The eta squared value of 0.096 indicates a medium effect size (Cohen, 1988), implying that the conventional teaching method accounted for 9.6% of the variance in the post-test scores. These findings support the claim by Hill and Uribe-Florez (2020) that incorporating ICT into our daily teaching and learning of mathematics can open new ways to promote students' learning and foster the acquisition of mathematical knowledge and skills. The above findings resonate with the study conducted by Haciomeroglu et al. (2009), who found that using technology in mathematics teaching benefits students immensely by enabling them to master complex or abstract concepts. It also allows for dynamic linkage, which is crucial since it helps students see and understand concepts. This view is supported by Wu and Rau (2019), who emphasised that technology could be used to create more interactive and relevant mathematics instruction.

### **6.3 Conclusion of Study**

The most significant contribution of this study is a new understanding of the effects of incorporating Maple Software into integral calculus instruction in senior high school mathematics to increase academic achievement. Participants who were taught utilising the Maple Software integration approach did better than those who were given a similar lesson using traditional methods, according to the results of this study. Finally, the findings revealed that integrating Maple Software into the teaching and learning of mathematics increases student motivation in the following ways: it induces learning practically, it arouses, motivates, sustains

the interest of students in the teaching process, and it presents concepts in a multi-dimensional way, it promotes real-life connections with concepts, it brings the students' immediate environment closer to the classroom, it promotes retention through its visualising qualities. Incorporating Maple Software into the teaching of mathematics in general and integral calculus, in particular, has a favourable impact on the motivation and achievement of senior high school students.

#### **6.4 Recommendation of the Study**

The study recommends that technology and mathematical software should be used in the teaching and learning of integration. Mathematics instructors must get through in-service training in technology and software application to enhance practices in the mathematics classroom. Pre-service teacher education should include considerable practice in fundamental and higher-order mathematical process abilities so that new teachers are more confident in using computer software when teaching mathematics. Furthermore, the integration of mathematics software can be used to attract students' attention, which has ramifications for mathematics teachers, especially in schools where students' passion for mathematics is limited. This teaching strategy can also be utilised to increase performance in low-performing schools, most of which are under-resourced.

Furthermore, through the Ghana Education Service (GES), the government should use mathematical software such as Maple to teach mathematics at senior high schools practicable. Further, it is recommended that senior high school stakeholders consider periodic seminars/workshops for math teachers on using adequate technological resources like Maple Software in learning and teaching mathematical concepts.

## REFERENCES

- Abdurrahman, A., Setyaningsih, C. A., & Jalmo, T. (2019). Implementating multiple representation-based worksheet to develop critical thinking skills. *Journal of Turkish Science Education*, 16(1), 138-155.
- Abukari, Z., Kuyini, A. B., & Kuyini-Mohammed, A. (2015). Education and health care policies in Ghana: examining the prospects and challenges of recent provisions. *Sage Open*, 5(4), 215-228.
- Accurso, K., Gebhard, M., & Purington, S. B. (2017). Analysing diverse learners' writing in mathematics: Systemic functional linguistics in secondary pre-service teacher education. *International Journal for Mathematics Teaching and Learning*, 18(1).
- Adu-Agyem, J., & Osei-Poku, P. (2012). Quality education in Ghana: The way forward. *International Journal of Innovative Research and Development (ISSN 2278-0211)*, 1(9), 164-177.
- Afriyani, D., & Sa'dijah, C. (2018). Characteristics of Students' Mathematical Understanding in Solving Multiple Representation Task Based on Solo Taxonomy. *International Electronic Journal of Mathematics Education*, 13(3), 281-287.
- Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and instruction*, 16(3), 183-198.
- Akgül, M. B. (2014). The Effect of using dynamic geometry software on eight grade students' achievement in transformation geometry, geometric thinking and attitudes toward mathematics and technology (Master's thesis, Middle East Technical University).
- Alacacı, C., & McDonald, G. (2012). The impact of technology on high school mathematics curriculum. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, 3(1).
- Ampadu, E., & Danso, A. (2018). Constructivism in mathematics classrooms: Listening to

- Ghanaian teachers' and students' views. *Africa Education Review*, 15(3), 49-71.
- Anderson, I. M., & Torre, C. G. (2012). New symbolic tools for differential geometry, gravitation, and field theory. *Journal of mathematical physics*, 53(1), 013511.
- Anderson, J. R., Lee, H. S., & Fincham, J. M. (2014). Discovering the structure of mathematical problem solving. *NeuroImage*, 97, 163–177.
- Asiamah, N., Mensah, H. K., & Oteng-Abayie, E. F. (2017). General, target, and accessible population: Demystifying the concepts for effective sampling. *The Qualitative Report*, 22(6), 1607.
- Atuahene, F., & Russell, T. (2016). Mathematics readiness of first-year university students. *Journal of Developmental Education*, 39(3), 12-32.
- Awang, T. S., & Zakaria, E. (2012). Module for Learning Integral Calculus with Maple: Lecturers' Views. *Turkish Online Journal of Educational Technology-TOJET*, 11(3), 234-245.
- Aydos, M. (2015). The impact of teaching mathematics with Geogebra on the conceptual understanding of limits and continuity: The case of Turkish gifted and talented students (Master's thesis, Eğitim Bilimleri Enstitüsü).
- Ayub, A. F., Mokhtar, M. Z., Luan, W. S., & Tarmizi, R. A. (2010). A comparison of two different technologies tools in tutoring calculus. *Procedia Social and Behavioral Sciences*, 2, 481-486.
- Bali, J. S., Kumar, A., & Nandi, V. (2016). An experience, using software based tools for teaching and learning mathematically intensive signal processing theory concepts. *Innovation and Technology in Education (MITE)*, 100-104.
- Bandura, A. (1997). *Self-Efficacy: The exercise of control*. New York, NY: W. H. Freeman.
- Baron, M. A. (2008). *Guidelines for writing research proposals and dissertations*. Division of



Educational Administration: University of South Dakota, 1, 1-52.

Blazar, D., & Kraft, M. A. (2017). Teacher and teaching effects on students' attitudes and behaviors. *Educational evaluation and policy analysis*, 39(1), 146-170.

Bowers, A. (2019). Teaching the Fundamental Theorem of Calculus. *The Mathematics Teacher*, 112(6), 418-421.

Brantlinger, A. (2014). Critical mathematics discourse in a high school classroom: Examining patterns of student engagement and resistance. *Educational Studies in Mathematics*, 85(2), 201-220.

Bray, A., & Tangney, B. (2016). Enhancing student engagement through the affordances of mobile technology: a 21st century learning perspective on Realistic Mathematics Education. *Mathematics Education Research Journal*, 28(1), 173-197.

Britannica, T. Editors of Encyclopaedia (2021, September 3). function. *Encyclopedia Britannica*. <https://www.britannica.com/science/function-mathematics>

Bruner, J. S. (1966). *Toward a theory of instruction* (Vol. 59). Cambridge, MA, USA: Harvard University Press.

Bryant, R., Weiss, R., Orr, G., & Yerion, K. (2011). Using the context of algorithmic art to change attitudes in introductory programming. *The Journal of Computing Sciences in Colleges*, 27(1), 112-119.

Buelin, J., Clark, A. C., & Ernst, J. V. (2016). Engineering's Grand Challenges: Priorities and Integration Recommendations for Technology Education Curriculum Development. *Journal of Technology Education*, 28(1), 37-52.

Buneci, M. R. (2014). Using Maple to represent the subgroups of trivial groupoid  $xxzxx$ . *Fiability & Durability / Fiabilitate Si Durabilitate*, (1), 446-454.

Carlson, M., Oehrtman, M., & Engelke, N. (2010). The precalculus concept assessment: A tool

- for assessing students' reasoning abilities and understandings. *Cognition and Instruction*, 28(2),113-145.
- Carlson, M. P., Smith, N., & Persson, J. (2003). Developing and Connecting Calculus Students' Notions of Rate-of Change and Accumulation: The Fundamental Theorem of Calculus. *International Group for the Psychology of Mathematics Education*, 2, 165-172.
- Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically*. Portsmouth, NH: Heinemann.
- Chen, C. L., & Wu, C. C. (2020). Students' behavioral intention to use and achievements in ICT-Integrated mathematics remedial instruction: Case study of a calculus course. *Computers & Education*, 145, 103740.
- Chimuka, A. (2017). *The effect of integration of geogebra software in the teaching of circle geometry on grade 11 students' achievement (Masters Thesis, University of South Africa)*. Cape Town: University of South Africa Press.
- Chrysanthou, I., Hohenwarter, J., Hohenwarter, M., Hreinsdóttir, F., Kreis, Y., Lavicza, Z., et al. (2010). *First International GeoGebra Conference 2009*. First International GeoGebra Conference 2009, (pp. 5-16). Hagenberg.
- Clark-Wilson, A., & Hoyles, C. (2019). A research-informed web-based professional development toolkit to support technology-enhanced mathematics teaching at scale. *Educational Studies in Mathematics*, 102(3), 343-359.
- Costley, K. C. (2014). *The Positive Effects of Technology on Teaching and Student Learning*. Online submission, 21-35.
- Covington-Ward, Y. (2017). Back home, people say America is heaven: Pre-migration expectations and post-migration adjustment for Liberians in Pittsburgh. *Journal of*

- International Migration and Integration, 18(4), 1013-1032.
- Creswell, J. W. (2014). *Qualitative, quantitative and mixed methods approaches*. Thousand Oaks, California: Sage.
- Cumino, C., Pavignano, M., Spreafico, M. L., & Zich, U. (2021). Geometry to Build Models, Models to Visualise Geometry. *Digital Experiences in Mathematics Education*, 7(1), 149-166.
- Demetriou, A., Kazi, S., Spanoudis, G., & Makris, N. (2019). Predicting school performance from cognitive ability, self-representation, and personality from primary school to senior high school. *Intelligence*, 76, 101381.
- Dreher, A., Kuntze, S., & Lerman, S. (2016). Why use multiple representations in the mathematics classroom? Views of English and German pre-service teachers. *International Journal of Science and Mathematics Education*, 14(2), 363-382.
- Emaikwu, S. O. (2012). Assessing the relative effectiveness of the three teaching methods in the measurement of students' achievement in mathematics. *Journal of Emerging Trends in Educational Research and Policy Studies (JETERAPS)*, 3(4), 479-486.
- Ertekin, E. (2014). Is Cabri 3D effective for the teaching of special planes in analytic geometry? *International Journal of Educational Studies in Mathematics*, 1(1), 27-36.
- Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American Journal of Theoretical and Applied Statistics*, 5(1), 1-4.
- Eyyam, R., & Yaratan, H. S. (2014). Impact of use of technology in mathematics lessons on student achievement and attitudes. *International Journal of Social Behavior and Personality*, 42(1), 31S-42S.
- Evbuomwan, D. (2013). *An investigation into the difficulties faced by Form C students in the learning of transformation geometry in Lesotho secondary schools (Doctoral dissertation)*.

- Fletcher, J. A. (2005). Constructivism and mathematics education in Ghana. *Mathematics connection*, 5, 29-36.
- Foster, A., & Shah, M. (2015). The play curricular activity reflection discussion model for game-based learning. *Journal of Research on Technology in Education*, 47(2), 71-88.
- Fowler, L. (2020). Using the multiple streams framework to connect policy adoption to implementation. *Policy Studies Journal*, 25-33.
- García-García, J., & Dolores-Flores, C. (2018). Intra-mathematical connections made by high school students in performing Calculus tasks. *International Journal of Mathematical Education in Science and Technology*, 49(2), 227-252.
- Genlott, A. A., & Grönlund, A. (2016). Closing the gaps—Improving literacy and mathematics by ict-enhanced collaboration. *Computers & Education*, 99, 68-80.
- Grove, M. (2012). The mathematical transition: A multi-stage problem? . *MSOR Connections*, 12(1), 15-18.
- Guetterman, T. C., & Fetters, M. D. (2018). Two methodological approaches to the integration of mixed methods and case study designs: A systematic review. *American Behavioral Scientist*, 62(7), 900-918.
- Guillén-Gámez, F. D., & Mayorga-Fernández, M. J. (2020). Identification of variables that predict teachers' attitudes toward ICT in higher education for teaching and research: A study with regression. *Sustainability*, 12(4), 1312.
- Gyasi-Agyei, K. A., Gyasi-Agyei, A., & Obeng-Denteh, W. (2014). Mathematical modeling of the epidemiology of tuberculosis in the Ashanti region of Ghana. *Journal of Advances in Mathematics and Computer Science*, 375-393.
- Gyedu, A. A., Owusu-Darko, I., & Ofori, E. K. (2020). Effect of Geometer's Sketchpad on Senior High School Students' Performance in Quadratic Graphing. *European Journal of Education and Pedagogy*, 1(1).

- Haciomeroglu, E. S., Bu, L., Schoen, R. C., & Hohenwarter, M. (2009). Learning to develop mathematics lessons with GeoGebra. *MSOR Connections*, 9(2), 24-26.
- Haripersad, R. (2011). Deep and surface learning of elementary calculus concepts in a blended learning environment. *International Journal of Mathematics and Computers in Simulation*, 5(4), 291-298.
- Henderson, S., & Broadbridge, P. (2007). Mathematics for 21st-century engineering students. Paper presented at the AaeE Conference. *AaeE Conference* (pp. 33-41). Melbourne, Australia: AaeE Conference.
- Herawaty, D., Widada, W., Adhitya, A., Sari, R. D., Novianita, L., & Anggoro, A. F. (2020). Students' ability to simplify the concept of function through realistic mathematics learning with the ethno-mathematics approach. *Journal of Physics*, 1470(1), 012031.
- Hill, J. E., & Uribe-Florez, L. (2020). Understanding Secondary School Teachers' TPACK and Technology Implementation in Mathematics Classrooms. *International Journal of Technology in Education*, 3(1), 1-13.
- Hodara, M., & Xu, D. (2016). Does developmental education improve labor market outcomes? Evidence from two states. *American Educational Research Journal*, 53(3), 781-813.
- Hong, Y. Y., & Thomas, M. O. (2015). Graphical construction of a local perspective on differentiation and integration. *Mathematics Education Research Journal*, 27(2), 183-200.
- Ifenthaler, D., & Schweinbenz, V. (2016). Students' acceptance of tablet PCs in the classroom. *Journal of Research on Technology in Education*, 48(4), 306-321.
- In, J. (2017). Introduction of a pilot study. *Korean journal of anesthesiology*, 70(6), 601.
- Jahanshahi, S., Babolian, E., Torres, D. F., & Vahidi, A. (2015). Original article: Solving Abel integral equations of first kind via fractional calculus. *Journal of King Saud University - Science*, 27,161-167.

- Jahanshahi, S., Babolian, E., Torres, D. F., & Vahidi, A. (2015). Solving Abel integral equations of first kind via fractional calculus. *Journal of King Saud University-Science*, 27(2), 161-167.
- Kang, R., & Liu, D. (2018). The importance of multiple representations of mathematical problems: Evidence from Chinese pre-service elementary teachers' analysis of a learning goal. *International Journal of Science and Mathematics Education*, 16(1), 125-143.
- Karaibryamov, S., Tsareva, B., & Zlatanov, B. (2013). Optimisation of the Courses in Geometry by the Usage of Dynamic Geometry Software Sam. *Electronic Journal of Mathematics & Technology*, 7(1).
- Kashefi, H., Ismail, Z., Yusof, Y. M., & Rahman, R. A. (2012). Fostering mathematical thinking in the learning of multivariable calculus through computer-based tools. *Procedia Social and Behavioral Sciences*, 46, 5534-5540.
- Kossivi, S. K. (2020). *Understanding Calculus Through Maple-Based Dynamic Visualisation Tools* (Doctoral dissertation, Walden University). Walden : Walden University Press.
- Kumar, K., Pandey, R. K., & Sharma, S. (2019). Approximations of fractional integrals and Caputo derivatives with application in solving Abel's integral equations. *Journal of King Saud University-Science*, 31(4), 692-700.
- Lavicza, Z. (2010). Integrating technology into mathematics teaching at the university level. *ZDM Mathematics Education*, 42(1), 105-119.
- LeBeau, B., Harwell, M., Monson, D., Dupuis, D., Medhanie, A., & Post, T. R. (2012). Student and high-school characteristics related to completing a science, technology, engineering or mathematics (STEM) major in college. *Research in Science & Technological Education*, 30(1), 17-28.
- Leikin, R., Zazkis, R., & Meller, M. (2018). Research mathematicians as teacher educators:

- Focusing on mathematics for secondary mathematics teachers. *Journal of Mathematics Teacher Education*, 21(5), 451-473.
- Leung, A. (2017). Exploring techno-pedagogic task design in the mathematics classroom. Springer, Cham, 3-16.
- Lim, S. Y., & Chapman, E. (2015). Effects of using history as a tool to teach mathematics on students' attitudes, anxiety, motivation and achievement in grade 11 classrooms. *Educational studies in mathematics*, 90(2), 189-212.
- Lohr, S. L. (2021). *Sampling: design and analysis*. Chapman and Hall/CRC.
- Lucero, J., Wallerstein, N., Duran, B., Alegria, M., Greene-Moton, E., Israel, B., et al. (2018). Development of a mixed methods investigation of process and outcomes of community-based participatory research. *Journal of Mixed Methods Research*, 12(1), 55- 74.
- Mac an Bhaird, C., Nolan, B. C., O'Shea, A., & Pfeiffer, K. (2017). A study of creative reasoning opportunities in assessments in undergraduate calculus courses. *Research in Mathematics Education*, 19(2), 147-162.
- MacBride, R., & Luehmann, A. (2016). Capitalising on Emerging Technologies: A Case Study of Classroom Blogging. *School Science & Mathematics*, 108(5), 173-183.
- Maciejewski, W., & Star, J. R. (2016). Developing flexible procedural knowledge in undergraduate calculus. *Research in Mathematics Education*, 18(3), 299-316.
- Mahmudi, A. (2010). Membelajarkan geometri dengan program GeoGebra. *Seminar Nasional Matematika dan Pendidikan Matematika*, 469-477.
- Majid, U. (2018). Research fundamentals: Study design, population, and sample size. *Undergraduate research in natural and clinical science and technology journal*, 2, 1-7.
- Martinovic, D., & Manizade, A. G. (2018). The challenges in the assessment of knowledge for teaching geometry. *ZDM*, 50(4), 613-629.
- McAndrew, A. (2015). Using CAS calculators to teach and explore numerical methods.

- Electronic Journal of Mathematics & Technology, 9(1).
- Meikle, L. I., & Fleuriot, J. D. (2012). Integrating Systems around the User: Combining Isabelle, Maple, and QEPCAD in the Prover's Palette. *Electronic Notes in Theoretical Computer Science*, 285, 115-119.
- Mereku, D. K. (2010). Five decades of school mathematics in Ghana. *Mathematics Connections*, 9(8), 73-86.
- Milovanović, M., Perišić, J., Vukotić, S., Bugarčić, M., Radovanović, L., & Ristić, M. (2016). Learning mathematics using multimedia in engineering education. *Acta Technica Corviniensis-Bulletin Of Engineering*, 9(1), 45-49.
- Ministry of Education (MOE). (2015). *ICT IN EDUCATION POLICY*. Accra, Ghana: MOE.
- Montijo, E. (2017). The effects of Desmos and TI-83 plus graphing calculators on the problem-solving confidence of middle and high school mathematics students. *Liberty University*, 20-29.
- Morin, K. H. (2013). Value of a pilot study. *Journal of Nursing Education*, 52(10), 547-548.
- Murphy, S. J. (2009). The power of visual learning in secondary mathematics education. *Research into practice mathematics*, 1-8.
- Mynbaev, D. K., Bo, C. C., Rashvili, R., Ya, K., & Liou-Mark, J. (2008). Support of study on engineering technology from Physics and Mathematics. Paper presented at the ASEE Mid-Atlantic Conference. *ASEE Mid-Atlantic Conference* (pp. 19-29). ASEE Mid-Atlantic Conference.
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring*. Reston, VA: Author.



- National Council of Teachers of Mathematics (NCTM). (2015). *Strategic Use of Technology in Teaching and Learning Mathematics*. Reston, VA: Author.
- Nedaei, M., Radmehr, F., & Drake, M. (2021). Exploring undergraduate engineering students' mathematical problem-posing: the case of integral-area relationships in integral calculus. *Mathematical Thinking and Learning*, 1-27.
- Ningsih, Y. L., & Paradesa, R. (2018). Improving students' understanding of mathematical concept using Maple. *Journal of Physics*, 948 (1), 012034.
- Nobre, C. N., De Resende, M. N., Da Costa, L. E., & Da Rocha, R. C. (2016). The Use of Geogebra Software as a Calculus Teaching and Learning Tool. *Informatics in Education*, 15(2), 253-267.
- Norton, P., Bridges, W., & High, K. (2018). Impact of course policy changes on calculus I DFW rates. *Journal of STEM Education*, 19(1).
- Nurzannah, N., Muliana, M., Herizal, H., Fajriana, F., & Mursalin, M. (2021). The effect of REACT strategy assisted by GeoGebra software on students' mathematical representation ability. *Malikussaleh Journal of Mathematics Learning (MJML)*, 4(2), 90-97.
- Obodo, G. C. (2004). Principles and practice of mathematics education in Nigeria . *Academic Forum* (pp. 2-7). Enugu: Organisers of Academic Forum.
- Ocal, M. F. (2017). The Effect of Geogebra on Students' Conceptual and Procedural Knowledge: The Case of Applications of Derivative. *Higher Education Studies*, 7(2), 67-78.
- Önal, N. (2017). Use of Interactive Whiteboard in the Mathematics Classroom: Students' Perceptions within the Framework of the Technology Acceptance Model. *International Journal of Instruction*, 10(4), 67-86.
- Opach, T., Gołębiowska, I., & Fabrikant, S. I. (2014). How do people view multi- component

- animated maps? . *Cartographic Journal*, 51(4), 330-342.
- Osborne, J. (2007). Science education for the twenty first century. *Eurasia Journal of Mathematics, Science and Technology Education*, 3(3), 173-184.
- Ott, N., Brünken, R., Vogel, M., & Malone, S. (2018). Multiple symbolic representations: The combination of formula and text supports problem solving in the mathematical field of propositional logic. *Learning and Instruction*, 58, 88-105.
- Özkan, E. M., & Ünal, H. (2009). Misconception in Calculus-I: Engineering students' misconceptions in the process of finding domain of functions. *Procedia-Social and Behavioral Sciences*, 1(1), 1792-1796.
- Öztürk, Y., Anapalı, A., Gülsu, M., & Sezer, M. (2013). A collocation method for solving fractional Riccati differential equation. *Journal of Applied Mathematics*, 72-95.
- Padilla-López, J. R., Charaoui, A. A., & Flórez-Revuelta, F. (2015). Visual privacy protection methods: A survey. *Expert Systems with Applications*, 42(9), 4177-4195.
- Panaoura, A., Michael-Chrysanthou, P., Gagatsis, A., Elia, I., & Philippou, A. (2017). A structural model related to the understanding of the concept of function: definition and problem solving . *International Journal of Science and Mathematics Education*, 15(4), 723-740.
- Perbowo, K. S., & Anjarwati, R. (2017). Analysis of Students' Learning Obstacles on Learning Invers Function Material. *Infinity Journal*, 6(2), 169-176.
- Perjési-Hámori, I. (2015). Teaching numerical methods using CAS. *Electronic Journal of Mathematics & Technology*, 9(3).
- Persson, P. E. (2014). How teachers can use TI-Nspire CAS with laptops in an upper secondary course. *The Electronic Journal of Mathematics & Technology*, 8(4), 257–273.
- Pyke, A., Betts, S., Fincham, J. M., & Anderson, J. R. (2015). Visuospatial referents facilitate the learning and transfer of mathematical operations: Extending the role of the angular

- gyrus. *Cognitive, Affective and Behavioral Neuroscience*, 15(1), 229-252.
- Quarles, C. L., & Davis, M. (2017). Is learning in developmental math associated with community college outcomes? *Community College Review*, 45(1), 33-51.
- Quintero, E., Salinas, P., González-Mendivil, E., & Ramírez, H. (2015). Augmented reality app for calculus: A proposal for the development of spatial visualization. *Procedia Computer Science*, 75, 301-305.
- Radmehr, F., & Drake, M. (2017). Exploring students' mathematical performance, metacognitive experiences and skills in relation to fundamental theorem of calculus. *International Journal of Mathematical Education in Science and Technology*, 48(7), 1043-1071.
- Radmehr, F., & Drake, M. (2020). Exploring students' metacognitive knowledge: The case of integral calculus. *Education Sciences*, 10(3), 55.
- Reinholz, D. L., Cox, M., & Croke, R. (2015). Supporting Graduate Student Instructors in Calculus. *International Journal for the Scholarship of Teaching and Learning*, 9(2), n2.
- Sallah, E. K. (2021). Use of Maple Software to Reduce Senior High School Students' Errors in Integral Calculus. *African Journal of Educational Studies in Mathematics and Sciences*, 17(2), 51-59.
- Sallah, E. K., Sogli, J. K., Owusu, A., & Edekor, L. K. (2020). Effective Application of Maple Software to Reduce Student Teachers' Errors in Integral Calculus. *Statistics*, 4(3), 64-78.
- Salleh, T. S., & Zakaria, E. (2016). The Effects of Maple Integrated Strategy on Engineering Technology Students' Understanding of Integral Calculus. *Turkish Online Journal of Educational Technology-TOJET*, 15(3), 183-194.
- Samková, L. (2012). Calculus of one and more variables with Maple. *International Journal Of Mathematical Education In Science & Technology*, 43(2), 230-244.

- Samsuddin, A. F., & Retnawati, H. (2018). Mathematical representation: the roles, challenges and implication on instruction. *Journal of Physics*, 1097 (1), 012152.
- Serhan, D. (2015). Students' Understanding of the Definite Integral Concept. *International Journal of Research in Education and Science*, 1(1), 84-88.
- Shaheen, M., & Pradhan, S. (2019). Sampling in qualitative research. IGI Global, 25-51.
- Shannon-Baker, P. (2016). Making paradigms meaningful in mixed methods research. *Journal of mixed methods research*, 10(4), 319-334.
- Shirley, M. L., Irving, K. E., Sanalan, V. A., Pape, S. J., & Owens, D. T. (2011). The practicality of implementing connected classroom technology in secondary mathematics and science classrooms. *International Journal of Science and Mathematics Education*, 9(2), 459-481.
- Sileyew, K. J. (2019). *Research design and methodology* . Rijeka: IntechOpen.
- Singh, A. S., & Masuku, M. B. (2014). Sampling techniques & determination of sample size in applied statistics research: An overview. *International Journal of economics, commerce and management*, 2(11), 1-22.
- Solares, A., & Kieran, C. (2013). Articulating syntactic and numeric perspectives on equivalence: The case of rational expressions. *Educational Studies In Mathematics*, 84(1), 115-148.
- Soon, W. M., & Ang, K. C. (2015). Introducing queuing theory through simulations. *Google Scholar*, 16-29.
- Soni, N. K., & Patel, T. P. *International Journal of Scientific and Research Publications. Quality Teaching & Higher Education System in India*” SSN, 2250-3153.
- Sözcü, Ö. F., İpek, İ., & Taşkın, E. (2013). A history of computer-based instruction and its effects on developing instructional technologies. *European Researcher*, (9-2), 2341-2347.

- Sozcu, O. F., Ziatdinov, R., & Ipek, I. (2013). The effects of computer-assisted and distance learning of geometric modeling. *European Researcher*, 39(1 -2), 175 – 181.
- Speiser, R., Maher, C. A., & Walter, C. N. (2001). Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Snowbird, Utah, October 18-21, 2001). Volume 1 [and] Volume 2. ERIC/CSMEE Publications, 1929 Kenny Road, Columbus, OH 43210-1080.
- Stein, H., Gurevich, I., & Gorev, D. (2020). Integration of technology by novice mathematics teachers—what facilitates such integration and what makes it difficult? *Education and Information Technologies*, 25(1), 141-161.
- Stols, G., & Kriek, J. (2011). Why don't all maths teachers use dynamic geometry software in their classrooms? *Australasian Journal of Educational Technology*, 27(1).
- Štrausová, I., & Hašek, R. (2013). "Dynamic visual proofs" using DGS. *Electronic Journal of Mathematics & Technology*, 7(2).
- Swallow, M. (2015). The year-two decline: Exploring the incremental experiences of a 1: 1 technology initiative. *Journal of Research on Technology in Education*, 47(2), 122-137.
- Tai, K. W., & Wei, L. (2020). Constructing playful talk through translanguaging in the English medium instruction mathematics classrooms. *Applied Linguistics*, 1, 34.
- Takyi, P. O., & Obeng, C. K. (2013). Determinants of financial development in Ghana. *International Journal of Development and Sustainability*, 2(4), 2324-2336.
- Tang, T. L., & Austin, M. J. (2009). Students' perceptions of teaching technologies, application of technologies, and academic performance. *Computers & education*, 53(4), 1241-1255.
- Thompson, P. W., & Silverman, J. (2008). The concept of accumulation in calculus. Making the connection: Research and teaching in undergraduate mathematics, 73, 43-52.

- Thompson, S. (2013). Systematic Calculation of Loop Points for Parametric Curves. *Electronic Journal of Mathematics & Technology*, 7(3).
- Tobi, H., & Kampen, J. K. (2018). Research design: the methodology for interdisciplinary research framework. *Quality & quantity*, 52(3), 1209-1225.
- Toeplitz, O. (2018). *The calculus*. Chicago: University of Chicago Press.
- Turner, S. F., Cardinal, L. B., & Burton, R. M. (2017). Research design for mixed methods: A triangulation-based framework and roadmap. *Organizational Research Methods*, 20(2), 243-267.
- Turner, S. F., Cardinal, L. B., & Burton, R. M. (2017). Research design for mixed methods: A triangulation-based framework and roadmap. *Organizational Research Methods*, 20(2), 243-267.
- Unodiaku, S. S. (2013). Effect of ethno-mathematics teaching materials on students' achievement in mathematics in Enugu State. *Journal of Education and Practice*, 4(23), 70-77.
- Vajda, R. (2015). Chebyshev Polynomials as Extremal Polynomials with Computer Algebra. *Electronic Journal of Mathematics & Technology*, 9(3).
- Vakili, M. M., & Jahangiri, N. (2018). Content validity and reliability of the measurement tools in educational, behavioral, and health sciences research. *Journal of Medical Education Development*, 10(28), 106-118.
- Vasquez, Daisy. "Enhancing student achievement using GeoGebra in a technology rich environment." PhD diss., California State Polytechnic University, Pomona, 2015.
- Vieira, A. F. (2015). Viewing the roots of polynomial functions in complex variable: The use of GeoGebra and the CAS Maple. *Acta Didactica Napocensia*, 6(4), 45-58.
- West African Examination Council (WAEC). (2017). Chief Examiner's Report. Accra, Ghana:

WAEC.

West African Examination Council (WAEC). (2018). Chief Examiner's Report. Accra, Ghana:

WAEC.

West African Examination Council (WAEC). (2019). Chief Examiner's Report. Accra, Ghana:

WAEC.

Wu, S. P., & Rau, M. A. (2019). How students learn content in science, technology, engineering, and mathematics (STEM) through drawing activities. *Educational Psychology Review*, 31(1), 87-120.

Young, J., Gorumek, F., & Hamilton, C. (2018). Technology effectiveness in the mathematics classroom: A systematic review of meta-analytic research. *Journal of Computers in Education*, 5(2), 133-148.

Yusri, M. Y., Ahmad, S. N., Ong, M. H., & Khalil, N. (2020). The Effectiveness of Collaborative Teaching Method in Teaching Calculus I. *Environment-Behaviour Proceedings Journal*, 5(SI1), 187-191.

Zainuddin, Z., Chu, S. K., Shujahat, M., & Perera, C. J. (2020). The impact of gamification on learning and instruction: A systematic review of empirical evidence. *Educational Research Review*, 30, 100326.

Zamuda, A., & Brest, J. (2013). Environmental framework to visualise emergent artificial forest ecosystems. *Information Sciences*, 220, 522-540.

Ziatdinov, R., & Valles, J. R. (2022). Synthesis of Modeling, Visualisation, and Programming in GeoGebra as an Effective Approach for Teaching and Learning STEM Topics. *Mathematics*, 10(3), 398.

Ziggah, Y. Y., & Youjian, H. (2013). Determination of GPS Coordinate Tra between Reference Datums-A Case. *International Journal of Engineering Sciences & Research Technology*, 27-35.

Ziggah, Y. Y., Youjian, H., Amans, C. O., & Kumi-Boateng, B. (2012). Regression models for 2-dimensional cartesian coordinates prediction: a case study at University of Mines and Technology (UMaT), Tarkwa-Ghana. *International Journal of Computer Science and Engineering Survey*, 3(6), 61.



**APPENDIX A**

**PRE-TEST**

QUESTION NUMBERS	QUESTIONS
1	Given the function, $f(x)$ , defined by $f(x) = 3x^2 + 2x - 4$ , find $f(x + a)$ .
2	Find the integral of $\int \frac{1}{2}(3x^2 - 2x)dx$
3	Evaluate $\int_1^3 (3x^2 + 2x + 4)dx$
4	Calculate to one decimal place, the area of the finite region enclosed by the curve $P(x) = x^3 - 7x^2 + 15x - 9$ and the lines $y = 0$ , $x = 1$ and $x = 3$ .
5	The gradient function of a curve is given by $3 + 2x - x^2$ . Find the equation of the curve, if $(2, 3)$ lies on it.

6

a. What do you understand by  $F(x) = \int f(x)dx$  ?

b. What do you understand by  $F(x) = \int_a^x f(t)dt$  ?

c. What do you understand by  $\int_a^b f(x)dx = F(b) - F(a)$  ? When do you use this formula? Can you justify how it is derived?

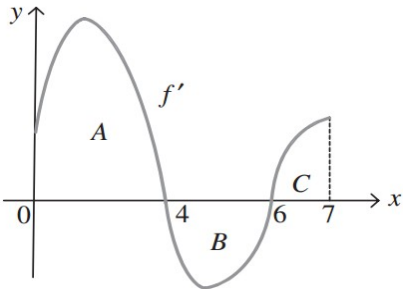
d. What do you understand by  $\frac{d}{dx} \left( \int_a^x f(t)dt \right) = f(x)$  ? When do you use this formula? Can you justify how it is derived?

**APPENDIX B****POST-TEST**

QUESTION NUMBERS	QUESTIONS
1	<p>a. Find the integral of <math>\int(4x^3 + 5x^2 + 4)dx</math></p> <p>b. Find an expression for <math>y</math> if <math>\frac{dy}{dx} = 6x^2 - 4x + 2</math> and sketch the graph of <math>y</math></p>
2	<p>Find, if possible, the area between the curve <math>y = x^2 - 4x</math> and the x-axis from <math>x = 0</math> to <math>x = 5</math></p>
3	<p>Calculate the area enclosed between the curve <math>x = y^2</math> and <math>y = x - 2</math></p>
4	<p>What do you understand by <math>A = \int_b^a [f(x) - g(x)]dx</math> and <math>B = \int_c^d [h(y) - p(y)]dy</math>? Can you justify how these formulas are derived? Can you justify when each one is used?</p>

5

The graph of  $f'(x)$ , the derivative of  $f(x)$ , is sketched below. The area of the regions  $A$ ,  $B$ , and  $C$  are 20, 8, and 5 square units, respectively. Given that  $f(0) = -5$ , find the value of  $f(6)$ .



## APPENDIX C

### PRE-TEST MARKING SCHEME

S/N	SOLUTIONS	MARKS
1.	$f(x + a) = 3(x + a)^2 + 2(x + a) = 4$ $f(x + a) = 3(x^2 + 2xa + a^2) + 2x + 2a - 4$ $f(x + a) = 3x^2 + (6a + 2)x + 3a^2 + 2a - 4$	2 marks
2.	$\int \frac{1}{2}(3x^2 - 2x)dx = \int \frac{3x^2}{2}dx - \int xdx$ $= \frac{3x^3}{2 \times 3} - \frac{x^2}{2} + C = \frac{x^3}{2} - \frac{x^2}{2} + C$	5 marks
3.	$\int_1^3 (3x^2 + 2x + 4)dx = [x^3 + x^2 + 4x]_1^3$ $= [(3)^2 + (3)^2 + 4(3) - ((1)^3 + (1)^2 + 4(1))]$ $= 42$	3 marks
4.	$A = \int_1^3 (x^2 - 7x^2 + 15x - 9)dx$ $= 4 \left[ \frac{x^4}{4} - 7 \frac{x^3}{3} + \frac{15x^2}{2} - 9x \right]_1^3$ $A = 1.3 \text{ units square}$	4 marks
5.	$y = \int f(x)dx$ $y = \int (3 + 2x - x^2)dx$ $y = 3x + x^2 - \frac{x^3}{3} + C$ <p><i>At the point (2,3)</i></p> $3 = 3(2) + (2)^2 - \frac{(2)^3}{3} + C$ $C = -\frac{13}{3}$ $y = -\frac{13}{3} + 3x + x^2 - \frac{x^3}{3}$	6 marks

**APPENDIX D**  
**INTERVIEW GUIDE AND QUESTIONS**

Interview Introduction

I would like to thank you for taking the time to meet with me today.

I would want to hear your thoughts regarding your encounters with utilising Maple to tackle integral calculus problems.

Specifically, as the major components of our overall program evaluation, we are assessing program effectiveness.

The interview should take less than 15 minutes. I will be writing notes during the discussion so that I don't forget any of your remarks.

While I will be writing notes during the meeting, I may not be capable of writing swiftly to capture all your thoughts. Kindly be prepared to talk out so I don't skip your opinions.

Your submissions will indeed be kept strictly anonymous and no data in my study will identifies you as the participant. Keep in mind that you are under no obligation to disclose anything you do not need to, and you have the option to terminate the interview at any moment.

Are there any questions about what I have just explained?

Are you willing to participate in this interview?

## INTERVIEW QUESTIONS

1. Have you ever used any software in studying mathematics? If yes, which software have you ever used?
2. To what extent did participation in using Maple Software in solving integral calculus questions aided or hindered your mathematics learning? Please explain.
3. Will you recommend the use of Maple software in learning mathematics? If yes, why? (What are the benefits of the Maple software) Please elaborate.

Is there anything more you would like to add?

Thank you for your time.

## APPENDIX E

### PERMISSION TO CONDUCT PILOT STUDY

THE HEADMASTER

FIASEMAN SENIOR HIGH SCHOOL

P.O. BOX KF 48

TARKWA, WESTERN REGION

Dear Sir,

#### AUTHORIZATION TO CONDUCT RESEARCH STUDIES IN SCHOOL

Presently, I am pursuing a Master of Science (MSc) in Mathematics Education at the University of Agder, in Norway.

As a requirement for completing my Graduate studies, I am required to carry out a research project with the following working title: "An investigation of Students' Learning of Integral Calculus with Maple Software and Paper-Pencil Strategies in the Western Region of Ghana."

I will need to interview and test students of Form two as research project. Because the research will take place only after school time, it will not interfere with regular classroom instruction. I can absolutely guarantee you that all relevant data discovered during the research will be held in the strictest confidence and will only be utilized for research purpose.

I anticipate your usual assistance.

Yours faithfully,

Samuel Boateng

Sammytuga1985@gmail.com



## APPENDIX F

### PERMISSION TO CONDUCT RESEARCH STUDY

THE HEADMASTER

TARKWA SENIOR HIGH SCHOOL

P.O. BOX 47, TARKWA, WESTERN REGION

Dear Sir,

#### AUTHORIZATION TO CONDUCT RESEARCH STUDIES IN SCHOOL

Presently, I am pursuing a Master of Science (MSc) in Mathematics Education at the University of Agder, in Norway.

As a requirement for completing my Graduate studies, I am required to carry out a research project with the following working title: "An investigation of Students' Learning of Integral Calculus with Maple Software and Paper-Pencil Strategies in the Western Region of Ghana."

I will need to interview and test students of Form two as research project. Because the research will take place only after school time, it will not interfere with regular classroom instruction. I can absolutely guarantee you that all relevant data discovered during the research will be held in the strictest confidence and will only be utilized for research purpose

I anticipate your usual assistance.

Yours faithfully,

Samuel Boateng

Sammytuga1985@gmail.com

## APPENDIX G

THE WESTERN REGIONAL DIRECTOR

GHANA EDUCATION SERVICE

TAKORADI, WESTERN REGION

Sir/Madam

### AUTHORIZATION TO CONDUCT RESEARCH STUDIES IN SCHOOLS

Presently, I am pursuing a Master of Science (MSc) in Mathematics Education at the University of Agder, in Norway. As a requirement for completing my Graduate studies, I am required to carry out a research project with the following working title: "An investigation of Students' Learning of Integral Calculus with Maple Software and Paper-Pencil Strategies in the Western Region of Ghana."

I would like to submit this letter to request the office of the Director to obtain approval to carry out my studies at Tarkwa Senior High School as well as a pilot study at Fiaseman Senior High School.

I can absolutely guarantee you that all relevant data discovered during the research will be held in the strictest confidence and will only be utilized for research purpose

I anticipate your usual assistance.

Yours faithfully,

Samuel Boateng

Sammytuga1985@gmail.com