
Developing RME-Based Learning Trajectory for Teaching Addition to A Dyscalculia Student in Elementary School

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Abstract. *This research aimed to design an RME-based learning trajectory for a dyscalculia student to learn the addition of whole numbers. The research used design research approach that consists of three phases: preparing for the experiment, conducting the experiment, and retrospective analysis. This research's data collection techniques were observations, interviews, videotaping, and analyzing the student's works. The main result of this research is the learning trajectory for teaching addition of whole numbers to a dyscalculia student using RME approach. The series of activities in the learning trajectory are addition of whole numbers between 1 and 10 by combining the objects, addition of whole numbers between 1 and 10 using number relations, addition of whole numbers between 1 and 20 using number relations, and finding the concept of place value of tens and ones in addition of numbers. This research also shows the cognitive improvement of dyscalculia student in learning the addition of whole numbers. Learning activities carried out by the dyscalculia student help him to shift from informal knowledge to formal mathematical knowledge in order to understand the concept of addition of whole numbers. It makes dyscalculia student has number sense, number construction, and number relation abilities which increase significantly in the learning process.*

Keywords: *design research, hypothetical learning trajectory (HLT), realistic mathematics education (RME), local instructional theory (LIT), dyscalculia.*

Introduction

The main difficulty experienced by teachers in learning mathematics is teaching students who have problems with their level of intelligence and ability. This crucial problem is seen in student disability in various writing, reading, and mathematics (American Psychological Association [APA], 1994; Rajaie, et.al, 2011). Particular disabilities in learning mathematics can interfere with students' cognitive intelligence, which refers to slow learners (Ahmad, et.al, 2015; Gifford & Rockliffe, 2012). There are unique characteristics in students who are slow to learn mathematics. For example, students who are slow to learn mathematics have a poor understanding of simple numerical concepts and have difficulty learning facts and processes related to numbers (Koh, 2020). This condition of slow learning children is categorized as dyscalculia. Dyscalculia is a condition that affects students' inability to acquire numeracy skills and solve complex arithmetic problems and difficulties in remembering facts and basic arithmetic concepts (APA, 2013; Butterworth, 2011; Geary & Hoard, 2001). Dyscalculia students have significant difficulties in learning math skills in everyday life, such as having

difficulty understanding numbers, extracting facts, or performing mathematical operations (Attout, et.al, 2015; Gillum, 2012). Dyscalculia students also have difficulty applying terms, facts, and algebraic procedures in solving mathematical problems (APA, 2013; Bishara & Kaplan 2018).

Dyscalculia students experience cognitive decline in processing numerical and arithmetic information ranging from 5–7% of their brain population. However, dyscalculia students have average intelligence, and there are no abnormalities in their nerves or brain (Chodura, et.al, 2015; McCloskey, et.al, 1985; Shalev, Manor, & Gross-Tsur, 2005; Temple, 1992). Many studies say that the leading cause of dyscalculia is the cognitive decline that occurs in the brain, especially in the working memory process (Kucian, Loenneker, Martin, & von Aster, 2011; Mammarella, Hill, Devine, Caviola, & Szűcs, 2015). What often happens to dyscalculia students causes loss of confidence, creates anxiety in learning mathematics and prevents the information from working in memory. As a result, students experience a decrease in their ability to count academically and in everyday life (Henik, Rubinsten & Ashkenazi, 2011; Gifford & Rockliffe, 2012). The capacity to process information in the brain is reduced, so students use other methods such as counting on their fingers more often. It causes students to need other steps and tends to increase the occurrence of calculation errors. Therefore, wrong answers tend to be found more often in arithmetic and arithmetic problems. To learn arithmetic and numeracy concepts, students need to have high enough self-efficacy as learners and pay attention effectively in learning activities (Attout & Majerus, 2015; Geary & Hoard, 2001; Munro, 2003).

Addition of whole numbers is a crucial subject matter in elementary school. There are many problems in everyday life whose solution requires arithmetic operations to add natural numbers. There is much other mathematics subject matter that dyscalculia students cannot follow if they do not understand the concept of addition well. As a hierarchical subject, every material in mathematics is related to other material, so students must master it to move on to more complex material in the next chapter (Ananda, 2018).

The competencies that dyscalculia students want to achieve in learning mathematics are providing knowledge, understanding concepts, and sensitivity to numbers. Students with number sensitivity can apply their understanding of numbers and number operations to solve problems flexibly by using effective, efficient, and practical strategies based on logical, critical, and creative reasoning. It aligns with NCTM (2000), which explains that students must understand the meaning or value of numbers, number symbolization, and the relationship between numbers in a number system. In addition, students must also understand the definition of a number operation and its impact and the relationship between number operations in a

number system. It makes number sensitivity a crucial aspect of the principles of learning mathematics (Fosnot & Dolk, 2001; Putrawangsa & Hasanah, 2018).

One of the crucial things that can help dyscalculia students learn mathematics is by creating a sense of sensitivity in these students, making it easier for them to understand mathematical concepts useful for meaningful learning. According to previous research, the understanding of concepts in mathematics to be the basis and significantly affect the cognitive abilities of dyscalculia students in solving problems in everyday life (Neergaard, 2013; Fosnot & Dolk, 2001).

The ability level of dyscalculia students in elementary schools is at the concrete operational stage. It is in line with Piaget's opinion that the mindset of elementary school students is still at the concrete active stage. Hence, students need to be directed to understand mathematics by paying attention to mental aspects. A person's cognitive development is obtained through a series of processes in the activities he experiences. For students who do not understand the law of conservation of numbers, then it is not the time for them to get the concept of addition or other arithmetic operations. If they were taught about addition operations, they would most likely not understand (Bishara & Kaplan, 2018). Meanwhile, students in the concrete operational stage can understand the concept of number operations (Karso, 2019). However, this is in contrast to dyscalculia students who experience delays in processing information related to learning arithmetic and arithmetic as well as the learning process of mathematics in elementary schools, which still emphasizes practical skills in the form of giving formulas and formulas (Henik, et. al, 2011; Gifford & Rockliffe, 2012; Guardian, 2013).

In improving the understanding of the dyscalculia students in addition of whole number, teachers need an exceptional approach to the needs, learning strategies, lesson design, and knowledge of what and how to teach students with dyscalculia. However, teachers do not yet understand and recognize the characteristics of dyscalculia students and how to teach these students (Morsanyi, van Bers, O'Connor, & McCormack, 2018; Tran, Nguyen, T., Le & Phan, 2019). It makes it difficult for mathematics teachers to teach arithmetic operations to dyscalculia students from childhood to adulthood (Attout, et. al, 2015; Gillum, 2012; Williams, 2013). Several studies suggest that further studies are needed through instructional design with an instructional format regarding numeracy skills (computational operations) in dyscalculia students that can be implemented in elementary schools (Monei & Pedro, 2017; Nelwan, Friso-van den Bos, Vissers, & Kroesbergen, 2021). It is supported by several studies that teachers must predict how it is likely that students with dyscalculia experience learning and the steps that

are prepared by the teacher in developing the thinking skills and understanding of these students from learning activities designed by the teacher (Dahar, 2018; Wijaya, 2009).

Fauzan, Yerizon, and Yolanda (2020) states that mathematical concepts in elementary schools must be introduced procedurally and formally in line with this statement. Mathematics instruction is generally conducted by introducing formulas and solving mathematical problems, including numbers and number operations. The use of procedures and the introduction of number concepts without understanding is one of the causes of the weak number sense of dyscalculia students. It causes dyscalculia students to lag behind other typical students in understanding more complex material (Arini, 2018; Putrawangsa & Hasanah, 2018).

Efforts made by teachers to help students with dyscalculia problems include creating a learning trajectory based on realistic mathematics education (RME). The RME approach helps students solve mathematics problems using processes or activities such as understanding contextual issues, solving these problems, comparing and discussing answers, and drawing conclusions about the concepts being studied (Gravemeijer, 2020).

The earlier version of RME-based learning trajectory is designed in form of a hypothetical learning trajectory (HLT) (Ayunika, 2011; Bustang, 2013; Syafriandi, Fauzan, Lufri, & Armiati, 2020; van den Heuvel-Panhuizen & Drijvers, 2020; Yeni, Yarmis, & Tarmansyah, 2013). HLT has three critical components: 1) the learning goal, the goal to be achieved, and 2) the learning activities. Learning activities are the design of the learning flow that students will pass to achieve the learning objectives that have been set. Learning activities are usually given in the form of tasks (mathematical tasks) and 3) hypothetical learning processes. The theoretical learning process is the predictions of students' understanding and reasoning that will develop in the learning process and the prepared anticipations to help students achieve their goals (Dickinson, Eade, Gough, Hough, & Solomon, 2020; Fauzan & Diana, 2020; Gravemeijer, 2020; Simon & Tzur, 2004). After the try out of the HLT through a cycle of design research, it become a local instructional theory (LIT). LIT is a learning process theory that describes the learning trajectory on a particular topic with a series of supporting activities (Gravemeijer & Van Eerde, 2009; Simon, Kara, Placa & Avitzur, 2018). This study aims to produce a LIT that is called an RME-based learning trajectory for teaching addition of whole number to a dyscalculia student. The RME-based learning trajectory is also expected to lead the dyscalculia student to have a good ability in number sense, number constructions, and number relations.

Method

This study is a design research of three phases of activities, namely preparing for the experiment, conducting the experiment, and retrospective analysis (Gravemeijer & Cob, 2006). These phases are used to develop an HLT into RME-based learning trajectory. In designing the HLT, the activity begins with a thought experiment in which the researcher thought about the trajectory of learning that the students will go through and then reflecting it on the results of the conducting the experiment phase. If the goal has not been reached, then the next thought experiment and instruction experiment will proceed. This cycle is shown on the Figure 1.

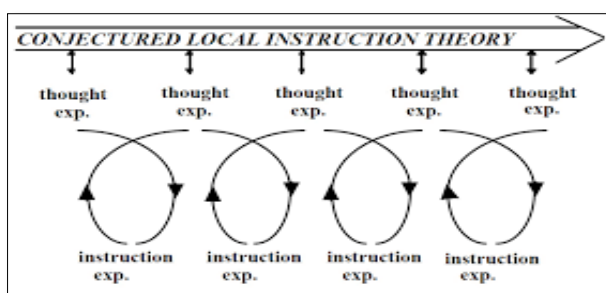


Figure 1. Gravemeijer and cobb model of research design cycle (Gravemeijer & Cobb, 2006)

When *preparing for the experiment*, the researcher conducts observations on how the students at grade three in an elementary school in Musi Rawas, Indonesia learn about number operations. Then, it is studied the literatures regarding the criteria for dyscalculia students who were the subject of research and how far these students had mastered the addition of whole numbers. To identify the dyscalculia students, a test developed by Marlina (2019) was given to the students. These activities intended as a basic idea in developing HLT, which includes learning objectives, activities, predictions/conjectures and anticipations.

Based on results of the test, it was indentified one dyscalculia student in the classroom. Although the students is already at grade three but he still have problems in addition of whole numbers. Therefore, the HLT was designed for topic addition of whole numbers. Before the HLT was tried out in the conducting experiment phases, it was validated by three math educators.

As the subject of the research was only one student, *the conducting experiment phases* were carried out through one-to-one meetings. In every meeting of 70 minutes, the researcher started the activity by giving a contextual problem regarding addition of whole numbers. Based on the student’s answers or responses, the researcher explored the student’s thinking and understanding by doing the interviews which were in line to the predictions and anticipations prepared in the HLT. These activities were videotaped.

The results of each meeting in the conducting the experiment phase were analyzed during the retrospective analysis phase. The analysis is focused on answering the questions: has the goal(s) of HLT been achieved or not and has the students learned as intended or not? If the answer is no, then the researcher will revise or change the activity in the HLT, then try out it again in the classroom. Otherwise, the next activities will be carried out.

Data collection in this research involves observations, test, interviews, videotaping, and analyzing the student's works. Collected data were analyzed descriptively to answer the research question about the impact of the RME-based learning trajectory toward the dyscalculia student's understanding on addition of whole numbers.

Results and Discussion

Result of preparing for the experiment phase

The results of the identification of dyscalculia student showed several unique characteristics such as (1) the student was often late when doing assignments compared to other typical students; (2) the comprehension of mathematics lessons given by the teacher is low and requires repeated explanations to understand them; (3) the ability to count is far below the ability of other typical students; (4) the student is unable to determine which units are and which are tens, and are unable to add up two-digit numbers. It is in line with Munro (2003) and Attout and Majerus (2015) on criteria and characteristics of dyscalculia students who refer to difficulty counting or performing mathematical operations.

Dyscalculia students aged 7-10 have good numeracy skills and can recognize numbers by counting, mentioning, sorting, and matching numbers with many objects. However, dyscalculia students show disturbances in the rules of substitution or number operations (Saga, Rkhaila, Ounine, & Oubaha, 2021). Further identification was carried out by providing an assessment on adding whole numbers to student who was identified as having dyscalculia. It aims to see the basic knowledge of the students about mathematics. The study results showed that the student still looked hesitant and very slow in performing addition operations from 1 to 10. The student also cannot solve problems about the addition of one missing term.

Based on the conditions of the dyscalculia student, it was designed the HLT for topic addition of whole numbers. The series of the activities in the HLT can be seen in Figure 2.

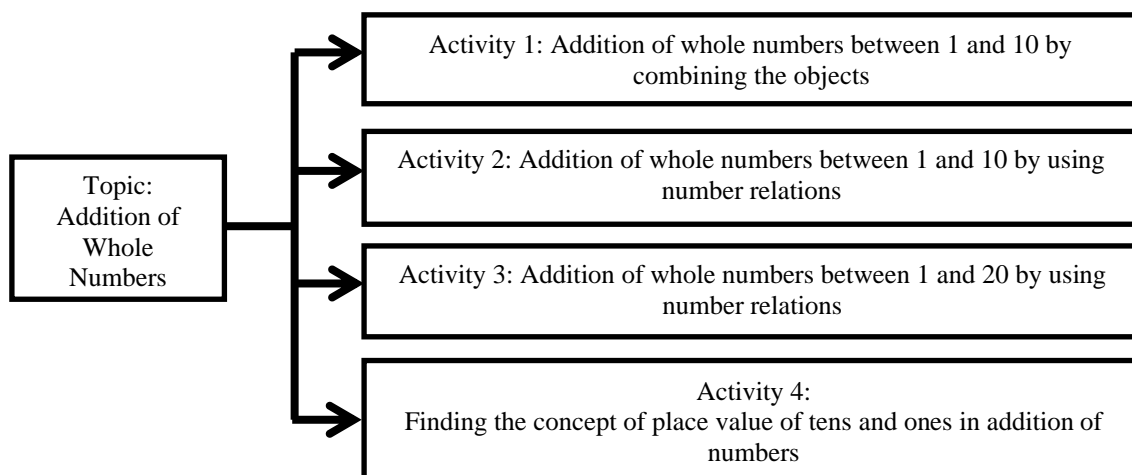


Figure 2. Learning activities topic addition of whole numbers

The goal and rational of each activity, the predictions of student's thinking, and the anticipations are elaborated further in this section. In the first activity, students are expected to be able to find the sum of 1-10 by combining many objects and being able to compare many things in a container. The activity provided is that student is asked to count the number of balls in container A and container B and determine the total number of balls. It is estimated that student will solve the given problem with several predictions, including that (1) student will answer correctly; (2) besides that student will look confused and silent to solve the given problem, (3) students will solve by pointing one by one the balls in the container A and balls in container B then determine the total number, (4) students will add up the results of counting the balls in container A and container B (with sideways addition). If student come with inaccurate answers, the teacher will direct them by providing anticipation in the form of the following questions: "Can we use the props provided to solve the problem?"; "Do we need to count the balls one by one in each container?"; "Is there any other easier way than counting the balls one by one in the container A and container B?"; "How about combining the balls in container A and container B into container C, which way is easier than counting the balls one by one?" "Do we need to count the balls one by one in each container?"; "Is there any other easier way than counting the balls one by one in the container a and container b?"; "How about combining the balls in container A and container B into container C, which way is easier than counting the balls one by one?" "Do we need to count the balls one by one in each container?"; "Is there any other easier way than counting the balls one by one in the container A and container B?"; "How about combining the balls in container a and container B into container C, which way is easier than counting the balls one by one?"

Furthermore, this activity is still related to compare many balls in A container, such as the question, "which are more balls in container A than balls in container B? Give the reason!".

Students with dyscalculia are expected to answer with several predictions, including (1) students will answer correctly; (2) students will be silent and cannot answer; (3) students will guess the answer while aligning their fingers forward to find out more balls compared. Based on the prediction of the answer, students who answered incorrectly will be given anticipation by the teacher in the form of the following questions: "Try to observe the number of balls in the container A and container B, then pair the balls in container A and container B"; "Observe, how many balls do not have a partner?"; "Raise the ball that has no partner, and consider the balls that have no such pairs as more balls than you compare"; "What do you think, is it still necessary to put your fingers forward to find out how much more a number you compare?"

In the second activity, the expected goal is that students can find the addition of numbers 1-10 with one of the digits omitted. The activity demonstrated the rabbit jumping on the numbered abacus pole, then asked the students how many times the rabbit had to add more jumps to reach a curtain pole. Based on the problems given, it is estimated that students will answer with predictions, namely, (1) the correct answer is as expected, (2) students will look confused and cannot answer in solving the given problem, (3) students will answer without demonstrating using the provided abacus media. Furthermore, the teacher provides anticipation in the form of questions that will lead students to arrive at the expected goals as follows: "Can we use the media that has been provided?"; "What position is the rabbit currently in?"; "Can you demonstrate how many times the rabbit has to add more jumps to reach the pole that is being asked in the question?"; "Why do you answer like that? Give the reason!"

To improve the ability of dyscalculia students in this activity, the teacher gives a different problem, namely, "at first, the rabbit is on pole 3. How many ways can the rabbit add more jumps to get to pole 3?". From the problems given, it is predicted that students will look confused and cannot answer. Students will respond, but the method mentioned is still not correct. Next, the teacher provides anticipation in the form of the following questions: "Try to demonstrate and observe, if the rabbit jumps on the pole 1, how many more jumps does the rabbit have to make to reach pole 3?"; "Can you demonstrate it again by moving different jumps so that the rabbit reaches pole 3?"; "How many ways to increase the jump did you find?"

The third activity aims for students to find the sum of 1-20. This activity is equipped with picture story questions that contain simple problems in everyday life. The researcher predicts that students will look confused in solving the problem based on these problems. Next, the students were asked the following anticipatory questions: "Let us read about the story. What do you know in the story?"; "What is asked in the story?"; "How can you calculate the completion of the story problem?"; "So, how much did you get?"

At the fourth meeting, the goal is that students can determine the place value of tens and units in the addition of numbers. The activity given is that students are asked to observe the abacus pole. Then students determine how many tens and how many units value from the numbers written. The researcher predicts that students will look confused and silent based on the problems given. Furthermore, the anticipation is: "How many abacus poles are full of 10 abacus seeds?"; "What is the value of the poles?"; "How many abacus seeds are worth a unit?"; "So the number asked in the question consists of how many tens and how many units".

The final result at this stage is a series of activities on HLT that have been declared feasible and can be applied to dyscalculia students. These activities aim to assist the cognitive development of dyscalculia students to understand the procedures, facts, and concepts of adding whole numbers.

Results of conducting the experiment phase and retrospective analysis.

The concept of adding many objects

Activity 1 aims to find the sum of the numbers 1-10 by combining objects. The student with dyscalculia is given a real contextual problem: doing calculations by combining balls into a container, starting with discussion and question and answer as follow.

The teacher (as the researcher) asked,

- T : What do you see on this contatable? Can you count the number of objects in this mother's container?*
S : Ball ma'am. Students observe and count carefully and again answer: there are ten balls.
T : If you hold one ball in your left hand and two balls in your right hand, can you count all of your balls?.
S : Yes, ma'am, there are three balls.

Teacher dig deeper into how these students can calculate,

- T : How do you count the number of balls you hold?*
S : By combining the balls in the right-hand and the left-hand ma'am.
T : Yes, that is right, so one ball combined with two balls is equal to three balls. Which hand that held more balls?
S : Left-hand ma'am.
T : How many more balls in the left hand than one ball in the right hand?
S : One ball, ma'am.

Then, the teacher asked,

- T : How many more balls in the left hand than one ball in the right hand?*
S : one ball, ma'am.
T : How many more balls in the left hand than one ball in the right hand?
S : one ball, ma'am

Next, the researcher gave a new problem with a more significant number than the previous question. It aims to determine whether dyscalculia student can perform additional

operations with different numbers. The students is given questions that direct him to be able to construct informal knowledge into formal knowledge with the student being asked to write down the number symbol from the number of balls that have been counted. Based on the analysis of the answers in Figure 3, the dyscalculia student has understood the concept of addition by combining many different balls into one container. It shows that student can explore concrete objects by determining the number of things. This activity provides an opportunity for the student to experience and observe firsthand the many balls that are combined in a container. It is in line with Hans Freudenthal's view of a realistic mathematical approach, namely, "mathematics should be connected to the reality" (Dickinson et.al, 2020; van den Heuvel-Panhuizen & Drijvers, 2020). Through these concrete objects (toy balls), the dyscalculia student can understand the concept of addition to find the results of adding these numbers more quickly. He can bring this informal knowledge into formal knowledge to be achieved as seen in Figure 3.

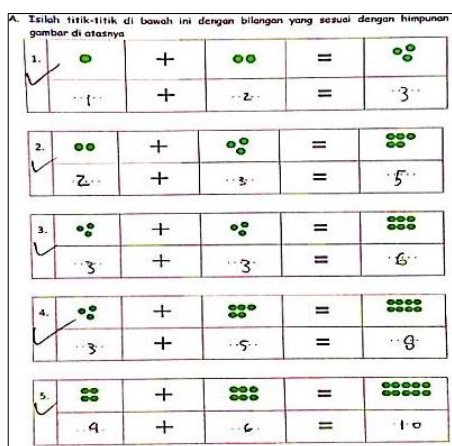


Figure 3. Student’s work on addition of whole numbers by combining objects

In working on the given problem, sometimes the student is not careful in counting the number of balls. There is a need for guided reinvention by the teacher to direct the student to count correctly. The principle of guided reinvention is one of the three key characteristics of RME (Simon et. al, 2018; van den Heuvel-Panhuizen & Drijvers, 2020). If the student experiences errors or doubts, the teacher asks, "Are you sure? Is that the answer? Try to observe the number of balls in container A and container B again!". With this question, student can reinvent the concept of addition of whole numbers.

The concept of adding numbers 1-10 with one digit omitted

Activity 2 aims to find the sum of the numbers 1-10 with one digit omitted. In this activity, the abacus and media with toy rabbits were used by the teacher to explore the student’s understanding. At first, the teacher asks queries using the abacus media as follows.

- T : How many abacus poles are there? Which pole has the least amount of content? Give a reason!, Then, which pole has the most contents? Give the reason!
- S : There are ten poles and 0 at least, because there is no content, and ten poles are the most because there are the most of the other poles (student demonstrates a rabbit jumping on an abacus pole and answered the following questions)
- S : At first, the mother rabbit jumps on pole 2.
- T : How many times does the rabbit have to add more jumps to reach pole 4?
- S : (Student is silent while observing the abacus pole, and then answer) add two jumps.
- T : Why should I add two jumps?
- S : Bcause four is two more than 2.
- T : How much is two less than 4?
- S : (student observe the abacus pole and answer) two more than four, ma'am.

Furthermore, the student's answers are written on the worksheets shown in Figure 4 below.

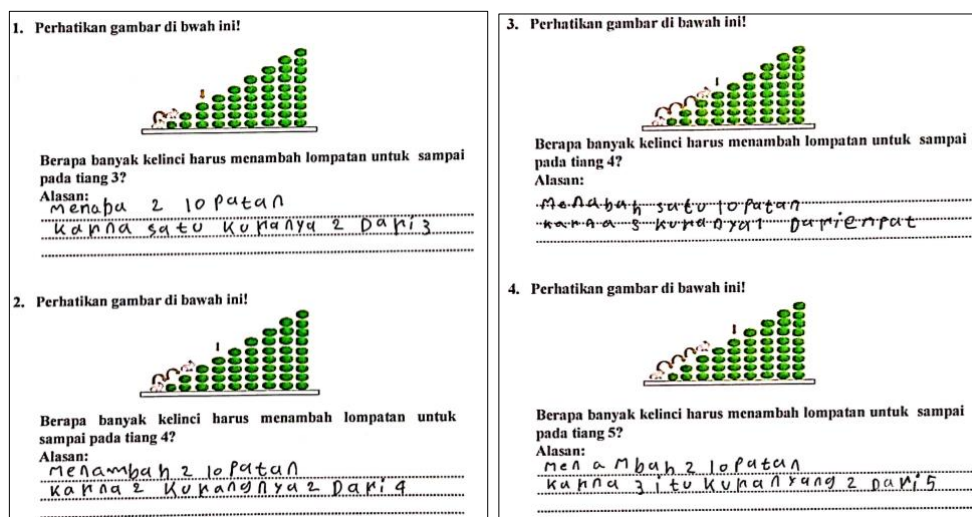


Figure 4. Students work on addition of whole number in which one number is omitted

Based on the answers written by student on the worksheet, it is known that student can understand addition in which one of the numbers is omitted with the help of the abacus media. At first, dyscalculia student had difficulty giving reasons why the student answered that way. With the use of anticipatory questions that direct student in solving the problems presented, such as "Let us see if you look at the abacus, before number 1, which number is the smallest?"; "At first, the rabbit jumps on pole 2, how many times must the rabbit add more jumps to reach pole 5?". In the end, student can give reasons for their answers even though students still need anticipatory questions in helping the thinking process of dyscalculia students' brains (Monei & Pedro, 2017). Anticipatory questions help to improve *mathematical cognition* students so that students are more sensitive to the operation of adding whole numbers (Bishara & Kaplan, 2018).

In achieving the objectives in activity 2, dyscalculia student must be able to construct the informal knowledge that they get into formal knowledge, which is written on student worksheets. Therefore, the learning activity is continued by describing other contextual problems. The results of student's answers related to the addition of numbers with one digit omitted can be seen in Figure 5.

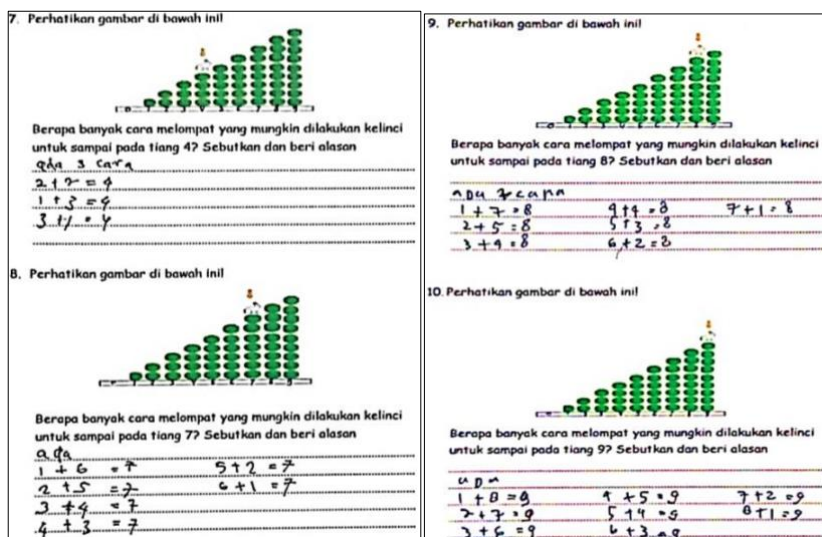


Figure 5. Student's work on the worksheet

Student' answers to the questions show that student can find various ways of adding. At first, student only gave three alternative solutions. The teacher directed the students to a new question:

T : At first, the rabbit is on pole 3. How many ways can the rabbit add more jumps to get to pole 3?

By observing and demonstrating the rabbit jumping on the abacus pole

S : 2 + 1.

T : Besides 2+1, is there another way?

Student again experiences blocking memory while thinking and answering “1+2” (silence while observing). It is a predictive solution that emerges from HLT. When student demonstrated a rabbit jumping on an abacus pole, the student answered that there were two ways to get to pole 3, i.e., 2 jumps + 1 jump and 1 jump + 2 jumps. It is in line with Gravemeijer's (2020) opinion that student experience a process of mathematization. Student begins to solve problems with an informal approach and gradually construct their knowledge into formal knowledge. Next, student use the formal knowledge and write their answers on student worksheets. By moving the toy rabbit on another pole, students have found a different way of finding the sum whose sum is 4, 7, 8, or 9.

The concept of addition of whole numbers 1-20

Activity 3 aims to find the sum of the numbers 1-20 through picture story problems. The teacher gives apperception as a first step to help dyscalculia student achieve learning objectives. The results of the answers in the initial step in activity 3 can be seen in Figure 6 below.

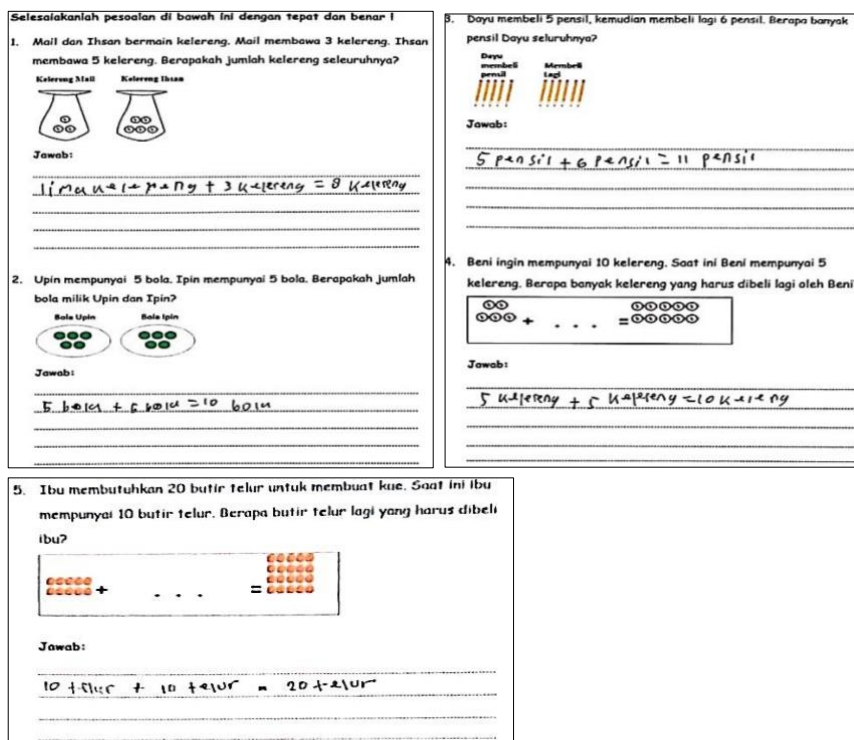


Figure 6. Student's answer on addition of whole numbers 1 - 20

The implementation of student activities showed progress compared to the previous activity. Student can count additions under ten without using their fingers again. However, dyscalculia student experiences a brief pause (preventing information from working in their memory) and can then find the sum. It means that the memory capacity of dyscalculia student in processing summation information under 10 has progressed, and the level of sensitivity to numbers has increased (Attout & Majerus, 2015). Next, student is given problems with addition operations above 10-20. However, dyscalculia student experiences a cognitive decline in making additions above 10-20. It causes the working memory to slow down, and the process of processing information in the brain is reduced so that errors in counting often occur. (Gifford et.al, 2012; Saga et. al , 2021).

To overcome the students' difficulties, the teacher provides questions that trigger the student to be able to think mathematically by themselves. It is in line with research Rahayu (2010) that the importance of anticipating questions by the teacher can improve students' ways and thinking processes. Student process information with the help of simple story illustrations accompanied by pictures and the following questions.

T : Adit wants to have 20 marbles. Currently, Adit has 10 marbles in the first container. How many marbles must Adit add again in the second container so that Adit has 20 marbles?

S : 10 marbles

Then to direct students to their formal knowledge, the question is continued:

T : Try Adit combining and counting. Is it true that 10 marbles plus 10 marbles get 20 marbles?

Students observe and count "10 marbles + 10 marbles = 20 marbles". These questions become the basis for student to answer more complex questions, as shown in Figure 6.

Discussion and questions and answers are needed to re-clarify student answers on the student worksheet. This interaction is one of the characteristics of RME, namely interactivity which aims to achieve forms of formal mathematical knowledge from forms of informal mathematical knowledge that students find themselves (Hadi, 2017; Maryati & Prahmana, 2021). Anticipatory questions can lead dyscalculia students to find the right reasons and write down the mathematical model of the given problem correctly. When students were asked to write a mathematical model of adding 5 pencils plus 6 pencils, students answered correctly that $5 \text{ pencils} + 6 \text{ pencils} = 11 \text{ pencils}$. It is proven that in activity 3, there is also a horizontal mathematization process and a vertical mathematization process. In horizontal mathematization, students with their knowledge can organize and solve real problems in everyday life. In other words, horizontal mathematization moves from the real world to the world of symbols. Horizontal mathematization occurs when dyscalculia students count concrete objects and write down the symbols for the numbers resulting from these operations. At the same time, vertical mathematization is a process of reorganizing using mathematics itself. So, vertical mathematization moves from the world of symbols (Fitri, 2016; Treffers, 1991).

The concept of the place value of tens and ones in addition of whole numbers

Activity 4 aims to find the place value of tens and units of a number using abacus media. The media used was an abacus, each of which had the same pole height and contained 10 abacus seeds. Based on the learning that has been carried out, several things are of concern to dyscalculia students in understanding place value. When student discuss with the teacher, a question and answer process occurs using the abacus media, as shown in the following illustration:

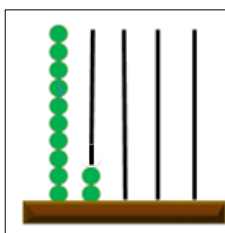


Figure 7. Media abacus

T : How many abacus seeds in total?
 S : There are 12
 T : How many places do the 12 consist of?
 Students observe and lower their voice slightly, then answer:
 S : 1 tens and 2 units
 T : How to write the place value of 12?
 S : $11 = 10 + 2 = 1 \text{ Tens} + 2 \text{ units}$

The results of the answers in this activity are shown in Figure 8. There is still a momentary pause in this activity, so student often forget to mention the numbers. However, with anticipation questions, student can state their place value. In the case of the number 12, which consists of $10 + 2$, the student answered correctly that the place value is 1 tens and 2 units. It means that the HLT can be generated from a whole series of learning processes where student with dyscalculia can provide the right ideas, thoughts, and answers and go through the entire series of activities in the HLT. It proves that the activities designed have involved horizontal mathematization and vertical mathematization. So, there are four activity levels in designing learning trajectories: situation, model level, model level for, and formal knowledge (Gravemeijer, 2020).

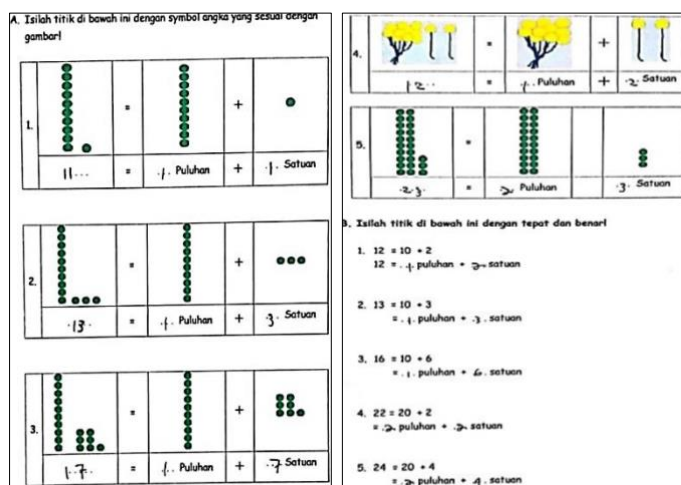


Figure 8. Student's work on the place value of tens and ones in addition of whole numbers

This HLT helps the dyscalculia student explores mathematical cognition and experience cognitive improvement on addition of whole numbers 1-20. This mathematical cognition process occurs in every series of learning activities. Beginning with perceiving dyscalculia, student can do their adding activities and rediscover the concept of addition with concrete objects by using toy balls, abacus, or other concrete objects. Then, student experiences a recognizing process done by giving symbols under a set of pictures related to the addition problem and followed by the conceiving process, namely writing down the numbers directly from the addition problem without the help of concrete objects and pictures. Finally, student

experiences reasoning activities that are carried out by giving an additional issue illustrated in the problem story (Campbell, 2005).

The sequences of activities in HLT aims to improve the reasoning and understanding of dyscalculia student in having number sense, number construction, and number relations (Luneta, 2016). It is in line with research by Morsanyi et. al (2018) that dyscalculia student need in-depth guidance using more explicit instructions in teaching numeracy concepts and procedures. In this case, the principles in question are the fundamental principles of RME, namely 1) use of context, 2) use of models for progressive mathematization, 3) utilization of student construction results, 4) interactivity, 5) intertwinement (Fessakis, Karta, & Kozas, 2017; Hadi, 2017; Taufina, Chandra, Fauzan, & Syarif, 2019).

Conclusion

This research developed the HLT for teaching addition of whole numbers to a dyscalculia student using RME approach. The series of activities in the HLT are addition of whole numbers between 1 and 10 by combining the objects, addition of whole numbers between 1 and 10 by using number relations, addition of whole numbers between 1 and 20 by using number relations, and finding the concept of place value of tens and ones in addition of numbers. The HLT has been validated and tried out in the classroom through the cyclic process of preparing the experiments, conducting the experiments, and retrospective analysis so that it becomes a local instructional theory (LIT) for teaching addition of whole numbers to the dyscalculia student.

This research also shows the cognitive improvement of dyscalculia student in learning the addition of whole numbers. Learning activities carried out by the dyscalculia student, help him to shif from informal knowledge to formal mathematical knowledge in order to understand the concept of addition of whole numbers 1 - 20. It makes dyscalculia student has number sense, number construction, and number relation abilities which increase significantly in the learning process.

The characteristics of each discalculia student varies between one and another. Therefore, the RME-based learning trajectory developed in this research can not be used directly for the other dyscalculia students. Some adjustments on the activities, predictions or conjectures, and anticipation are required, in order to make the HLT suits the characteristics of the students. The researchers will conduct further research on how the RME-based learning trajectory works on the other dyscalculia students after adjusting the HLT.

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