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Chapter 1
Dynamics, Adaptation and Control for Mental Models: A Cognitive Architecture

Laila van Ments and Jan Treur

Abstract In this chapter, an overview of the wide variety of occurrences of mental models in the literature is discussed. They are classified according to two dimensions obtaining four categories of mental models: static-dynamic and world-mental, where static refers to mental models for static world states or for static mental states and dynamic refers to mental models for world processes or for mental processes. In addition, distinctions are made for what can be done by mental models: they can, for example, be (1) used for internal simulation, they can be (2) adapted, and these processes can be (3) controlled. This leads to a global three-level cognitive architecture covering these three ways of handling mental models. It is discussed that in this cognitive architecture reflection principles play an important role to define the interactions between the different levels.

Keywords Mental model · Cognitive architecture · Dynamics · Adaptation · Control

1.1 Introduction

Mental models are a kind of blueprints or pictures in the mind that can occur in various forms; e.g., Craik (1943), Evans (2006), Furlough and Gillan (2018), Gentner and Stevens (1983), Halford (1993), Johnson-Laird (1983). One relatively simple example is that you perceive the world state in front of you and after closing your eyes you still see a picture of this world state in your mind. Another, more dynamic example is that you perceive an impressive course of events in front of you and after closing your eyes you see a kind of movie replay in your mind that replays this course of events. Although the notions of 'picture' or 'movie' provide an intuitive

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way to imagine what a mental model can be, for the general case such notions should not be taken literally but more in a metaphorical sense. For example, in a wider sense you can imagine a situation that you have never seen. Humans often use some form of mental model, as a blueprint or manual to handle situations. Well-known examples are operating a device or machine or software system, but also how to handle somebody else who needs to be handled based on some special personal 'user manual'. Still other examples are standard patterns learnt to solve certain types of problems in the context of certain disciplines, as so often are learnt at school.

All these examples show the wide variety of possibilities for mental models, usually described as structures consisting of collections or *networks* of certain *relations* that can be of various types. In this chapter this variety will be discussed, analysed and structured in some more detail in such a way that a basis is obtained for a cognitive architecture to handle mental models.

1.2 Mental Models and What They Model

In this section, part of the extensive literature on mental models is discussed and a structured overview is made based on distinguishing whether they consider an external world or an internal mental world and whether they model a static situation or a dynamic process.

1.2.1 Mental Models as Small-Scale Models Within the Head

For the history of the mental models area, often Kenneth Craik is mentioned as a central person. In his book (Craik 1943) he describes a mental model as a *small-scale model* that is carried by an organism within its head as follows:

> If the organism carries a "small-scale model" of external reality and of its own possible actions within its head, it is able to try out various alternatives, conclude which is the best of them, react to future situations before they arise, utilise the knowledge of past events in dealing with the present and future, and in every way to react in a much fuller, safer, and more competent manner to the emergencies which face it. (Craik 1943, p. 61)

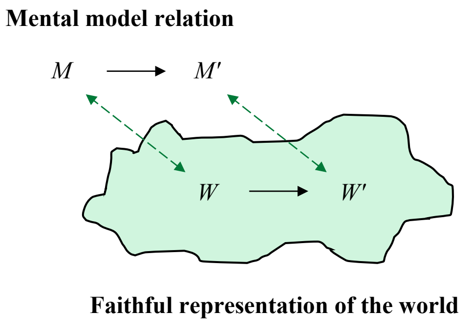
He emphasizes that such internal models use certain types of *relation-structure* that makes the mental model work in a way similar to how the real world works:

> By "relation-structure" I do not mean some obscure non-physical entity which attends the model, but the fact that it is a physical working model which works in the same way as the process it parallels…Thus, the model need not resemble the real object pictorially; Kelvins' tide-predictor, which consists of a number of pulleys on levers, does not resemble a tide in appearance, but it works in the same way in certain essential respects… (Craik 1943, p. 51).

This similarity is depicted in Fig. 1.1, where the relation $M \rightarrow M'$ within the mental model corresponds to a similar relation $W \rightarrow W'$ in the world; by this similarity, the mental model faithfully represents the world.

In his book (Craik 1966) he emphasizes the benefits of being able to model and simulate processes from the world in the brain.

Fig. 1.1 A relation $M \to M'$ in a mental model and its correspondence to a similar relation $W \to W'$ in the world, thus providing a faithful representation of the world



> "...the "modelling", by the brain, of the sequence of events whose consequence is sought, so that this model may predict the answer earlier than it occurs in the course of external nature. In other words, by the ability to model… the brain is able to outrun the physical processes which are too rapid for it and so can, on the average, forestall and anticipate the course of nature. (Craik 1966, p. 27).

For more discussion on Craik's work, see, for example Williams (2018a, b).

Other authors also have formulated what mental models are. For example, with an emphasis on causal relations, Shih and Alessi (1993, p. 157) explain that

> By a mental model we mean a person's understanding of the environment. It can represent different states of the problem and the causal relationships among states.

De Kleer and Brown (1983) describe it in the following way:

- The envisioning of the system, including a topological representation of the system components, the possible states of each of the components, and the structural relations between these components
- The running or execution of the causal model based on basic operational rules and on general scientific principles.

Moreover, after an extensive analysis, Doyle and Ford (1998) formulate the following definition, where the focus is on a dynamic system:

> A mental model of a dynamic system is a relatively enduring and accessible but limited internal conceptual representation of an external system whose structure maintains the perceived structure of that system.

All these descriptions strongly focus on how the world functions based on certain dynamic or temporal causal relations (sometimes called the *dynamic system view*) and that in a mental model similar relations are used to simulate a similar process. This idea of running a simulation inside the head is also called *internal simulation*; e.g., Damasio (1994), Goldman (2006), Hesslow (2002, 2012). For example, in Bhalwankar and Treur (2021a), the functioning of a car in interaction with its driver is internally simulated and in Van Ments and Treur (2021a) addressing PTSD, as a flashback experience a movie based on a mental model of a course of traumatic events is replayed in the brain; see also Treur and Van Ments (2022, Chap. 3), and Treur and Van Ments (2022, Chap. 5), respectively (this volume).

1.2.2 Mental Models for Individual Processes

In principle there are two ways in which a mental model can be considered to describe reasoning: to describe a reasoning state or to describe a reasoning process as a whole.

1.2.2.1 Describing Reasoning States by Mental Models

For example, Gentner and Stevens (1983), Johnson-Laird (1983), Halford (1993) put an emphasis on the use of mental models to reasoning states. Here *reasoning states* are the snapshots of a reasoning process at specific time points (sometimes also called information states or knowledge states). Each of these time-dependent reasoning states is conceptualized by a mental model or by a set of mental models. Such mental models used to describe reasoning states have a slightly different appearance, compared to the mental models according to the perspective discussed above:

- In Sect. 1.2.1 dynamics is described within the mental model: the mental model represents the dynamics by relations defining a dynamical system
- In the current section the dynamics is not represented within a mental model: in contrast, a reasoning step is described as a transition step for mental models by which every step a new mental model is created.

So, based on this perspective where reasoning states are mental models, *reasoning steps* are considered transitions (maybe by standard generic inference rules, maybe based on other things) of one reasoning state to another one. For example, one reasoning state is described by a mental model

$$a$$
$$a \rightarrow b \qquad \text{where } \rightarrow \text{ denotes logical implication}$$

including the two items a and $a \rightarrow b$ both describing a static world situation and by a reasoning step (based on modus ponens in this case), this mental model is transformed into a new mental model

$$a$$
$$b$$
$$a \rightarrow b$$

including the three items a, b and $a \rightarrow b$ all describing the same static world situation. Thus reasoning steps are conceptualised as adaptations of the mental models representing these reasoning states, which can also have the form that two or more mental models are used (as antecedents) in combination. In a reasoning process as a whole, these transitions are executed in succession, resulting in sequences of reasoning states (also called *reasoning traces*). These are then conceptualised by sequences of mental models over time. See Johnson-Laird (2004) for more history of this perspective. This view on dynamics of reasoning also has been addressed within AI for different types of reasoning from a formal logical and computational perspective; for example, see Brazier et al. (1999), Gavrila and Treur (1994), Jonker and Treur (2002), Jonker and Treur (2003), Meyer and Treur (2001), Treur (1994).

1.2.2.2 Describing a Reasoning Process by One Mental Model

A different perspective on conceptualising reasoning processes can be obtained from the dynamical system view, more closely related to the view discussed in Sect. 1.2.1. From this view not a reasoning state like above, but instead a reasoning *process* is described as one mental model by temporal (or causal) relations relating one reasoning state to another one. In other words, in this case a mental model describes the reasoning process by temporal relations defining a dynamical system, similar to how world processes can be modeled by a dynamical system mental model as discussed in Sect. 1.2.1. So, this time reasoning steps from one reasoning state to another one are not transitions between different mental models but are described by temporal relations within one fixed overall mental model. For example, such a mental model for a reasoning process can be described by relations of the form

$$a \rightarrow b \qquad \text{(where this time } \rightarrow \text{ is interpreted as a temporal casual relation)}$$

expressing that within a reasoning process knowing *a* will causally affect knowing *b*, like how, for example, in philosophy of mind, in general mental states are assumed to causally affect each other over time; e.g., Kim (1996). This view on reasoning provides a description of reasoning like any other mental process as a dynamical system as described in Sect. 1.2.1. This perspective on reasoning is considered within literature in AI such as Engelfriet and Treur (1994), Engelfriet and Treur (1995), Meyer and Treur (2001), where a 'temporal theory of reasoning' plays the role of a mental model according to a dynamical system view.

So, in summary, in principle there are two different ways to model reasoning by mental models: (1) mental models model the reasoning states (as snapshots in a reasoning process), and reasoning steps are adaptations or transitions of these mental models over time, and (2) mental models model the dynamics of the reasoning process by modeling reasoning steps as temporal relations within one overall mental model for the entire reasoning process.

1.2.2.3 Mental Models Used to Model Cognitive Metaphors

Cognitive metaphors can also be considered a type of mental model (Cardillo et al. 2012; Carroll and Thomas 1982; Kuang 2003; Leary 1994; Ponterotto 2000; Romero and Soria 2005). Cognitive metaphors are a way to explain our conceptualization and mapping of new concepts on existing knowledge, and how we communicate this to others (Lakoff and Johnson 2003).

Or, in other words, one mental domain is understood in terms of a mental model of another phenomenon. Imagine encountering a novel animal: we will immediately compare its behaviors and looks to the bank of animals that exists in our brain, and try to map an understanding of this new animal, based on the knowledge we already have in our brain.

Another way to explain cognitive metaphors is as analogy making a mapping between a source and a target inside the brain (Gentner 1983; Gentner and Stevens 1983; Vosniadou and Ortony 1989), based on features the source and domain have in common. For example, we often hear the catchphrase 'Love is a Journey'. Of course, literally, love is not a journey, but rather an abstract concept. However, because of the complexity of love as a concept, we use more concrete concepts, like a journey, to understand and communicate our understanding of love. By using the journey metaphor, we unconsciously map concepts like roadblocks and the fact that a journey is something to embark on onto our concept of love, and thus come closer to a (mutual) understanding. Lakoff (1993) stresses that metaphors are an essential mechanism that is systematically mapped in our brain for humans to understand the world, and be able to think and reason, without us even noticing. Furthermore, bodily changes can unconsciously affect our metaphorical thoughts, see Barsalou (2008), Landau et al. (2010), Williams et al. (2009). Even more so, our mental models can be influenced by the metaphors we use, as constant repetition of particular metaphors will lead to our unconscious acceptance of that particular metaphor as a normal way of seeing that situation El Refaie (2003). Thereby, a metaphor subconsciously constructs how we perceive situations; see Barsalou (2008), Landau et al. (2010), Williams et al. (2009). Several studies have shown that our actions are subconsciously influenced by the automated activation of motives (Bargh et al. 2001; Bargh and Morsella 2008). Therefore, through this route cognitive metaphors also affect the way humans make decisions. As an example, in (Van Ments and Treur 2021b) metaphors for cooperative and competitive joint decision making are modeled as a second-order adaptive mental model; see also (Treur and Van Ments 2022, Chap. 10) (this volume).

1.2.3 Mental Models in Social Processes

In this section the focus is on mental models used in a social context. These can concern mental models for bonding and attachment in dyadic relationships or mental models for groups, teams or organisations. A well-known social type of mental model occurs when one has some 'image' of the mental state of another person, or of oneself. If the dynamics of mental processes are also considered, one can, for example, have a mental model of how your partner will get angry or disappointed after you undertake some specific action. Also more in general in social life, humans often use some mental model to understand each other and interact in an adequate manner based on that mental model, for example, to get something done. And the same even applies to having a user manual for handling oneself. In this section, in particular mental models for attachment in dyadic relationships are briefly discussed, and mental God-models, mental models for bonding by homophily, and team mental models.

1.2.3.1 Mental Models for Attachment

The way an individual forms relationships with others can be explained by the *Attachment Theory*, constructed by Ainsworth and Bowlby. This theory is based on its predecessor, the 'Security Theory', developed by William Blatz and Mary Salter Ainsworth; e.g., Blatz (1966), Salter (1940), Salter Ainsworth (2010), Salter Ainsworth and Bowlby (1965). The attachment theory explains how a child develops a set of emotions, memories, thoughts, expectations, behaviours and beliefs about itself and others, based on its early experiences with its primary caregiver. This set is called the 'internal working model of social relationships', which continues to change with age and experience (Mercer 2006). More specifically, this 'model of self' and 'model of other' that the child initially develops, is based on experiences with the primary caregiver and their behaviour (Bretherton 1992). Using their internal 'model of other' children can predict the primary caregiver's behaviour, and using their internal 'model of self', they can plan their own behaviour accordingly (Bretherton 1992), and the same happens later in life in interaction with significant others.

In Hermans et al. (2021) a second-order adaptive network model is presented for development of mental models of self and others according to Attachment Theory; see also (Treur and Van Ments 2022, Chap. 12) (this volume).

1.2.3.2 Mental God-Models

Another interesting place where we can find place we can find is double mental models is a person's relationship with God. When a person prays, the same brain regions that are used for interactions with other people become activated, enabling a person to generate an internal representation of 'the other', in this case the image they have of God. This allows people to form a real, meaningful relationship with God, and to construct a mental model of an image of God (Schjoedt et al. 2009). This mental God-model that an individual has of God, and how this image has impact on the individual, can involve many aspects. For example, the attachment style discussed in the previous section can be studied in combination with a person's God-model, and how these two influence each other (Granqvist and Kirkpatrick 2008). The relationship and mental image of God can also be explained from a mental model or mentalizing perspective, as introduced by Schaap-Jonker and Corveleyn (2014). Mentalizing is the capacity of thinking about thinking and feeling. It provides awareness that one's own and others' behaviour is driven by mental states, and gives the ability to selectively activate internal states that fit the individual's particular. Mentalizing also involves a process of internal simulation, where an individual internally simulates mind states to predict effects in the external world or other persons. In other words, a mental model is an interesting way to describe an individual's relationship with God.

In Van Ments et al. (2022) an adaptive network model for developing and using a mental God-model is described.

1.2.3.3 Mental Models for Bonding Based on Homophily

Social networks often are adaptive, for example based on a bonding by a homophily principle for the adaptation of the weights of the network connections between persons over time. A bonding by homophily adaptation principle expresses how 'being alike' strengthens the connection between two persons, also explained as 'birds of a feather flock together'; e.g., McPherson et al. (2001). Usually, in literature such adaptation processes are considered without taking into account subjective elements for the persons involved. For example, do the persons themselves actually know in how far they are alike? Or are they just will-less victims of objective social laws independent of what they know or what they want? Such subjective aspects are often lacking in (computational) research on bonding by homophily, as usually these processes are addressed exclusively from the perspective of an assumed objective social world. However, a more realistic bonding by homophily principle can be obtained if the bonding is not assumed to be based on an objective form of homophily but on the mental models both persons have of each other. If two persons both have a mental model of themselves and the other that show that they are alike, then that will clearly affect their bonding, even if these mental models are not correct and, for example, based on fake information. This subjective mental model based perspective on bonding by homophily is addressed in Treur (2021b); see also Treur and Van Ments (2022, Chap. 13), (this volume), which also includes an example scenario where one of the persons on purpose fakes incorrect personal characteristics or properties in order to make bonding happen.

1.2.3.4 Team Mental Models

A team mental model is based on the assumption that high performing teams need to have team members that are on the same page in order to perform complex tasks well; e.g., Burtscher and Manser (2012), Langan-Fox et al. (2000), Mohammed et al. (2010). This requires team members to have a shared understanding of the relevant elements to perform a specific task. A team mental model is an emergent team level concept which is generated by each team member's cognition up to the level that it becomes a shared mental model: so, the origin and basis of a team mental model is formed by the individual team members. More specifically, the team mental model itself is an emerging collective phenomenon which is created bottom-up from each team member's cognition in a dynamic manner (DeChurch and Mesmer-Magnus 2010a, b). The main functions of team mental models are improved planning, coordination and alignment (Nini 2019). Two types of team mental models are distinguished (Mohammed et al. 2010):

- task-related team mental models
- team-related team mental models.

The first type provides a team's cognitive representation of task-related elements such as goals and subtasks, subtask dependencies, subtask durations, milestones, and resources required for task coordination. The second type covers the team's mental model for the knowledge, skills, competencies and relationships of team members.

In Van Ments et al. (2021) an example of an adaptive network model for handling a team mental model in a medical context is presented; see also Treur and Van Ments (2022, Chap 14) (this volume).

1.2.4 A Mental Models Overview According to Mental Versus World and Static Versus Dynamic

In the above Sects. 1.2.1–1.2.3, mental models have been described as consisting of a collection or network of relations. In some cases these relations describe static relationships for a world situation or state (such as 'Joe is taller than Kamala') or for a mental state (such as 'Joe does not believe in complot theories'). In other cases, these relations describe temporal or causal relationships according to a dynamic system view of a world process (e.g., 'human action causes climate change') or a reasoning process (e.g., 'because I believe I have no time left, I now decide to do this action'). In all such examples, that can be represented by a mental model, two dimensions of variation can be recognized. The first dimension is the dimension *static-dynamic*, where static refers to representing static situations, and dynamic to representing a process. The second dimension is the dimension *world-mental* where world refers to the external world and mental to mental states or processes. Distinctions according to these dimensions have been used in Table 1.1 to get a structured overview of the options.

Note that this table is not the end of the story, as several important aspects that occur in relation to mental models are not covered yet. As mental models are usually described as networks of certain types of relations, one characteristic of mental models that also varies is which types of relation are used exactly. Causal relations are often used, especially from a dynamic system view, but also other types of relations often occur in mental models. In addition, also relations of higher-order, as used among others, in analogical reasoning have not been distinguished yet. Moreover, the adaptation of mental models as takes place in learning or development still has to be addressed, and the same holds for the control over such adaptation. These topics will be addressed in next sections.

Table 1.1 The variety of mental models structured for what is modeled according to state vs process and world versus mental; this provides a summary of the concepts discussed in more detail in the text of Sect. 1.2

State versus process	World versus mental	Example mental models
Process description	World process	• A mental model of a dynamical system for world dynamics • A mental model of a how the water level changes with tide • A mental model of how the climate of the earth changes due to human action • A step-by-step description of a route to follow to get from A to B in a city; e.g., 'when you reach the cinema on your right hand, turn left and get into that street to the supermarket'
	Mental process	• A mental model of how your partner will get angry or disappointed after you undertake a specific action • A mental model of how you yourself will get angry or disappointed after your partner undertakes a specific action • A step-by-step algorithm to calculate the area of a rectangle or a long division
State description	World state	• A mental model of a city in the form of a map; e.g., 'the supermarket is in the street opposite the cinema' • A mental model of the current climate in different regions • A mental model of a rectangle
	Mental state	• A mental model of beliefs someone else has on the world • A mental model of desires or goals someone else has • A mental model of the emotions someone else has • A mental model of the knowledge and skills someone else has • A mental model of any of the above for yourself instead of 'someone else'

1.3 Learning and Development of Mental Models

Within educational science, mental models are often considered an important vehicle for learning; for just a few of the many contributions, see Benbassat (2014), Buckley (2000), Doll et al. (2012), Du Plooy (2016), Greca and Moreira (2000), Halloun (1996), Hurley (2008), Koedinger and Terao (2002), Larbi and Mavis (2016), Mayer (1989), Seel (2006), Skemp (1971), Van Gog et al. (2009), Yi and Davis (2003). The focus in this area is usually on how mental models can be formed (learnt) and adapted over time. In this section this perspective from educational science is discussed in some detail.

1.3.1 Learning and Development as Adaptation of Mental Models

Within educational science, sometimes the term *model-based learning* is used for learning described as constructing coherent mental models; for example, Buckley (2000) formulates this as:

> Model-based learning is a dynamic, recursive process of learning by building mental models. It incorporates the formation, testing, and subsequent reinforcement, revision, or rejection of mental models of some phenomenon.

However, note that in most cases that mental models are considered for learning, the term model-based learning is not explicitly used. This view on learning was also described by Piaget. Although he did not use the term mental model, the ideas he put forward do apply to mental models. Within the literature also the term schema or schemata is often used; this concept has no sharp boundary with the concept mental model and both concepts have much in common. Following the ideas of Piaget (1936, 1954) , formation and adaptation of mental models during learning or development can occur in two forms: by *assimilation* (extension or refinement of a mental model) or by *accommodation* (revision of a mental model). As an example, suppose that a mental model includes the relation.

$$\text{need something} \rightarrow \text{go to shopping area}$$

By assimilation, this can be refined into a mental model including the following relations:

$$\begin{aligned}&\text{need something} \rightarrow \text{go to shopping area}\\&\text{need book} \rightarrow \text{need something}\\&\text{need book \& in shopping area} \rightarrow \text{look for book shop}\end{aligned}$$

This is a refinement and not a revision, as the previous relation still applies. In contrast, accommodation takes place, for example, when due to a lockdown the shops are closed for a long time. Then the mental model including.

$$\text{need something} \rightarrow \text{go to shopping area}$$

can be revised into a mental model including.

$$\text{need something} \rightarrow \text{go to web shop}$$

This is indeed a revision and not a refinement as the previous relation does not apply anymore. Such types of examples illustrate how mental models can change over time due to learning or development, as also described by the quote above from Buckley (2000). Next, some elements of learning processes are addressed in more detail and the importance of control over the learning is discussed.

1.3.2 Learning of Mental Models by Observation and by Instruction

Observational learning indicates when observation is important for the learning or development of a mental model. This can be observation of others but also observation of oneself while 'learning by doing' or 'learning by discovery'. Learners may see someone perform a t type of behavior and then start to imitate it; e.g., Benbassat (2014), Yi and Davis (2003). This is often used to make others learn a specific motor task. A mechanism based on mirror neurons underly the ability to learn by observing and imitating others; e.g., Hurley (2008), Rizzolatti and Craighero (2004) Van Gog et al. (2009). An example of an adaptive network model for learning by observation a mental model of how a car works and how to drive it can be found in Bhalwankar and Treur (2021a); see also Treur and Van Ments (2022, Chap. 3) (this volume). Another example showing how a mental model is learned by counterfactual thinking and observation can be found in Bhalwankar and Treur (2021c); see also Treur and Van Ments (2022, Chap. 6) (this volume).

Instructional learning describes how information provided by an expert instructor can be an important source for the learning. Only learning based on observation often may lead to processes of trial and error; e.g., Seel (2006). Instructions from an expert are a useful addition to develop mental models in an effective manner. A format of scaffolded model-based learning in which many supporting actions such as prompts, questions, hints, stories, conceptual models, visualizations are performed, facilitates a learner's progress; e.g., Hogan and Pressley (1997). An example of an adaptive network model for learning by instruction a mental model of how a car works and how to drive it can be found in Bhalwankar and Treur (2021a); see also Treur and Van Ments (2022, Chap. 3) (this volume).

1.3.3 Control for Learning of Mental Models Based on Metacognition

To handle mental models and in particular the learning of them, *control* is important; e.g., Gibbons and Gray (2002) claim that instructions are most effective for learning processes when the learner controls them. The already mentioned scaffolded model-based learning format in the previous section supports this (Hogan and Pressley

1997). As another example, Kozma (1991) claims that persons actively pick external sources for mental model learning. So, the learner's initiatives for instruction and information acquisition are important for mental model learning. The learner has (to be able) to be proactive and in control of the learning. As yet another example, Meela, and Yuenyong (2019) have shown that Model-Based Inquiry (MBI) can support a student's mental model formation in scientific learning; see also Neilson et al. (2010).

An example of an adaptive network model for controlled learning of a mental model of how a car works and how to drive it can be found in Bhalwankar and Treur (2021b); see also Treur and Van Ments (2022, Chap. 9).

Metacognition is described in Darling-Hammond et al. (2008), Shannon (2008), Mahdavi (2014), Flavell (1979), Koriat (2007), Pintrich (2000) as cognition about cognition. More specifically, Koriat (2007) presents it as what people know about their own cognitive processes and how they put that knowledge to use in regulating their cognitive processing and behavior. Sometimes the term self-regulation and self-regulated learning are used. In Pintrich (2000), this is formulated as an active, constructive process whereby learners set goals for their learning and then monitor, regulate, and control their cognition, motivation, and behavior, guided by these goals.

Also in learning complex tasks using mental models, control is a crucial element; see Treur (2021c) for an example network model for this; see also Treur and Van Ments (2022, Chap. 7) (this volume).

In learning, often different mental models play a role; e.g., Gentner and Stevens (1983), Greca and Moreira (2000), Skemp (1971), Seel (2006). An example can be the learning of subtracting numbers. The learner can use a more visual model, drawing out the numbers on a line, or a more abstract model, using formulas to represent the subtraction e.g., Bruner (1966), Du Plooy (2016). Here, metacognition plays an important role for the decisions about when to *switch* from one mental model to another one. In Treur (2021a) more can be found on this case, particularly for learning arithmetic or algebraic skills in primary or secondary schools supported by visualisation; see also Treur and Van Ments (2022, Chap. 4) (this volume).

1.4 A Cognitive Architecture for Mental Models

In this section several aspects of mental models are discussed that are important to obtain a cognitive architecture to handle mental models. In particular, the following aspects are addressed:

- higher-order relations in mental models
- adaptation of mental models
- control of adaptation of mental models.

Finally, it will be pointed out how an overall cognitive architecture can be designed covering these aspects.

1.4.1 Higher-Order Relations

Higher-order relations are relations between relations. In Fig. 1.2 an example is depicted of a first-order relation R and a second-order relation T. In this example, this second-order relation T expresses that the first-order relation R is transitive. Below the dashed purple line, a first-order mental model is depicted based on relation R. Above this dashed purple line a second-order mental model is depicted based on transitivity relation T.

In the first-order self-model, the relation R is a relation for objects *X*, *Y* and *Z,* where, for example, R denotes the relation 'is taller than'. Linguistically or logically, such a first-order relation can also be expressed as $X{:}Y$ or $X{:}_{\mathrm{R}}Y$ or $X\ \mathrm{R}\ Y$ or $\mathrm{R}(X, Y)$. In the second-order mental model, the relation T is also between certain objects, but this time the objects are indicated by terms r(*X,Y*), r(*Y,Z*), and r(*X,Z*) which are names for the relation instances $X \xrightarrow{\mathrm{R}} Y$, $Y \xrightarrow{\mathrm{R}} Z$, and $X \xrightarrow{\mathrm{R}} Z$, respectively, one level lower. These objects can be considered reifications of the relation instances represented in the first-order mental model: they are now represented by objects like r(*X*, *Y*) that refer to relational expression R(*X, Y*); e.g., see Galton (2006). This is similar to, for example, how Gödel used a representation of logical statements by natural numbers to obtain his famous incompleteness theorems for mathematical logic; e.g., see Hofstadter (1979), Nagel and Newman (1965), Smorynski (1977). The upward and downward relations between the two levels can be described by so-called *reflection principles*; see also Sect. 1.4.3 below and Treur (1991, 1994), Weyhrauch (1980).

Another example of a second-order relation in a slightly different notation is the relation *A*:*B*::*C*:*D* where the symbol : denotes the first-order relation and the symbol :: denotes a second-order relation between the two first-order relational expressions *A*:*B* and *C*:*D*. This is often used in experiments concerning analogical inference as also discussed in Sect. 1.4.2; e.g., Alfred et al. (2020), Holyoak and Monti (2020), Whitaker et al. (2018). For such a second-order relation *A*:*B*::*C*:*D*, a picture similar

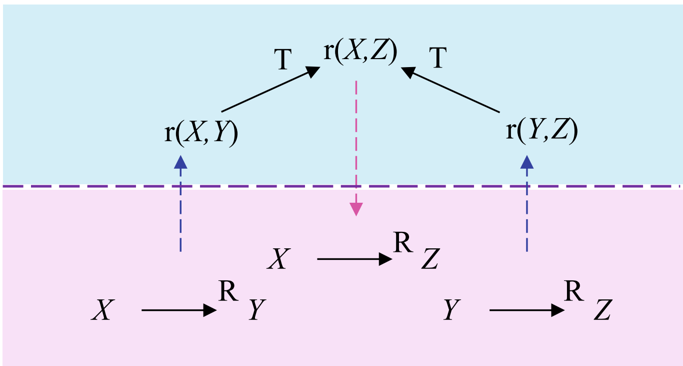


Fig. 1.2 Second-order relation *T* expressing transitivity of first-order relation *R*

to Fig. 1.2 can be drawn. In principle, also third- and higher-order relations may be possible; the use of third-order relations for control is discussed in Sect. 1.4.3.

The above shows that in addition to the distinctions made in Table 1.1, also a distinction between mental models according to the orders of the relations they use can be made, where in one mental process multiple mental models of different orders may be used in an integrative manner.

1.4.2 What Exactly Do Mental Models Do?

Next, distinctions are made for what mental models actually do. In different sections, different types of processes were encountered that in one way or the other relate to mental models. The following overview of these processes can be made.

- **Simulation: Mental Models Simulate**
  As discussed in Sects. 1.2 and 1.4, mental models are often used for a form of inferencing or internal or mental simulation to relate known facts to unknown facts about world or mental states or processes. This occurs in many forms, varying from prediction, visualisation in sport, flashback movies in PTSD, dreaming, reasoning and many more cases.
- **Adaptation: Mental Models Adapt**
  Mental models often are adapted; they can be formed or learned and they can be revised, as Piaget (1936, 1954) already pointed out. This has been discussed in some detail in Sect. 1.3, thereby addressing observational learning and instructional learning in particular.
- **Control: Mental Models Respond to Control**
  Using mental models and adapting them is in principle done in a coordinated manner by some form of control by a form of metacognition. This also has been discussed in some detail in Sect. 1.3 in particular for the timing of observational learning and instructional learning.

This shows that in addition to the distinctions made in Table 1.1 and in Sect. 1.4.1, also distinctions have to be made between what mental models actually do, where in one mental process often multiple mental models of different levels will be used in interaction with each other. In Sect. 1.4.3, it is pointed out how a cognitive architecture for this may be obtained.

1.4.3 A Cognitive Architecture for Handling Mental Models

Based on the different processes in which mental models are used as summarised in Sects. 1.4.1 and 1.4.2, it can be assumed that a cognitive architecture for handling

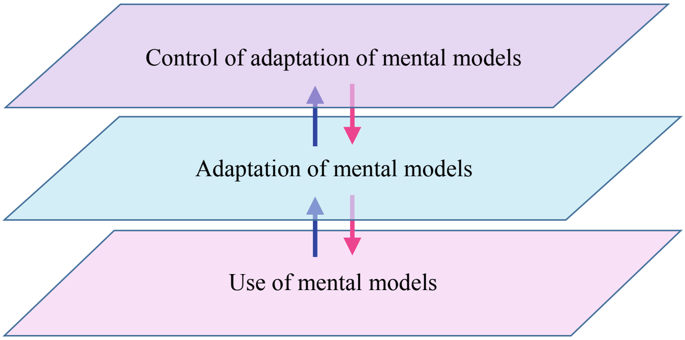


Fig. 1.3 Cognitive architecture for mental model handling with three levels of mental processing for mental models where each next level is modeled by relations one order higher than at the level below it

mental models has to cover the following three types of processes in an integrated manner (see also Fig. 1.3):

Level 1: Use (Base Level)
This level covers mental models described by relations that can be used to generate internal simulation.

Level 2: Adaptivity (First-Order Adaptation Level)
This level covers adaptation of Level 1 mental models by learning, revision, or other change; this can be described by a mental model using relations for changing the relations of the mental models at Level 1. In principle, this will involve a mental model with relations of one order higher than the relations used at Level 1.

Level 3: Control (Second-Order Adaptation Level)
This level covers control of adaptation processes described by a mental model using relations for changing the relations used at Level 2 for change of the Level 1 mental models. In principle, this will involve relations of one order higher than the relations used at Level 2, and two orders higher than the relations used at Level 1.

Here the second and third level are higher-order levels (involving higher-order relations; see Sect. 1.4.1) compared to the first level. This architecture was inspired by literature on metalevel architectures and reflection such as Bowen and Kowalski (1982), Bowen (1985), Galton (2006), Sterling and Beer (1989), Treur (1991), Treur (1994), Weyhrauch (1980). To illustrate the levels in Fig. 1.3 and their relations by an abstract mini-example, assume at the three levels 1 to 3 relations R, S and T (denoted by $\xrightarrow{R}$, $\xrightarrow{S}$, $\xrightarrow{T}$, respectively) are used as shown in Table 1.2 (columns 2–4) and Fig. 1.4; here *V, W*, *X* (column 5) model some contextual or situational factors. These relations may be causal relations, but they can also be of any other type of relation. An important notion to describe the interaction between the different levels of such an architecture is the notion of *reflection principle* (Treur 1991, 1994; Weyhrauch

Table 1.2 Overview of the mini-example for the three levels

Level	Relations	Relation instances	Object terms	Context	Reflection principles
3	T	$V \xrightarrow{\text{T}} \text{s}(W, \text{r}(X, Y))$	V, $\text{s}(W, \text{r}(X, Y))$	V	$\text{s}(W, \text{r}(X, Y))$ ↑↓
2	S	$W \xrightarrow{\text{S}} r(X, Y)$	W, $\text{r}(X, Y)$	W	
1	R	$X \xrightarrow{\text{R}} Y$	X, Y	X	$\text{r}(X, Y)$ $W \xrightarrow{\text{S}} r(X, Y)$ ↑↓ $X \xrightarrow{\text{R}} Y$

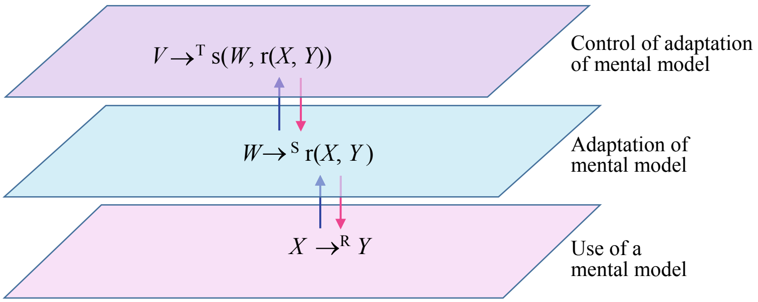


Fig. 1.4 The mini-example within the cognitive architecture

1980); this type of principle (see also column 6 in Table 1.2) will also be explained below by the mini-example

The explanation of this mini-example is as follows. At the base level the mental model includes an instance of relation R from X to Y, represented as

$$X \xrightarrow{\text{R}} Y$$

This relation R is usually called a first-order relation. By an upward reflection principle from level 1 to level 2, at the second level (for adaptation) this R-relation instance relates to an object denoted by the term

$$\text{r}(X, Y)$$

referring to relation $X \xrightarrow{\text{R}} Y$; so, $\text{r}(X, Y)$ is a name to refer to relation instance $X \xrightarrow{\text{R}} Y$ (alternatively, sometimes the notation '$X \xrightarrow{\text{R}} Y$' is used for such a name). For this object at level 2, in turn an instance of relation S applies that relates the object $\text{r}(X, Y)$ to context factor W:

$$W \xrightarrow{\text{S}} \text{r}(X, Y)$$

This relation S is usually called a second-order relation. By a downward reflection principle from level 2 to level 1, this makes first-order relation R adaptive, as via the relation $W \xrightarrow{\mathrm{S}} \mathrm{r}(X, Y)$ the object $\mathrm{r}(X, Y)$ representing $X \xrightarrow{\mathrm{R}} Y$ depends on circumstances modeled by context factor W and by the downward reflection principle, this affects the relation instance $X \xrightarrow{\mathrm{R}} Y$ at level 1 accordingly. Note that for this cognitive architecture, this is called first-order *adaptation*, as it concerns adaptation of the first-order relation. But note that the term used in the literature for the relation S involved is *second-order relation.*

However, also the second-order relation S is adaptive, because similarly by an upward reflection principle from level 2 to level 3 it relates to an object denoted by the term

$$\mathrm{s}(W, \mathrm{r}(X, Y))$$

at level 3 referring to relation $W \xrightarrow{\mathrm{S}} \mathrm{r}(X, Y)$, and this object also depends on circumstances (modeled by context factor V), as at level 3 a third-order relation T is applied:

$$V \xrightarrow{\mathrm{T}} \mathrm{s}(W, \mathrm{r}(X, Y))$$

Therefore, $\mathrm{s}(W, \mathrm{r}(X, Y))$ depends on context factor V and by a downward reflection principle from level 3 to level 2, this affects S-relation instance $W \xrightarrow{\mathrm{S}} \mathrm{r}(X, Y)$ at level 2 accordingly. As second-order relation S models the first-order adaptation of first-order relation R, by this control over the first-order adaptation can be exerted. In summary, second-order relation S models adaptation of first-order relation R using context factor W, whereas third-order relation T models control of this adaptation using context factor V. Note that for this cognitive architecture, this is called *second-order adaptation*, as it concerns adaptation of the second-order relation. But the term used in the literature for the relation T involved is *third-order relation.*

This simple example illustrates how the adaptation of a mental model and its control can be modeled, and it points out how reflection principles can connect the levels and enable the transfer between the levels.

This structure of three levels for handling mental models can be used in conjunction with the structure of Table 1.1 in Sect. 1.2 to obtain an overview of the many possible occurrences and uses of mental models. Note that due to the relationships between the different levels explained above where objects at each higher level refer to relations at the next lower level, the higher levels can be interpreted as *self-models* of part of the architecture itself, namely self-models of the part at the next lower level. In this sense it can be considered a *self-modeling architecture.* In Treur and Van Ments (2022, Chap. 2) (this volume), using the notion of multi-level *self-modeling network* (also called reified network), it is described in more detail how this cognitive architecture with three description levels indeed can be modeled based on a three-level self-modeling network model. Moreover, in Treur and Van

Ments (2022, Chap. 21) (this volume) a more in depth analysis is presented on what the self-modeling network format can offer for modeling the cognitive architecture introduced here and its applications.

1.5 Discussion

In this chapter, an overview of the wide variety of occurrences of mental models in the literature was discussed. They were classified according to two dimensions obtaining four categories of them: static-dynamic and world-mental, where static refers to mental models for static world states or for static mental states and dynamic refers to mental models for world processes or for mental processes. In addition, distinctions were made for what can be done by mental models: they can, for example, be (1) used for internal simulation, they can be (2) adapted, and these processes can be (3) controlled. This has led to a global three-level cognitive architecture covering these three ways of handling mental models. It has been pointed out that in this cognitive architecture reflection principles play an important role to define the interactions between the different levels. In Treur and Van Ments (2022, Chap. 2), the notion of self-modeling network is used to work this architecture out in more detail based on the self-modeling network modeling approach described in Treur (2020). For this modeling approach, further details on design using the modeling environment can be found in Treur and Van Ments (2022, Chap. 17) (this volume), on verification by analysis of stationary points and analysis in Treur and Van Ments (2022, Chap. 18), validation using parameter tuning in Treur and Van Ments (2022, Chap. 19), and the scope of applicability in Treur and Van Ments (2022, Chap. 20). Note that in many cases the three-level cognitive architecture described in the current chapter is sufficient. However, sometimes a model with more than three levels fits better, as, for example, shown in Treur and Van Ments (2022, Chap. 8).

Some more philosophically focused background for mental models and their modeling can be found in Treur (2021d) about neural correlates for mental models and (Treur 2021e) about the emerging informational content of mental models; see also Treur and Van Ments (2022, Chap. 15), and Treur and Van Ments (2022, Chap 16), respectively.

References

Alfred, K.L., Connolly, A.C., Cetron, J.S., Kraemer, D.J.M.: Mental models use common neural spatial structure for spatial and abstract content. Commun. Biol. **3**, 17 (2020)

Barsalou, L.W.: Grounded cognition. Annu. Rev. Psychol. **59**(1), 617–645 (2008)

Bargh, J.A., Gollwitzer, P.M., Lee-Chai, A., Barndollar, K., Trötschel, R.: The automated will: nonconscious activation and pursuit of behavioral goals. J. Pers. Soc. Psychol. **81**(6), 1014–1027 (2001)

Bargh, J.A., Morsella, E.: The Unconscious mind. Perspect. Psychol. Sci. **3**(1), 73–79 (2008)

Benbassat, J.: Role modeling in medical education: the importance of a reflective imitation. Acad. Med. **89**(4), 550–554 (2014)
Bhalwankar, R., Treur, J.: Modeling the development of internal mental models by an adaptive network model. In: Proceedings of the 11th Annual International Conference on Brain-Inspired Cognitive Architectures for AI, BICA*AI'20. Procedia Computer Science, Elsevier, vol. 190, issue 4, pp. 90–101 (2021a)
Bhalwankar, R., Treur, J.: A second-order adaptive network model for learner-controlled mental model learning processes. In: Proceedings of the 9th International Conference on Complex Networks and their Applications, vol. 2. Studies in Computational Intelligence, vol. 944, pp. 245–259. Springer, Switzerland AG (2021b)
Bhalwankar, R., Treur, J.: If only i would have done that...: A controlled adaptive network model for learning by counterfactual thinking. In: Proceedings of the 17th International Conference on Artificial Intelligence Applications and Innovations, AIAI'21. Advances in Information and Communication Technology, vol. 627, pp. 3–16. Springer (2021c)
Blatz, W.E.: Human Security: Some Reflections. University of Toronto Press, Toronto, Canada (1966)
Bowen, K.A., Kowalski, R.: Amalgamating language and meta-language in logic programming. In: Clark, K., Tarnlund, S. (eds.) Logic Programming, pp. 153–172. Academic Press, New York (1982)
Bowen, K.A.: Meta-level programming and knowledge representation. N. Gener. Comput. **3**, 359–383 (1985)
Brazier, F.M.T., Treur, J., Wijngaards, N.J.E., Willems, M.: Temporal semantics of compositional task models and problem solving methods. Data Knowl. Eng. **29**(1), 17–42 (1999)
Buckley, B.C.: Interactive multimedia and model-based learning in biology. Int. J. Sci. Educ. **22**(9), 895–935 (2000)
Bretherton, I.: The origins of attachment theory: John Bowlby and Mary Ainsworth. Dev. Psychol. **28**, 759–775 (1992)
Bruner, J.S.: Towards a Theory of Instruction. Harvard University, Cambridge, Mass (1966)
Burtscher, M.J., Manser, T.: Team mental models and their potential to improve teamwork and safety. A review and implications for future research in healthcare. Safety Sci. **50**(5), 1344–1354 (2012). https://doi.org/10.1016/j.ssci.2011.12.033
Cardillo, E.R., Watson, C.E., Schmidt, G.L., Kranjec, A., Chatterjee, A.: From novel to familiar: tuning the brain for metaphors. Neuroimage **59**(4), 3212–3221 (2012)
Carroll, J.M., Thomas, J.C.: Metaphor and the cognitive representation of computing systems. IEEE Trans. Syst. Man Cybern. **12**(2), 107–116 (1982)
Craik, K.J.W.: The Nature of Explanation. University Press, Cambridge, MA (1943)
Craik, K.J.W.: In: Sherwood, S.L (ed.) The Nature of Psychology. Cambridge University Press, Cambridge (1966)
Damasio, A.R.: Descartes Error: Emotion, Reason and the Human Brain. Vintage Books, London (1994)
Darling-Hammond, L., Austin, K., Cheung, M., Martin, D.: Thinking about thinking: metacognition. In: The Learning Classroom: Theory into Practice, pp. 157–172. Stanford University School of Education (2008)
DeChurch, L.A.; Mesmer-Magnus, J.R.: Measuring shared team mental models. A meta-analysis. Group Dyn.: Theory Res. Practice **14**(1), 1–14 (2010a). https://doi.org/10.1037/a0017455
DeChurch, L.A.; Mesmer-Magnus, J.R.: The cognitive underpinnings of effective teamwork. A meta-analysis. J. Appl. Psychol. **95**(1), 32–53 (2010b). https://doi.org/10.1037/a0017328
De Kleer, J., Brown, J.: Assumptions and ambiguities in mechanistic mental models. In: Gentner, D., Stevens, A. (eds.) Mental Models, pp. 155–190. Lawrence Erlbaum Associates, Hillsdale, NJ (1983)
Doll, B.B., Simon, D.A., Daw, N.D.: The ubiquity of model-based reinforcement learning. Curr. Opin. Neurobiol. **22**, 1075–1081 (2012)

Doyle, J.K., Ford, D.N.: Mental models concepts for system dynamics research. Syst. Dyn. Rev. **14**(1), 3–29 (1998)
Du Plooy, M.C.: Visualisation as a metacognitive strategy in learning multiplicative concepts: a design research intervention. Ph.D. thesis, Department of Mathematics Education, University of Pretoria. https://repository.up.ac.za/handle/2263/51258 (2016)
Engelfriet, J., Treur, J.: A temporal model theory for default logic. In: Clarke, M., Kruse, R., Moral, S. (eds.) Proceedings of 2nd European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty, ECSQARU'93, pp. 91–96, Springer (1994)
Engelfriet, J., Treur, J.: Temporal theories of reasoning. J. Appl. Non-Class. Logics **5**(1), 97–119 (1995). See also in: MacNish, C., Pearce, D., Pereira L.M. (eds.) Logics in Artifical Intelligence, Proceedings of the 4th European Workshop on Logics in Artificial Intelligence, JELIA'94, pp. 279–299. Springer (1994)
Evans, J.: The heuristic-analytic theory of reasoning: extension and evaluation. Psychon Bull. Rev. **13**(3), 378–395 (2006)
Flavell, J.H.: Metacognition and cognitive monitoring: a new area of cognitive–developmental inquiry. Am. Psychol. **34**(10), 906–911 (1979)
Furlough, C.S., Gillan, D.J.: Mental models: structural differences and the role of experience. J. Cogn. Eng. Decis. Making **12**(4), 269–287 (2018). https://doi.org/10.1177/1555343418773236
Galton, A.: Operators versus arguments: the ins and outs of reification. Synthese **150**, 415–441 (2006)
Gavrila, I.S., Treur, J.: A formal model for the dynamics of compositional reasoning systems. In: Cohn, A.G. (ed.) Proceedings of the 11th European Conference on Artificial Intelligence, ECAI'94, pp. 307–311. Wiley, Chichester (1994)
Gentner, D.: Structure-mapping: a theoretical framework for analogy. Cogn. Sci. **7**(2), 155–170 (1983)
Gentner, D., Stevens, A. (eds.) Mental Models. Lawrence Erlbaum Associates, Hillsdale, NJ (1983)
Gibbons, J., Gray, M.: An integrated and experience-based approach to social work education: the Newcastle model. Soc. Work. Educ. **21**(5), 529–549 (2002)
Goldman, A.I.: Simulating Minds: The Philosophy, Psychology, and Neuroscience of Mindreading. Oxford University Press, New York (2006)
Granqvist, P., Kirkpatrick, L.A.: Attachment and religious representations and behavior. In: Cassidy, J., Shaver, P.R. (eds.) Handbook of Attachment: Theory, Research, and Clinical Applications, 2nd edn., pp. 906–933. Guilford, New York (2008)
Greca, I.M., Moreira, M.A.: Mental models, conceptual models, and modelling. Int. J. Sci. Educ. **22**(1), 1–11 (2000)
Halford, G.S.: Children's Understanding: The Development of Mental Models. Lawrence Erlbaum Inc. (1993)
Halloun, I.: Schematic modelling for meaningful learning of physics. J. Res. Sci. Teach. **33**, 1019–1041 (1996)
Hermans, A., Muhammad, S., Treur, J.: A second-order adaptive network model for attachment theory. In: Proceedings of the 21th International Conference on Computational Science, ICCS'21. Lecture Notes in Computer Science, vol. 12744, pp. 462–475. Springer (2021).
Hesslow, G.: Conscious thought as simulation of behaviour and perception. Trends Cogn. Sci. **6**, 242–247 (2002)
Hesslow, G.: The current status of the simulation theory of cognition. Brain Res. **1428**, 71–79 (2012)
Hofstadter, D.R.: Gödel, Escher, Bach. Basic Books, New York (1979)
Hogan, K.E., Pressley, M.E.: Scaffolding Student Learning: Instructional Approaches and Issues. Brookline Books (1997)
Holyoak, K.J., Monti, M.M.: Relational integration in the human brain: a review and synthesis. J. Cogn. Neurosci. (2020)
Hurley, S.: The shared circuits model (SCM): How control, mirroring, and simulation can enable imitation, deliberation, and mindreading. Behav. Brain Sci. **31**(1), 1–22 (2008)

Johnson-Laird, P.N.: Mental Models: Towards a Cognitive Science of Language, Inference, and Consciousness. Harvard University Press (1983)

Johnson-Laird, P.: The history of mental models. In: Manktelow, K., Chung, M.C. (eds.) Psychology of Reasoning: Theoretical and Historical Perspectives. Psychology Press, New York (2004)

Jonker, C.M., Treur, J.: Analysis of the dynamics of reasoning using multiple representations. In: Gray, W.D., Schunn, C.D. (eds.) Proceedings of the 24th Annual Conference of the Cognitive Science Society, CogSci 2002, pp. 512–517. Lawrence Erlbaum Associates, Inc., Mahwah, NJ (2002)

Jonker, C.M., Treur, J.: Modelling the dynamics of reasoning processes: reasoning by assumption. Cogn. Syst. Res. J. **4**, 119–136 (2003)

Kim, J.: Philosophy of Mind. Westview Press (1996)

Koedinger, K.R., Terao, A.: A cognitive task analysis of using pictures to support pre-algebraic reasoning. In: Gray, W.D., Schunn, C.D. (eds.) Proceedings of the 24th Annual Conference of the Cognitive Science Society, CogSci'02, pp. 542–547. Lawrence Erlbaum Associates, Mahwah, NJ (2002)

Koriat, A.: Metacognition and consciousness. In: Zelavo, P.D., Moscovitch, M., Thompson, E. (eds.). Cambridge Handbook of Consciousness. Cambridge University Press, New York (2007)

Kozma, R.B.: Learning with media. Rev. Educ. Res. **61**(2), 179–211 (1991)

Kuang, W.X.Y.: The systematicity and coherence of conceptual metaphor. Foreign Lang. Res. 3 (2003)

Lakoff, G.: The contemporary theory of metaphor. In: Ortony, A. (ed.) Metaphor and Thought, pp. 202–251. Cambridge University Press (1993)

Lakoff, G., Johnson, M.: Metaphors We Live By. University of Chicago Press, Chicago (2003)

Landau, M.J., Meier, B.P., Keefer, L.A.: A metaphor-enriched social cognition. Psychol. Bull. **136**(6), 1045–1067 (2010)

Langan-Fox, J., Code, S., Langfield-Smith, K.: Team mental models. Techniques, methods, and analytic approaches. Hum. Factors **42**(2), 242–271 (2000). https://doi.org/10.1518/001872000779656534

Larbi, E., Mavis, O.: The Use of Manipulatives in mathematics education. J. Educ. Pract. **7**(36), 53–61 (2016)

Leary, D.E. (ed.): Metaphors in the history of psychology. Paperback (ed.) Cambridge University Press, Cambridge (1994)

Mahdavi, M.: An overview: metacognition in education. Int. J. Multidiscip. Curr. Res. **2**, 529–535 (2014)

Mayer, R.E.: Models for understanding. Rev. Educ. Res. **59**(1), 43–64 (1989)

McPherson, M., Smith-Lovin, L., Cook, J.M.: Birds of a feather: homophily in social networks. Ann. Rev. Sociol. **27**(1), 415–444 (2001)

Meela, P., Yuenyong, C.: The study of grade 7 mental model about properties of gas in science learning through model based inquiry (MBI). In: Proceedings of the International Conference for Science Educators and Teachers, pp. 1–6. AIP Conference Proceedings, vol. 2081(030028). AIP Publishing LLC (2019)

Mercer, J.: Understanding Attachment: Parenting, Child Care, and Emotional Development. Greenwood Publishing Group (2006)

Mohammed, S., Ferzandi, L., Hamilton, K.: Metaphor no more. A 15-year review of the team mental model construct. In: J. Manage. **36**(4), 876–910 (2010). https://doi.org/10.1177/0149206309356804

Meyer, J.-J. Ch, Treur, J. (eds.): Dynamics and Management of Reasoning Processes. Springer (2001)

Nagel, E., Newman, J.: Gödel's Proof. New York University Press, New York (1965)

Neilson, D., Campbell, T., Allred, B.: Model-based inquiry: a buoyant force module for high school physics classes. Sci. Teach. **77**(8), 38–43 (2010)

Nini, M.: All on the same page: how Team Mental Models (TMM) increase team performance. CQ Net (2019). https://www.ckju.net/en/dossier/team-mental-models-increase-team-performance

Piaget, J.: Origins of Intelligence in the Child (La Naissance de l'intelligence chez l'enfant). Routledge & Kegan Paul, London (1936)

Piaget, J.: The Construction of Reality in the Child. Basic Books Inc., New York (1954)

Pintrich, P.R.: The role of goal orientation in self-regulated learning. In: Boekaerts, M., Pintrich, P., Zeidner, M. (eds.) Handbook of Self-regulation Research and Applications, pp. 451–502. Academic Press, Orlando, FL (2000)

Ponterotto, D.: The cohesive role of cognitive metaphor in discourse and conversation. In: Metaphor and Metonymy at the Crossroads: A Cognitive Perspective, pp. 283–298 (2000)

Refaie, E.E.: Understanding visual metaphor: the example of newspaper cartoons. Vis. Commun. **2**(1), 75–95 (2003)

Rizzolatti, G., Craighero, L.: The mirror-neuron system. Annu. Rev. Neurosci. **27**, 169–192 (2004)

Romero, E., Soria, B.: Cognitive metaphor theory revisited. J. Lit. Semant. **34**(1), 1–20 (2005)

Salter, M.D.: An evaluation of adjustment based on the concept of security. Ph.D. thesis, vol 18, p. 72. University of Toronto Studies, Child Development Series (1940)

Salter Ainsworth, M.D.: Security and attachment. In: Volpe, R. (ed.) The Secure Child: Timeless Lessons in Parenting and Childhood Education, pp. 43–53. Information Age Publishing, Charlotte, NC (2010)

Salter Ainsworth, M.D., Bowlby, J.: Child Care and the Growth of Love. Penguin Books, London (1965)

Schaap-Jonker, H., Corveleyn, J.M.: Mentalizing and religion. Arch. Psychol. Relig. **36**(3), 303–322 (2014)

Schjoedt, U., Stodkilde-Jorgensen, H., Geerts, A.W., Roepstorff, A.: Highly religious participants recruit areas of social cognition in personal prayer. SocCog Affect. Neurosci. **4**, 199–207 (2009)

Seel, N.M.: Mental models in learning situations. In: Advances in Psychology, vol. 138, pp. 85–107. North-Holland, Amsterdam (2006)

Shannon, S.V.: Using metacognitive strategies and learning styles to create self-directed learners. Inst. Learning Styles J. **1**, 14–28 (2008)

Shih, Y.F., Alessi, S.M.: Mental models and transfer of learning in computer programming. J. Res. Comput. Educ. **26**(2), 154–175 (1993)

Skemp, R.R.: The Psychology of Learning Mathematics. Penguin Books, Harmondsworth (1971)

Smorynski, C.: The incompleteness theorems. In: Barwise, J. (ed.) Handbook of Mathematical Logic, vol. 4, pp. 821–865. North-Holland, Amsterdam (1977)

Sterling, L., Beer, R.: Metainterpreters for expert system construction. J. Log. Program. **6**, 163–178 (1989)

Treur, J.: On the use of reflection principles in modelling complex reasoning. Int. J. Intell. Syst. **6**, 277–294 (1991)

Treur, J.: Temporal semantics of meta-level architectures for dynamic control of reasoning. In: Fribourg, L., Turini, F. (ed.) Logic Program Synthesis and Transformation-Meta-Programming in Logic, Proceedings of the Fourth International Workshop on Meta-Programming in Logic, META'94. Lecture Notes in Computer Science, vol. 883, pp. 353–376. Springer (1994)

Treur, J.: Network-Oriented Modeling for Adaptive Networks: Designing Higher-Order Adaptive Biological, Mental and Social Network Models. Springer Nature (2020)

Treur, J.: An adaptive network model covering metacognition to control adaptation for multiple mental models. Cogn. Syst. Res. **67**, 18–27 (2021a)

Treur, J.: Controlled social network adaptation: subjective elements in an objective social world. In: Proceedings of the 7th International Congress on Information and Communication Technology, ICICT'21. Advances in Intelligent Systems and Computing, vol. 235, pp. 263–274. Springer Nature (2021b)

Treur, J.: Self-modeling networks using adaptive internal mental models for cognitive analysis and support processes. In: Proceedings of the 9th International Conference on Complex Networks and Their Applications, vol. 2. Studies in Computational Intelligence, vol. 944, pp. 260–274. Springer (2021c)

Treur, J.: Mental models in the brain: on context-dependent neural correlates of mental models. Cogn. Syst. Res. **79**, 83–90 (2021d)

Treur, J.: Modeling the emergence of informational content by adaptive networks for temporal factorisation and criterial causation. Cogn. Syst. Res. **68**, 34–52 (2021e)

Treur, J., Van Ments, L. (eds.): Mental Models and their Dynamics, Adaptation and Control: A Self-Modeling Network Modeling Approach. Springer, Cham, Switzerland (2022) (this volume)

Van Gog, T., Paas, F., Marcus, N., Ayres, P., Sweller, J.: The mirror neuron system and observational learning: implications for the effectiveness of dynamic visualizations. Educ. Psychol. Rev. **21**(1), 21–30 (2009)

Van Ments, L., Treur, J.: A higher-order adaptive network model to simulate development of and recovery from PTSD. In: Proceedings of the 11the International Conference on Computational Science, ICCS'21. Lecture Notes in Computer Science, vol. 12743, pp. 154–166. Springer (2021a)

Van Ments, L., Treur, J.: Modeling adaptive cooperative and competitive metaphors as mental models for joint decision making. Cogn. Syst. Res. **69**, 67–82 (2021b)

Van Ments, L., Treur, J., Klein, J., Roelofsma, P.H.M.P.: A second-order adaptive network model for shared mental models in hospital teamwork. In: Nguyen, N.T., et al. (eds.) Proceedings of the 13th International Conference on Computational Collective Intelligence, ICCCI'21. Lecture Notes in AI, vol. 12876, pp. 126–140. Springer Nature (2021)

Van Ments, L., Treur, J., Roelofsma, P.H.M.P.: An adaptive network model for formation and use of a mental god-model and its effect on human empathy. In: Treur and Van Ments, 2022, Chap. 11 (this volume) (2022)

Vosniadou, S., Ortony, A. (eds.): Similarity and Analogical Reasoning. Cambridge University Press, New York (1989)

Weyhrauch, R.W.: Prolegomena to a theory of mechanized formal reasoning. Artif. Intell. **13**, 133–170 (1980)

Williams, D.: Predictive minds and small-scale models: Kenneth Craik's contribution to cognitive science. Philos. Explor. **21**(2), 245–263 (2018a)

Williams, D.: The mind as a predictive modelling engine: generative models, structural similarity, and mental representation. Ph.D. thesis, University of Cambridge, UK. (2018)

Williams, L.E., Huang, J.Y., Bargh, J.A.: The scaffolded mind: higher mental processes are grounded in early experience of the physical world. Eur. J. Soc. Psychol. **39**(7), 1257–1267 (2009)

Whitaker, K.J., Vendetti, M.S., Wendelken, C., Bunge, S.A.: Neuroscientific insights into the development of analogical reasoning. Dev. Sci. **21**, e12531 (2018). https://doi.org/10.1111/desc.12531

Yi, M.Y., Davis, F.D.: Developing and validating an observational learning model of computer software training and skill acquisition. Inf. Syst. Res. **14**(2), 146–169 (2003)