

Students' Critical Thinking Ability in Solving the Application of the Derivative of Algebraic Function Problems

Marzuki^{1*}, Saiman², Wahyudin³

¹Mathematics Education Department, IAIN Langsa, Indonesia

²Mathematics Education Department, Universitas Samudra, Indonesia

³Mathematics Education Department, Universitas Pendidikan Indonesia, Indonesia

*Email: marzuki@iainlangsa.ac.id

Received: 15 March 2022; Revised: 28 March 2022; Accepted: 27 April 2022

Abstract. *Critical thinking (CT) is convergent thinking that can be explored when students learn mathematics. Yet, CT based on the level of mathematical ability has not been widely explored. The participants involved 30 Year 11 students in Bandung, Indonesia. The research instruments were a written CT ability test and a semi-structured interview guide. Interviews were conducted in-depth to capture very high mathematical abilities until saturated data were obtained. Data saturation occurred during the interview with the 10th participant, where new categories and sub-categories were no longer found. Data was analyzed following an intrinsic case study design based on the post-positivist paradigm through NVivo 12 plus. This study found that students' CT ability level was very high, their thinking abilities varied, and they able to provide solutions to anyone in solving similar problems. This finding will have implications for the variety of students' critical thinking patterns in learning mathematics on the application of the derivatives of algebraic function in high school.*

Keywords: *critical thinking ability, very high group mathematical ability, algebra function derivative application, NVivo 12 plus*

Introduction

Thinking is an activity to process the power of reasoning that a person does when faced with a problem that needs to be solved (Ennis, 1993; McGregor, 2007). According to (Sobur, 2010), thinking is a process that affects the interpretation of stimuli that involves the process of sensation, perception, and memorialization. In the scope of thinking, there is the term Critical Thinking (CT), namely convergent thinking. Convergent thinking itself means thinking that is done with an emphasis on the single most logical or appropriate answer (Ennis, 1991; Fisher, 2001; Halpern, 2014; McGregor, 2007; Munandar, 2016).

CT is one of the most important skills in the 21st century (Miller & Toppo, 2020; Sustekova et al., 2019). CT is very important because it allows a person to solve problems in difficult situations, and has effective and accurate communication skills (Marzuki et al., 2019; Setiyani et al., 2020). In the context of the new paradigm of learning curriculum (free learning) in Indonesia, CT is one of the parts that is developed in the profile of Pancasila students (Kementrian Pendidikan dan Kebudayaan, 2020), thus teaching and learning activities should direct students to develop critical thinking skills.

CT capabilities have been identified as an area for development (Association of American Colleges and Universities, 2005; Australian Council for Educational Research, 2002; Basri et

al., 2019). Learning CT skills is a challenge for educators around the world (Wang & Woo, 2010), including in Indonesia. We realize that this is not an easy thing for teachers because basically, critical thinking skills are not something that can be honed in a day or two, but it is an ability that must be trained continuously. However, what is not easy does not mean it cannot. We need to take this more seriously.

The importance of students' CT abilities in mathematics studied in educational settings is that it allows individuals to go beyond simply storing information, to gain a more complex understanding of the information presented to them (Dwyer et al., 2012; Halpern, 2014; Widyatiningtyas et al., 2015). CT skills are also important in social and interpersonal contexts where good decision-making and problem-solving are needed daily (As'ari et al., 2019; Hage, 2020; Liu & Hsueh, 2016). This shows that CT capabilities are important as a provision to face the challenges of the 21st century.

In addition to the importance of CT skills for individual interests, and social interests, this ability is also important in the child's learning process (Ennis, 1991; Fisher, 2001). CT abilities can be explored when students learn mathematics, but in fact, the group's mathematical CT ability is very high in solving mathematical problems, it has not been explored much, especially in the application material of algebraic function derivatives. Where the function derivative application material is a mandatory material that must be mastered by high school students. Although there are several studies in the previous literature that have examined mathematical CT abilities (Karpouza & Emvalotis, 2019; Marzuki et al., 2021; Miller & Topple, 2020; Sadeghi, 2020), they only considered only a few strategies, but they have not provided a comprehensive source of CT capabilities in the application material of algebraic function derivatives based on very high levels of mathematical ability.

The purpose of this study was to obtain a comprehensive picture of students' mathematical CT abilities based on a very high level of mathematical ability in solving mathematical problems in the application of derivatives of algebraic functions. The difference between this researcher and previous research is that there are three new things; first, tracing students' CT abilities through the treatment of derivative functions, and their application, observation, CT skills tests, and in-depth semi-structured interviews; second, the data processing uses a qualitative case study methodology assisted by the NVivo 12 plus software; and third, the researcher collaborates with the teacher in the field of mathematics studies in exploring the level of mathematical ability of students with a very high level of mathematical ability, to achieve this goal, this research researcher uses a qualitative methodology with the type of case study, aiming to obtain comprehensive information about the ability The

mathematical CT of students is based on a very high level of mathematical ability in solving algebraic function derivative application problems at the high school level.

Method

Participants involved in this study amounted to 30 participants. To collect data on very high level math skills, the researcher collaborated with a math teacher in class XI of senior high school based on the results of daily tests, and report cards in the previous semester, then the researchers grouped the students so that 10 students had very high levels of mathematical ability, consisting of 4 students. boys, and 6 girls.

Instruments and materials used in data collection and analysis include: (1) CT ability test, which consists of 4 questions that have been validated by experts. Before the critical thinking ability test questions are used, validation, and reliability tests are carried out with the aim of obtaining accurate data from the instruments used (Taherdoost & Group, 2017). Validity, and reliability tests are carried out with reference to the opinion (Cohen et al., 2020). Content validation was chosen to see the suitability of the context in the application material for derivatives of algebraic functions both theoretically and practically, and to see the suitability of Core Competence (CC), and Basic Competencies (BC). While the reliability test was carried out to see how the context of the material provided affected the performance of students in answering questions, so the researchers wanted to see the level of readability of critical thinking skills, mathematics, (2) semi-structured interviews, which aims to find out how the CT ability of group students is very high in completing mathematical problems in application material derived from algebraic functions, (3) digital voice recorder, aims to record the voices of interviewed students, researchers use a digital voice recorder in the form of a zoom application (Archibald, Ambagtsheer, Casey, & Lawless, 2019). (4) NVivo 12 plus software, aims to analyze qualitative data, whereas NVivo 12 plus can increase the efficiency of qualitative data analysis and facilitate complex data management and analysis (Feng & Behar-Horenstein, 2019; Violetta Wilk, Geoffrey N. Soutar, 2019). (5) Ethical considerations, permission to conduct this research was granted by universities, schools, teachers in the field of mathematics studies, parents/guardians of participants, and participating students. Consent letters were provided and signed by all participants involved in this study. After the transcription was completed, the students had read each transcript to ascertain whether the results of the interviews matched their agreement, and the initials of the participants were used to disguise identity in this study.

This research is a qualitative, case study type with an intrinsic case study design based on the post-positivist paradigm. According to (Stake, 1995), the purpose of intrinsic case studies is not to find out general phenomena, but intrinsic interest in specific phenomena, so although theory

can be built through this research, the main goal is not that, the case that wants to be disclosed is by detecting how the individual ability of students to think critically whose level of student ability is very high in solving algebraic function derivative application questions, here are the steps:

1. The first step, the researcher gave first treatment in class XI senior high school on the application material of algebraic function derivatives, with the aim of getting closer to the participants, so that they could get more information from them.
2. The second step, the researcher gave 4 questions on the CT ability test to 30 participants.
3. The third step, based on test result data, and data from collaboration with mathematicians for class XI senior high school, namely the daily test results, and report cards in the previous semester, obtained 10 students with a very high level of mathematical ability.
4. The fourth step, the researcher traced the answers of 10 participants, to be interviewed in depth based on the criteria for the group's very high level of mathematical ability. For interview data, researchers conducted in-depth interviews one by one, until the data reached the saturated category (Charmaz, 2019). Data saturation occurred after interviewing the 10th participant, which coincidentally, the number of participants with a very high level of thinking ability was 10 people. The interview started by asking easy factual and personal questions, such as after-school activities, studying outside of school, and students' daily activities. Then conduct follow-up interviews, because some new questions can be detected each time the interview. So we consider these questions in the next interview (Corbin & Strauss, 1990). Researchers have conducted data analysis since the beginning of data collection, because the results obtained lead to further interviews. If the data and information extracted from the interview are not found in new categories, then the data is saturated, and the theory obtained is sufficient (Eisenhardt, 1989).

Results and Discussion

The mathematical CT ability of group students is very high in solving mathematical problems in the application material of algebraic function derivatives.

After analyzing sentence by sentence, word by word, on a written test of CT ability, and semi-structured interviews, with the help of the Nvivo 12 Plus application software, the research findings were obtained. In general, the findings on students' CT abilities are very high in solving mathematical problems in the application material of algebraic function derivatives which include 9 categories, namely: (1) problem solving, (2) arguments, (3) derivative interpretation, (4) strategy development, (5) reasoning, (6) analysis, (7) constructing meaning, (8) understanding basic concepts, and (9) evaluating. The following is an analysis of the answers of 10 participants

with very high levels of critical thinking skills with codes A1, A2, A3, A4, A5, A6, A7, A8, A9, and A10.

1. Category problem-solving

Problem-solving carried out by participant A1, by looking for derivatives using the concept of limit (A6, A7, & A10), then the result of the first derivative is seen as the gradient of the tangent line (A7, & A8), then analyzing the results of $f'(x) \forall x, x \neq 0$. Furthermore, knowing the concept where the square number is always positive, the concept of the result of the operation of multiplying and dividing between two negative and positive numbers, and finally, participant A1 concludes (A2) that the gradient is always negative (A2, A3, A4, A5, A6, A7, A8, A9, & A10).

In Figure 1, it can be seen from the beginning that participant A1 solved the problem by confirming a conclusion, namely "the gradient of the tangent is always negative", then participant A1 explained the solution that was carried out using the concept of limit so that the first derivative was $f'(x) = -\frac{2}{x^2}, x \neq 0$. The next step is to analyze the results of the first derivative, namely $f'(x) = -\frac{2}{x^2}, x \neq 0$, and in the end participant A1 concludes that, the gradient of the tangent to the curve $f(x)$ when $x \neq 0$ is always negative.

① Ditetahui $f(x) = \frac{2}{x}, x \neq 0$
 a) Perlihatkan gradien garis singgung selalu negatif.

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
 $f(x+h) = \frac{2}{x+h}$ dan $f(x) = \frac{2}{x}$
 $f(x+h) - f(x) = \frac{2}{x+h} - \frac{2}{x} = \frac{2(x) - 2(x+h)}{(x+h)(x)} = \frac{-2h}{(x+h)(x)}$
 $f'(x) = \lim_{h \rightarrow 0} \frac{-2h}{(x+h)(x)} = \frac{-2}{(x+h)(x)}$

$f'(x) = -\frac{2}{x^2}$
 $f'(x)$ adalah fungsi gradien garis singgung $f(x)$ pada $x=x, x \neq 0$.

Perhatikan bahwa x^2 selalu bernilai positif untuk $x \neq 0$,
 $\frac{2}{x^2}$ selalu positif, }
 $-\frac{2}{x^2}$ selalu negatif } untuk $x \neq 0$

\therefore Gradien garis singgung $f(x)$ pada saat $x \neq 0$ selalu negatif.

$f'(x)$ is a function of the slope of the tangent line $f(x)$ on $X = x, x \neq 0$.
 Note that x^2 always positive value for $x \neq 0$,
 $\frac{2}{x^2}$ always positive,
 $-\frac{2}{x^2}$ always negative } $x \neq 0$
 \therefore the gradient of the tangent to the curve $f(x)$ when $X \neq 0$ always negative.

Known $f(x) = \frac{2}{x}, x \neq 0$
 a) Prove that the gradient of a tangent is always negative

Figure1. Participant answer A-1 to question number 1

2. Category Argument

Participant A2 was asked to provide an argument regarding the answer saying that, the gradient of the tangent to the curve f is always negative. Participant A2 gives an argument about $f'(-2) = -\frac{1}{2}$, meaning that at the time of abscissa ($x = -2$), the gradient is equal to -2 (A2, A4, A5, & A7). The graph of $f(x)$ when the value of $x = -2$, if from the sketch of the tangent from the point abscissa -2 , then it has a negative half gradient, and the graph on the left is slightly

sloping towards the horizontal. Participant A2 also said that the graphic image $f(x) = -\frac{2}{x}, x \neq 0$ is in the quadrant I and III regions (See Figure 2). Furthermore, participants A2 were asked to provide further arguments about the value of $f'(-2) = -\frac{1}{2}$ in question number 1. Participants A2 and A3 said when the value of $x = -2$ and 2 , the slope of the slope (A1, A4, A5, & A7) at that point is $-\frac{1}{2}$. This means that the gradient is negative (A1, A3, A4, A5, A6, A7, A8, A9, & A10), and is always going down. Furthermore, A3's participation says, at time $x = -\frac{1}{2}$, the function decreases, because the gradient of the tangent line is negative ($-\frac{1}{2}$), and at $x = -\frac{1}{2}$, the first derivative is equal to $-\frac{1}{2}$ meaning that there is no stationary point because $-\frac{1}{2}$, not equal to zero.

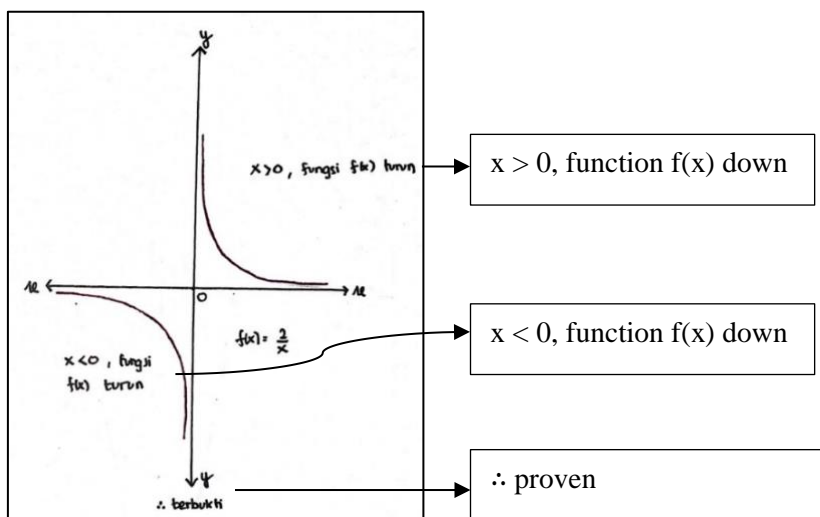


Figure 2. Answers of Participant A-2 to question number 1

3. Category derivative interpretation

Participant A3, in question number 1, part a, was asked about a conclusion about the graph rising or falling of the function f when $x > 0$ or $x < 0$. Participant A3 said that $f'(x)$ or the gradient of the tangent line is always negative (A2, A5, A7, A9, & A10), when drawn (A2, & A4) the gradient of each tangent line will point to the lower right, at every value of x in the function f . This explains that the function $f(x)$ always decreases for $x > 0$ or $x < 0$ (A3, A5, A6, A7, A8, A9, & A10). Furthermore, participant A3 said that for any value of x (except zero) the gradient will always be negative, so the graph will always go down (A1, A3, A5, A6, A7, A8, A9, & A10). So, for $x < 0$ or $x > 0$, it will not affect the monotony of the function or the rise and fall of the function $f(x)$. In addition, participant A2 also said that the graph of the function $f(x) = \frac{2}{x}, x \neq 0$ in quadrant I where the values of x and y are positive and in quadrant III the x value is negative and the y value is negative (see Figure 2).

4. Category Strategy development

The development of a problem-solving strategy carried out by participant A4 in question number 1, looks like the following expression of the interview conclusions:

"Choosing a strategy in reducing functions by using derivative formulas (A2, A3, A4, A5, A6, A7, A8, A9, & A10), decreasing functions will be much faster when compared to using the limit concept (A2, A3, A4, A6, A8, A9, & A10). Then, choosing a strategy in concluding $f'(x)$ for those who do not know the concept of a quadratic number is always positive, may have to use a multiple-point test (A5, & A10), and in the end, conclude some of the samples that have been given".

Meanwhile, participant A2 said that in developing the solution strategy, they used the concept of limits (A1, A3, A4, A6, A8, A9, & A10), and the definition of derivatives (A1, A2, A4, A6, A8, & A9). Besides that, you can also draw graphs (A1, & A3). From the graph it will be seen that the function goes down or up, then proceeds to analyze the shape of the function (A1, & A5) to determine the negative definite nature. Furthermore, participant A3 said that the graph of the function was displayed, then the tangent lines were drawn (the tangent line drawn at the point of tangency with components $x < 0$ and $x > 0$). If you do this, you will see that the tangents are all descending. This means that the slope of the tangent to $f'(x)$ is negative and the function decreases for both $x < 0$ and $x > 0$.

5. Category reasoning

The reasoning is a flow of thinking or student thinking that is used to produce statements in concluding solving problems. The line of thinking used by participant A1 in question number 2 includes; knowing where the function $f(x) = y$ (A2), knowing that the roots on the graph of the quadratic function are not only 3, but -3 as well (see Figure 3). Then participant A1 looks for the derivative, either by using the formula for the derivative of a polynomial or by using the concept of limit (A2, A3, A9, & A10), after finding it, participant A1 continues by looking for the equation of the line that passes through the point (x_1, y_1) with a gradient of m (A2, A3, A4, A5, A6, A7, A9, & A10), then investigate how to sketch graphs (A4, A5, & A10) and their analysis, both for graphs of linear equations and quadratic equations. Participant A1 in question number 2 was able to explain in more detail the process of solving the problem, it can be seen from the following interview snippet:

"Especially for sketching a quadratic equation graph, knowing the concept of a stationary point from the graph to determine the absolute minimum/maximum stationary point of the graph".

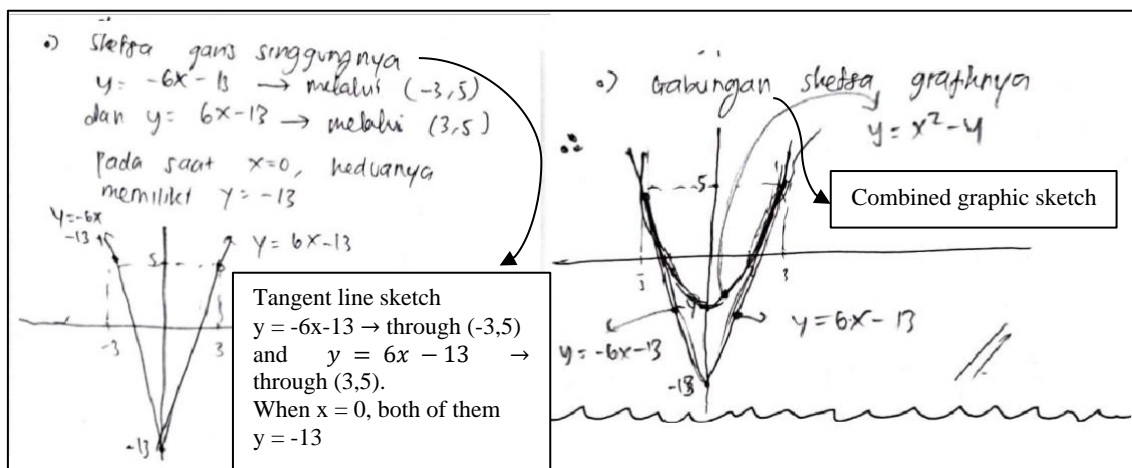


Figure 3. Participant A1 question number 2

The thinking flow of A2 participants includes the concept of a quadratic function, using the equation formula for the tangent line (A1, A3, A4, A5, A6, A7, A9, & A10), the concept of limit (A1, A3, A9, & A10) and the definition of derivative (A1, A3, A6, A7, A9, & A10) to determine the gradient of the tangent equation. Then proceed with sketching the graph of the function (A1, A4, A5, & A10), and the equation of the tangent line.

A3 participants' thinking flow includes; (1) determining the properties of derivatives or the concept of limits (A1, A2, A3, A9, & A10), in finding the first and second derivatives; (2) using the linear equation formula if the gradient and point are known $(y - y_1) = m(x - x_1)$; (3) to get a stationary point, $f'(x) = 0$, is used, while the second derivative is used to determine the type of stationary point. If $f''(x) > 0$ means the type of point is a relative minimum turning point if $f''(x) < 0$ means the type of point is a relative maximum turning point (A1); (4) to get the inflection point, $f''(x) = 0$ is used; (5) find the x-intercept of the graph, use $f(x) = 0$ and (6) find the y-intercept of the graph, use $x = 0$.

6. Category analysis

In question number 2, participants are asked to analyze the graph of the y curve along with the equations of the tangent lines, the analysis carried out by participant A1 in question number 2 is based on the graph that has been sketched by dividing the three interval regions, as seen in participant A1 (see Fig. 3), the combined graph of the quadratic function, and the two equations of the tangent line. When $x < 0$, the y graph falls (A1, A2, A6, & A9). The downward y graph is explained by the slope of the tangent line which is negative, when $x = 0$, the y graph experiences a minimum. The graph of y experiencing a minimum can be explained by the slope of the tangent line which is 0, and when $x > 0$, the graph of y increases (A1, A2, A6, & A9). The rising y graph is described by the gradient of the tangent line which is positive. As an example

of the answer, the tangent at $x = -3$ has a gradient of -6 , the function descends at x . The tangent line at $x = 3$ has a gradient of 6 , the function goes up at that x .

7. Category building definition

Constructing meaning is to explain how the first derivative of a function can be viewed as the rate of change. Participant A4 gave his opinion because the derivative formula itself comes from the problem, for the example given a function, $V(r)$, and suppose that $V(r)$ is the volume of a sphere determined by the radius. If the radius is increased by h , and his very small (towards zero), we get $V(r + h)$ is the final volume sebagai of the sphere. To calculate the average rate of change of volume concerning the radius, we can use the concept of dividing the change in volume by the change in radius, so that we get something like $(\frac{V(r+h)-V(r)}{h})$. Since h is very small, towards zero, we give a limit with $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ which represents the instantaneous rate of change of volume concerning radius. Based on the example argument above, it can be concluded that the first derivative of a function can be viewed as the rate of change.

Participant A5 said; For example, there is a variable y that depends on the variable x where $y = f(x)$. The change in x is $x_2 - x_1$, while the change in y is $y_2 - y_1$. Broadly speaking, the rate of change is divided into two, namely the average rate of change and the instantaneous rate of change. To distinguish them, we can look at the value of x proposed in the problem, whether it is an interval or a specific value. If it is an interval, then the rate of change is the average rate of change. If it is a specific value, then the rate of change is the instantaneous rate of change. The instantaneous rate of change can be expressed as $\frac{f(x+h)-f(x)}{h}$, so that $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ can be viewed as the formula for the first derivative of a function, as explained by participant A5 below.

<p>A) Turunan/diferensial suatu fungsi $y = f(x)$ terhadap x dapat diartikan sebagai laju perubahan fungsi terhadap suatu variabel bebas x. Maka dari itu, laju perubahan volume balon juga dapat diartikan sebagai turunan pertama persamaan volume balon.</p> <p>B) Laju perubahan suatu fungsi sama dengan turunan pertamanya, dengan syarat nilai limit x ada. Semakin garis besar, laju perubahan fungsi dibagi menjadi dua, yaitu rata-rata dan sesaat. Karena di soal tidak disebutkan interval x, tetapi menyebutkan nilai x ($r = 25$ cm) yang spesifik, laju yang dimaksud adalah laju sesaat, artinya perubahan pada interval h mendekati nol ($h \rightarrow 0$) dengan catatan nilai limitnya ada.</p> $f(x) = \frac{4}{3}\pi x^3$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= 4\pi x^2 //$ <p>$4\pi x^2$ juga merupakan rumus luas permukaan balon (bola). Maka dari itu, karena perubahan V sesaat pada saat $r = 25$ cm dapat dikatakan sangat kecil ($h \rightarrow 0$), laju perubahan volume sama dengan luas permukaannya.</p>	<p>A. The derivative/differential of a function $y = f(x)$ with respect to x can be interpreted as the rate of change of function Against an independent variable x, therefore, the rate of change of balloon volume can also be interpreted as the first derivative of the balloon volume equation</p> <p>B. The rate of change of a function is the same as its first derivative, provided that the limit value of x exists. Broadly speaking, the rate of change of the function is divided into two, namely the average, and the instantaneous, because the question does not mention the interval x, but mentions the specific value of x ($r = 25$ cm), the rate in question is the instantaneous rate, meaning the change in the interval h is close to zero ($h \rightarrow 0$), with a note that the limit value is</p> <p>$4\pi x^2$ is also a formula for the surface area of a balloon (ball). Therefore, because the instantaneous change in V at $r = 25$ cm can be said to be very small ($h \rightarrow 0$), the rate of change of volume is equal to the surface area.</p>
---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Figure 4. Participant A5 answered question number 3

8. Category understanding basic concepts

There are several steps taken by participant A5 in examining the basic concepts used to answer question number 4, following the answers of participant A5.

<p>③ Diberikan fungsi: $f(x) = 2x^3 + 3x^2 - 12x - 4$ menyelidiki hasil operasi $f'(-1), f''(-1)$ dan $f'''(-1)$ → Menyelidiki apakah $f(x)$ kontinu di $x = -1$ $\lim_{x \rightarrow -1^-} 2x^3 + 3x^2 - 12x - 4 = 9$ $\lim_{x \rightarrow -1^+} 2x^3 + 3x^2 - 12x - 4 = 9$ $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} f(x) \Rightarrow \lim_{x \rightarrow -1} f(x) = 9$ $f(-1) = 9$ $\lim_{x \rightarrow -1} f(x) = f(-1) \Rightarrow f(x)$ kontinu di $x = -1$. → Mencari $f'(x), f''(x)$ dan $f'''(x)$ $f'(x) = 6x^2 + 6x - 12$ $f''(x) = 12x + 6$</p>	<p>Give function $f(x) = 2x^3 + 3x^2 - 12x - 4$ Investigating the results of the operation Investigate whether $f(x)$ is continuous at $x = -1$</p>
	<p>→ Mencari nilai $f'(-1), f''(-1)$ dan $f'''(-1)$ → Find value $f'(-1) = 6(-1)^2 + 6(-1) - 12 = -12$ $f''(-1) = 12(-1) + 6 = -6$ $f'''(-1) = 12$</p>
	<p>$f(x)$ is continuous at $x = -1$ Find $f'(x), f''(x)$, and $f'''(x)$</p>

Figure 4. Answers from Participant A-5 to question number 4

Participant A5 performs the basic concept used by investigating whether $f(x)$ can be derived for $x = -1$, by testing for continuity, $f(x)$ at $x = -1$, using the concept that the left limit is the same as the right limit. If the limits are the same, then the function can be derived. So the next step is to reduce $f(x)$ three times (A2, A4, A5, A6, A7, A8, A9, & A10), to find the first, second, and third derivatives. The concept used by participant A5 here is by using the basic properties of addition and subtraction derivatives (A3, & A4).

9. Category evaluating

To evaluate the statement in question number 4 by participant A5 (See Fig. 6).

<p>→ selidiki pernyataan. ① $f'(-1) \cdot f''(-1) = -12 \cdot 6 = -72$ (sejati) ② $\frac{f'''(-1)}{f''(-1)} = \frac{12}{-6} = -2$ (sejati) ③ $\frac{f''(-1)}{f'(-1)} = \frac{-6}{-12} = \frac{1}{2}$ (sejati) ④ $f'''(-1) - f''(-1) + f'(-1)$ $= 12 - (-6) + (-12) = 6$ (tidak sejati) ∴ pernyataan yang SALAH adalah pernyataan 4.</p>	<p>Investigating Statement. 1. $f'(-1) \cdot f''(-1) = 12 \cdot 6 = -72$ (match) 2. $\frac{f'''(-1)}{f''(-1)} = \frac{12}{-6} = -2$ (match) 3. $\frac{f''(-1)}{f'(-1)} = \frac{-6}{-12} = \frac{1}{2}$ (match) 4. $f'''(-1) - f''(-1) + f'(-1)$ $= 12 - (-6) + (-12) = 6$ (Not match) ∴ The incorrect statement is statement number 4</p>
--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Figure 6. Participant Answers A-5

Participant A5 evaluates the derivative by substituting (A3, A4, A5, A6, A8, A9, & A10) the value of $x = -1$ in the equations obtained from the first, second, and third derivatives, then operates it (A2) according to with the operations given to statements 1, 2, 3, and 4. Next, look for which statements are wrong, so that the answers are fulfilled (A3, A4, & A5).

From the exploration of the answers of the participants whose mathematical abilities are very high on the ability of the application material for the derivative of algebraic functions above, conclusions are obtained about the subcategories and subcategories found in the extraction process of the participants' answers which include, written tests, google forms, semi-interviews. structured through the zoom application, obtained 9 categories, and 40 subcategories of students' critical thinking skills that are correlated with each other. For the problem-solving category, 6 subcategories were obtained including the definition of the derivative, the concept of limit, the gradient is always negative, substitution, the gradient is equivalent to the first derivative, must be negative. The argument categories obtained are 6 sub-categories including definitely negative, descending function, graph sketch, substitution, always gradient, negative $f'(x)$ function is negative. The category of derivative interpretation obtained 6 subcategories including graph function $f(x)$ always descends, the gradient of the tangent line, graph sketch, the gradient is always negative, function $f'(x)$ is always negative, graphic sketch. The strategy development category obtained 8 sub-categories including graph sketch, mathematical induction, double point test, derivative properties, the definition of derivative, graph method, limit concept, and functional form analysis.

The categories obtained from 7 sub-categories include a definition of the derivative, the concept of limit, relative maximum turning point, gradient equal to the first derivative, the concept of quadratic function, x -axis, and y -axis intersection point, stationary point. Category analysis obtained 6 sub-categories including positive and negative tangent gradient, minimum tangent gradient, minimum relative turning point, decreasing function, the axis of symmetry, ascending function. The category that builds meaning is obtained by 5 subcategories including the concept of limit, the rate of change of the interpretation of the first derivative, the gradient of the first derivative, the gradient of interpretation of the rate of change, the average rate of change. The basic categories of the examination obtained are 4 sub-categories including substitution, (first, second, and third derivative concepts), limit continuity, and conclusions. Categories obtained from 8 sub-categories include a definition of the derivative, concept of limit, substitution, finding false statements, operations, calculations, equations, and definitions of derivatives, to be clear, see picture 7, NVivo 12 plus analysis balloon below.

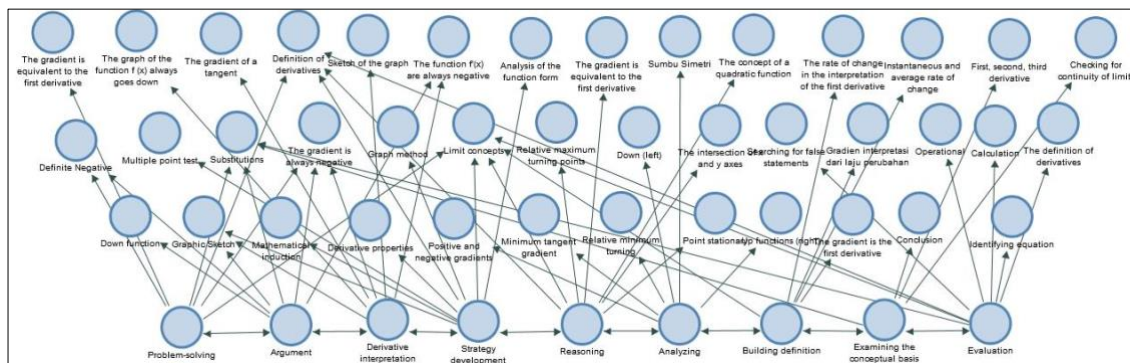


Figure 7. Categories and sub-categories of CT abilities of very high group students in solving algebraic function derivative application problems.

The findings in the group of students with very high mathematical ability, students mastered the problem better. All aspects in the category experienced a significant increase. Students at a very high level of mathematical ability, students master the problem very well, this can be seen from the category of argument and reasoning that appears with very significant changes in the student problem-solving process. In addition, the strategies used are more varied, and the process of examining the basic concepts and many sub-categories is found.

However, two aspects stand out the most at a very high level, namely argument and reasoning. This is in line with several previous studies (Facione, 1990; Halpern, 2014; Lipman, 1988; Sadeghi, 2020; Sheromova et al., 2020) which emphasize the most important characteristics of a critical thinker, namely argument, and reasoning, where the argument is described as a cognitive activity, and reasoning is the key in developing critical thinking skills.

Many views from other researchers who say reasoning is the core of critical thinking (Singer, 2018; Stella Cottrell, 2005; Sternberg, 2012), but in the results of this study, problem solving skills are an important part of the process of critical thinking skills, the deeper the problem solving process is, the more subcategories that can be explored from the participants' answers.

Furthermore, (Papp et al., 2014; Sadeghi, 2020) Critical thinking ability is very dependent on the intellectual capacity of students, meaning that the higher the level of students' thinking skills, the deeper the problem solving process. The process of developing students' critical thinking skills should start from the learning process that uploads students' thoughts (Fahim & Mirzaii, 2013). If the learning process inspires students' minds, and is supported by students' intellectual capacity, it is certain that a variety of student problem solving strategies will be found. Besides that, (Markey et al., 2020; Moon, 2007) said problem solving strategies are the most important thing to strengthen critical thinking, where participants are required to see problems from different perspectives.

Conclusion

The findings in this study are expected to help educational practitioners in developing students' CT abilities in the learning process, by paying attention to nine aspects of the researchers' findings in CT abilities which include: problem-solving, arguments, derivational interpretation, strategy development, reasoning, analysis, constructing meaning, a basic understanding of concepts, and evaluation.

The critical thinking skills of group students are very high, critical thinking skills vary widely in solving problems, and students can provide solutions to anyone who solves similar problems. Aspects of problem-solving, argument, reasoning and analysis lead to the level of creative thinking skills. This can be seen from the vocabulary that appears when students solve problems. For example, analysis of the form of a function, continuity test, and identification of each given equation.

The researcher's findings were limited to a very high level of mathematical ability, so they did not find CT abilities in students with low, medium, and high mathematical ability levels in the application material of algebraic function derivatives. Given the importance of exporting CT abilities based on mathematical ability levels, further research on low, medium, and high mathematical ability levels needs to be carried out further.

References

- As'ari, A. R., Kurniati, D., & Subanji. (2019). Teachers expectation of students' thinking processes in written works: A survey of teachers' readiness in making thinking visible. *Journal on Mathematics Education*, 10(3), 409–424. <https://doi.org/10.22342/jme.10.3.7978.409-424>
- Association of American Colleges and Universities. (2005). *Liberal education outcomes: A preliminary report on student achievement in college*. AAC&U. https://www.aacu.org/sites/default/files/files/LEAP/LEAP_Report_2005.pdf
- Australian Council for Educational Research. (2002). *Graduate skills assessment: Stage one validity study*. Department of Education, Science and Training. https://research.acer.edu.au/higher_education/27
- Basri, H., Purwanto, As'ari, A. R., & Sisworo. (2019). Investigating critical thinking skill of junior high school in solving mathematical problem. *International Journal of Instruction*, 12(3), 745–758. <https://doi.org/10.29333/iji.2019.12345a>
- Charmaz, K. (2019). Thinking about data with grounded theory. *Qualitative Inquiry*, 25(8), 743–753. <https://doi.org/10.1177/1077800418809455>
- Cohen, L., Manion, L., & Morrison, K. (2020). Experiments, quasi-experiments, single-case research and meta-analysis. In *Research Methods in Education*. <https://doi.org/10.4324/9780203029053-23>
- Corbin, J. M., & Strauss, A. (1990). Grounded theory research: Procedures, canons, and evaluative criteria. *Qualitative Sociology*, 13(1), 3–21. <https://doi.org/10.1007/BF00988593>

- Dwyer, C. P., Hogan, M. J., & Stewart, I. (2012). An evaluation of argument mapping as a method of enhancing critical thinking performance in e-learning environments. *Metacognition and Learning*, 7(3), 219–244. <https://doi.org/10.1007/s11409-012-9092-1>
- Eisenhardt, K. M. (1989). Building Theories from case study research. *Academy of Management Review*, 14(4), 532–550. <https://doi.org/10.5465/amr.1989.4308385>
- Ennis, R. H. (1991). Critical thinking: a streamlined conception. *Teaching Philosophy*, 14(1), 5–25.
- Ennis, R. H. (1993). Critical thinking assessment. *Theory Into Practice*, 32(3), 179–186. <https://doi.org/10.1080/00405849309543594>
- Facione. (1990). *Critical thinking: a statement of expert consensus for purposes of educational assessment and instruction. Research findings and recommendations*. The California Academic Press.
- Fahim, M., & Mirzaii, M. (2013). Improving EFL argumentative writing: A dialogic critical thinking approach. *International Journal of Research Studies in Language Learning*, 2(3), 3–20. <https://doi.org/10.5861/ijrsl.2013.313>
- Feng, X., & Behar-Horenstein, L. (2019). Maximizing NVivo utilities to analyze open-ended responses. *Qualitative Report*, 24(3), 563–571.
- Fisher, A. (2001). *An Introduction to Critical Thinking*. Cambridge.
- Hage, G. (2020). The haunting figure of the useless academic: Critical thinking in coronavirus time. *European Journal of Cultural Studies*, 23(4), 662–666. <https://doi.org/10.1177/1367549420926182>
- Halpern, D. F. (2014). Thought and Knowledge: An introduction to critical thinking. In *Educational Psychology in Practice* (Fifth Edit, Vol. 30, Issue 4). Taylor & Francis Group. <https://doi.org/10.1080/02667363.2014.934516>
- Karpouza, E., & Emvalotis, A. (2019). Exploring the teacher-student relationship in graduate education: a constructivist grounded theory. *Teaching in Higher Education*, 24(2), 121–140. <https://doi.org/10.1080/13562517.2018.1468319>
- Kementrian Pendidikan dan Kebudayaan. (2020). *Projek penguatan profil pelajar pancasila dan budaya kerja (P5BK)*. 1–40.
- Lipman, M. (1988). *Critical thinking- what can it be ? education leadership, the association for supervision and curriculum development*. 38–43.
- Liu, K. S., & Hsueh, S. L. (2016). Effects of digital teaching on the thinking styles and the transfer of learning of the students in department of interior design. *Eurasia Journal of Mathematics, Science and Technology Education*, 12(6), 1697–1706. <https://doi.org/10.12973/eurasia.2016.1563a>
- Markey, K., Tilki, M., & Taylor, G. (2020). Practicalities in doctorate research of using grounded theory methodology in understanding nurses' behaviours when caring for culturally diverse patients. *Nurse Education in Practice*, 44(June 2018), 102751. <https://doi.org/10.1016/j.nepr.2020.102751>
- Marzuki, Asih, E. C. M., & Wahyudin. (2019). Creative thinking ability based on learning styles reviewed from mathematical communication skills. *Journal of Physics: Conference Series*, 1315(1). <https://doi.org/10.1088/1742-6596/1315/1/012066>
- Marzuki, Wahyudin, Cahya, E., & Juandi, D. (2021). *Students' critical thinking skills in solving mathematical problems ; a systematic procedure of grounded theory study*. 14(4), 529–548.

- McGregor, D. (2007). *Developing thinking ; developing learning : a guide to thinking skills in education*. Open University Press. http://vct.qums.ac.ir/portal/file/?180494/Developing-thinking_-developing-learning.pdf
- Miller, S. E., & Topple, T. A. (2020). Thinking and thinking about thinking: a qualitative study of learning in a process-centric teaching model. *Journal of Social Work Education, 56*(1), 115–130. <https://doi.org/10.1080/10437797.2019.1648224>
- Moon, J. (2007). Critical thinking: An exploration of theory and practice. In *Critical Thinking: An Exploration of Theory and Practice*. <https://doi.org/10.4324/9780203944882>
- Munandar. (2016). *Pengembangan Kreativitas Anak Berbakat*. Rineka Cipta.
- Papp, K. K., Huang, G. C., Lauzon Clabo, L. M., Delva, D., Fischer, M., Konopasek, L., Schwartzstein, R. M., & Gusic, M. (2014). Milestones of critical thinking: A developmental model for medicine and nursing. *Academic Medicine, 89*(5), 715–720. <https://doi.org/10.1097/ACM.0000000000000220>
- Sadeghi, G. Z. & M. D. (2020). Iranian EFL teachers' and learners' perceptions of the principles of critical thinking: A constructivist grounded theory study. *Iranian Journal of Language Teaching Research, 8*(2), 63–81.
- Setiyani, Putri, D. P., Ferdianto, F., & Fauji, S. H. (2020). Designing a digital teaching module based on mathematical communication in relation and function. *Journal on Mathematics Education, 11*(2), 223–236. <https://doi.org/10.22342/jme.11.2.7320.223-236>
- Sheromova, T. S., Khuziakhmetov, A. N., Kazinets, V. A., Sizova, Z. M., Buslaev, S. I., & Borodianskaia, E. A. (2020). Learning styles and development of cognitive skills in mathematics learning. *Eurasia Journal of Mathematics, Science and Technology Education, 16*(11). <https://doi.org/10.29333/EJMSTE/8538>
- Singer, F. M. (2018). *Mathematical creativity and mathematical giftedness*. <http://link.springer.com/10.1007/978-3-319-73156-8>
- Sobur, A. (2010). *Psikologi umum*. Pustaka Setia.
- Stake, R. E. (1995). *The art of case study research*. SAGE Publications, Inc.
- Stella Cottrell. (2005). Critical thinking skills. developing effective analysis and argument. In *Contemporary Nurse* (Vol. 25, Issues 1–2). Palgrave.
- Sternberg, R. J. (2012). Critical thinking in psychology. In *Critical Thinking in Psychology*. <https://doi.org/10.1017/cbo9780511804632.018>
- Sustekova, E., Kubiato, M., & Usak, M. (2019). Validation of critical thinking test on Slovak conditions. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(12). <https://doi.org/10.29333/ejmste/112295>
- Taherdoost, H., & Group, H. (2017). Validity and reliability of the research instrument ; how to test the validation of a questionnaire / survey in a research. *International Journal of Academic Research in Managemen (IJARM), 28–36*. <https://doi.org/10.2139/ssrn.3205040>
- Violetta Wilk, Geoffrey N. Soutar, P. H. (2019). Tackling social media data analysis: Comparing and contrasting QSR NVivo and Leximancer. *Qualitative Market Research, 22*(2), 94–113. <https://doi.org/10.1108 / QMR-01-2017-0021>
- Widyatiningtyas, R., Kusumah, Y. S., Sumarmo, U., & Sabandar, J. (2015). The impact of problem-based learning approach tosenior high school students' mathematics critical thinking ability. *Journal on Mathematics Education, 6*(2), 30–38. <https://doi.org/10.22342/jme.6.2.2165.107-116>