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A new approach to model predictive control based on two degrees of freedom control and B-splines input shaping

Leopoldo Jetto, Valentina Orsini, Raffaele Romagnoli

***Abstract*—The purpose of this paper is to reduce some technical difficulties related to the complexity of stability and feasibility analysis of MPC as well as to reduce the complexity of the relative optimization procedure. The new approach is based on a two Degrees of Freedom (2DoF) control scheme where the output $r(k)$ of a feedforward Input Estimator (IE) is used as input forcing the closed loop system Σ_f. This latter is given by the feedback connection of a plant with a dynamic output controller. The task of the controller is to guarantee the stability of Σ_f, as well as the fulfillment of hard constraints for any $r(k)$ satisfying an "a priori" determined admissibility condition. The input $r(k)$ is computed through the on-line minimization of a quadratic cost functional and is applied to Σ_f according to the usual MPC strategy. To simplify the constrained optimization problem, the input $r(k)$ forcing Σ_f is assumed to be given by a B-spline function. This greatly decreases the number of decision variables of the on-line optimization procedure because B-splines are universal approximators which admit a parsimonious parametric representation. Moreover such parametrization allows us to reformulate the minimization of the cost functional as a box constrained least square problem. It is shown that stability and recursive feasibility of the adopted MPC strategy are guaranteed in advance, regardless the chosen prediction horizon.**

***Index Terms*—Predictive control for linear systems, Constrained control, Linear systems, LMIs**

I. INTRODUCTION

The complexity of stability and feasibility analysis and the demanding on-line computational burden are the major issues of MPC. As for the stability problem, two main approaches exist: finite, sufficiently large, prediction horizon with terminal constraints (see e.g. [1]- [4]), infinite prediction horizon (see e.g. [5]- [8]). A comprehensive review of research dealing with stability and feasibility is reported in [9]. As the length of the prediction horizon is a predominant factor determining the on-line numerical effort required by MPC, several authors dealt with the problem of estimating a "suitable length" for the prediction horizon [10]- [13]. A review of practical guidelines for tuning the prediction horizon is given in [14].

A drastic reduction of the on-line computational burden is obtained using explicit MPC [15]. By exploiting multiparametric programming techniques, the explicit MPC approach computes the optimal control action off line and the on-line operations reduce to a simple function evaluation. However, explicit formulation of MPC requires the partition of the state space into polyhedral regions whose number can grow exponentially with the problem size, thus implying a very rapidly increasing memory usage.

The purpose of the alternative approach proposed in this paper is to simplify dealing with the three aforementioned fundamental issues. The two basic features of the new method are the adoption of an MPC strategy in a 2DOF control scheme and the parametrization as a B-spline function of the input $r(k)$. In practice, the present method works according to the following two steps procedure:

Step 1. Let Σ_p be a discrete time LTI plant affected by an unmeasurable constant disturbance $d(k)$ with $\|d(k)\|_2^2 \leq \gamma_d$. An LTI feedback controller Σ_g is designed to guarantee the asymptotic stability of the closed-loop system Σ_f. Moreover Σ_g must guarantee that for any admissible norm bounded input $w(k) \triangleq \begin{bmatrix} r(k) \\ d(k) \end{bmatrix}$ (i.e. $\|w(k)\|_2^2 \leq \gamma$ for a suitably computed γ) forcing Σ_f, the hard constraints imposed on some physical variables are satisfied.

Step 2. The MPC strategy is applied to Σ_f. Over each prediction interval, the admissible sequence $r(k)$ minimizing a quadratic cost functional is searched in the linear space generated by B-spline functions of a fixed degree. This second step is executed by the feedforward IE.

The above two steps approach implies the following remarkable advantages:

1) Stability and recursive feasibility of the adopted MPC strategy are guaranteed in advance, regardless the chosen prediction horizon. In fact, the internal stability of Σ_f and the admissibility condition on $r(k)$ assure both the uniform boundedness of any internal variable of the 2DoF control scheme Σ_{2DoF} and the fulfillment of all constraints at any time instant.

2) Endowing Σ_g of a suitable internal model guarantees exact asymptotic tracking of the desired reference $\bar{y}_d(k)$ to be tracked even in the case of plant-model mismatch (provided stability is preserved). This greatly simplifies the alternative solutions which are mostly based on augmenting the model of the plant, which in turn implies an increase of the decision variables involved in the optimization problem (see e.g. [16] and references therein). The internal model also yields a Σ_f with a diagonal static gain matrix, so that it guarantees the remarkable advantage of an exact static decoupling [17].

3) Modeling $r(k)$ as a B-spline decreases the number of decision variables involved in the optimization because these functions admit a parsimonious parametric representation [18].

L. Jetto and V. Orsini are with Department of Information Engineering University Politecnica delle Marche, Ancona, Italy. R. Romagnoli is with Department of Electrical and Computer Engineering, Carnegie Mellon University. Corresponding Author: vorsini@univpm.it

Under the assumption of a measurable state vector, pre-stabilizing state feedback strategies are often used in the MPC. They are exploited to improve numerical conditioning in the optimization procedure (see e.g. [19], [20] and references therein) but do not guarantee either stability or recursive feasibility of the overall MPC scheme.

Improving numerical efficiency of MPC through a parametric representation of the control effort directly forcing the plant has been proposed in [21], where Laguerre functions are used. B-splines are used here because of their significant advantages: first, they admit a more parsimonious parametric representation and are more suitable to fit curves which exhibit different shapes over different time-intervals; second, B-splines belong to the convex hull defined by the relative control points [18]. This property allows the transfer of any amplitude constraint defined on a B-spline function to its control points. As a consequence (see Section VI), the constrained minimization of the cost functional can be formulated as a least square estimation problem with only box constraints on the unknowns (the control points defining the admissible B-spline function $r(k)$). Furthermore, the convexity property enables us to directly prove the internal stability of Σ_{2DoF} (see Theorem 3), while in [21], further assumptions are needed.

It is also mentioned that, in a different context, the use of B-splines together with MPC has been proposed in [22]. This reference deals with the trajectory generation problem and uses B-spline functions to parametrize the reference trajectory to be tracked. MPC is used to iteratively correct the initially chosen reference trajectory with the purpose of ensuring constraints satisfaction.

The paper is organized in the following way. A brief glossary is given in Section II. Some mathematical preliminaries are recalled in Section III, the problem setting is defined in Section IV, the design of the internal feedback controller is illustrated in Section V. The constrained on line estimation of the input $r(k)$ is explained in Section VI. A numerical example is reported in Section VII. Some concluding remarks are given in Section VIII.

II. Glossary

- $\mathbb{R}$ denotes the set of real number
- diag$[A\ B\ \cdots]$ denotes a block diagonal matrix
- the apex $'$ denotes transposition
- $\|\cdot\|_2$ denotes the euclidean norm
- $\lambda_{min}\{A\}$ ($\lambda_{max}\{A\}$) denotes the minimum (maximum) eigenvalue of A
- the bold symbol $\mathbf{0}$ denotes a null matrix of suitable dimensions, 0 denotes the null scalar.

III. Mathematical Background

A. B-spline functions [18]

Analytic B-splines are defined in the following way:

$$s(v) = \sum_{i=1}^{\ell} c_i B_{i,d}(v), \qquad v \in [\hat{v}_1, \hat{v}_{\ell+d+1}] \subseteq \mathbb{R}, \tag{1}$$

where the c_i's are real numbers representing the control points of $s(v)$, d is the degree of the spline, the $(\hat{v}_i)_{i=1}^{\ell+d+1}$ are the non decreasing knot points, and the $B_{i,d}(v)$ are given by the Cox-de Boor recursion formula [18].

Convex hull property. Any value assumed by $s(v)$, $\forall v \in [\hat{v}_j, \hat{v}_{j+1}]$, $\ j > d$, lies in the convex hull of its $d+1$ control points $c_{j-d}, \cdots, c_j$.

Smoothness property. Suppose that $\hat{v}_i < \hat{v}_{i+1} = \cdots = \hat{v}_{i+m} < \hat{v}_{i+m+1}$, with $1 \leq m \leq d+1$ then the B-spline function $s(v)$ has continuous derivative up to order $d-m$ at knot $\hat{v}_{i+1}$. This property implies that the spline smoothness can be changed using multiple knot points. It is common choice to set $m = d+1$ multiple knot points for the initial and the last knot points and to evenly distribute the other ones. In this way (1) assumes the first and the final control points as initial and final values.

Identifying the parameter v of (1) with the time instant t, the sampled B-spline $s(kT_c)$ is obtained by direct uniform sampling of the corresponding analytic B-spline.

The discrete B-spline $s(k)$ (omitting the explicit dependence on T_c) can be used to represent a scalar discrete time signal. Defining

$$\mathbf{c} \triangleq [c_1 \cdots c_\ell]', \quad \mathbf{B}_d(k) \triangleq [B_{1,d}(k) \cdots B_{\ell,d}(k)], \tag{2}$$

where each $B_{i,d}(k)$ is obtained setting $v = k$ and $\hat{v}_i = \hat{k}_i$, $i = 1, \cdots, d+\ell+1$, the sampled B-spline $s(k)$ can be represented as

$$s(k) = \mathbf{B}_d(k)\mathbf{c}, \quad k \in [\hat{k}_1, \hat{k}_{\ell+d+1}]. \tag{3}$$

For a q-component vector $\mathbf{s}(k) = [s_1(k) \cdots s_q(k)]'$, a compact B-splines representation can be used

$$\mathbf{s}(k) = \bar{\mathbf{B}}_d(k)\bar{\mathbf{c}}, \quad k \in [\hat{k}_1, \hat{k}_{\ell+d+1}], \tag{4}$$

where: $\bar{\mathbf{c}} \triangleq [\mathbf{c}_1' \cdots \mathbf{c}_q']'$, $\bar{\mathbf{B}}_d(k) \triangleq$ diag$[\mathbf{B}_d(k) \cdots \mathbf{B}_d(k)]$. Each $\mathbf{c}_i \triangleq [c_{i,1} \cdots c_{i,\ell}]'$, $i = 1, \cdots, q$, is defined as in (2). The dimensions of $\bar{\mathbf{c}}$ are $(q\ell \times 1)$. The dimensions of the block diagonal matrix $\bar{\mathbf{B}}_d(k)$ are $(q \times q\ell)$.

Remark 1. From (3) it is apparent that, once the degree d and the knot points $\hat{k}_i$ have been fixed, the scalar B-spline $s(k)$, $k \in [\hat{k}_1, \hat{k}_{\ell+d+1}]$, is completely determined by the corresponding vector $\mathbf{c}$ of ℓ control points. As, in general, $\ell << k_M$, where k_M is the number of sampled instants of $[\hat{k}_1, \hat{k}_{\ell+d+1}]$, B-splines are said to admit a parsimonious parametric representation.

B. Constrained Least Squares

The discrete-time constrained least square problem has the general form

$$\min_f J \triangleq \min_f \|e\|_2^2 = \min_f \|b - Df\|_2^2 \tag{5}$$

where $e \in \mathbb{R}^r$ is the residual vector, $b \in \mathbb{R}^r$ is the observation vector, $D \in \mathbb{R}^{r \times s}$, $r \geq s$, is the design matrix and $f \in \mathbb{R}^s$ is the vector of model parameters.

Box constraints on the parameters vector to be estimated are represented as: $f_{min} \leq f \leq f_{max}$, where f_{min} (f_{max}) is the lower (upper) bound of the parameter vector f.

The constrained least square problem does not admit the well-known closed-form solution given by the pseudo-inverse (see e.g. [23]). Specific numeric algorithms can be found in [24], [25]. The numerical algorithm given in [25] is implemented by the MatLab function *lsqlin*.

C. System constraints and invariant sets

Consider the LTI system (Σ, x)

$$x(k+1) = Ax(k) + Bw(k), y(k) = C\,x(k) + D\,w(k), \quad (6)$$

where: $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^{n_y}$ and $w(k) \in \mathbb{R}^{n_w}$. The input $w(k) \triangleq [r(k)', d(k)']'$ includes a manipulable component $r(k)$ and an unmeasurable disturbance $d(k)$. An invariant γ-feasible set of (6) is a convex compact set $\mathcal{X}$ containing the origin, such that, for every input $w(k)$, $k \geq 0$ satisfying the following admissibility condition

$$\|w(k)\|_2^2 = \|r(k)\|_2^2 + \|d(k)\|_2^2 \leq \gamma, \quad (7)$$

one has $x(k) \in \mathcal{X} \Rightarrow Ax(k) + Bw(k) \in \mathcal{X}$ and the following constraints are satisfied

$$|z_i(k)| = \|z_i(k)\|_2 \leq \bar{z}_i,\ i = 1, \cdots, h, \quad (8)$$

where $z_i(k)$ is the i-th element of the constrained variables vector $z(k) \in \mathbb{R}^h$ and $\bar{z}_i$ is the corresponding pre-specified hard constraint. The vector $z(k) \triangleq C_z x(k)$ represents any vector of variables linearly depending on the state.

Here $\mathcal{X}$ is assumed to be an ellipsoid set defined as $\mathcal{E}(P, \gamma) = \{x(k)\,|\,x'(k)Px(k) \leq \gamma\}$, where $P \triangleq Q^{-1}$ is a positive definite matrix.

IV. Problem setup

The new approach makes reference to the 2DoF control scheme Σ_{2DoF} shown in Fig. 1 where: $\bar{y}_d(k)$ is the piecewise

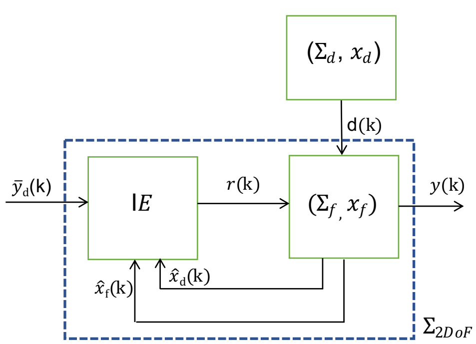

Fig. 1. The 2DoF control scheme

constant desired reference to be tracked, $y(k)$ is the output, $r(k)$ is the manipulable input and $d(k)$ is an unmeasurable constant disturbance satisfying $\|d(k)\|_2^2 \leq \gamma_d$ for some known γ_d. It is produced as free output response of (Σ_d, x_d) starting from an unknown initial state $x_d(0)$. The input of Σ_f is $w(k) \triangleq [r'(k)\,d'(k)]'$.

The block (Σ_f, x_f) is the feedback connection of a discrete-time LTI plant (Σ_p, x_p) with an LTI dynamic output controller (Σ_g, x_g) including a linear observer $(\Sigma_o, \begin{bmatrix}\hat{x}_p \\ \hat{x}_d\end{bmatrix})$ and an internal model (Σ_c, x_c) of constant signals.

The purpose of Σ_g is to guarantee the fulfillment of the following requirements: r1) asymptotic stability of Σ_f; r2) the existence of an invariant γ-feasible set $\mathcal{X}$ for $\Sigma_f \equiv (A_f, B_f, C_f, D_f)$, such that $x_f(k) \in \mathcal{X} \Rightarrow A_f x_f(k) + B_f w(k) \in \mathcal{X}$, and constraints like (8) are satisfied by each component of the vector of constrained variables $z_f(k) = C_{z_f} x_f(k)$, for any admissible input $w(k)$ satisfying (7). The inputs of IE are $\bar{y}_d(k)$, the current state estimates $\hat{x}_f(k)$ (defined later), and $\hat{x}_d(k)$. This information is exploited by the IE to compute the manipulable input $r(k)$. The output of the IE is obtained solving the following constrained optimization problem (COP) at each k

$$(\mathbf{COP}) \qquad \min_{[r(k),\cdots,r(k+N_y-1)]} J \quad (9)$$

$$J \triangleq \lambda_1(k) \sum_{i=1}^{N_y} e_y'(k+i|k) Q_y(k) e_y(k+i|k) + \lambda_2(k) \sum_{i=0}^{N_y-1} e_r'(k+i|k) Q_r(k) e_r(k+i|k),$$

where

$$e_y(k+i|k) \triangleq \bar{y}_d(k) - y(k+i|k), \quad (10)$$

$$e_r(k+i|k) \triangleq \bar{y}_d(k) - r(k+i), \quad (11)$$

subject to

$$r_{min} \leq r(k+i) \leq r_{max},\ i = 0, \cdots, N_y - 1 \quad (12)$$

where: $\lambda_1(k) \geq 0$, $\lambda_2(k) \geq 0$, $\forall k \geq 0$, r_{min} and r_{max} are computed so as to satisfy (7), $Q_r(k)$ and $Q_y(k)$ are positive definite matrices, $y(k+i|k)$ is the predicted output and N_y is the length of the prediction horizon.

Remark 2. Some considerations on the cost functional J are in order.

− No term on the control effort $u(k)$ forcing the plant is included in J because, as shown in the next section, Σ_g is designed so that $u(k)$ obey constraint of the kind (8).

− In (10),(11), the desired reference is evaluated at time instant k to avoid undesired anticipative effects on $y(k)$ due to possible set point changes inside the prediction horizon.

− Parameters $\lambda_1(k)$ and $\lambda_2(k)$ drive the convergence of $y(k)$ and $r(k)$ respectively towards the current desired set point. At the first time instants following any set point reset, $\lambda_1(k)$ and $\lambda_2(k)$ should be chosen so that $\lambda(k) \triangleq \lambda_1(k)/\lambda_2(k) >> 1$, because this allows $r(k)$ to freely vary over all the admissible range. After the transition period has elapsed, $\lambda_2(k)$ should be increased to speed up the convergence of $r(k)$ to the new desired fixed set point. In fact, owing to the presence of the internal model Σ_c, an $r(k)$ converging to the actual set point guarantees exact asymptotic tracking. This is particularly important in the case of piecewise constant signals $\bar{y}_d(k)$ which are not frozen on a fixed set point for a sufficiently long time interval and tracking precision is the dominant criterion. In practice, a good practical tuning rule is to put $\lambda_1(k) = 1$, $\forall k$, and to choose a $\lambda_2(k)$ increasing according to an S-shaped membership function starting from $\lambda_2(0) = 0$ and converging to an $\bar{\lambda}_2 > 0$ such that $1/\bar{\lambda}_2 \leq 1$.

– The positive definite matrices $Q_y(k)$ and $Q_r(k)$ have a diagonal form. Different values of the entries on the main diagonal are useful to modulate the effect of $\lambda_1(k)$ and $\lambda_2(k)$ on the components of $e_y(k+i|k)$ and $e_r(k+i|k)$ respectively.
– As the length N_y of the prediction horizon is decoupled from stability and feasibility considerations, it can be mainly chosen on the basis of considerations on the computational burden. △

The considerations developed in this section clearly show the idea underlying the present approach and the relative advantages of the resulting MPC procedure. Designing Σ_g according to r1 guarantees the uniform boundedness of $x_f(k)$ for any uniformly bounded $r(k)$, independently of N_y, $\lambda_1(k)$, $\lambda_2(k)$, $Q_r(k)$ and $Q_y(k)$. This releases the stability issue from the prediction horizon and other tuning parameters. Requirement r2 allows us to transfer any constraint on $z_f(k)$ of the kind (8) on a corresponding upper bound γ on $\|w(k)\|_2^2$. As $\|d(k)\|_2^2 \leq \gamma_d$, $\gamma_r \triangleq \gamma - \gamma_d$ represents the admissibility bound to be imposed on $\|r(k)\|_2^2$. Such γ_r is explicitly taken into account in the COP by suitably defining r_{min} and r_{max} in (12). As it will be formally stated in Theorem 3 of Section VI, the above implies that the proposed two-step procedure yields a recursively feasible MPC strategy with guarantee of internal stability for the system Σ_{2DoF} shown in Fig. 1.

V. STEP 1: DESIGN OF Σ_g

Let Σ_p be the LTI plant given by

$$x_p(k+1) = A_p x_p(k) + B_p u(k) + B_d d(k), \quad (13)$$
$$y(k) = C_p\, x_p(k) + D_d\, d(k), \quad (14)$$

where $x_p(k) \in \mathbb{R}^{n_p}$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, and $y(k) \in \mathbb{R}^q$ is the output. The triplet (A_p, B_p, C_p) is reachable and observable. The unknown constant disturbance $d(k)$ is generated as free output response of Σ_d according to:

$$x_d(k+1) = x_d(k),\ d(k) = x_d(k),\ x_d(k) \in \mathbb{R}^{n_d}. \quad (15)$$

In view of an exact asymptotic tracking requirement for constant signals, the following assumptions on Σ_p are made: A1) Σ_p has not a transmission zero at $z = 1$; A2) $q \leq m$; A3) rank $\begin{bmatrix} I - A_p & -B_d \\ C_p & D_d \end{bmatrix} = n_p + n_d$.

Let $(\bar{\Sigma}_p, \bar{x}_p)$ with $\bar{x}_p = \begin{bmatrix} x_p' & x_d' \end{bmatrix}'$ be the augmented plant

$$\bar{x}_p(k+1) = \bar{A}_p \bar{x}_p(k) + \bar{B}_p u(k) \quad (16)$$
$$y(k) = \bar{C}_p \bar{x}_p(k) \quad (17)$$

where $\bar{A}_p = \begin{bmatrix} A_p & B_d \\ \mathbf{0} & I \end{bmatrix}$, $\bar{B}_p = \begin{bmatrix} B_p \\ \mathbf{0} \end{bmatrix}$ and $\bar{C}_p = \begin{bmatrix} C_p & D_d \end{bmatrix}$.

By A3) and the observability of (C_p, A_p), also the pair $(\bar{C}_p, \bar{A}_p)$ is observable [26] and the following observer $(\Sigma_o, \hat{\bar{x}}_p)$ of $\bar{\Sigma}_p$ with $\hat{\bar{x}}_p \triangleq \begin{bmatrix} \hat{x}_p' & \hat{x}_d' \end{bmatrix}'$ can be defined:

$$\hat{\bar{x}}_p(k+1) = \bar{A}_p \hat{\bar{x}}_p(k) + \bar{B}_p u(k) + \begin{bmatrix} L_p \\ L_d \end{bmatrix} \left(y(k) - \bar{C}_p \hat{\bar{x}}_p(k) \right). \quad (18)$$

The state space representation of Σ_c is $x_c(k+1) = x_c(k) + (r(k) - y(k))$ with $x_c(k) \in \mathbb{R}^{n_c}$, $r(k) \in \mathbb{R}^q$ and $n_c = q$. According to [27], the control input forcing Σ_p is given by

$$u(k) = -K_p \hat{x}_p(k) + K_c x_c(k). \quad (19)$$

The state space representation (A_f, B_f, C_f, D_f) of the closed loop system Σ_f is

$$x_f(k+1) \quad (20)$$
$$= \begin{bmatrix} A_p - B_p K_p & B_p K_c & L_p C_p & L_p D_d - B_d \\ -C_p & I & -C_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A_p - L_p C_p & B_d - L_p D_d \\ \mathbf{0} & \mathbf{0} & -L_d C_p & I - L_d D_d \end{bmatrix} x_f(k)$$
$$+ \begin{bmatrix} \mathbf{0} & B_d \\ I & -D_d \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} w(k),$$
$$y(k) = \begin{bmatrix} C_p & \mathbf{0} & C_p & \mathbf{0} \end{bmatrix} x_f(k) + \begin{bmatrix} \mathbf{0} & D_d \end{bmatrix} w(k), \quad (21)$$

with $x_f(k) \triangleq [\hat{x}_p'(k)\ \ x_c'(k)\ \ x_p'(k) - \hat{x}_p'(k)\ \ x_d'(k) - \hat{x}_d'(k)]' \in \mathbb{R}^n$ and $n \triangleq 2n_p + n_d + n_c$.

The constrained variables vector is

$$z_f(k) \triangleq [z_u'(k)\, z_{x_f}'(k)]',\ z_u(k) \in \mathbb{R}^{n_u},\ z_{x_f}(k) \in \mathbb{R}^{n_{x_f}} \quad (22)$$

where the respective components $z_{u,r}(k)$ and $z_{x_f,l}(k)$ ($r = 1, \cdots, n_u$, $l = 1, \cdots, n_{x_f}$) have to satisfy constraints like (8) for some given $\bar{z}_{u,r}$ and $\bar{z}_{x_f,l}$ respectively. Typically $z_u(k) = C_{z_u} x_f(k) = u(k)$, so that, by (19), $n_u = m$ and $C_{z_u} = \begin{bmatrix} -K_p & K_c & \mathbf{0} & \mathbf{0} \end{bmatrix} \triangleq \hat{K}$ while $z_{x_f}(k) = C_{z_{x_f}} x_f(k)$ represents any vector of variables linearly depending on the state.

It is remarked that the above distinction between $z_u(k)$ and $z_{x_f}(k)$, is necessary because, unlike $z_{x_f}(k)$, $z_u(k)$ depends on $x_f(k)$ through of a matrix which is a design parameter. Such a matrix has to be determined imposing the fulfillment of the control specifications.

Once Σ_c has been designed according to the internal model principle, the controller gain matrices are computed as specified beneath.

A. Design of the observer gain

The observer gain $\bar{L} \triangleq \begin{bmatrix} L_p \\ L_d \end{bmatrix}$ is designed so as to impose a suitable decay rate to the state estimation error $\bar{x}_p(k) - \hat{\bar{x}}_p(k) \triangleq \varepsilon(k)$ which satisfies $\varepsilon(k+1) = (\bar{A}_p - \bar{L}\bar{C}_p)\varepsilon(k)$. Let $V(\varepsilon(k)) = \varepsilon'(k) S \varepsilon(k)$, with $S = S' > 0$, be a Lyapunov function such that

$$V(\varepsilon(k+1)) - \rho V(\varepsilon(k)) < 0, \text{ for some } \rho \in (0,\, 1). \quad (23)$$

From (23) it directly follows that $V(\varepsilon(k)) < \rho^k V(\varepsilon(0))$, whence $\|\varepsilon(k)\|_2^2 < \rho^k \frac{\lambda_{\max}\{S\}}{\lambda_{\min}\{S\}} \|\varepsilon(0)\|_2^2 = \rho^k \kappa(S) \|\varepsilon(0)\|_2^2$, where $\kappa(S)$ is the condition number of S.

The observer gain $\bar{L}$ is computed solving the following optimization problem:

$$\text{(OP)}: \quad \text{minimize } \kappa(S) \quad \text{subject to (23)}$$

which can be formulated in terms of LMIs as shown in the following theorem.

Theorem 1 For any fixed $\rho \in (0,\,1)$, OP is equivalent to the following semidefinite programming problem

$$\text{minimize} \quad \zeta \quad \text{subject to:}$$

$$\begin{bmatrix} -S_1 & S_1\,\bar{A}_p - Z_1\bar{C}_p \\ \bar{A}_p'\,S_1 - \bar{C}_p'\,Z_1' & -\rho S_1 \end{bmatrix} < 0,\ I \leq S_1 \leq \zeta\, I \quad (24)$$

in the variables $\zeta \in \mathbb{R}$, $S_1 = S_1' \in \mathbb{R}^{(n_p+n_d)\times(n_p+n_d)}$ and $Z_1 \in \mathbb{R}^{(n_p+n_d)\times q}$. The observer gain $\bar{L}$ is given by $\bar{L} = S_1^{-1}\,Z_1$.

Proof of Theorem 1 Applying the Schur complement and the change of variable $S\bar{L} \triangleq Z$, one has

$$(23)\ \text{holds} \iff (\bar{A}_p - \bar{L}\bar{C}_p)'SS^{-1}S(\bar{A}_p - \bar{L}\bar{C}_p) - \rho S < 0$$

$$\iff \begin{bmatrix} -S & S\,\bar{A}_p - Z\bar{C}_p \\ \bar{A}_p'\,S - \bar{C}_p'\,Z' & -\rho S \end{bmatrix} < 0. \quad (25)$$

By (25), (OP) can be reformulated as:

$$\text{minimize} \quad \zeta \quad \text{subject to:}$$

$$\begin{bmatrix} -S & S\,\bar{A}_p - Z\bar{C}_p \\ \bar{A}_p'\,S - \bar{C}_p'\,Z' & -\rho S \end{bmatrix} < 0 \quad (26)$$

$$\mu > 0,\ \ \mu I \leq S \leq \mu\zeta I \quad (27)$$

where $\mu \triangleq \lambda_{min}\{S\}$. Defining the new variables $\nu = \frac{1}{\mu}$, $S_1 = \nu S$ and $Z_1 = \nu Z$, conditions (26)-(27) are equivalent to (24) ([28], p. 38) and $\bar{L} \triangleq S^{-1}Z$ can be written as $\bar{L} = S_1^{-1}Z_1$. $\triangle$

B. Design of the feedback gain

For any fixed matrix $\bar{L}$ of the observer, the gain matrix $\begin{bmatrix} -K_p & K_c & \mathbf{0} & \mathbf{0} \end{bmatrix} \triangleq \hat{K}$, defining the control law $u(k) = \hat{K}x_f(k)$, can be computed observing that by (20) the closed loop dynamical matrix A_f can be rewritten as $A_f \triangleq \hat{A} + \hat{B}\hat{K}$, where the pair

$$\hat{A} \triangleq \begin{bmatrix} A_p & \mathbf{0} & L_pC_p & L_pD_d - B_d \\ -C_p & I & -C_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A_p - L_pC_p & B_d - L_pD_d \\ \mathbf{0} & \mathbf{0} & -L_dC_p & I - L_dD_d \end{bmatrix},\ \hat{B} \triangleq \begin{bmatrix} B_p \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (28)$$

is stabilizable.

The following problem is now formulated.

P1 Given the pair $(\hat{A}, \hat{B})$ in (28) with $\bar{L} = S_1^{-1}\,Z_1$, find a matrix $\hat{K}$ and the maximum invariant γ-feasible set $\mathcal{X}$ (where also γ is maximized), such that the following conditions are satisfied: i) $\Sigma_f \equiv (\hat{A} + \hat{B}\hat{K}, B_f, C_f, D_f)$ is internally stable, ii) constraints on $z_f(k)$ are fulfilled $\forall\, x_f(0) \in \mathcal{X}$ and every admissible input $w(k)$ satisfying (7). $\triangle$

Remark 3 Since in the augmented state x_f only the plant state, x_p, is of interest, instead of maximizing the entire ellipsoid volume only the ellipsoid projection on x_p subspace is maximized. The projection of $\mathcal{X}$ onto x_p is given by $\mathcal{X}_{x_p} \triangleq \mathcal{E}_{x_p}(P,\gamma) = \{x_p(k)\,|\, x_p'(k)(T_{x_p}QT_{x_p}')^{-1}x_p(k) \leq \gamma\}$ with T_{x_p} defined by $x_p(k) = T_{x_p}x_f(k)$. $\triangle$

Theorem 2 Consider the pair $(\hat{A}, \hat{B})$ in (28) and define η as $\eta \triangleq \gamma^{-1}$. Stability and the invariant γ feasible set $\mathcal{X}$ (where both $\mathcal{X}_{x_p}$ and γ are maximized) for Σ_f subject to constraints on $z_f(k)$ of the kind (8) and forced by any $w(k)$ satisfying (7), are obtained by solving the following semidefinite programming problem:

$$\text{minimize} \quad (\text{-log}(\det(T_{x_p}QT_{x_p}'))\beta_1 + \eta\beta_2) \quad \text{subject to} \quad (29)$$

$$\begin{bmatrix} Q & \mathbf{0} & \alpha Q & Q\hat{A}' + Y'\hat{B}' \\ \mathbf{0} & \alpha I & \mathbf{0} & B_f' \\ \alpha Q & \mathbf{0} & \alpha Q & \mathbf{0} \\ \hat{A}Q + \hat{B}Y & B_f & \mathbf{0} & Q \end{bmatrix} \geq 0 \quad (30)$$

$$\begin{bmatrix} Q & Y'I_r' \\ I_rY & \bar{z}_{u,r}^2\eta \end{bmatrix} \geq 0,\ r = 1,\cdots,m \quad (31)$$

$$\begin{bmatrix} Q & (Q\hat{A}' + Y'\hat{B}')C_{z_{x_f}}'\,I_l' \\ I_l\,C_{z_{x_f}}(\hat{A}Q + \hat{B}Y) & \Sigma \end{bmatrix} \geq 0 \quad (32)$$

$$l = 1,\cdots,n_{x_f}$$

where: $\Sigma \triangleq \bar{z}_{x_f,l}^2\eta - I_lC_{z_{x_f}}B_fB_f'C_{z_{x_f}}'I_l'$, β_1 and β_2 are fixed positive weighting scalars, $0 < \alpha < 1$, $Q = Q' = \text{diag}[Q_1\ Q_2] \in \mathbb{R}^{n\times n}$, $Y = [Y_1\ \mathbf{0}] \in \mathbb{R}^{m\times n}$ are the variables where $n = 2n_p + n_c + n_d$. The row vector I_r (I_l) is composed of all null elements save the element 1 in the r-th (l-th) position.

If the set of inequalities admits a solution then the stabilizing feedback gain $\hat{K} = YQ^{-1}$ is found. The maximum admissible value $\gamma = \eta^{-1}$ is found for $w(k)$ and the invariant γ-feasible set $\mathcal{X} \equiv \mathcal{E}(P,\gamma)$ with $P = Q^{-1}$ for Σ_f is obtained.

Proof of Theorem 2. For the sake of brevity, the details of the proof are not reported. The theorem can be proved along the lines provided in [29] (see Theorem 1) with some modifications due to: 1) according to Remark 4, a projected ellipsoid is here used, 2) in [29] $\|w\|_2^2$ is overbounded by 1, here $\|w\|_2^2$ is overbounded by a scalar γ which is not fixed "a priori" but is maximized including $\eta = \gamma^{-1}$ in the functional to be minimized; 3) in [29] an euclidean norm bound is imposed to the constrained variables vector $z(k)$, here component-wise bounds are more realistically considered. $\triangle$

Remark 4 The presence of α makes inequality (30) a BMI, which can be transformed into a LMI through a gridding over the interval $(0,\,1)$ where α takes values. $\triangle$

Remark 5 Some considerations on the above synthesis procedure are now in order. Ellipsoidal invariant sets have been used because of their closed relation to quadratic Lyapunov functions leading to LMIs based conditions. Due to the need to satisfy hard constraints, only sufficient conditions have been derived for the computation of the feedback gain $\hat{K}$. Pre-computing $\bar{L}$ permits the definition of an augmented open-loop plant with a stabilizable pair $(\hat{A}, \hat{B})$. This, in turn, directly allows the derivation of sufficient conditions for the existence of the invariant set $\mathcal{X}$ and the stabilizing state-feedback gain $\hat{K}$ in presence of hard constraints. The optimum criterion (29) has been introduced to maximize the value of γ ($\gamma = \eta^{-1}$) and the volume of $\mathcal{X}_{x_p}$ compatibly with (30)-(32). This allows enlarging the sets of admissible $w(k)$ and of admissible initial conditions thus reducing the conservatism due to: 1) the off line computation of the invariant set; 2) the "a priori"

guarantee of recursive feasibility.
It is mentioned that sufficient conditions for the simultaneous design of observer and controller gains have recently been proposed in [30]- [31] for continuous-time Takagi-Sugeno fuzzy affine dynamic model and discrete-time semi-Markov jump linear plants respectively. These methods are not directly applicable in the present case where also the fulfillment of hard constraints on some physical variables and the construction of the invariant γ feasible set are required. △

VI. STEP 2: COMPUTATION OF $r(k)$

The computation of the manipulable input $r(k)$ is performed solving the COP defined in Section IV.
Let $\hat{x}_f(k) = \begin{bmatrix}\hat{x}'_p(k) & x'_c(k) & \mathbf{0} & \mathbf{0}\end{bmatrix}'$ be the current estimate of $x_f(k)$. Expressing the manipulable input $r(j)$ as $r(j) = \bar{\mathbf{B}}_d\ (j)\bar{\mathbf{c}}$ according to (4), the predicted output is given by

$$y(k+i|k) = C_f A_f^i \hat{x}_f(k) + \sum_{j=k}^{k+i-1} C_f A_f^{k+i-j-1} B_{f,1} \bar{\mathbf{B}}_d\ (j)\bar{\mathbf{c}} + \sum_{j=k}^{k+i-1} C_f A_f^{k+i-j-1} B_{f,2} d(j|k) + D_d d(k+i|k), \quad (33)$$

where: $d(\cdot|k)$ is the predicted disturbance and $B_{f,i},\ i=1,2$ denotes the i-th column of B_f.

Taking into account that, by (15) $d(j|k) = d(k+i|k) = \hat{x}_d(k)$, the predicted errors $e_y(k+i|k)$ and $e_r(k+i|k)$, given by (10)-(11) respectively, can be rewritten as

$$e_y(k+i|k) = b_y(k+i|k) - D_y(k+i|k)f \quad (34)$$
$$e_r(k+i|k) = b_r(k+i|k) - D_r(k+i|k)f \quad (35)$$

where

$$\begin{aligned} b_y(k+i|k) &\triangleq \bar{y}_d(k) - C_f A_f^i \hat{x}_f(k) \\ &- \sum_{j=k}^{k+i-1} C_f A_f^{k+i-j-1} B_{f,2} \hat{x}_d(k) \\ &- D_d \hat{x}_d(k) \\ D_y(k+i|k) &\triangleq \sum_{j=k}^{k+i-1} C_f A_f^{k+i-j-1} B_f \bar{\mathbf{B}}_d(j), \\ b_r(k+i|k) &\triangleq \bar{y}_d(k), \quad D_r(k+i|k) \triangleq \bar{\mathbf{B}}_d(k+i), \\ f &\triangleq \bar{\mathbf{c}} \end{aligned}$$

Define the following vectors $e \triangleq [e'_y\ e'_r]'$, $g \triangleq [g'_y\ g'_r]'$, and matrices $D \triangleq \begin{bmatrix} D_y \\ D_r \end{bmatrix}$, $Q_e \triangleq \begin{bmatrix} Q_y & \mathbf{0} \\ \mathbf{0} & Q_r \end{bmatrix}$ where:

$$\begin{aligned} e_y &\triangleq \begin{bmatrix} e'_y(k+1|k) & \cdot & \cdot & e'_y(k+N_y|k) \end{bmatrix}', \\ b_y &\triangleq \begin{bmatrix} b'_y(k+1|k) & \cdot & \cdot & b'_y(k+N_y|k) \end{bmatrix}', \\ D_y &\triangleq \begin{bmatrix} D'_y(k+1|k) & \cdot & \cdot & D'_y(k+N_y|k) \end{bmatrix}', \\ Q_y &\triangleq \lambda_1(k)\,\mathrm{diag}[Q_y(k) \cdots Q_y(k)]. \end{aligned}$$

An analogous definition applies to vectors e_r, b_r and matrices D_r and Q_r.
From the above definitions, it directly follows that the $2qN_y$ scalar equations (34)-(35) can be written in the compact form $e = b - Df$ and functional (9) can be written as $J \triangleq J(e_q) = \|e_q\|_2^2$, where $e_q = Q_e^{1/2} e$. Recalling the convex hull property of B-splines and defining $b_q \triangleq Q_e^{1/2} b$ and $D_q \triangleq Q_e^{1/2} D$, it is evident that the COP is equivalent to solve the following box-constrained least square problem

$$\min_f \ \|D_q f - b_q\|_2^2, \quad (36)$$
$$\text{subject to} \quad f_{min} \leq f \leq f_{max} \quad (37)$$

The bounds $f_{min} \triangleq c_{min}$ and $f_{max} \triangleq c_{max}$ relative to the vector $\bar{\mathbf{c}} \triangleq f$ of control points are chosen so as to satisfy $\|r(k)\|_2^2 \leq \gamma_r \triangleq \gamma - \gamma_d$ where $\gamma = \eta^{-1}$ results from the minimization of functional (29).
At each k, the parameter vector $\bar{\mathbf{c}}$ of control points is estimated as explained in Section III B. The corresponding B-spline input $r(k)$ results to be known over $[k, k+N_y-1]$, but only the first sample is applied to Σ_f according to the usual MPC strategy.

Feasibility and stability properties of the whole control strategy proposed here can be now formally stated in the following theorem.
Theorem 3. Assume that the problem P1 stated in Section V is solvable and that the input $r(k)$ of Σ_f is computed as the solution of the box-constrained RLS problem (36),(37), then the proposed two-step procedure yields a recursively feasible MPC strategy and an asymptotically internally stable Σ_{2DoF}.
Proof of Theorem 3. Recursive feasibility is a direct consequence of computing $r(k)$ as the solution of an optimization problem where the feasible box-constraints (37) are imposed on a vector of variables which is the same one with respect to the optimization problem has to be solved. Moreover the fulfillment of (37) directly implies that also the components of $z_f(k)$ satisfy constraints like (8). Internal asymptotic stability of the resulting overall control system Σ_{2DoF} is a direct consequence of the internal asymptotic stability of Σ_f and of the uniform boundedness of $r(k)$ resulting from (37).

VII. DC SERVOMOTOR WITH CONSTRAINT ON THE SHAFT TORSIONAL TORQUE.

The position servomechanism considered in [32] consists of a DC motor, gearbox, elastic shaft and load. This example is extended here to the case of an unmeasurable constant disturbance affecting the plant. Denoting by θ_M and θ_L, respectively, the motor and the load angle, and by setting $x_p \triangleq [\theta_L, \dot{\theta}_L, \theta_M, \dot{\theta}_M]^T$, the following state space representation is derived from the model equations:

$$\dot{x}_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_T}{J_L} & -\frac{\beta_L}{J_L} & \frac{k_T}{\rho J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_T}{\rho J_M} & 0 & \frac{k_T}{\rho^2 J_M} & -\frac{\beta_M + \frac{k_M^2}{R}}{J_M} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_M}{R J_M} \end{bmatrix} u + 0.5 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} d \quad (38)$$

$$\theta_L = C_p x_p + D_d d = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x_p + 0.1\, d \quad (39)$$

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where $u = V$ (voltage) is the control input, θ_L (load angular position) is the output and d is the constant disturbance satisfying $\|d(k)\|_2^2 \leq \gamma_d = 0.04$.

The physical parameter values are reported in [32] (Section 1, page 90). It is easily seen that plant (38)-(39) satisfies the same assumptions of plant (13)-(14).

The controller must set the angular position of the load, θ_L, at the desired set-point value (1 rad).

The sampled data representation of the DC servomotor is obtained by sampling with $T_c = 0.1$ sec and using a zero order holder on the input voltage V. Technical specifications involves constraints on the input voltage $|V| \leq 220$ V as well as on the shaft torsional torque $T = C_T x_p = \begin{bmatrix} k_T & 0 & -\frac{k_T}{\rho} & 0 \end{bmatrix} x_p$ which is required to satisfy the constraint $|T| \leq 78.5398\,\text{Nm}$. According to (22), the constrained variables are $z_u = V \in \mathbb{R}$ and $z_{x_f} = T \in \mathbb{R}$, with bounds $\bar{z}_{u,1} = 220$ and $\bar{z}_{x_f,1} = 78.5398$, respectively.

The first step is to design Σ_g. According to the procedure described in Section V.A, the observer gain $\bar{L}$ is computed solving OP.

By Theorem 1, choosing $\rho = 0.5$ a feasible solution for OP is given by $L = S_1^{-1} Z_1 = \begin{bmatrix} -14.6037 & 3.9338 & -337.5358 & 29.7487 & 162.1898 \end{bmatrix}'$.

Given the pair $(\hat{A}, \hat{B})$ in (28), the feedback gain $\hat{K} = [-K_p \quad K_c \quad \mathbf{0} \quad \mathbf{0}]$ and the invariant γ-feasible set $\mathcal{X}$ for Σ_f are determined solving the semidefinite programming problem of Theorem 2. Taking into account that $z_{x_f} = C_{z_{x_f}} x_f = T$, the matrix $C_{z_{x_f}}$ in (32) is defined as $C_{z_{x_f}} = \begin{bmatrix} C_T & \mathbf{0} & C_T & \mathbf{0} \end{bmatrix}$. According to Remark 4, the scalar parameter α is fixed to transform (30) in an LMI. Choosing $\beta_1 = 1$ and $\beta_2 = 100$ in (29), it is found that, for $\alpha = 0.04$, the stabilizing feedback gain is $\hat{K} = \begin{bmatrix} 921.212 & 144.842 & -52.493 & -5.393 & 2.411 & \mathbf{0} & \mathbf{0} \end{bmatrix}$.

The invariant γ-feasible set $\mathcal{X} \equiv \mathcal{E}(P, \gamma)$ with $P = Q^{-1}$ (not reported for brevity) and $\gamma = \eta^{-1} = 4.6515$ are computed for the resulting closed loop system Σ_f.

Next step is to determine the trajectory of the manipulable input $r(k)$, subject to $\|r(k)\|_2^2 \leq \gamma_r \triangleq \gamma - \gamma_d = 4.6115$, optimally driving the output transition from 0 to the set point value 1 at time $t = 1s$. To this end $r(k) \in \mathbb{R}^q$ $(q = 1)$ is modeled as a scalar sampled B-spline. The following parameters are set: $d = 1$ (order of B-spline), $\ell = 3$ (number of control points), $5 \triangleq \ell + d + 1$ (number of knot points $\hat{k}_i$), $Q_y(k) \triangleq I$, $Q_r(k) \triangleq I$, $\forall\, k \geq 0$, $N_y = 10$ and $\lambda_1(k) = 1$ $\forall k \geq 0$. According to Remark 2, the S-shaped membership function chosen for $\lambda_2(k)$ starts from a null initial value and converges to $\bar{\lambda}_2 = 1$ over the transient period following any set point reset. The vector $\bar{\mathbf{c}}$ of decision variables to be estimated at each k is composed by $\ell q = 3$ control points. As $\gamma_r = 4.6115$ and $r(k)$ is scalar, the bounds of inequalities (37) are $|c_{min}| = c_{max} = \begin{bmatrix} 2.1474 & 2.1474 & 2.1474 \end{bmatrix}'$.

The simulation has been carried out assuming: $x_p(0) = 0$ and $d(k) = 0.2$, $\forall k \geq 0$. The obtained input $r(k)$ and the controlled output θ_L are shown in figures 2 and 3 (solid line) respectively. The behaviors of both constrained variables V and T are shown in figures 4-5 respectively. The design of an MPC controller for the same position servomechanism in the simpler disturbance free case is proposed in [32] (Section 1 page 96) setting: $T_c = 0.1$ (sampling time), $N_y = 20$ and $N_u = 5$.

For a comparison with [32] from the computational point of view, the following considerations should be taken into account.

- The choice of an halved prediction horizon ($N_y = 10$) emphasizes two significant aspects of the proposed algorithm: the number of operations to be executed at each k in the on-line optimization procedure is significantly reduced and the stability of the overall 2DoF scheme is not compromised.
- At each k the number of decision variables (control points of $r(k)$) involved in the proposed on-line optimization procedure is $q\ell = 3 < qN_y = 10$, and the total number of constraints (37) to be imposed is $2(q\ell) = 6$. For the traditional MPC controller implemented in [32], the number of decision variables involved at each k is $mN_u = 5$ and the number of constraints to be imposed is $2N_y + 2mN_u + 2mN_u = 60$ (namely $2N_y = 40$ for $|T| \leq 78.5398$, $2mN_u = 10$ for $|V| \leq 220$ and $2mN_u = 10$ for the input voltage rate).

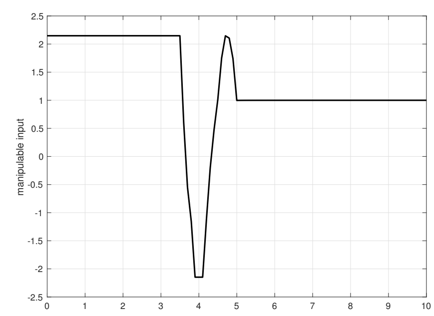

Fig. 2. The manipulable input $r(k)$ (scalar B-spline of order $d = 1$ with 3 control points and $5 \triangleq \ell + d+$ knots points).

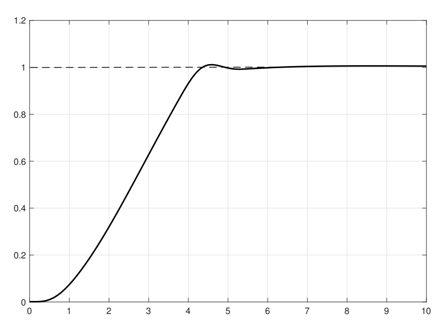

Fig. 3. The desired set point (dashed line) and the controlled output θ_L (solid line).

VIII. CONCLUSIONS

This paper has shown how to reduce some technical difficulties of MPC related to the complexity of stability and feasibility analysis as well as to the demanding computational effort. Stability and feasibility problems are essentially solved by the formulation of the MPC through a 2DoF control scheme. Also the tracking problem is greatly simplified and an exact static

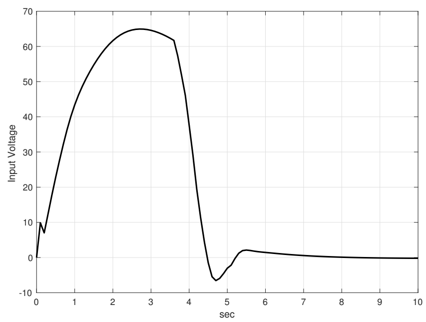

Fig. 4. The constrained input voltage V.

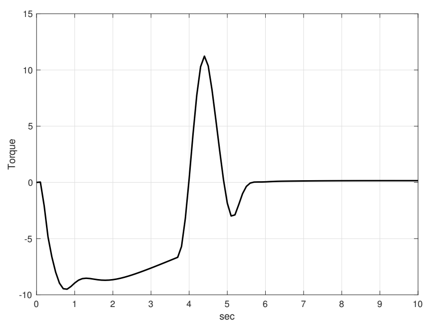

Fig. 5. The constrained shaft torsional torque T.

decoupling is achieved. This latter feature is of a noticeable importance in industrial control because allows multiple set points to meet their target independently of each other. The numerical effort is significantly curtailed for the two following reasons: 1) the closed-loop formulation releases the choice of the prediction horizon from stability considerations, 2) the use of B-splines transforms the constrained optimization problem in the constrained least-square estimation of a largely reduced number of parameters. The numerical example confirmed the validity of the approach.

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