

The Splash Deposition Transition Limits of a Biofuel Droplet Wall Impact with a and without Crossflow

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Abstract

Over the last years, there have been many investigations into new ways to obtain clean and efficient energy production. In this spirit, this study aims to successfully adapt low emission aero-engines combustors to burn biofuels. Biofuels are the immediate alternative to fossil-fuel powered aero-engines, given that by regulation is possible to apply today, a biofuel into the aeronautical industry. There are some regulations to have into account, being the biggest one the fact that the new blend must be constituted of at least 50% of conventional jet fuel (JF). In this work were selected four blends to study: 100% JF; 75% JF and 25% NEXBTL (biofuel); 50% JF and 50% NEXBTL and H₂O. A smooth, dry aluminium plate was used as the impact surface and the objective was to observe the splash-deposition limits and characterize the dynamic behaviour of the droplets with and without crossflow. Different velocities of the crossflow were chosen and the impact angle of the droplet was also analysed. The splash-deposition threshold was also compared with those proposed by other authors.

Keywords: Biofuels, Aero Engines, Crossflow; Droplets

1. Introduction

The behaviour of droplets impinging upon dry surfaces is not fully understood. It involves a great number of influencing parameters and the outcomes are normally classified observing the morphological characteristics of the impinging drop. In his experimental work, Riobbo et al. [1] identified six possible outcomes for droplet impact upon dry surfaces: deposition, prompt splash, corona splash, receding breakup, partial rebound and rebound. The outcome is governed by several parameters such as the droplet diameter, the impact velocity, the impact angle, the fluid physical properties (density, viscosity, and surface tension), the surface roughness and even if a gas boundary layer is applied in the near-wall region [2].

Jayarathne and Mason [3] found that the droplet outcomes depend on the impact angle (θ_i). This angle also influences the direction of the secondary droplets for smooth surfaces. A small incident angle of the droplet leads to a small reflection angle, while a large incident angle leads to a large deflection angle [4]. According to Yao and Cai [5] if the impact angle was not 90°, the droplet velocity tangential to the surface destabilizes the liquid film and enhances the fragmentation of the droplet after the impact. The impact angle differs from the contact angle (θ_c). The contact angle is influenced by the liquid physical properties and the surface topography. When a droplet impinges upon a solid surface, a liquid film is formed, and due to the influence of the surface tension and surface forces, this film reaches a maximum diameter and then recoils. When this process achieves the equilibrium, the static contact angle can be measured. This angle is also known as the wettability of the surface. The static contact angle varies between 90° and 180° for wetting systems, while for non-wetting systems vary between 0° and 90° [6].

The surface roughness is also an important parameter in the droplet impingement dynamics and it is normally characterized by its average height (τ_s). Mundo et al. [4] observed that the surface roughness altered the impact angle of the impinging droplets. For small droplets, this effect becomes more evident. This also affects the volume, the number, and the size distribution of the secondary droplets. For rough surfaces, the critical threshold for splash is lower than for smooth surfaces [7]. In addition, the mass of the splash decreases with the increase of the surface roughness [8]. Mundo et al. [8] also showed that the droplets with high kinetic energy presented a more irregular behaviour for rough surfaces compared to smooth surfaces. For rough surfaces, the high tangential momentum leads to the disintegration into secondary droplets and the normal phenomena are no longer observed. Droplets with low kinetic energy impinging upon rough surfaces did not present different results than those impacted on smooth surfaces.

There are other parameters that affect the dynamic behaviour of the droplets prior to impact, namely a crossflow. The presence of a crossflow affects the direction and the outcome of the droplets before impact [9]. The effect of a crossflow on the impinging droplets can be attributed to the aerodynamic forces exerted by the gas flow. When the droplet enters the gas flow it may deform and be oriented by the direction of the flow. This flow will also apply an additional force to the droplet which can vary the outcome of the impingement.

The outcome of a droplet impinging on a wall depends on a number of conditions. To be able to establish the transition criteria between impingement regimes a number of empirical correlations have been proposed (Table 1). In the Bai and Gosman [2] work, different impact regimes were identified for dry and wetted walls depending on

the Weber number and wall roughness. For dry surfaces, the empirical correlation was derived using the Stow and Hadfield [10] experimental data and the influence of the surface roughness was considered. The A coefficient depends on the surface roughness. For a rough surface the A coefficient assumes the value of 1322 while for very smooth surfaces assume the value 5264. Mundo et al [8] used the splashing parameter (K_c) to define the transition criteria. Their experiments showed that the critical point between deposition and splash was $K=57.7$. In their study two discs were used, one with a smooth surface and other with a rough surface. Vander Wal et al. [11] used a large number of hydrocarbon fuels and alcohols to develop their criteria. Although Vander Wal et al. [12] observed different disintegration mechanisms, a single threshold was developed to fit all their data. Their experiments were conducted with an aluminium disk with very low surface roughness.

Table 1. Transition criteria between spread/splash regimes.

Authors	Reference	Transition Criteria
Bai and Gosman	[2]	$We_c = A.La^{0.18}$
Mundo et al.	[8]	$Oh.Re^{1.25}=57.7$
Vander Wal et al.	[11]	$Oh.Re^{0.609}=0.85$

The present work aims to analyse the phenomena obtained by the impact of a single droplet of mixtures with biofuel and conventional jet fuel on a dry surface with and without a crossflow. An experimental set-up was designed to allow the study of the normal impact and the impact with a crossflow. For the present study the droplet was exposed to the influence 7m/s of a crossflow velocity and four substances were considered 100% JF; 75% JF-25% HVO; 50% JF-50% HVO and H₂O as a reference substance. The different impact phenomena were visualized and transition conditions between impact regimes were identified. Lastly, the results were compared to the transition criteria proposed by other authors.

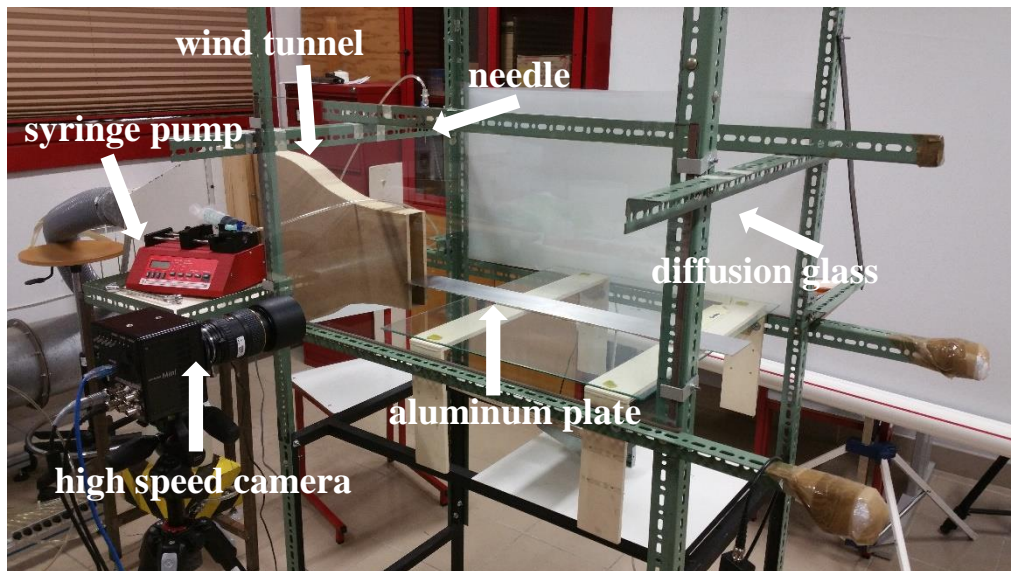


Figure 1. Experimental set-up.

2. Experimental Method

Details of the experimental setup can be found in Cunha [13] and only a summary is given here. The experimental facility is presented in figure 1. A ventilator with 15kW was used to provide a crossflow with 3000 m³/h through a low-speed wind tunnel. The wind tunnel has a rectangular exit nozzle with 200x40 mm² and the impact surface is a smooth aluminium plate with 700x80 mm². This plate is placed 70 mm above the base of the wind tunnel exit nozzle and it is equidistant to two glasses, one of them is a diffusion glass used to spread the light evenly. The wind tunnel is used to provide a crossflow of 7 m/s. Figure 2 shows the velocity profiles 1mm downstream of the wind tunnel exit. The light is provided by a 20W led light (behind the diffusion glass) and it is aligned with the impact surface and directly across from the high-speed camera. The droplets are formed by a needle with an inner diameter of 1.5 mm connected to a syringe pump with a pumping rate of 0.5 ml/min. The different impact velocities were achieved by placing the syringe at different heights above the impact surface. Four fluids were used: 100% JF, 75% JF - 25% HVO, 50% JF - 50% HVO and H₂O. Table 2 presents the physical properties measured for these fluids.

A high-speed camera, FASTCAM mini UX50, was used to capture the motion of the droplets. It has a 1.3 Megapixel image resolution at frame rates up to 2000fps and it can go up to 160000fps with a resolution reduction. Also, a Macro Lens Tokina AT-X M100 AF PRO D was used, it has a minimum focus distance of 300mm, a focal

length of 100mm, a 1:1 macro ratio and a filter size of 55mm. To be able to observe the phenomena the images were taken with a frame rate of 10000fps and a shutter of 1/10240s.

The quantitative characterization of the droplets characteristics was made through image data processing. An algorithm was developed using the MATLAB Software to measure the diameter and the impact velocity. After the image binarization and with the pixel size value the droplet diameter was obtained (Table 3). To calculate the impact velocity, a function to find the centroid of the droplet was added to the algorithm and two frames were considered, at the instant of the droplet impact ($\tau = 0ms$) and the instant $0.5\mu s$ before the droplet impact ($\tau = -0.5ms$). The impact velocity range considered was between 1.78m/s and 4.76m/s. For the present study, the maximum error obtained for the droplet diameter and impact velocity were $20.3\mu m$ and $0.0406m/s$, respectively.

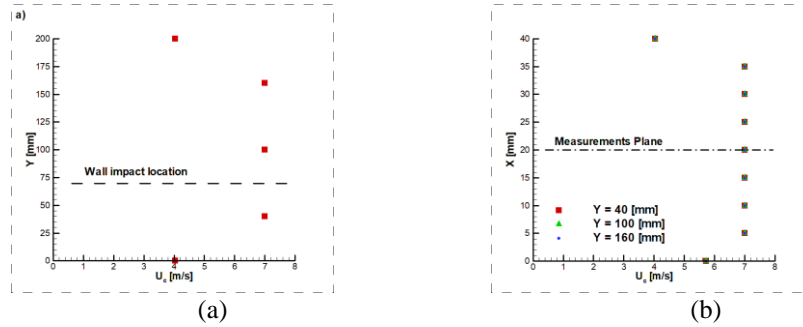


Figure 2. Velocity profiles 1mm downstream of the wind tunnel exit: a) vertical profile; b) horizontal profile.

Table 2. Physical properties of the substances.

Substances	ρ [kg/m ³]	$\sigma \cdot 10^3$ [N/m]	$\mu \cdot 10^3$ [Pa.s]
H ₂ O (*literature)	1000.0	72.0	1.00
100% Jet A-1	798.3	25.4	1.12
75% JF-25% HVO	794.9	25.5	1.44
50% JF-50% HVO	792.3	24.6	1.79

Table 3. Droplet diameter.

Substances	D_0 [mm]
H ₂ O (*literature)	4.1
100% Jet A-1	3.0
75% JF-25% HVO	3.1
50% JF-50% HVO	3.1

3. Results and Discussion

3.1 Phenomena Visualization

In this work, for the normal impact of a single droplet upon a dry wall three different phenomena were spotted: deposition, prompt splash and fingering. In figure 3 (a) there is a sequence of images which shows deposition for the 75%JF - 25%HVO mixture. As can be seen, the droplet impinges the surface ($\tau = 0ms$) and prompt splash did not occurred ($\tau = 0.1ms$), none droplet were ejected. The droplet spreads radially in the surface ($\tau = 0.5ms$) until it reaches the maximum spreading diameter ($\tau = 4.4ms$). After this point, the liquid film started to recoil ($\tau = 5ms$) to its minimum diameter ($\tau = 10ms$). Figure 3 (b) shows a sequence of images where prompt splash occurred for the 50%JF - 50%HVO mixture. The droplet impinges the surface ($\tau = 0ms$) and immediately after the impact it is possible to see the creation of secondary droplets ($\tau = 0.1ms$), therefore, prompt splash. The droplet continues its spreading ($\tau = 0.5ms$) until reaches its maximum spreading diameter ($\tau = 4.5ms$). After that, the liquid film started to recoil ($\tau = 5ms$) until its minimum diameter ($\tau = 10ms$). Finally, figure 3 (c) shows a sequence of images of a droplet H₂O where no prompt splash was spotted but fingering occurred. Similarly to (a), after the droplet impingement, none droplets were ejected ($\tau = 0.1ms$) and the droplet spreads radially in the surface ($\tau = 0.5ms$). At $\tau = 0.7ms$ it is possible to see the formation of tiny fingers in the liquid lamella. With time they become more evident and the maximum spreading diameter is reached ($\tau = 4ms$). After that, the liquid film started to recoil towards the impact point ($\tau = 10ms$) until reaches its minimum diameter ($\tau = 19ms$). It is important to mention that none finger broke.

In the impact influenced by the crossflow, it was possible to see prompt splash, ‘‘crown’’ splash and also fingering (figure 4). As can be seen, the shape of the droplet was clearly influenced by the crossflow ($\tau = -0.5ms$). The droplet impinges the surface with a certain impact angle ($\tau = 0ms$) and prompt splash immediately occurred ($\tau = 0.1ms$). Due to the crossflow influence, the secondary droplets ejected seemed to be lift upstream in a crown-like shape, similarly to the observed for crown splash but only in the upstream side of the impinging droplet ($\tau = 0.5ms$). Droplets continued to be ejected and fingers started forming at the lamella ($\tau = 1ms$). At $\tau = 2ms$ the crown-like sheet was completely disintegrated into secondary atomisation. The liquid film spreads until reaches its maximum spreading diameter at the upstream side of the impact point ($\tau = 3.5ms$) and starts to recoil ($\tau = 3.6ms$). In this case, some fingers broke originating the separation of small portions of the liquid film. At the downstream side of the impact point the liquid film spreads continually and reaches its maximum at $\tau = 6.7ms$ after the droplet impact. Finally, the liquid film recoils to its minimum diameter ($\tau = 17.5ms$).

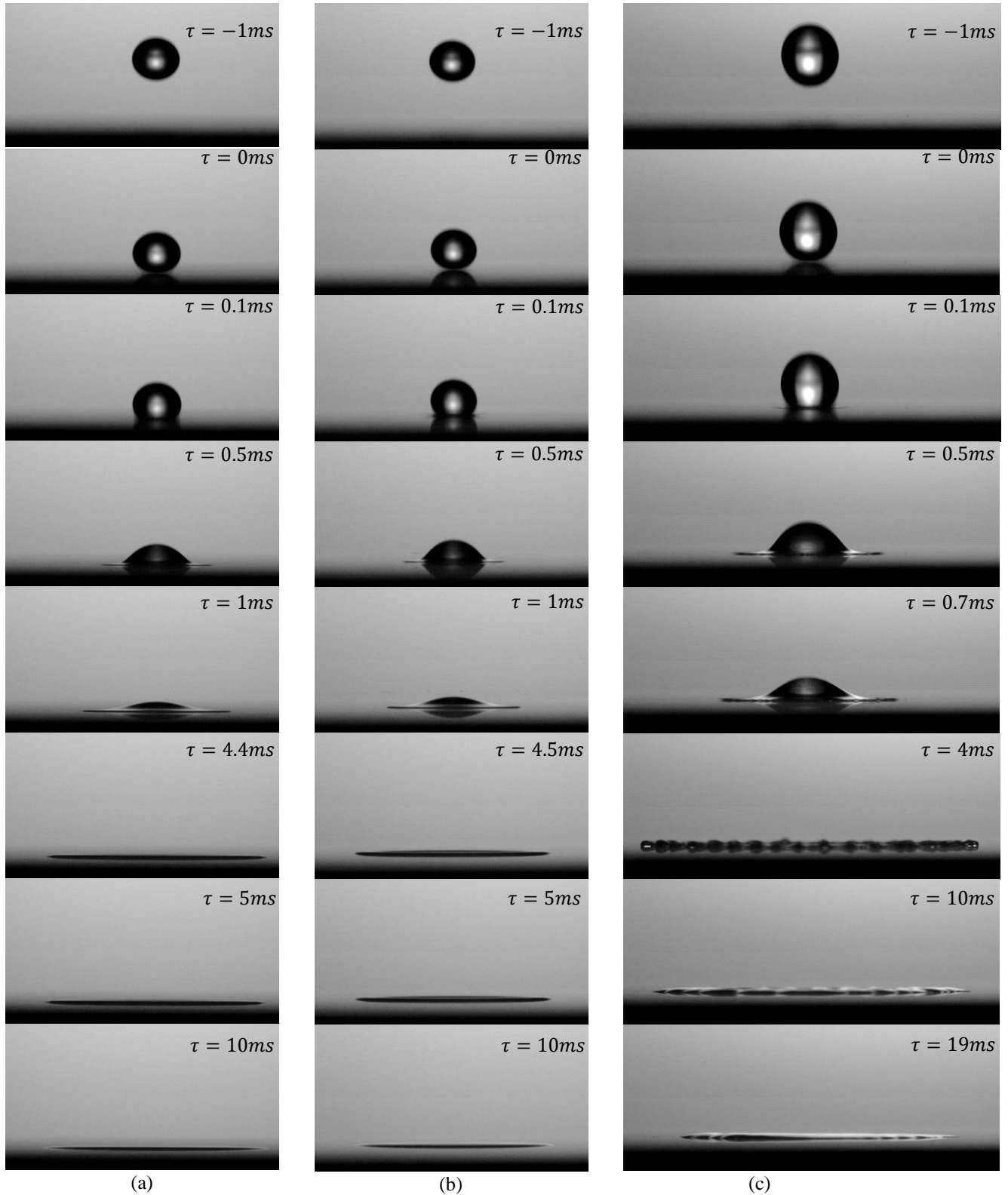


Figure 3. Image sequences of the normal impact of a single droplet upon a dry wall: a) the spreading of the droplet for the 75%JF-25%HVO mixture ($D_0 = 3.1mm$, $Re = 4967$, $We = 825$, $La = 29912$); b) the prompt splashing of the droplet for the 50%JF-50%HVO mixture ($D_0 = 3.1mm$, $Re = 3863$, $We = 798$, $La = 18708$); c) the fingering of the droplet for H₂O ($D_0 = 4.1mm$, $Re = 15588$, $We = 828$, $La = 293454$).

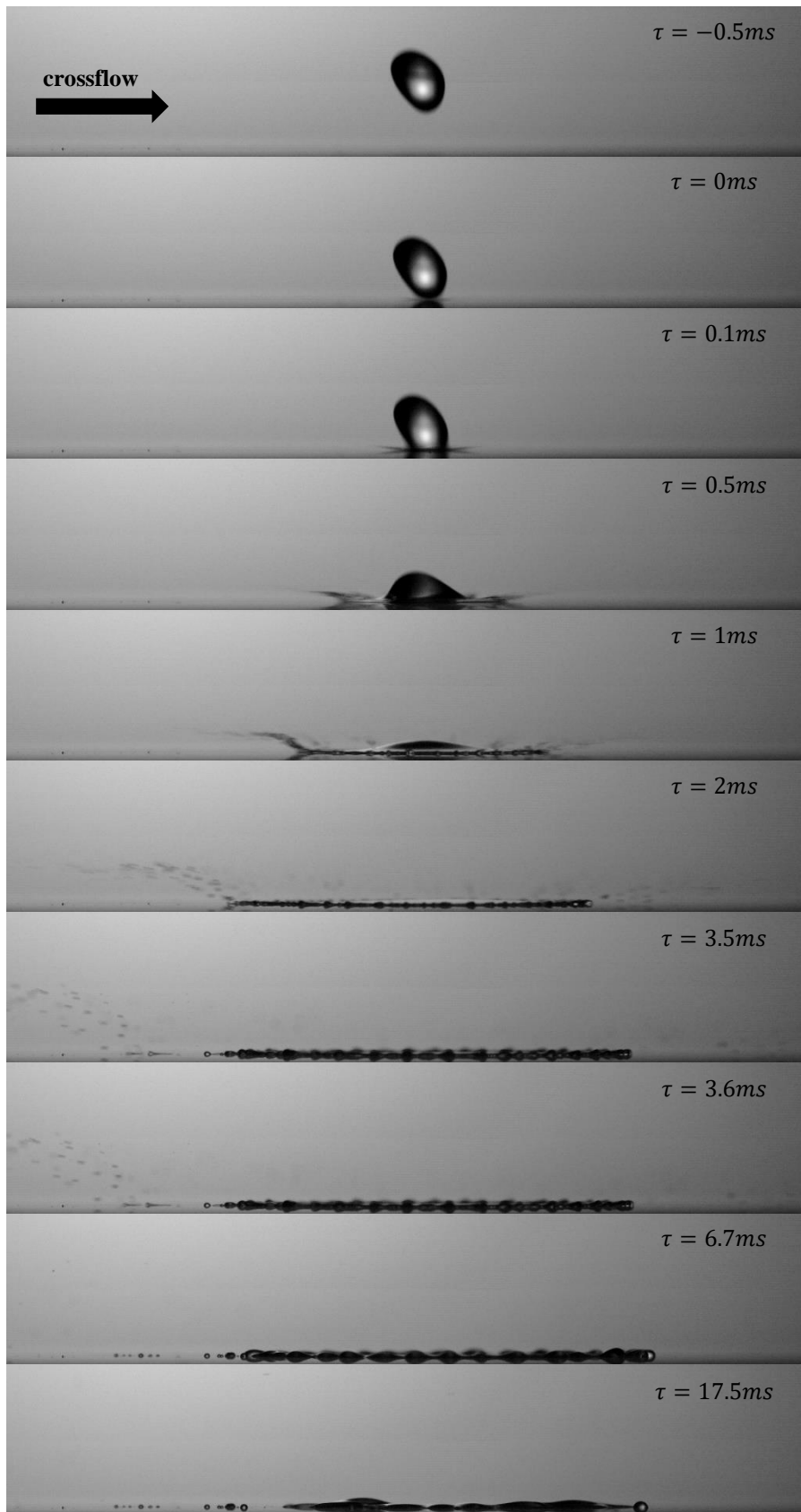


Figure 4. An H₂O droplet with 4.1 mm original diameter impinging onto an aluminum plate in a crossflow of 7 m/s ($Re = 18618$, $We = 1181$, $La = 293454$).

3.2 Splash/Deposition Threshold Analysis

Two different types of impact were analysed, normal impact and crossflow impact. For normal impact the crossflow was 0m/s and for crossflow impact the crossflow was 7m/s. The difference between deposition and splash is considered here to be the minimum interval obtainable with the experimental facility. The impact velocity, droplet size, and dimensionless numbers are presented in Table 4 for the normal impact. For the mixtures, JF-HVO, it was needed a higher impact velocity for the transition between deposition and splash could be observed at similar impact conditions to the 100% JF. The surface tension and density values are almost constant for these mixtures. Comparing the H₂O to the remaining results, it is noticed that this fluid presents a different range of all the variables studied (droplet diameter, impact velocity, fluid physical properties, etc).

Table 4. Deposition/Splash threshold analysis for a normal impact.

Substances	100% Jet A-1		75% JF–25% HVO		50% JF–50% HVO		H ₂ O	
	Deposition	Splash	Deposition	Splash	Deposition	Splash	Deposition	Splash
Regime								
U_0 [m/s]	2.03	2.12	2.94	3.01	2.70	2.84	3.86	4.01
Re	4361	4572	4966	5075	3671	3863	15588	16211
We	390	429	825	861	720	798	828	898
$Oh \cdot 10^3$	4.528		5.782		7.311		1.85	
La	48765		29912		18708		293454	
D_0 [mm]	3.0		3.1		3.1		4.1	

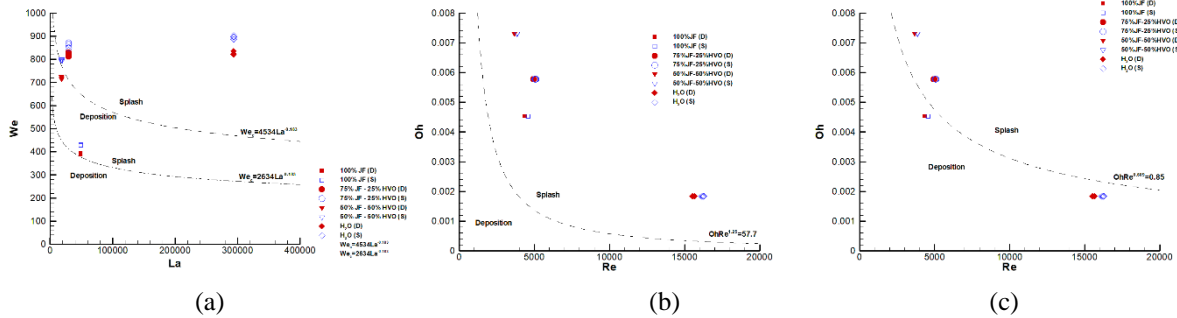


Figure 5. Droplets behaviour for normal impact compared a) to Bai and Gosman’s [2] criterion with different A coefficient values; b) to Mundo et al. [4] criterion; c) to Vander Wal et al. [11] criterion.

These experimental results were compared to the empirical correlations mentioned before (Table 1) and presented in figure 5. The dashed lines represent the empirical correlation proposed, the blue data points represent the cases where splash occurred and the red ones where splash was not observed. Bai and Gosman’s [2] correlation enables the use of different surface roughness values (figure 5 a). The impact surface of the present study was considered smooth, the values of the A coefficient reported in the literature for smooth surfaces were $A=4534$ and $A=2634$. Figure 5 (a) shows a difference between the behaviours of the 100% JF and H₂O and the two mixtures. For the normal impact, the 100% JF seems to have some proximity to the boundary proposed with the higher surface roughness value. On the other hand, the mixtures have a close proximity to the boundary with the lower surface roughness. This might suggest that the empirical correlation proposed with the lower surface roughness, could be used to predict the transition criteria for the mixtures.

Figure 5b) presents the experimental data compared to the Mundo et al. [4] boundary and it was observed that all the experimental results obtained were plotted in the splash area. Despite this, the results seem to have the same tendency as the empirical correlation proposed. Vander Wal et al. [11] criterion (figure 5 c) seems to give the better correlation if all substances were considered. Despite the fact that the results for 100% JF and H₂O are in the deposition area and the mixtures in the splash area, there is an overall proximity to the boundary line suggested.

Mundo et al. [4] correlation seems to be the one that differs more from the behaviour observed but, it could be adjusted with further experimental studies. Vander Wal et al. [11] used an aluminium disk with a mean surface roughness of less than 10 nm which could explain the proximity between the experimental data and the correlation. However, to be able to conclude this, it would be necessary to measure the surface roughness of the aluminium plate used in the present work. This is also supported by observing figure 5 (a), the experimental results are closer to the boundary line with lower surface roughness.

Table 5 presents the impact velocity, droplet size, impact angle and dimensionless numbers for the crossflow impact. The crossflow velocity was 7m/s and the impact angle was defined as the angle between the impact surface and the impact velocity vector. Two studies were considered, first it was verified which regime occurred in the presence of the crossflow with the same conditions where splash first occurred for the normal impact. From the results it was possible to observe that deposition occurred for all the substances. Subsequently, the conditions of the experimental facility were varied until the splash regime be observed in the presence of a crossflow. Results

show that the impact angle increases with the increase of the droplets impact velocity. However, the vertical velocity also increases, which might suggest that when the droplet impact velocity was higher the effect of the crossflow in the droplet was diminished. The splash phenomenon seems to require higher impact velocities for the 75% JF-25% HVO mixture to occur. The results also suggest that the influence of the horizontal velocity was small for the existence of splash regime.

Figure 7 compared the experimental results with crossflow with the empirical correlations from Table 1. As in figure 5, the dashed lines represent the empirical correlation proposed, the blue data points represent the cases where splash occurred and the red ones where splash was not observed. Figure 7 (a) shows that the criteria proposed by Bait and Gosman [2] fails to predict the transition regime between the deposition and splash outcomes for the impact of the water droplets, whether with or without the presence of crossflow. However, the 100% JF seems to have some proximity to the boundary proposed with the higher surface roughness value, and the mixtures have a close proximity to the boundary with the lower surface roughness. Mundo et al. criterion [4], shown in figure 7 (b), seems to do not give a good correlation for the experimental results obtained. Considering that every point of the experimental results is presented in the splash area. Although, if a trend line was created with the results of the present study it would be almost parallel to the one proposed. Vander Wal et al. [11] correlation seems to be the closer boundary line if all the substances are considered (figure 7 c).

Table 5. Deposition/Splash threshold analysis for a 7 m/s crossflow impact.

Substances	100% Jet A-1		75% JF-25% HVO		50% JF-50% HVO		H ₂ O	
	Deposition	Splash	Deposition	Splash	Deposition	Splash	Deposition	Splash
Regime								
U_0 [m/s]	2.21	2.29	2.86	3.50	2.73	3.14	4.01	4.61
Re	4760	4928	4834	5903	3706	4264	16226	18618
We	465	498	781	1165	734	972	897	1181
θ_c [°]	60.4	62.2	72.6	75.7	69.8	72.1	85.2	86.9
U_y [m/s]	1.92	2.03	2.73	3.39	2.56	2.99	4.00	4.60
U_x [m/s]	1.09	1.06	0.86	0.86	0.94	0.97	0.34	0.25
$Oh \cdot 10^3$	4.528		5.782		7.311		1.85	
La	48765		29912		18708		293454	
D_0 [mm]	3.0		3.1		3.1		4.1	

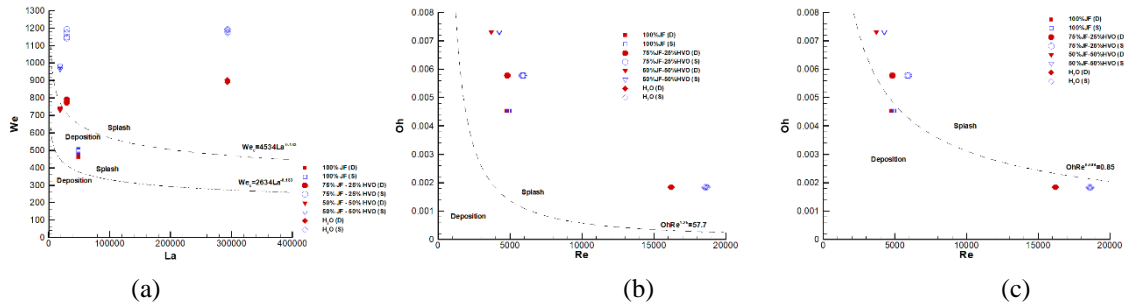


Figure 7. Droplets behaviour for the impact with a crossflow compared a) to Bai and Gosman's [2] criterion with different A coefficient values; b) to Mundo et al. [4] criterion; c) to Vander Wal et al. [11] criterion.

Table 6. Same initial parameters for a normal impact and a crossflow impact.

Substances	100% Jet A-1		75% JF-25% HVO		50% JF-50% HVO		H ₂ O	
	U_c [m/s]	Regime	U_c [m/s]	Regime	U_c [m/s]	Regime	U_c [m/s]	Regime
U_c [m/s]	0	7	0	7	0	7	0	7
Regime	Splash	Deposition	Splash	Deposition	Splash	Deposition	Splash	Deposition
U_0 [m/s]	2.12	2.21	3.01	2.86	2.84	2.73	4.01	4.01
Re	4572	4760	5075	4834	3863	3706	16211	16226
We	390	465	861	781	798	734	898	897
θ_c [°]	90	60.4	90	72.6	69.8	69.8	90	85.2
U_y [m/s]	2.12	1.92	3.01	2.73	2.84	2.56	4.01	4.00
U_x [m/s]	0	1.09	0	0.86	0	0.94	0	0.34
D_0 [mm]	3.0		3.1		3.1		4.1	

Table 6 presents the results for the same initial parameters for a normal impact and a crossflow impact. However, the impact conditions and outcome phenomena differ. This table presents the results for the splash occurrence for the normal impact and the deposition occurrence for the crossflow impact. Through this table it is observable that the crossflow affects the droplet impact outcome. For the same initial conditions, the impact velocity of the mixtures is lower for the crossflow impact. This suggests that the crossflow decreases the impact velocity. Nonetheless, for 100% JF and H₂O, this was not observed. In the 100% JF case, the absolute velocity was higher.

This might suggest that the impact angle affects the droplet outcome since splash was not observed for the crossflow impact. Also, the results show that the tangential component to the surface is larger for droplets with smaller diameters, which can be an explanation for the impact velocity increase. Droplets with smaller size are more quickly affected by the crossflow. The analysis of the H₂O case might support this since the impact velocity for this fluid did not have a significant variation, neither comparing the type of impact nor the two velocity components. This fluid had the largest droplet size which suggests that the crossflow velocity was not high enough to influence the droplet impact. Other explanation could be that the crossflow cause deformation in the droplet, delaying the splash phenomenon.

Table 7 presents the results for the parameters where splash occurred both for normal impact and crossflow impact. In this case, both the initial parameters and impact conditions differ. These results suggest that the presence of a crossflow delays the occurrence of splash. For the majority of the substances, both the velocity impact and the vertical velocity component had to increase so the splash phenomenon could be observed, except for the 100% JF case. In this last case, only the impact velocity was higher than in the normal impact. This could suggest that the impact velocity has a bigger influence on the impact outcome. Still, the 100% JF is the only substance where this happened. The remaining results suggest that the vertical velocity components need to be higher in the crossflow impact than in the normal impact for the same outcome to occur. Excluding the 100% JF, the experimental data suggests that the vertical velocity could have a major role in the impact outcome.

Table 7. Splash occurred both for a normal impact and a crossflow impact.

Substances	100% Jet A-1		75% JF-25% HVO		50% JF-50% HVO		H ₂ O	
U_c [m/s]	0	7	0	7	0	7	0	7
U_0 [m/s]	2.12	2.29	3.01	3.50	2.84	3.14	4.01	4.61
Re	4572	4928	5075	5903	3863	4264	16211	18618
We	390	498	861	1165	798	972	898	1181
θ_c [°]	90	62.2	90	75.7	69.8	72.1	90	86.9
U_y [m/s]	2.12	2.03	3.01	3.39	2.84	2.99	4.01	4.60
U_x [m/s]	0	1.06	0	0.86	0	0.97	0	0.25
D_0 [mm]	3.0		3.1		3.1		4.1	

Summary and Conclusions

The goal of the present work was to analyse the phenomena obtained by the impact of a single droplet of mixtures with biofuel and conventional jet fuel on a dry surface with and without a crossflow. An experimental setup was built and four substances were considered 100% JF; 75% JF-25% HVO; 50% JF-50% HVO and H₂O. The same initial parameters had different outcomes regarding normal impact and crossflow impact. For the majority of the substances, the impact velocity and the vertical velocity had to be increased in order to observe the splash phenomenon (comparing to the normal impact). Vander Wal et al. [11] empirical correlation for the deposition/splashing boundary gives a better approximation for all the substances. The two mixtures presented different behaviours compared to the 100% JF and H₂O suggesting that adding a biofuel to a convention jet fuel alters the impact outcome.

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References

- [1] R. Rioboo, C. Tropea, M. Marengo, *Atomization and Sprays*, vol.11, no.2, p.12 (2001)
- [2] C. X. Bai and A. D. Gosman, *Society of Automotive Engineers* (1995)
- [3] O. Jayaratne and B. J. Mason, *Proceedings of the Royal Society of London Series A-Mathematical and Physical*, vol. 280, no. 138, pp. 545–565 (1964)
- [4] C. Mundo, M. Sommerfeld, C. Tropea, *Int. Journal of Multiphase Flow*, vol. 21, no. 2, pp. 151–173 (1995)
- [5] S. C. Yao and K. Y. Cai, *Experimental Thermal and Fluid Science*, vol. 1, no. 4, pp. 363–371 (1988)
- [6] A. Moita and A. L. N. Moreira, Proc. 9th ICLASS-Europe, 2003
- [7] A. Yarin, *Annual Review of Fluid Mechanics*, vol. 38, no. 1, pp. 159–192 (2006)
- [8] C. Mundo, M. Sommerfeld, and C. Tropea, *Atomization and Sprays*, vol. 8, pp. 625–652 (1998)
- [9] H. Zhang, B. Bai, L. Liu, H. Sun, J. Yan, *Experimental Thermal and Fluid Science*, vol. 45, pp.25–33 (2013)
- [10] C. D. Stow and M. Hadfield, *Proceedings of The Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 373, pp. 419–441 (1981)
- [11] R. L. Vander Wal, G. M. Berger, and S. D. Mozes, *Experiments in Fluids*, vol. 40, no. 1, pp. 53–59 (2006)
- [12] R. L. Vander Wal, G. M. Berger, and S. D. Mozes, *Experiments in Fluids*, vol. 40, no. 1, pp. 33–52 (2006)
- [13] N. F. C. Cunha, *Master Dissertation*, University of Beira Interior, (2018)