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The friction of a sliding plate on a thin immersed granular layer obeys Amonton-Coulomb law. We bring to the fore a large set of experimental results which indicate that, over a few decades of values, the effective dynamical friction-coefficient depends neither on the viscosity of the interstitial fluid nor on the size of beads in the sheared layer, which bears out the analogy with the solid-solid friction in a wide range of experimental parameters. We accurately determine the granular-layer dilatancy, which dependance on the grain size and slider velocity can be qualitatively accounted by considering the rheological behaviour of the whole slurry. However, additional results, obtained after modification of the grain surface by a chemical treatment, demonstrate that the theoretical description of the flow properties of dense granular matter, even immersed, requires the detailed properties of the grain surface to be taken into account.

PACS: 47.57.Gc: Granular flow; 83.50.Ax: Steady shear flows, viscometric flow; 83.80.Hj: Suspensions, dispersions, pastes, slurries, colloids; 81.40.Pq: Friction, lubrication, and wear.

Conducting studies on immersed granular flows remains of primary interest. A host of geophysical or industrial issues such as submarine avalanches [1] and snow flows or clay suspensions [2] deals with mixtures of grains and fluid. Also fundamental issues are at stake: one would like to extend the empirical friction law proposed for dense and dry granular flows [3] to immersed ones [1]. And, in the limit of the low shearing-rates, the control parameters of the jamming-unjamming transition remains to be clearly identified [4]. Enlightening previous studies of sheared and immersed granular media are numerous, and different devices have been developed to describe mixtures of grains and fluids (meaning air or liquids). Among them we choose to focus on the three following.

Studying immersed granular matter flowing down an inclined plane, C. Cassar *et al.* measured the dynamical friction-coefficient, μ, for different flow configurations [1]. Their results were analyzed using an approach inspired by recent results obtained for dry and dense granular flows [3]: They report the friction coefficient as a function of the dimensionless parameter I, first introduced by Da Cruz *et al.* [5], defined to be the ratio of an apt microscopic time scale (inertial, viscous, ...) to the relevant macroscopic time scale $\dot{\gamma}^{-1}$, where $\dot{\gamma}$ denotes the shear rate. For immersed granular-matter in the viscous regime [6], $I \equiv (\dot{\gamma}\,\eta_f)/(\alpha\,P_g)$, where η_f denotes the viscosity of the interstitial fluid, P_g the pressure exerted on the sheared media and α the normalized permeability of the granular packing [1]. They propose a semi-empirical law for $\mu(I)$ which describes the whole set of data they report for both aerial and immersed granular flows.

Using a Couette geometry Bocquet *et al.* tuned the pressure within the granular material by applying an upward air flow between the rotating and the stationnary cylinder [7]. They found out that mean-flow properties and fluctuations in particle motion are coupled. They introduced an hydrodynamic model which quantitatively describes their experiments: The shear force obtained from this model is found to be proportional to the pressure and approximately independent of the shear velocity. This model does not include any frictional forces between grains, but contains a phenomenological relationship between the viscosity and the dilation of the media.

Using an experimental setup first designed to perform sensitive and fast force-measurements in the dry case [8], Géminard *et al.* brought to the fore a dynamic friction-coefficient μ in the case of an immersed granular layer sheared by means of a sliding plate [9]. At low imposed normal-stress, the friction force is shown to be independent of the plate velocity, which holds true as long as the granular material is allowed to dilate [10]. The main difference with the dry case lays in the fact that the slider usually exhibits a continuous sliding instead of the stick-slip motion and in the value of the friction coefficient which is roughly cut down by half [9, 11]. The dependance of the frictional coefficient on the fluid viscosity and of the associated dependance of the dilatancy on the slider velocity were not reported.

Here we report a set of experimental measurements of the friction coefficient and dilatancy in a wide range of fluid viscosities and grain sizes at very low I. Such a study is relevant for several reasons: First, the quasi-static regime is unaccessible to the free-surface-flow geometry as size effects crop up in this limit [1]. In addition, there is a strong discrepancy between the limit of μ for vanishing I reported in [1] ($\mu \simeq 0,43$ with $I \simeq 4.10^{-3}$) and those reported for the plane-shear geometry by Géminard *et al.* ($\mu \simeq 0.23$, [9, 11]) and S. Siavoshi *et al.* ($\mu \simeq 0.54$, [12]), both for $I \simeq 2.10^{-4}$. How can be explained such discrepancies between those three results? We also raise the following questions: What does happen when the fluid viscosity or the bead size are changed? How far does the analogy with the Amonton-Coulomb laws remain relevant? What does the effective friction-coefficient depend on? We choose to stick to the canonical plane-shear geometry for which we know that there is a strong analogy between the friction of a slider on an immersed granular layer and the Amonton-Coulomb law [9]. In the chosen geometry the layer is free to dilate, which makes it possible to measure both the friction coefficient and the dilation of the granular layer at imposed normal stress.

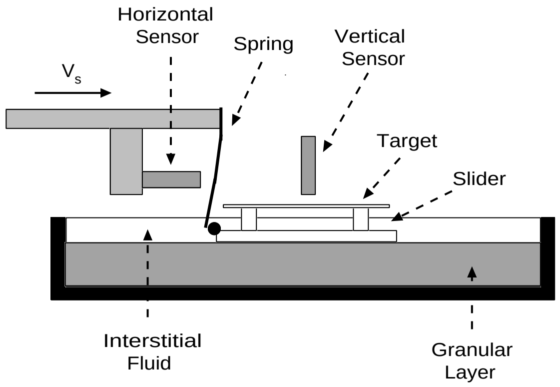

FIG. 1: **Sketch of the experimental setup.**

Experimental setup. - The experimental setup (Fig. 1) is very similar to the one described in [9, 11]. A thin plate, the slider, is pushed at the free surface of an immersed granular layer by means of a steel leaf-spring ($k = 129 \pm 2$ N.m^{-1}) connected to a translation stage driven at constant velocity, V_s, by a computer-controlled stepping motor (V_s ranging from 0.1 to 100 μm.s^{-1}). The coupling between the spring and the plate is insured by a metal bead, which avoids applying a torque. The frictional force is monitored by measuring the receding of the spring from its rest position with an inductive sensor (ElectroCorp, EMD1053). The dilatancy is obtained from the vertical displacement of the slider: A second inductive sensor, at rest in the laboratory referential, monitors the distance to a metallic target which endows the slider, which consists in a thin (5 mm) PMMA plate (76×53, 53×51, or 53×24 mm^2). The granular material consists in spherical glass beads (Matrasur Corp.) sieved in order to obtain the three following mean diameters $d = (100 \pm 11)$, (215 ± 20) and (451 ± 40) μm, with a relative standard deviation almost independant of the characteristic grain size. The interstitial fluid consists in distilled water, water and sugar mixtures (viscosity η ranging from 1 to 76 mPa.s), or Rhodorsil silicon oil (Rhodorsil, viscosity η ranging from 71 mPa.s to 500 mPa.s). All viscosities were measured, in addition, using an Ubbelohde viscosimeter. The thickness of the granular bed (6.0 mm) is always larger than ten bead-diameters so that the sheared zone is not limited by the bottom of the container and, thus, that edge effects are not at stake [12]. Finally, the contact of the slider with the granular layer is insured by gluing a layer of the largest beads (451 μm) onto the lower surface. We checked, for a layer of 215 μm-in-diameter beads, that the friction coefficient is independent of the size of the glued beads as long as it remains larger than that of the beads in the granular bed (table I, Top).

d (μm)	100	215	451
μ_d (μm)	0.33 ± 0.02	0.38 ± 0.02	0.37 ± 0.02
S (mm^2)	53×24	53×51	76×53
μ_d (μm)	0.38 ± 0.02	0.42 ± 0.02	0.38 ± 0.02

TABLE I: Top: Measured friction coefficient μ as a function of the diameter of the beads that insure the contact at the bottom surface of the slider (The sample consists of 215 μm beads in water); Bottom: Friction coefficient μ measured with sliders having different surface area and aspect ratio (100 μm beads in a water-sugar mixture, vicosity $\eta = 4.3$ mPa.s.)

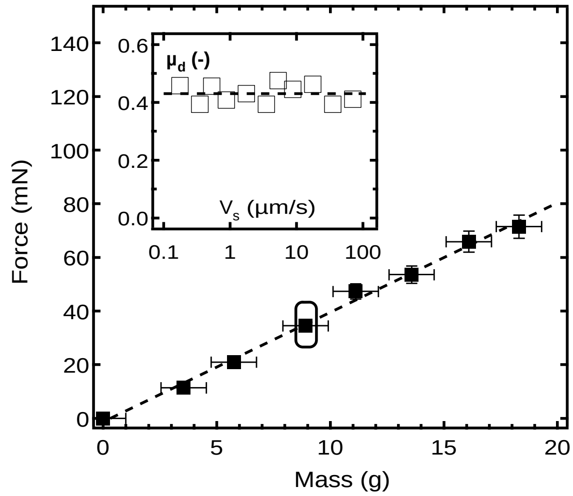

FIG. 2: **Dynamic frictional force vs. effective mass of the slider**. The effective mass of the slider is obtained by reducing the weight by the buoyancy force. From the slope one can infer $\mu = 0.41 \pm 0.02$ ($V_s = 3.5$ μm.s^{-1}, 451 μm beads in silicon oil, viscosity $\eta = 500$ mPa.s). Inset: Friction coefficient vs slider velocity V_s in the same experimental conditions for the mass pointed by the rectangle. The size of the symbols indicates the error bars.

Friction coefficient. - In our experimental conditions, the value of the spring constant k is chosen so as to observe the continuous motion of the slider in the whole accessible range of the driving velocity V_s. After a transient regime, the frictional force reaches a steady-state value which is observed to scale up with the slider mass m (Fig. 2), provided that the buoyancy force is taken into account, and to be independant of the slider velocity V_s (inset, Fig. 2). In addition, we checked that μ does not significantly depend on the slider surface-area or aspect-ratio (Table I, Bottom), as already known for the dry case [13]. We repeated the procedure for different bead-diameter (d from 100 μm to 450 μm) and fluid viscosity (η from 1 mPa.s to 500 mPa.s). We found out that μ neither depends on η nor on d in the whole experimental range. In order to encompass those two results and the independence on the slider velocity V_s, we report μ as a function of the Reynolds number $Re \equiv \rho\, d\, V_s/\eta$, where ρ stands for the fluid density. We estimate $\mu = 0.38 \pm 0.03$ for $10^{-5} \leq I \leq 5.10^{-3}$, which nicely supplements the data reported for the free-surface-flow configuration in [1] that limited to $I \geq 4.10^{-3}$.

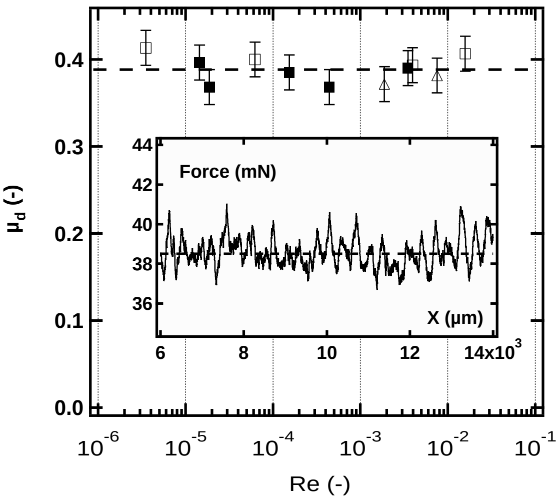

FIG. 3: **Friction coefficient vs. the Reynolds number**. We report data for three different diameters d: ■: 100 μm; △: 215 μm; □: 451 μm (η ranging from 1 to 500 mPa.s). Note that μ is constant in a range of Re covering more than 4 orders of magnitude. Inset: Frictional force in the steady state regime vs slider position. The dynamic friction-coefficient is defined to be the mean value of the frictional force ($V_s = 8.8\,\mu$m.s^{-1}, $\eta = 1$ mPa.s and $m = 10.1$ g).

Layer dilation. - Experimentally, the moot point consists in obtaining a reproducible reference state. The chosen procedure is as follows: In order to obtain a well-defined state of compaction, we initially push the slider over a distance of approximately 10 bead-diameters in the steady regime at a given velocity, henceforth denoted V_{ref} (usually $8.8\,\mu$m.s^{-1}, excepted when specified). We then stop the translation stage and move it backwards until the spring goes back to its rest position without loosing contact with the slider, which remains at rest (contact loss could make the slider surf over the granular layer as we push it forth at large velocity, meaning above $40\,\mu$m.s^{-1}). We then immediately push the slider forwards at various driving velocities, V_s, over a few millimeters and monitor the vertical position of the plate. We observe that the total variation of the vertical position of the plate Δh, or total dilation, increases with the bead diameter d and the velocity V_s. By constrast, Δh does not significantly depend on the interstitial-fluid viscosity η (Fig. 4). We checked that these latter measurements neither depend on the preparation of the granular layer (by varying the velocity of reference V_{ref}), nor on the slider mass m (Fig. 4, inset).

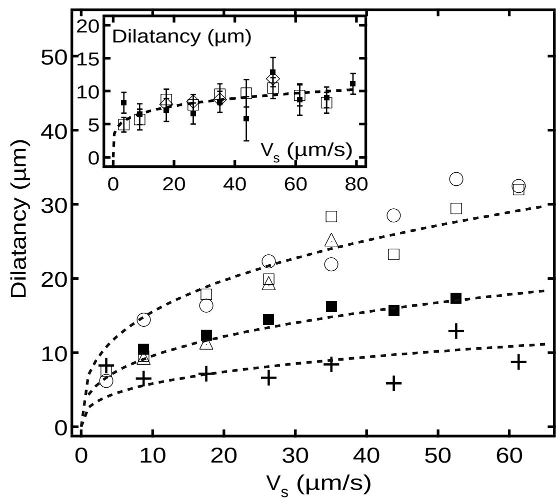

FIG. 4: **Total dilation of the layer Δh vs. slider velocity V_s.** (symbol, bead diameter, fluid viscosity): (+, $100\,\mu$m, 1 mPa.s); (■, $215\,\mu$m, 1 mPa.s); (△, $451\,\mu$m, 1 mPa.s); (□, $451\,\mu$m, 71 mPa.s); (∘, $451\,\mu$m, 500 mPa.s); The dashed lines correspond to the interpolation of the experimental data to Eq. 1 with $\beta = 2.9 \pm 0.3$. Inset: similar results for different reference velocity and normal stress: (■, $100\,\mu$m, $V_{ref} = 3.5\,\mu$m.s^{-1}); (□, $100\,\mu$m, $V_{ref} = 14\,\mu$m.s^{-1}); (⋄, $100\,\mu$m, $V_{ref} = 3.5\,\mu$m.s^{-1} and overloaded by 10.0 g).

Discussion. - The dependence of the total dilation on the velocity and on the bead diameter can be accounted for by the two following ingredients: First, we can guess that, due to the steric interaction between the grains (solid contact between grains or hydrodynamical interaction), the local shear-stress σ_s induces a local normal-stress $\sigma_n = \alpha(\phi)\,\sigma_s$. In a first approximation, the coefficient α, which describes a geometrical property, depends only on the volume fraction of the grains, ϕ, and not on the shear rate, $\dot{\gamma}$. The assumption is correct, at least for a dense suspension in the limit of small $\dot{\gamma}$ [14]. We point out that, from this local relation between σ_n and σ_s, we recover the apparent friction law, $F = \mu\, mg$ where $\mu = 1/\alpha$, independent of the shear rate $\dot{\gamma}$ and slider surface-area S provided that α does not significantly depend on ϕ [14]. Second, we assume, as already proposed by Bocquet *et al* [7], that the rheological behavior of the immersed granular-material can be accounted for by $\sigma_s = \eta(\phi)\,\dot{\gamma}$, where the effective viscosity η diverges algebraically as a function of ϕ near a critical volume fraction ϕ_c: $\eta = \eta_0/(1-\phi/\phi_c)^{\beta}$ (We discuss the limit viscosity η_0 below.) At this point, assuming that the total dilation Δh is mainly due to the dilation of a constant number N of layers underneath the slider and linearizing the velocity profile in this region where the dilation is the larger, we write $\dot{\gamma} = V_s/(Nd)$ and get the following scaling law:

$$\frac{\Delta h}{d} \propto \left(\frac{\eta_0}{m}\frac{V_s}{d}\right)^{1/\beta} \quad (1)$$

Experimental dependances of Δh on V_s and d lead to $\beta = 2.9 \pm 0.3$ (Fig. 4), which seems reasonable once compared to the values reported in [7] and references therein. Because of the narrow range of the values accessible to the mass, the scaling law $\Delta h \propto m^{-1/\beta}$ can not be tested (inset Fig. 4). Finally, one could be tempted to interpret η_0 as the viscosity η_f of the interstitial fluid but the experimental data clearly demonstrate that $\eta_0 \neq \eta_f$.

For a dense material ($\phi \simeq \phi_c$), the interaction between the beads is likely not to be purely hydrodynamical and solid contacts might be at stake and contribute to the value of η_0 which, thus, might significantly differ from η_f. A few recent observations, dealing with the influence of the surface properties on the friction coefficient, proved that the roughness can drastically alter the dynamic angles of repose for dry materials [15] and even be a motor for shear-induced segregation in immersed materials [16]. In order to prove that, in our experimental conditions, the surface properties alter the value of η_0, and thus that of the friction coefficient μ, we performed experiments with 451 μm-in-diameter beads previously immersed for 30 minutes in a 1.0 mol.L^{-1} sodium-carbonate solution [17] and subsequently thoroughly washed up with distilled water. We obtained $\mu = 0.30 \pm 0.03$ (to compare to $\mu = 0.38 \pm 0.03$ previously obtained for the same sample before treatment) corresponding to a significant decrease of about 20%. This variation of the friction coefficient is only due to a change in the surface properties which are thus proven to play a role in the limit of low shearing rates, even for immersed granular matter.

As a conclusion, at low shearing rate, the friction coefficient μ of dense and immersed granular materials depends neither on the grain size nor on the viscosity of the interstitial fluid. As a consequence, measurements of μ, defined from the mean value of the friction force in the steady-state regime, does not provide any piece of information about the grain- or fluid-characteristics. As an extension of this study, we are currently focusing on the fluctuations of the frictional force (inset, Fig. 3) and dilation in the steady-state regime, from which we hope to recover a signature of the components of the slurry. In addition, the local properties of the grain surface play a significant role in the rheological properties and might be responsible for the dispersion of μ values encountered in the literature. Thus, we are also extensively studying the dependence of the friction coefficient on the grains-surface roughness. Our findings might help to understand how the physical properties, at the scale of the grains, alter the granular-flows macroscopic-properties.

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