Endogenous Technology Spillovers in Dynamic R&D Networks

Konig, Michael; Liu, Xiaodong; Hsieh, Chih-Sheng

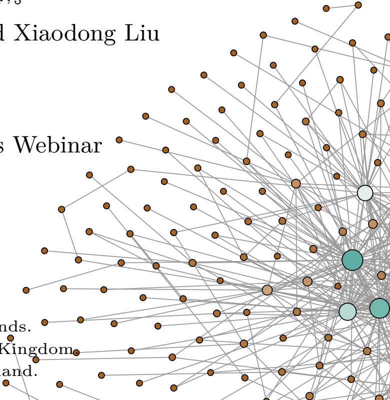
2021

Endogenous Technology Spillovers in Dynamic R&D Networks

Michael D. König[†,‡,§]
joint with Chih-Sheng Hsieh and Xiaodong Liu

Cambridge INET Networks Webinar

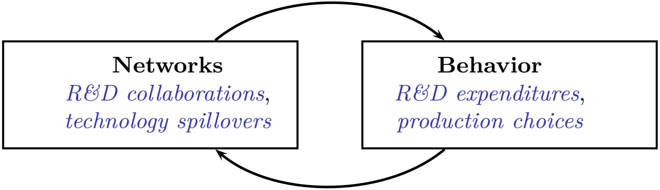
8 October 2021



† Tinbergen Institute and VU Amsterdam, The Netherlands.
‡ Centre for Economic Policy Research (CEPR), United Kingdom.
§ ETH Zurich, Swiss Economic Institute (KOF), Switzerland.

Introduction

- The many aspects that are governed by networks make it critical to understand:
  - how *networks impact behaviour* (and vice versa),
  - which *network structures* are likely to *emerge*, and
  - how they *affect welfare* in the society.



- We make three interrelated contributions to address these questions: (i) *theory*, (ii) *econometrics* and (iii) *policy*.

Contribution: Theory

- We provide an analytic characterization of both,
  - equilibrium *networks* and
  - endogenous *production* choices,

  by making the network in Ballester at al. (ECMA, 2006) endogenous.
- Equilibrium networks are particular *nested* structures,[1] while the firms' output levels and degrees follow a *Pareto distribution*, consistent with the data.
- Our efficiency analysis further reveals that equilibrium networks tend to be *under-connected* (with R&D policy implications).

---

[1] A network exhibits *nestedness* if the neighborhood of a node is contained in the neighborhoods of the nodes with higher degrees. See e.g. König et al. (TE, 2014).

Contribution: Econometrics

- We provide an *estimation framework* that can handle the endogeneity of both, the network structure and (either continuous or discrete) effort choices.[2]
- The analytic characterization allows us to design an estimation algorithm that can handle *large network datasets*.
- We estimate the model using a unique *dataset on R&D collaborations* matched to firm's balance sheets and patents.

[2] This generalizes previous works such as Mele (ECMA, 2017), where only the formation of the network was considered.

Contribution: Policy

- We provide the first (R&D) *policy analysis* with an *endogenous network* structure.
- Our analysis identifies which *collaborations should be subsidized*.
- We find that subsidizing an R&D collaboration can yield a welfare gain almost *five times larger than the cost* of the subsidy.
- Our framework could be used to assist governmental funding agencies that typically do not take into account the dynamic R&D network structure.[3]

[3] E.g. EUREKA's total subsidies for cooperative R&D accumulated to more than €37 billion in 2015.

Related Literature

Authors	Journal[a]	Year	Network	Action/Behavior
D'Aspremont & Jacquemin	AER	1988	exogenous[b]	endogneous
Goyal & Moraga-Gonzalez	RAND	2001	exogenous[c]	endogenous
Ballester et al.	ECMA	2006	exogenous	endogneous
Bramoullé et al.	AER	2014	exogenous	endogenous
Belhaj et al.	GEB	2014	exogenous	endogenous
Bimpikis et al.	MS	2016	exogenous	endogneous
König et al.	REStat	2019	exogenous	endogneous
Goyal & Joshi	GEB	2003	endogenous	none
Westbrock	RAND	2010	endogenous	none
Mele	ECMA	2017	endogenous	none
Chandrasekhar & Jackson	WP	2016	endogenous	none
König et al.	TE	2014	endogenous	no competition/ no linking cost random link decay
Hiller	GEB	2017	endogenous	no competition/ no characterization
Belhaj et al.	TE	2017	endogenous	no competition/ no characterization
Snijders	AAS	2001	endogenous	no competition/ no characterization
Badev	ECMA	2021	endogenous	binary choice/ no competition / no characterization

[a] Note: ECMA...Econometrica, AER...American Economic Review, TE...Theoretical Economics, GEB...Games and Economic Behavior, RAND...RAND Journal of Economics, AAS...Annals of Applied Statistics, MS...Management Science, WP...Working Paper.

[b] An endogenous network is considered restricted to 2 firms.

[c] An endogenous network is considered restricted to 4 firms.

The Model

- The *inverse demand* for firm i producing quantity q_i is

$$p_i = a - q_i - b \sum_{j \neq i} q_j. \tag{1}$$

- A firm i can reduce *marginal costs* c_i by investing e_i into R&D, or by benefiting from the R&D investment e_j of its collaboration partner j:

$$c_i = \bar{c}_i - \alpha e_i - \beta \sum_{j=1}^{n} a_{ij} e_j, \tag{2}$$

where $a_{ij} = 1$ if firms i and j set up a collaboration (0 otherwise) and $a_{ii} = 0$.

Profits

- Firm *i*'s *profit* π_i is then given by

$$\pi_i = (p_i - c_i)q_i - \gamma e_i^2 - \zeta d_i, \tag{3}$$

where γe_i^2 is the cost of R&D, $\gamma > 0$, and $\zeta \geq 0$ is a fixed cost of collaboration.

- Inserting marginal cost from Eq. (2) and inverse demand from Eq. (1) into Eq. (3) gives

$$\pi_i = (a - \bar{c}_i)q_i - q_i^2 - bq_i \sum_{j \neq i} q_j + \alpha q_i e_i + \beta q_i \sum_{j=1}^{n} a_{ij} e_j - \gamma e_i^2. \tag{4}$$

- The FOC with respect to R&D effort e_i yields $e_i = \lambda q_i$,[4] with $\lambda = \frac{\alpha}{2\gamma}$.

[4] Cf. Cohen & Klepper (EJ, 1996).

Potential

- Denoting by $\eta = a - \bar{c}_i$, $\nu = 1 + \lambda(\lambda\gamma - \alpha)$ and $\rho = \lambda\beta$, Eq. (4) becomes[5]

$$\pi_i = \underbrace{\eta_i q_i - \nu q_i^2}_{\text{own concavity}} \quad \underbrace{-b q_i \sum_{j \neq i}^{n} q_j}_{\text{global substitutability}} \quad + \quad \underbrace{\rho q_i \sum_{j=1}^{n} a_{ij} q_j}_{\text{local complementarity}} \quad - \zeta d_i. \qquad (5)$$

- **Proposition:** The profit function of Eq. (5) admits a *potential function* $\Phi \colon \mathbb{R}_+^n \times \mathcal{G}_n \to \mathbb{R}$ given by

$$\Phi(\mathbf{q}, G) = \sum_{i=1}^{n} (\eta_i q_i - \nu q_i^2) - \frac{b}{2} \sum_{i=1}^{n} \sum_{j \neq i} q_i q_j + \frac{\rho}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} q_i q_j - \zeta m \qquad (6)$$

where m is the number of links in G.

[5] Cf. Ballester et al. (ECMA, 2006).

Cournot Best Response Dynamics

- We consider a Markov chain, where opportunities for change (links/output) arrive as a Poisson process.[6]
- We follow the *best response* dynamics analyzed in Cournot (1838):[7]
  - Firms maximize profits by taking the output levels and collaborations of the other firms as given (myopic).[8]
- As R&D projects and collaborations are fraught with *uncertainty*,[9] we also introduce noise in this decision process.

[6] Similar to Calvo models of pricing (Calvo, JME, 1983).
[7] See Cournot (1838) and Daughety (2005).
[8] Cf. Jackson & Watts (JET, 2002).
[9] Cf. Kelly et al. (RDM, 2002) and Czarnitzki et al. (JIO, 2015).

- The evolution is characterized by a sequence $(\boldsymbol{\omega}_t)_{t \in \mathbb{R}_+}$, $\omega_t \in \Omega$, consisting of
  - a vector of firms' output levels $\mathbf{q}_t \in \mathcal{Q}^n$ and
  - a network of collaborations $G_t \in \mathcal{G}^n$.
- Then, in a short time interval $[t, t + \Delta t)$, $t \in \mathbb{R}_+$, one (and only one) of the following events happens:
  - *output adjustment*,
  - *link formation* or
  - *link removal*.

Output Adjustment

- At rate $\chi > 0$ a firm i receives an output adjustment opportunity.
- The profit of firm i from choosing an output level $q \in \mathcal{Q}$ is then given by $\pi_i(q, \mathbf{q}_{-i}, G) + \varepsilon_{it}$.
- When ε_{it} is i.i. type-I extreme value distributed with parameter ϑ, then[10]

$$\mathbb{P}\left(\boldsymbol{\omega}_{t+\Delta t} = (q, \mathbf{q}_{-it}, G_t) | \boldsymbol{\omega}_t = (\mathbf{q}_t, G_t)\right) = \chi \frac{e^{\vartheta \pi_i(q, \mathbf{q}_{-it}, G_t)}}{\int_{\mathcal{Q}} e^{\vartheta \pi_i(q', \mathbf{q}_{-it}, G_t)} \, dq'} \Delta t + o(\Delta t), \quad (7)$$

- When $\vartheta \to \infty$ the noise vanishes and the firm chooses the profit maximizing output level.

[10] That is a *multinomial logistic function* with choice set $\mathcal{Q}$ and parameter ϑ (cf. Anderson et al., GEB, 2001, and McFadden, 1976).

Link Formation

- With rate $\lambda > 0$ a pair of firms ij which is not already connected receives an opportunity to form a link.
- The formation of a link depends on the marginal profits plus a logistically distributed error term $\varepsilon_{ij,t}$.
- The *link ij is created* only if both firms find this profitable:[11]

$$\begin{aligned}
&\mathbb{P}\left(\boldsymbol{\omega}_{t+\Delta t} = (\mathbf{q}_t, G_t + ij) | \boldsymbol{\omega}_{t-1} = (\mathbf{q}, G_t)\right) \\
&= \lambda\, \mathbb{P}\left(\{\pi_i(\mathbf{q}_t, G_t + ij) + \varepsilon_{ij,t} > \pi_i(\mathbf{q}_t, G_t)\}\right. \\
&\quad \left. \cap \{\pi_j(\mathbf{q}_t, G_t + ij) + \varepsilon_{ij,t} > \pi_j(\mathbf{q}_t, G_t)\}\right) \Delta t + o(\Delta t) \\
&= \lambda \frac{e^{\vartheta \Phi(\mathbf{q}_t, G_t + ij)}}{e^{\vartheta \Phi(\mathbf{q}_t, G_t + ij)} + e^{\vartheta \Phi(\mathbf{q}_t, G_t)}} \Delta t + o(\Delta t).
\end{aligned}$$

[11] We have used the fact that $\pi_i(\mathbf{q}_t, G_t + ij) - \pi_i(\mathbf{q}_t, G_t) = \pi_j(\mathbf{q}_t, G_t + ij) - \pi_j(\mathbf{q}_t, G_t) = \Phi(\mathbf{q}_t, G_t + ij) - \Phi(\mathbf{q}_t, G_t)$.

Link Removal

- With rate $\xi > 0$ a pair of connected firms ij receives an opportunity to terminate their collaboration.
- The marginal profits from removing the link ij are perturbed by a logistically distributed error term $\varepsilon_{ij,t}$.
- The *link ij is removed* if at least one firm finds this profitable:[12]

$$\begin{aligned}
&\mathbb{P}\left(\boldsymbol{\omega}_{t+\Delta t} = (\mathbf{q}_t, G_t - ij) | \boldsymbol{\omega}_t = (\mathbf{q}, G_t)\right) \\
&= \xi\, \mathbb{P}\left(\{\pi_i(\mathbf{q}_t, G_t - ij) + \varepsilon_{ij,t} > \pi_i(\mathbf{q}_t, G_t)\}\right. \\
&\quad \left.\cup \{\pi_j(\mathbf{q}_t, G_t - ij) + \varepsilon_{ij,t} > \pi_j(\mathbf{q}_t, G_t)\}\right) \Delta t + o(\Delta t) \\
&= \xi \frac{e^{\vartheta \Phi(\mathbf{q}_t, G_t - ij)}}{e^{\vartheta \Phi(\mathbf{q}_t, G_t - ij)} + e^{\vartheta \Phi(\mathbf{q}_t, G_t)}} \Delta t + o(\Delta t).
\end{aligned}$$

[12] We have used the fact that $\pi_i(\mathbf{q}_t, G_t - ij) - \pi_i(\mathbf{q}_t, G_t) = \pi_j(\mathbf{q}_t, G_t - ij) - \pi_j(\mathbf{q}_t, G_t) = \Phi(\mathbf{q}_t, G_t - ij) - \Phi(\mathbf{q}_t, G_t)$.

Stationary States and Gibbs Measure

- **Proposition:** The ergodic Markov chain $(\boldsymbol{\omega}_t)_{t\in\mathbb{R}_+}$ has a unique stationary distribution $\mu^\vartheta : \mathcal{Q}^n \times \mathcal{G}^n \to [0,1]$ given by the *Gibbs measure*[13]

$$\mu^\vartheta(\mathbf{q}, G) = \frac{e^{\vartheta\left(\Phi(\mathbf{q},G) - m\ln\left(\frac{\xi}{\lambda}\right)\right)}}{\sum_{G'\in\mathcal{G}^n}\int_{\mathcal{Q}^n} d\mathbf{q}'\, e^{\vartheta\left(\Phi(\mathbf{q}',G') - m'\ln\left(\frac{\xi}{\lambda}\right)\right)}}. \tag{8}$$

- In the limit of vanishing noise $\vartheta \to \infty$, the *stochastically stable states*[14] are given by

$$\lim_{\vartheta\to\infty} \mu^\vartheta(\mathbf{q}, G) \begin{cases} > 0, & \text{if } \Phi(\mathbf{q}, G) \geq \Phi(\mathbf{q}', G'), \;\; \forall \mathbf{q}' \in \mathcal{Q}^n, \;\; G' \in \mathcal{G}^n, \\ = 0, & \text{otherwise}, \end{cases} \tag{9}$$

and we denote by $\mu^* = \lim_{\vartheta\to\infty} \mu^\vartheta$.

[13] Cf. Bisin et al. (JET, 2006).
[14] Cf. Kandori et al. (ECMA, 1993).

Homogeneous Firms

- **Proposition:** Consider homogeneous firms ($\bar{c}_i = \bar{c}_j = \bar{c}$ for all $i, j \in \mathcal{N}$) such that $\eta_i = \eta$, let $\eta^* \equiv \eta/(n-1)$ and $\nu^* \equiv \nu/(n-1)$. Then $\bar{q} = \frac{1}{n}\sum_{i=1}^{n} q_i \xrightarrow{\text{a.s.}} q^*$, where q^* is the root of

$$(b + 2\nu^*)q - \eta^* = \frac{\rho}{2}\left(1 + \tanh\left(\frac{\vartheta}{2}\left(\rho q^2 - \zeta\right)\right)\right) q, \tag{10}$$

with at least one solution if $b + 2\nu^* > \rho$, and for $\vartheta \to \infty$ (stochastically stable state)

$$q^* = \begin{cases} \frac{\eta^*}{b+2\nu^*-\rho}, & \text{if } \zeta < \frac{\rho(\eta^*)^2}{(b+2\nu^*)^2}, \\ \left\{\frac{\eta^*}{b+2\nu^*-\rho}, \frac{\eta^*}{b+2\nu^*}\right\}, & \text{if } \frac{\rho(\eta^*)^2}{(b+2\nu^*)^2} < \zeta < \frac{\rho(\eta^*)^2}{(b+2\nu^*-\rho)^2}, \\ \frac{\eta^*}{b+2\nu^*}, & \text{if } \frac{\rho(\eta^*)^2}{(b+2\nu^*-\rho)^2} < \zeta. \end{cases} \tag{11}$$

Equilibrium Output & Hysteresis

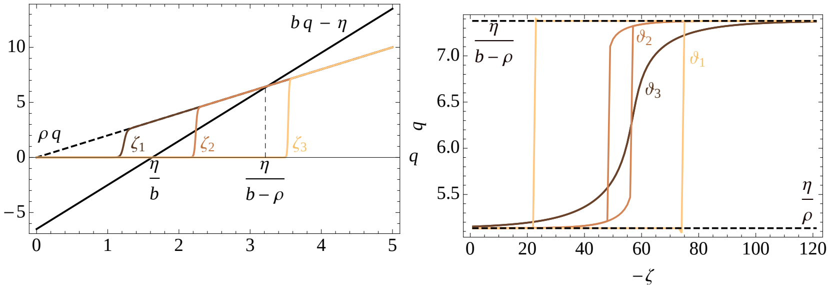


Figure: (Left panel) The right hand side of Eq. (10) for different values of the linking cost ζ, and (right panel) the corresponding values of q solving Eq. (10).

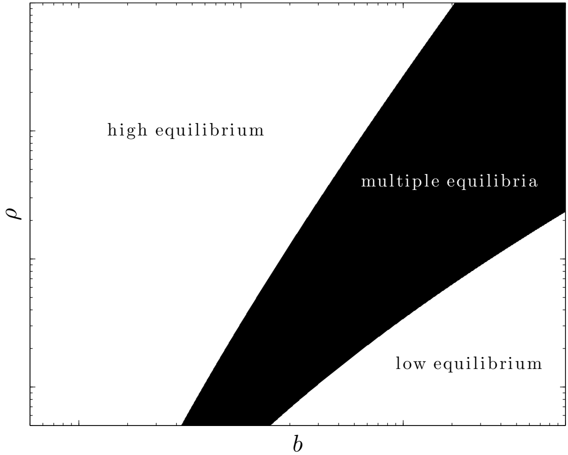


Figure: A phase diagram illustrating the regions with a unique and with multiple equilibria according to Eq. (10).

- **Proposition:** The firms' output levels become independent *Gaussian* random variables, $q_i \xrightarrow{\mathrm{d}} \mathcal{N}(q^*, \sigma^2)$, with mean q^* and variance σ^2.
- The degree d_i of firm i follows a (mixed) *Poisson* distribution

$$P^{\vartheta}(k) = \mathbb{E}_{\mu^{\vartheta}}\left(\frac{e^{-\bar{d}(q_1)}\bar{d}(q_1)^k}{k!}\right)(1 + o(1)), \tag{12}$$

where the expected degree is given by

$$\mathbb{E}_{\mu^{\vartheta}}\left(\bar{d}\right) = \frac{n-1}{2}\left(1 + \tanh\left(\frac{\vartheta}{2}\left(\rho(q^*)^2 - \zeta\right)\right)\right). \tag{13}$$

- In the limit $\vartheta \to \infty$ the stochastically stable network is either *empty or complete*.

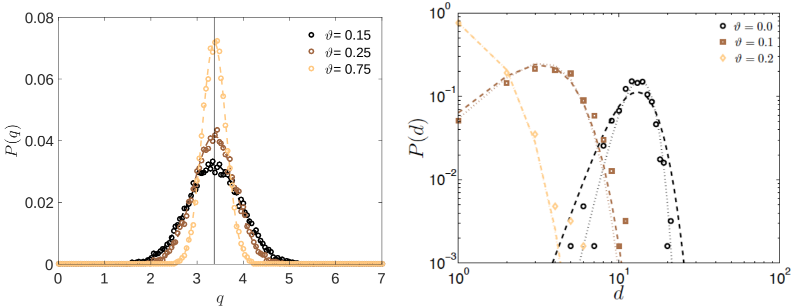


Figure: (Left panel) The stationary output distribution $P(q)$ with the dashed lines indicating a normal distribution $\mathcal{N}(q^*, \sigma^2)$. (Right panel) The stationary degree distribution $P(k)$ with the dashed lines indicating the solution of Eq. (12).

Heterogeneous Firms

- **Proposition:** For heterogeneous firms the stationary distribution of Eq. (8) can be written as $\mu^{\vartheta}(\mathbf{q}, G) = \mu^{\vartheta}(G|\mathbf{q})\mu^{\vartheta}(\mathbf{q})$.
- The marginal distribution $\mu^{\vartheta}(\mathbf{q})$ of the firms' output levels is *multivariate Gaussian*:[15]

$$\mu^{\vartheta}(\mathbf{q}) = \left(\frac{2\pi}{\vartheta}\right)^{-\frac{n}{2}} \left|-\Delta\mathscr{H}_{\vartheta}(\mathbf{q}^*)\right|^{\frac{1}{2}} \times$$
$$\exp\left\{-\frac{1}{2}\vartheta(\mathbf{q}-\mathbf{q}^*)^{\top}(-\Delta\mathscr{H}_{\vartheta}(\mathbf{q}^*))(\mathbf{q}-\mathbf{q}^*)\right\} + o\left(\|\mathbf{q}-\mathbf{q}^*\|^2\right),$$

with mean $\mathbf{q}^* \in \mathcal{Q}^n$ solving the following system of equations

$$q_i^* = \eta_i + \sum_{j \neq i}^{n} \left(\frac{\rho}{2}\left(1 + \tanh\left(\frac{\vartheta}{2}\left(\rho q_i^* q_j^* - \zeta\right)\right)\right) - b\right) q_j^* .$$

gaussian

[15] We have introduced the effective Hamiltonian, $\mathscr{H}(\mathbf{q})$, implicitly defined by $\sum_{G \in \mathcal{G}^n} e^{\vartheta \Phi(\mathbf{q}, G)} = e^{\vartheta \mathscr{H}(\mathbf{q})}$.

Nested Split Graphs

- **Proposition:** For $\vartheta \to \infty$ the stochastically stable network $G \in \mathcal{G}^n$ is a *nested split graph*[16] where i and j are connected iff $\rho q_i q_j > \zeta$.
- The output profile, $\mathbf{q} \in \mathcal{Q}^n$, solves

$$q_i = \frac{\eta_i}{2\nu} + \frac{1}{2\nu} \sum_{j \neq i}^{n} q_j \left(\rho \mathbb{1}_{\{\rho q_i q_j > \zeta\}} - b \right), \quad \mu^*\text{-a.s.} \tag{14}$$

- **Corollary:** If firms i and j are such that $\eta_i > \eta_j$ then i has a higher output than j, $q_i > q_j$ and a larger number of collaborations, $d_i > d_j$, μ^*-a.s..

[16] Cf. Mahadev & Peled (1995) and König et al. (TE, 2014).

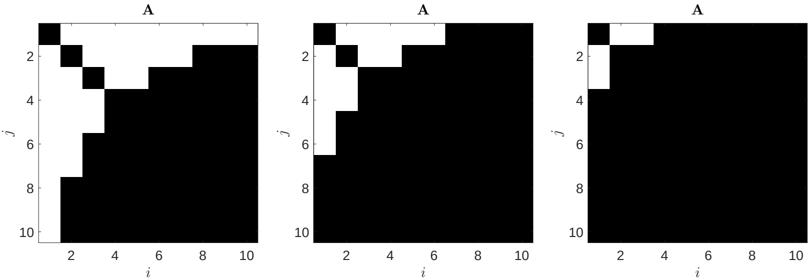


Figure: The (stepwise) adjacency matrix $\mathbf{A} = (a_{ij})_{1 \leq i,j,n}$, characteristic of a nested split graph, with elements $a_{ij} = 1$ iff $q_i q_j > \frac{\zeta}{\rho}$, where the vector $\mathbf{q}$ is the solution to Eq. (14). The panels from the left to the right correspond to increasing linking costs $\zeta \in \{0.0075, 0.01, 0.02\}$.

Pareto Output Distribution

- **Proposition:** Assume that $(\eta_i)_{i=1}^n$ are Pareto distributed with density $f(\eta) = (\gamma - 1)\eta^{-\gamma}$ for $\eta \geq 1$.
- Then the stochastically stable output distribution is given by

$$\mu^*(\mathbf{q}) = (\gamma - 1)^n |\det(\mathbf{M})| \prod_{i=1}^n (\mathbf{M}\mathbf{q})_i^{-\gamma},$$

  where $\mathbf{M} \equiv \mathbf{I}_n + b\mathbf{B} - \rho\mathbf{A}$, $\mathbf{B}$ is a matrix of ones with zero diagonal and $\mathbf{A}$ has elements $a_{ij} = 1$ iff $q_i q_j > \frac{\zeta}{\rho}$.
- In particular, for $\mathbf{q} = c\mathbf{u}$, with $c > 0$, and $\mathbf{u}$ being a vector of ones, we get a Pareto distribution

$$\mu^*(c\mathbf{u}) \sim \prod_{i=1}^n O\left(c^{-\gamma}\right)$$

  as $c \to \infty$.

Pareto Degree Distribution

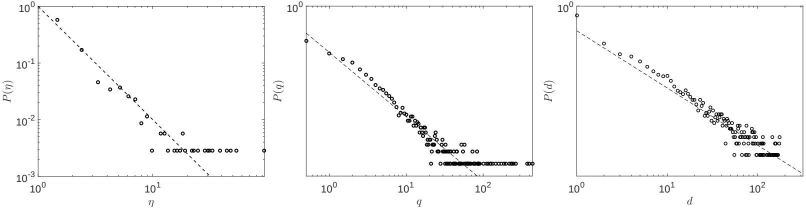


Figure: The Pareto distribution $P(\eta)$ of η (left panel), the resulting stationary output distribution $P(q)$ (middle panel) and the degree distribution $P(d)$ (right panel). Dashed lines indicate a power-law fit.

extensions

Efficiency

- Social *welfare*, W, is given by the sum of consumer surplus, U, and firms' profits, Π.
- Consumer surplus is given by

$$U(\mathbf{q}) = \frac{1}{2}\sum_{i=1}^{n} q_i^2 + \frac{b}{2}\sum_{i=1}^{n}\sum_{j\neq i}^{n} q_i q_j.$$

- Producer surplus is given by aggregate profits

$$\Pi(\mathbf{q}, G) = \sum_{i=1}^{n} \pi_i(\mathbf{q}, G).$$

- The *efficient* state $(\mathbf{q}^*, G^*)$ maximizes welfare $W(\mathbf{q}, G)$, that is, $W(\mathbf{q}^*, G^*) \geq W(\mathbf{q}, G)$ for all $G \in \mathcal{G}^n$ and $\mathbf{q} \in \mathcal{Q}^n$.

Stability vs. Efficiency

- **Proposition:** The efficient network $G^* \in \mathcal{G}^n$ is a *nested split graph*,[17] and $\mathbf{q}^*$ solves

$$q_i^* = \frac{\eta_i}{2\nu - 1} + \frac{1}{2\nu - 1} \sum_{j \neq i}^{n} q_j^* \left(\rho \mathbb{1}_{\{\rho q_i^* q_j^* > \zeta\}} - b \right). \tag{15}$$

- Further, the stochastically stable equilibrium output / R&D and the collaboration intensity are too low compared to the social optimum (μ^*-a.s.).
- Hence, equilibrium networks tend to be *under-connected*.[18]

[17] Cf. Belhaj et al. (TE, 2016).
[18] Cf. Buechel & Hellmann (RED, 2012).

Empirical Implications

- We merged the MERIT-CATI with the Thomson SDC *alliance databases*.[19]
- We use annual data about *balance sheets* and income statements from Standard & Poor's Compustat and Bureau Van Deijk's Orbis databases.
- We also obtained the firms' *patents* (PATSTAT), and computed the potential technology spillovers between collaborating firms using various patent proximity indices.

data

[19] These databases contain information about strategic technology agreements, including any alliance that involves some arrangements for mutual transfer of technology or joint research, such as joint research pacts, joint development agreements, cross licensing, R&D contracts, joint ventures and research corporations. Cf. Schilling (SMJ, 2009) and Hagedoorn (RP, 2002).

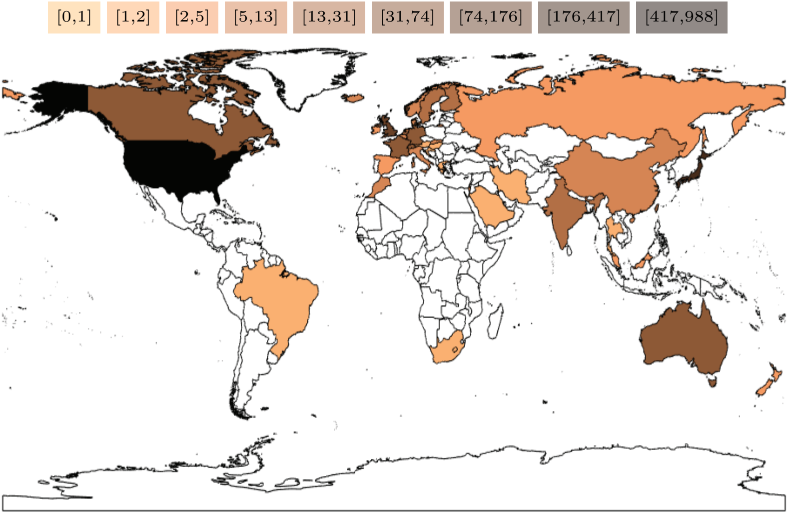


Figure: The number of firms in each country.

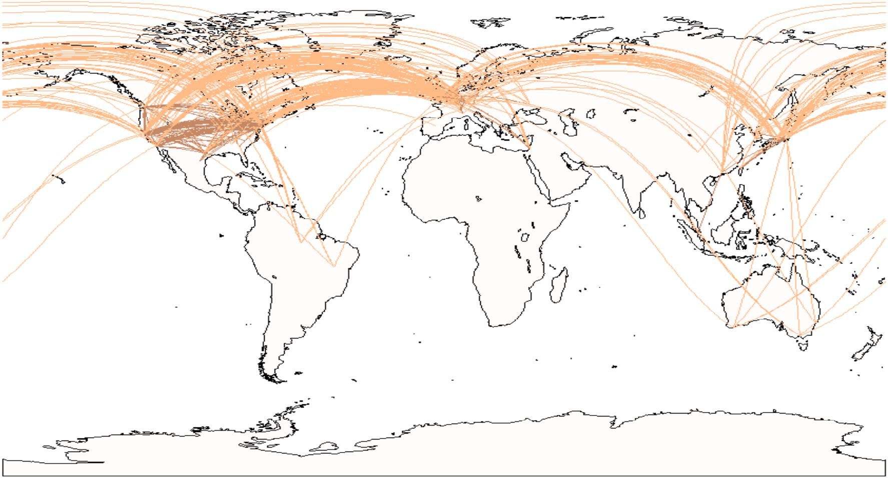
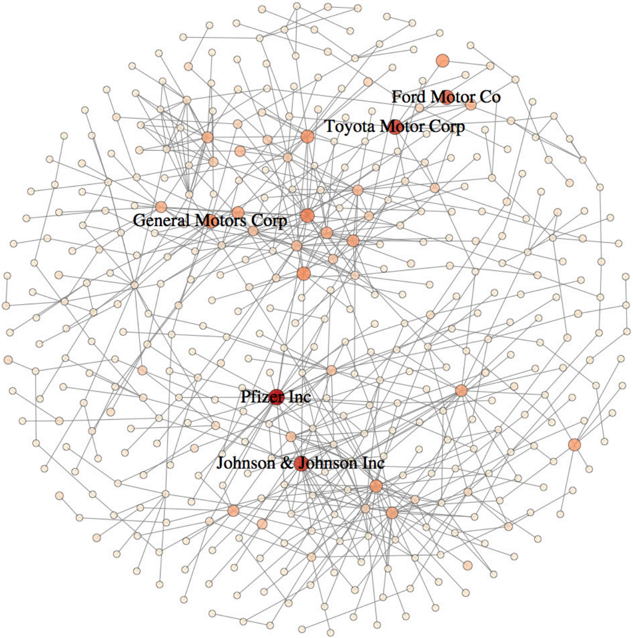
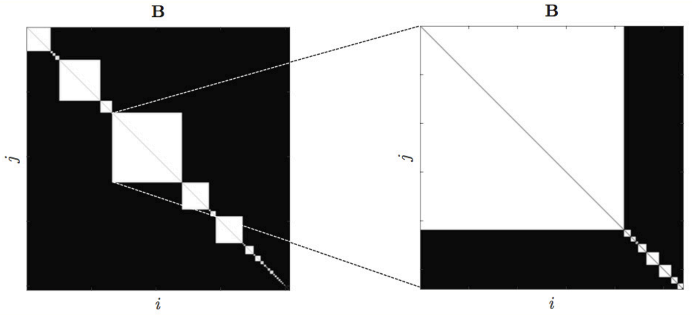


Figure: The locations and collaborations of the firms in the combined CATI-SDC database.


Ford Motor Co
Toyota Motor Corp
General Motors Corp
Pfizer Inc
Johnson & Johnson Inc



	20	33	87	37	73	35	38	36	28
20	0	0	1	1	0	0	1	0	3
33	0	1	0	4	0	2	1	3	1
87	1	0	0	0	1	2	4	3	14
37	1	4	0	17	5	2	7	2	1
73	0	0	1	5	4	17	7	17	6
35	0	2	2	2	17	9	5	26	2
38	1	1	4	7	7	5	6	13	25
36	0	3	3	2	17	26	13	29	3
28	3	1	14	1	6	2	25	3	141

	281	282	283	284	285	286	287	289
281	1	2	13	0	0	0	0	0
282	2	1	1	0	0	0	0	0
283	13	1	121	0	2	0	0	0
284	0	0	0	0	0	0	0	0
285	0	0	2	0	0	0	0	0
286	0	0	0	0	0	0	0	0
287	0	0	0	0	0	0	0	0
289	0	0	0	0	0	0	0	0

Figure: (Left panel) The competition matrix **B** across all 2-digit SIC sectors. (Right panel) The competition matrix **B** across all 3-digit SIC sectors within the SIC-28 sector (comprising 29.22% of all firms).

Firm Heterogeneity

- Accounting for heterogeneous marginal costs, substitution, and heterogeneous technology spillovers, *profits* are

$$\pi_i(\mathbf{q}, G) = \eta_i q_i - \frac{1}{2} q_i^2 - b \sum_{j \neq i}^{n} b_{ij} q_j q_i + \rho \sum_{j=1}^{n} f_{ij} a_{ij} q_j q_i - \sum_{j=1}^{n} a_{ij} \zeta_{ij}.$$

- The weights $(f_{ij})_{1 \leq i,j \leq n}$ capture heterogeneous technology spillovers across firms either using Jaffe's or the Mahalanobis *patent similarity* index (Bloom et al., 2013).
- The corresponding *potential* function $\Phi \colon \mathbb{R}_+^n \times \mathcal{G}^n \to \mathbb{R}$ is given by

$$\Phi(\mathbf{q}, G) = \sum_{i=1}^{n} \left(\eta_i q_i - \frac{1}{2} q_i^2 \right) - \frac{b}{2} \sum_{i=1}^{n} \sum_{j \neq i}^{n} b_{ij} q_i q_j + \frac{\rho}{2} \sum_{i=1}^{n} \sum_{j \neq i}^{n} f_{ij} a_{ij} q_i q_j - \frac{1}{2} \sum_{i=1}^{n} \sum_{j \neq i}^{n} a_{ij} \zeta_{ij} \quad (16)$$

Linking Costs and Homophily

- Linking costs are specified by the pairwise symmetric function

$$\zeta_{ij} = \boldsymbol{\gamma}^\top \mathbf{c}_{ij} + z_i + z_j.$$

- The r-dimensional vector of dyadic-specific variables, $\mathbf{c}_{ij}$, represents measures of similarity (homophily) between firms i and j regarding sector, location, technology, etc. (Lychagin et al., 2010).
- We also include individual *latent variables* z_i and z_j in ζ_{ij} to capture unobserved degree/collaboration heterogeneity (Graham, 2017).

Estimation

- The stationary distribution $\mu^{\vartheta}(G, \mathbf{q})$ of Equation (8) contains an intractable normalizing constant in the denominator.
- Furthermore, existing simulation-based estimation approaches are also not feasible for the network size we consider.
- However, we can overcome these issues by considering the *composite-likelihood* (Lindsay, 1988; Varin, 2011)

$$\mu^{\vartheta}(G|\mathbf{q})\mu^{\vartheta}(\mathbf{q}|G), \tag{17}$$

  where $\mu^{\vartheta}(G|\mathbf{q})$ and $\mu^{\vartheta}(\mathbf{q}|G)$ represent conditional probabilities of network and output.

Composite Likelihood

- The probability of observing a network $G \in \mathcal{G}^n$, conditional on an output distribution $\mathbf{q} \in \mathcal{Q}^n$, is

$$\mu^{\vartheta}(G|\mathbf{q}) = \frac{\mu^{\vartheta}(\mathbf{q}, G)}{\mu^{\vartheta}(\mathbf{q})} = \prod_{i<j}^{n} \frac{e^{\vartheta a_{ij}(\rho f_{ij} q_i q_j - \zeta_{ij})}}{1 + e^{\vartheta(\rho f_{ij} q_i q_j - \zeta_{ij})}}. \tag{18}$$

- The conditional probability $\mu^{\vartheta}(\mathbf{q}|G)$ of the output profile $\mathbf{q}$ given the network G is

$$\mu^{\vartheta}(\mathbf{q}|G) = \left(\frac{2\pi}{\vartheta}\right)^{-\frac{n}{2}} |\mathbf{M}(G)|^{\frac{1}{2}} e^{-\frac{\vartheta}{2}\left(\mathbf{q}-\mathbf{M}(G)^{-1}\mathbf{X}\boldsymbol{\delta}\right)^{\top}\mathbf{M}(G)(\mathbf{q}-\mathbf{M}(G)^{-1}\mathbf{X}\boldsymbol{\delta})}, \tag{19}$$

where $\mathbf{M}(G) \equiv \mathbf{I}_n + b\mathbf{B} - \rho(\mathbf{A} \circ \mathbf{F})$.

- Since the composite-likelihood in Equation (17) does not involve the intractable normalizing constant it is computationally simple to evaluate.

Estimation Results

Table: Estimation results for the homogeneous technology spillovers case.

		Model (1) W/o Unobs. Heterogeneity		Model (2) With Unobs. Heterogeneity	
Profits					
R&D Spillover	(ρ)	0.0174***	(0.0005)	0.0099***	(0.0007)
Substitutability	(b)	3.77e-5***	(1.35e-5)	3.45e-5***	(1.35e-5)
Productivity	(δ)	0.8475***	(0.0021)	0.8531***	(0.0022)
Unobs. Heterogeneity	(κ)	–		0.0103***	(0.0044)
Sector FE		Yes		Yes	
Linking Cost					
Constant	(γ_0)	6.8432***	(0.1795)	8.4542***	(0.2742)
Same Sector	(γ_1)	-1.1935***	(0.0546)	-1.4786***	(0.0834)
Same Country	(γ_2)	-0.3791***	(0.0484)	-0.6484***	(0.0766)
Diff-in-Prod.	(γ_3)	-0.0901***	(0.0110)	0.0020	(0.0137)
Noise/Uncertainty					
Noise in Decisions	(ϑ)	1.7364***	(0.0481)	1.4205***	(0.0432)
Unobs. Heterogeneity	(σ_z^2)	–		1.0196***	(0.1293)
Sample Size	(n)	1,738			

Notes: The asterisks ***(**,*) indicate that a parameter's 99% (95%, 90%) highest posterior density range does not cover zero.

fit

Heterogeneous Spillovers

Table: Estimation results for the heterogeneous technology spillovers case *à la* Jaffe.

		Model (1) W/o Unobs. Heterogeneity		Model (2) With Unobs. Heterogeneity	
Profits					
R&D Spillover	(ρ)	0.0401***	(0.0020)	0.0250***	(0.0021)
Substitutability	(b)	5.77e-5**	(1.82e-5)	3.68e-5***	(1.38e-5)
Productivity	(δ)	0.8605***	(0.0022)	0.8595***	(0.0023)
Unobs. Heterogeneity	(κ)	–		0.0753***	(0.0061)
Sector FE		Yes		Yes	
Linking Cost					
Constant	(γ_0)	6.6103***	(0.2285)	8.3960***	(0.2701)
Same Sector	(γ_1)	-0.9785***	(0.0778)	-1.2986***	(0.0910)
Same Country	(γ_2)	-0.5072***	(0.0601)	-0.6931***	(0.0844)
Diff-in-Prod.	(γ_3)	-0.1254***	(0.0147)	0.0151	(0.0139)
Noise/Uncertainty					
Noise in Decisions	(ϑ)	1.3748***	(0.0450)	1.3220***	(0.0393)
Unobs. Heterogeneity	(σ_z^2)	–		1.4864***	(0.1515)
Sample Size	(n)	1,738			

Notes: The asterisks ***(**,*) indicate that a parameter's 99% (95%, 90%) highest posterior density range does not cover zero.

mahalanobis

R&D Collaboration Subsidies

- We analyze a counterfactual policy that selects a specific firm-pair, (i, j), and compensates their collaboration costs through a subsidy, i.e., setting $\zeta_{ij} = 0$.
- The pair of firms for which the *subsidy* results in the largest gain in welfare is defined as

$$(i,j)^* = \underset{(i,j)\in\mathcal{E}}{\mathrm{argmax}} \left\{ \sum_{G\in\mathcal{G}^n} \int_{\mathcal{Q}^n} \left[W(\mathbf{q}, G|\zeta_{ij} = 0) - W(\mathbf{q}, G) \right] \mu^{\vartheta}(\mathbf{q}, G)\, d\mathbf{q} \right\}$$

- The probability measure $\mu^{\vartheta}(\mathbf{q}, G)$ is given by Eq. (8),
- $W(\mathbf{q}, G|\zeta_{ij} ij = 0)$ denotes the welfare function with firms i and j receiving a subsidy such that they do not incur a pair-specific collaboration cost (by setting $\zeta_{ij} = 0$ permanently).

R&D Subsidies Ranking

Table: R&D subsidy analysis for firms in the drugs development sector (SIC code 283).

Firm i	Firm j	Relat.[a] Prod. i	Relat.[a] Prod. j	Market[b] Share i (%)	Market[b] Share j (%)	Deg. d_i	Deg. d_j	Long Run[c] ΔW_{E}	Short Run[c] ΔW_{F}	R&D Sub.[d] Multiplier	Rank[e]
Novartis	Pfizer	1.592	1.653	2.069	2.768	19	16	11.387	2.387	4.778	1
Merck & Co.	Pfizer	1.579	1.653	1.300	2.768	16	16	11.185	2.195	5.096	2
Johnson & Johnson	Pfizer	1.617	1.653	3.055	2.768	11	16	10.650	2.352	4.534	3
Amgen	Pfizer	1.526	1.653	0.819	2.768	14	16	10.538	2.281	4.610	4
Merck & Co.	Novartis	1.579	1.592	1.300	2.069	16	19	10.460	3.066	4.139	5
GlaxoSmithKline	Novartis	1.509	1.592	0.724	2.069	14	19	10.222	2.180	4.697	6
GlaxoSmithKline	Pfizer	1.509	1.653	0.724	2.768	14	16	10.035	3.602	4.025	7
Novartis	Johnson & Johnson	1.592	1.617	2.069	3.055	19	11	9.998	5.168	3.731	8
Merck & Co.	Johnson & Johnson	1.579	1.617	1.300	3.055	16	11	9.908	3.547	3.977	9
Amgen	Novartis	1.526	1.592	0.819	2.069	14	19	9.838	5.242	3.758	10
Amgen	Merck & Co.	1.526	1.579	0.819	1.300	14	16	9.718	3.656	3.999	11
Amgen	Johnson & Johnson	1.526	1.617	0.819	3.055	14	11	9.575	5.313	3.708	12
GlaxoSmithKline	Merck & Co.	1.509	1.579	0.724	1.300	14	16	9.574	5.645	3.631	13
Bristol-Myers Squibb	Pfizer	1.564	1.653	1.029	2.768	7	16	9.440	5.668	3.504	14
GlaxoSmithKline	Johnson & Johnson	1.509	1.617	0.724	3.055	14	11	9.266	6.426	3.322	15
GlaxoSmithKline	Amgen	1.509	1.527	0.724	0.819	14	14	9.226	6.340	3.380	16
Bristol-Myers Squibb	Merck & Co.	1.564	1.579	1.029	1.300	7	16	9.100	6.660	3.207	17
Bristol-Myers Squibb	Johnson & Johnson	1.564	1.617	1.029	3.055	7	11	9.086	7.188	3.039	18
Abbott Laboratories	Pfizer	1.532	1.653	1.291	2.768	3	16	8.877	6.559	3.132	19
Abbott Laboratories	Merck & Co.	1.532	1.579	1.291	1.300	3	16	8.750	7.219	2.938	20

[a] Relative productivity shows the firm's productivity relative to the average productivity of all firms in the sample.

[b] Market share (%) is the market share measured in the primary 3-digit sector in which the firm is operating.

[c] The expected welfare gain due to subsidizing the R&D collaboration costs between firms i and j is computed as $\Delta W = \mathbb{E}_{\mu^\vartheta}[W(\mathbf{q}, G|\zeta_{ij} = 0) - W(\mathbf{q}, G)]$. Expected welfare without setting the linking cost to zero is $\mathbb{E}_{\mu^\vartheta}[W(\mathbf{q}, G)] = 33616.35$ in the long run and 33628.80 in the short run. In the short run the network is assumed to be fixed, while in the long run the network endogenously responds to the R&D collaboration subsidy.

[d] The R&D subsidy multiplier is defined as the ratio of the expected (long run) welfare gain to the cost of the subsidy.

[e] The rank is based on the long run welfare gain.

Conclusion

- We analyze the coevolution of *networks and behavior*, provide a complete equilibrium characterization and reproduce the observed patterns in real world networks.
- The model can be conveniently estimated even for *large networks*.
- The model is amenable to *policy analysis* (e.g. firm exit, M&As and R&D collaboration subsidies).
- Due to the *generality* of our payoff function the model can be applied to peer effects in education, crime, terrorist networks, risk sharing, financial contagion, scientific co-authorship, etc.
- Our methodology can also be applied to study *discrete choice* models and network games with *local substitutes*.

Additional Results

Multivariate Gaussian

- The variance is given by the inverse of $-\Delta\mathscr{H}_\vartheta(\mathbf{q}^*)$, where

$$(\Delta\mathscr{H}_\vartheta(\mathbf{q}))_{ii} = -1 + \frac{\vartheta\rho^2}{4}\sum_{j\neq i}^{n} q_j^2 \left(1 - \tanh\left(\frac{\vartheta}{2}\left(\rho q_i q_j - \zeta\right)\right)^2\right),$$

while for $j \neq i$ we have that

$$\begin{aligned}(\Delta\mathscr{H}_\vartheta(\mathbf{q}))_{ij} = -b + \frac{\rho}{2}&\left(1 + \tanh\left(\frac{\vartheta}{2}\left(\rho q_i q_j - \zeta\right)\right)\right)\\ &\times\left(1 + \frac{\vartheta\rho}{2} q_i q_j \left(1 - \tanh\left(\frac{\vartheta}{2}\left(\rho q_i q_j - \zeta\right)\right)\right)\right),\end{aligned}$$

- The conditional distribution $\mu^\vartheta(G|\mathbf{q})$ is given by

$$\mu^\vartheta(G|\mathbf{q}) = \prod_{i=1}^{n}\prod_{j=i+1}^{n} \frac{e^{\vartheta a_{ij}(\rho q_i q_j - \zeta)}}{1 + e^{\vartheta(\rho q_i q_j - \zeta)}}, \tag{20}$$

back

Extension: Heterogeneous collaboration costs

- Firms with higher productivity incur lower *collaboration costs*,

  $$\zeta_{ij} = \frac{\zeta}{s_i s_j},$$

  where $s_i > 0$ denotes the productivity of firm i.
- A similar equilibrium characterization using a *Gibbs measure* is possible.
- In the special case of s_i being Pareto distributed, one can show that the *degree distribution* also follows a *Pareto distribution*, confirming previous empirical studies of R&D networks.[20]
- For a power-law productivity distribution, we can generate two-vertex and three-vertex *degree correlations*.

[20] E.g. Powell et al. (AJS, 2005).

Extension: Heterogeneous spillovers

- Firms can only benefit from collaborations if they have at least one *technology in common*.
- Technologies are randomly distributed across firms.
- Then we obtain a generalized *random intersection graph*,[21] with
  - a power-law degree distribution,
  - a decaying clustering degree distribution and
  - positive degree correlations / *assortativity*.

back

[21] Cf. Deijfen & Kets (PEIS, 2009).

Data

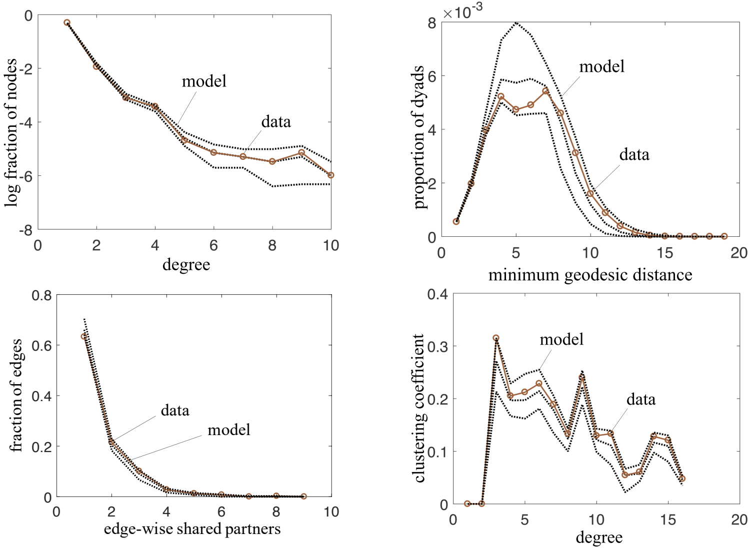
Table: Descriptive statistics.

	R&D effort			Productivity			R&D collaborations		
Num. of firms	mean	min	max	mean	min	max	mean	min	max
1738	9.1467	0	15.2467	10.5977	0.6427	17.0613	0.7273	0	24

Notes: R&D effort is measured by log R&D expenditure (in thousand U.S. dollars). The reference year is 2006. Firm's productivity is measured by its log-R&D capital stock (lagged by one year). To compute the R&D capital stocks we use a perpetual inventory method based on the firms' R&D expenditures with a 15% depreciation rate following Hall (2000) and Bloom (2013).

back

Model Fit



back

Heterogeneous Spillovers

Table: Estimation results for the heterogeneous technology spillovers case *à la* Mahalanobis.

		Model (1) W/o Unobs. Heterogeneity		Model (2) With Unobs. Heterogeneity	
Profits					
R&D Spillover	(ρ)	0.0192***	(0.0009)	0.0125***	(0.0010)
Substitutability	(b)	4.08e-5**	(1.43e-5)	7.05e-5***	(1.38e-5)
Productivity	(δ)	0.8602***	(0.0024)	0.8631***	(0.0024)
Unobs. Heterogeneity	(κ)	–		0.1050***	(0.0154)
Sector FE		Yes		Yes	
Linking Cost					
Constant	(γ_0)	6.6876***	(0.2288)	8.1432***	(0.3396)
Same Sector	(γ_1)	-1.0025***	(0.0806)	-1.2806***	(0.1041)
Same Country	(γ_2)	-0.5309***	(0.0616)	-0.6861***	(0.0803)
Diff-in-Prod.	(γ_3)	-0.1272***	(0.0143)	0.0141	(0.0142)
Noise/Uncertainty					
Noise in Decisions	(ϑ)	1.3604***	(0.0441)	1.3565***	(0.0472)
Unobs. Heterogeneity	(σ_z^2)	–		1.3507***	(0.1665)
Sample Size	(n)	1,738			

Notes: The asterisks ***(**,*) indicate that a parameter's 99% (95%, 90%) highest posterior density range does not cover zero.

back