

Article

Unbounded Energy Collisions inside and outside Black Holes

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Abstract: The possibility of on-horizon collisions of unbounded energy in the case of an extreme Kerr black hole is known as the BSW effect. It is also a widely accepted point of view that the energy collision of two identical particles of mass m near the horizon of a Schwarzschild black hole is limited to a value of $2\sqrt{5} m$. We show that there are two possible scenarios for unbounded energy collisions both for the exterior and for the interior of spherically symmetric black holes. Similar scenarios are found for axially symmetric black holes. It is shown that divergent (infinite) energy on-(inner)horizon collisions are excluded due to the anisotropic character of the dynamics of black hole interiors.

Keywords: Penrose mechanism; unbounded energy release; Schwarzschild; Reissner–Nordström and Kerr black holes; event horizon



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1. Introduction

Energy extraction due to the Penrose mechanism [1] is one of the most interesting aspects of the presence of the black holes (BHs). It became even more intriguing due to the work of Banados et al. [2] who established that near-horizon two-particle collisions may provide unbounded energy release (tending to an infinite value for on-horizon collisions) in the case of extreme Kerr BHs. There have been many subsequent claims that infinite energy release due to on-horizon collisions would not be strictly observable (see, e.g., [3]) or that they are simply impossible for fundamental reasons [4,5]. However, one finds observational data [6,7] indicating high-energy explosions within strong gravitational fields. The other interesting theoretical outcome of ultrahigh-energy collisions was the concept that extreme or near-to-extreme Kerr black holes may be regarded as particle accelerators. Consequently, a variety of extreme Kerr-like BHs have been considered (see, e.g., [8–10]). Yet, Schwarzschild BHs have been regarded as less interesting due to their well-defined energy collision limit, $\gamma = 2\sqrt{5} m_0$ (see, e.g., [2,6]). In very recent research, Zaslavskii [11] considered collisions of particles following non-geodesic trajectories, proving that, in such a case, unbounded energy may be produced in the vicinity of the Schwarzschild black hole horizon.

On the other hand, as claimed by Toporensky and Zaslavskii [12], the interior of a Schwarzschild black hole should be the scene of common unbounded energy collisions due to the existence of ‘zero-energy’ geodesics. Yet, Zaslavskii, in one of his former papers [13], scrutinized ultrahigh-energy collisions inside BHs near the inner horizon (i.e., for charged and for rotating BHs).

The aim of this paper is to discuss the following question: what (if any) are the common features of the ultrahigh-energy collisions undergone in the vicinity of black hole horizons? Going further, what is meaning of “kinematic censorship” [13], i.e., the fact that infinite energy collisions are impossible to occur?

Throughout the paper, we consider neutral two-particle collisions occurring in the vicinity of the horizon(s) of spherically symmetric Schwarzschild (S) and Reissner–Nordström (RN) BHs, and axially symmetric Kerr BHs, both in the exterior and in the interior of such BHs. One finds that, for spherically symmetric spacetimes, contrary to general belief, the limiting factor $\gamma = 2\sqrt{5} m_0$ may be overcome. In fact, one can show that there is no energy limit, and ultrahigh-energy collisions may occur in this case. Defining the horizon as $f = 0$ (see below), near-horizon collisions would result in an ultrahigh-energy outcome, of the order $f^{-1/2}$ or f^{-1} , for “in-in” and “in-out” trajectories. This result found for a Schwarzschild BH turns out to be a general property of spherically symmetric and axially symmetric BHs.

The interior of BHs may be considered from a specific but well-known perspective. Specifically, the coordinate system is the one used in the description of the exterior of BHs applied to the interior (see, e.g., [14–16]). This process of extension of the coordinate system, ill-defined on the event horizon, to the interior of BHs is accompanied by the interchange of the roles of temporal and radial coordinates, linked to the interchange of the Killing vector from being temporal to becoming spatial. Such an approach enables us to make a comparison between the character of unbounded energy collisions both outside and inside BHs leading to outcomes that appear to be of wider interest.

The paper is organized as follows: ultrahigh-energy collisions exterior to the event horizon of spherically symmetric and axially symmetric BHs are considered in Section 2. Extension of these considerations to the interior of BHs is undertaken in Section 3. The analysis of the results of Sections 2 and 3 is given in Section 4. Final remarks and comments are given in Section 5.

2. Ultrahigh-Energy Collisions Outside the Horizon

2.1. Spherically Symmetric Spacetimes

We begin our discussion with the case of two particles following their respective geodesics, colliding near the horizon of a spherically symmetric Schwarzschild or Reissner–Nordström BH. S and RN spacetimes are described (using the system of units where $c = G = 1$) by the following line element:

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta \equiv f_i dt^2 - f_i^{-1} dr^2 - r^2 d\Omega^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$; θ and φ are angular coordinates; $i = S$ and RN .

$$f_i(r) = \begin{cases} 1 - \frac{2M}{r} & i = S \\ 1 - \frac{2M}{r} + \frac{Q^2}{r^2} & i = RN' \end{cases} \quad (2)$$

and where M denotes the mass of the black hole; for a charged BH (RN), $Q \leq M$. Zeros of $f_i(r)$ define the BH's horizon(s); there is a single one, an event horizon, in the former case (S),

$$r_h^S = 2M. \quad (3)$$

There are two horizons in the latter one (RN).

$$r_{\pm}^{RN} = M \pm \sqrt{M^2 - Q^2}. \quad (4)$$

The outer solution of Equation (4), $r_h = r_+$, labels an event horizon, and the inner one, r_- , labels a Cauchy horizon (without loss of generality, we take $M = 1$). One of the interesting features of the exterior of a BH, $r > r_h$, $f_i(r) > 0$, is the possibility of high-energy (or even unbounded) collisions occurring in the vicinity of its horizon.

The energy of two-particle collisions in a center-of-mass framework,

$$E^2 = [p(1) + p(2)]^2, \quad (5)$$

is characterized by the scalar product of the momenta of the colliding particles,

$$p(1) \cdot p(2) = g_{\alpha\beta} p^\alpha(1) p^\beta(2). \tag{6}$$

Symmetry properties result in planar geodesics with conserved energy, $E_i = m_i e_i$, and angular momentum, $L_i = m_i l_i$, where e_i and l_i denote respective densities, representing the temporal and spatial Killing vectors, respectively. The geodesics themselves, regarded as belonging to the equatorial plane $\theta = \frac{\pi}{2}$, are determined by their velocity vectors and given as follows:

$$\{u^\alpha(i)\} = \left\{ \frac{dx^\alpha(i)}{d\tau} \right\} = \left\{ \frac{e_i}{f}, \pm \sqrt{e_i^2 - f \left(1 + \frac{l_i^2}{r^2} \right)}, 0, \frac{l_i}{r^2} \right\}. \tag{7}$$

Hence, considering colliding particles of equal masses $m_1 = m_2 = m$ within the same plane, one obtains

$$p(1) \cdot p(2) = m^2 \frac{e_1 e_2 - f \frac{l_1 l_2}{r^2} \mp \sqrt{N_1} \sqrt{N_2}}{f}, \tag{8}$$

where the signs $-/+$ represent the same “in-in” or “out-out” / opposite “in-out” directions of motion of colliding particles, and

$$N_i = e_i^2 - f \left(1 + \frac{l_i^2}{r^2} \right). \tag{9}$$

In the vicinity of the horizon, Equation (8) for “in-in” collision tends to the on-horizon value,

$$p(1) \cdot p(2) \xrightarrow{f \rightarrow 0} \frac{m^2}{2} \left[\frac{1}{r_h^2} \left(l_1 \sqrt{\frac{e_2}{e_1}} - l_2 \sqrt{\frac{e_1}{e_2}} \right)^2 + \frac{e_1}{e_2} + \frac{e_2}{e_1} \right], \tag{10}$$

which appears to be unbounded. Indeed, making e_2 arbitrarily small and taking a finite value for e_1 , one can obtain a (squared) collision energy,

$$E^2(r \rightarrow r_h) = m^2 \left[\frac{e_1}{e_2} + \frac{e_2}{e_1} + 2 + \frac{1}{r_h^2} \left(l_1 \sqrt{\frac{e_2}{e_1}} - l_2 \sqrt{\frac{e_1}{e_2}} \right)^2 \right], \tag{11}$$

greater than any fixed value. One must remember, however, that e relates to the initial conditions; in radial free fall, $e_2 = \sqrt{f(r_0)}$, where r_0 labels an initial position. The limiting value of energy collision in this case, Equation (11), is determined by the value of the ratio, $\frac{e_1}{e_2}$,

$$E^2(r \rightarrow r_h) \cong m^2 \frac{e_1}{\sqrt{f(r_0)}}, \tag{12}$$

and this may be, in principle, unbounded. This means that, if the initial position r_0 of the particle (2) is located arbitrarily close to the event horizon, the energy collision is arbitrary large. One can observe that the energy collision takes its largest value at the initial position of the particle (2), r_0 , and then it drops when the particle (2) approaches the horizon. The general property of this spacetime is as follows: an (ultra)high-energy collision of the particles (1) and (2) takes its maximal value at the initial position of the particle (2) being halved on the horizon.

A rather obvious conclusion is that, in the case when colliding particles start at infinity, $e_1 = 1 = e_2$, the expression for the collision energy is

$$E(r \rightarrow r_h = 2) = 2m \sqrt{\frac{1}{16} (l_1 - l_2)^2 + 1}, \tag{13}$$

i.e., the BSW result,

$$E(r_h = 2) = 2\sqrt{5} m, \tag{14}$$

is reproduced for Schwarzschild spacetime, and $l_1 - l_2 = 8$.

One can also consider the case of the two colliding particles following “in-out”-going geodesics, “+” in Equation (8), leading to an unbounded energy collision. There are, however, no natural sources of particles outgoing from BHs.

2.2. Kerr Spacetime

In the case of Kerr spacetime, the line element expressed in Lundquist–Boyer coordinates is

$$\begin{aligned} ds^2 &= g_{tt}dt^2 + 2g_{t\varphi}dtd\varphi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\varphi\varphi}d\varphi^2 \\ &= \left(1 - \frac{2r}{\Sigma}\right)dt^2 + \frac{4arsin^2\theta}{\Sigma}dtd\varphi - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 \\ &\quad - \left(r^2 + a^2 + \frac{2a^2rsin^2\theta}{\Sigma}\right)sin^2\theta d\varphi^2, \end{aligned} \tag{15}$$

where

$$\Sigma = r^2 + a^2cos^2\theta, \tag{16}$$

$$\Delta = r^2 - 2r + a^2, \tag{17}$$

and where $a \leq 1$ denotes the angular momentum of a BH, and the two horizons, the event horizon, r_+ , and Cauchy horizon, r_- , are defined as $\Delta = 0$,

$$r_{\pm}^K = 1 \pm \sqrt{1 - a^2}. \tag{18}$$

The conservation laws for energy and the φ -component of angular momentum are then given as

$$e_i = g_{tt}u^t(i) + g_{t\varphi}u^\varphi(i), \tag{19}$$

$$l_i = g_{t\varphi}u^t(i) + g_{\varphi\varphi}u^\varphi(i), \tag{20}$$

respectively. One can then consider the geodesics of the colliding particles as belonging to the equatorial plane (see [2,6]).

$$u^t(i) = \frac{1}{r^2\Delta} \left\{ (r^2 + a^2)A_i - \Delta aB_i \right\}, \tag{21}$$

$$u^\varphi(i) = \frac{1}{r^2\Delta} \{ aA_i - \Delta B_i \}, \tag{22}$$

$$u^r(i) = \pm \frac{1}{r^2} \sqrt{C_i}, \tag{23}$$

where

$$A_i(r) = (r^2 + a^2)e_i + al_i, \tag{24}$$

$$B_i = l_i + ae_i, \tag{25}$$

$$C_i = A_i^2(r) - (B_i^2 + r^2)\Delta. \tag{26}$$

The scalar product of the two “in” (“-”) momenta of the geodesics confined in the equatorial plane may be given in the following form (see also [2]):

$$p(1) \cdot p(2) = m^2 \frac{1}{r^2\Delta} \left(\{A_1A_2 - B_1B_2\Delta\} - \sqrt{C_1}\sqrt{C_2} \right) \tag{27}$$

Close to the event horizon, both the numerator and the denominator in Equation (27) tend to zero (see Equations (8)–(10)). In the general case of a nonextremal Kerr BH, $a < 1$, and one can express Δ as a function of r ,

$$\Delta \approx (r_+ - r_-)(r - r_+) \xrightarrow{r \rightarrow r_+} 0. \quad (28)$$

The scalar product in Equation (27) may then be expressed as follows:

$$p(1) \cdot p(2) = m^2 2 \frac{(A_1 e_2 + A_2 e_1)}{r_+(r_+ - r_-)} \left(1 - \frac{A_1 A_2}{|A_1 A_2|} \right) + \frac{m^2}{2} \left\{ \frac{A_2}{A_1} + \frac{A_1}{A_2} + \frac{1}{r_+^2} \left(B_1 \sqrt{\frac{A_2}{A_1}} - B_2 \sqrt{\frac{A_1}{A_2}} \right)^2 \right\}. \quad (29)$$

The first term in Equation (29) vanishes (both particles are ingoing), and the rhs takes a form similar to the scalar product for spherically symmetric spacetimes in Equation (9) with appropriate substitutions; the energy parameter e_i is substituted with an “effective energy” parameter $A_i(r)$ and angular momentum parameter l_i with an “effective angular momentum” parameter B_i . The consequences of a substitution of the energy parameter e_i by its combination with momentum, $A_i(r)$, which is r -dependent, result in an interesting possibility. In the case of spherically symmetric BHs, the smallness of the energy parameter reflects the particular choice of initial position being located close to the event horizon. In this case of an axially symmetric BH, the effective energy parameter $A_i(r)$ is a superposition of differently signed terms, i.e., it may take, in principle, an arbitrarily small value on the horizon of a Kerr BH, leading to an ultrahigh-energy collision. Moreover, for the *critical geodesic* determined by the condition of vanishing on-horizon effective energy,

$$A_1 = (r_+^2 + a^2)e_1 + a l_1 = 0, \quad (30)$$

i.e., for the energy–angular momentum relation,

$$e_1 = -\frac{a}{r_+^2 + a^2} l_1, \quad (31)$$

the on-horizon energy collision may reach an *infinite* limit. However, such a limit can hardly ever occur. In fact, it cannot be reached in the case of general, nonextreme Kerr BHs, $a < 1$.

Let us briefly show why. The condition in Equation (30), $A_1(r_h = r_+) = 0$, leads to the following form of C_1 as a function of r :

$$C_1(r) \cong (r - r_h) \left[g_0 + g_1(r - r_h) + g_2(r - r_h)^2 + g_3(r - r_h)^3 \right], \quad (32)$$

where

$$g_0(r_h) = -(r_h - r_-) \left(\frac{r_h^4}{a^2} e_1^2 + r_h^2 \right) \leq 0, \quad (33)$$

$$g_1(r_h) = 4r_h^2 e_1^2 - \left[\frac{r_h^4}{a^2} e_1^2 + r_h^2 + 2r_h(r_h - r_-) \right], \quad (34)$$

$$g_2(r_h) = 4r_h e_1^2 - [2r_h + (r_h - r_-)], \quad (35)$$

$$g_3(r_h) = e_1^2 - 1. \quad (36)$$

Therefore, $C_1(r) < 0$ for $r \rightarrow r_h$, i.e., such a (“critical”) trajectory cannot reach the horizon.

On the other hand, one finds that the critical geodesic in Equation (30) can reach the horizon for an extreme Kerr BH, $a = 1$, $r_+ = r_h = r_-$, when the energy parameter is appropriately tuned, $3e_1^2 - 1 > 0$

For near-to-extreme Kerr BHs, $a < 1$, particles tuned in a critical way according to Equation (30) would reach a closest point to the horizon r^* ,

$$r^* \cong r_h + 2\sqrt{1 - a^2} \frac{e_1^2 + 1}{3e_1^2 - 1}, \quad (37)$$

where the energy of the possible collision, as determined by Equation (29), however large, would be limited. Let us underline that, in the case of extreme Kerr BHs, $a = 1 = r_h$, and for a fall from infinity, $e_1 = 1 = e_2$, the results of Equations (29) and (30) reproduce the BSW result.

3. Ultrahigh-Energy Collisions inside the Horizon

Let us now consider hypothetical collisions inside the BH horizon. This problem was already discussed by Toporensky and Zaslavskii [12] in the case of spherically symmetric BHs from the specific perspective of the so-called “river model”. We generalize such considerations taking into account also axially symmetric BHs. More important, however, is to adopt a different perspective than that of the river model. Specifically, one can regard the interior of the S and RN BHs as anisotropic cosmological models, expanding and contracting along different directions (see [14–16]). Although such an interpretation does not of itself provide any new results, it allows us to identify interesting aspects leading to new outcomes not considered before. It should be underlined that the question of unbounded energy collisions inside BHs of different kinds was discussed by Zaslavskii [13]. Zaslavskii’s discussion illustrated with Penrose diagrams led to the ultimate conclusion that infinite-energy collisions are impossible. Our considerations referring to the picture of anisotropic, expanding and contracting spacetime appear to provide an interesting complement to the picture offered in [13].

Inside the horizon of BHs, one can apply the same system of coordinates as that used for the outer region (see also [14–16]). Hence, the interior of a spherically symmetric or axially symmetric BH can be described in terms of Equations (1) and (13), respectively, with one important caveat. Inside BHs, the coordinates t and r interchange their roles; t plays the role of the spatial coordinate, and r plays the role of the temporal coordinate.

Although the discussion of the properties of the interior of spherically symmetric and axially symmetric spacetimes is similar in various aspects, we nonetheless consider them separately.

3.1. Interior of Spherically Symmetric Spacetimes

The interior spacetime of the Schwarzschild BH described by Equations (1)–(3) for

$$r < r_h^S \quad (38)$$

may be regarded as analogous to a cylinder (with a sphere as the base of this cylinder), expanding along the axis of homogeneity, the t -direction, and contracting perpendicularly to it (see [15]). The interior of the Reissner–Nordström BH,

$$r_+^{RN} > r > r_-^{RN}, \quad (39)$$

being determined by an outer, r_+^{RN} , (event) horizon and an inner, r_-^{RN} , (Cauchy) horizon, first expands, $r_+^{RN} > r > r_{min}^{RN} = \frac{Q^2}{M}$, and then contracts, $r_{min}^{RN} > r > r_-^{RN}$, along the homogeneity axis. The forward-in-time condition, $dr < 0$, means that the temporal-like coordinate r runs in the case of Schwarzschild black hole from $r_h^S = 2$ to 0 (the ultimate singularity). In the case of a Reissner–Nordström BH, the temporal coordinate t runs from the initial instant r_+^{RN} until the exit instant (inner horizon) r_-^{RN} . Another consequence of the exchange of the roles of the t - and r -coordinates inside BHs is a re-interpretation of energy conservation. Now it is interpreted as conservation of the t -component of momentum, with the conservation law representing the appropriate symmetry: homogeneity of the system along the t -direction. Geodesics are described in a manner similar to Equation (6).

$$\{u^\alpha(i)\} = \left\{ \frac{dx^\alpha(i)}{d\tau} \right\} = \left\{ \pm \frac{e_i}{f}, -\sqrt{e_i^2 - f \left(1 + \frac{l_i^2}{r^2} \right)}, 0, \frac{l_i}{r^2} \right\}, \quad (40)$$

where one takes $e_i > 0$. The important difference is the rearrangement of the signs; the forward-in-time condition results in a minus sign in the temporal, r coordinate, whereas the plus/minus signs in the first term on the rhs of Equation (40) represent the two possibilities for motion along possible \pm directions along the t -axis. This difference affects the form of the scalar product of the momenta inside the horizon.

$$p(1) \cdot p(2) = m^2 \frac{\pm e_1 e_2 - f \frac{l_1 l_2}{r^2} - \sqrt{N_1} \sqrt{N_2}}{f}. \quad (41)$$

The interpretation of Equation (41) is as follows: inside the horizon of S and RN BHs, particles can move in both directions along the t -axis which provides a particular perspective when considering two-particle collisions. There are two possibilities for ultrahigh-, unbounded-, or even infinite-energy release in this case.

The first of these was indicated by Toporensky and Zaslavskii [12]. A particle (1) crossing the Schwarzschild BH's horizon, $u^t(1) = \frac{e_1}{f}$, could collide with particle (2), such that $u^t(2) = 0$, i.e., it follows a 'zero-momentum' geodesic. A collision of this kind, (a), occurring in the vicinity of the horizon, $f \rightarrow 0$, results in

$$p(1) \cdot p(2) \cong m^2 \frac{e_1 \sqrt{\left(1 + \frac{l_2^2}{r^2}\right)}}{\sqrt{-f}}, \quad (42)$$

i.e., in energy output tending as fast as $\frac{1}{\sqrt{-f}}$ to on-horizon infinity.

There is also another possibility with an energy output tending to infinity as f^{-1} in the vicinity of the horizon. Specifically, if particle (2) moves along the negative t -axis, i.e., with "negative momentum", then (b),

$$p(1) \cdot p(2) \cong m^2 \frac{2e_1 e_2}{-f}. \quad (43)$$

Hence, formally, in both cases, the energy output tends to infinity as one approaches the horizon: in the former one, (a), as $|f|^{-1/2}$, Equation (42), and in the latter one, (b), as $|f|^{-1}$, Equation (43).

There is, however, a more or less obvious problem here. At the instant when particle (1) crosses the horizon, the BH's interior starts its rapid expansion along the t -axis (see, e.g., [16,17]). In both S and RN cases, the other colliding particle, particle (2)—the one with zero or negative 'momentum'—resides within the horizon. One can then hardly identify the source of such particles, "zero" (a) or "negative" (b) momentum/energy residing inside the horizon at the initial instant when the t -axis starts its expansion. After that instant, due to the expansion of the BH's interior, the energy release of further collisions would drop dramatically. Therefore, even if one could expect high-energy collisions at later instants inside the horizon, one cannot expect an ultrahigh-energy collision on the event horizon with a particle residing there.

The situation in the case of a RN BH's interior differs from the S BH's interior. While, at the event horizon, $r = r_+^{RN}$, one cannot expect collisions leading to infinite energy release due to the reasons presented above, the presence of the inner horizon in this case makes a difference. During the time period between r_+^{RN} and r_-^{RN} , different processes can occur, leading to the production of a variety of particles, including those with zero and negative momentum. Due to the violent contraction of the BH's interior at the final stage of its evolution [15,16], the mutual speed of the particles increases. Hence, collisions of particle type (1) following the $+t$ (or $-t$) direction with particle (2) of (a) zero or (b) negative/positive momentum in the vicinity of the inner horizon, r_-^{RN} , would then lead to an energy outcome diverging as follows (see Equations (42) and (43)):

$$(a) |f|^{-1/2}, \quad (44)$$

$$(b) |f|^{-1}, \tag{45}$$

respectively, thus being singular on the inner horizon.

3.2. Interior of Axially Symmetric Spacetime—Kerr BH

The interior of the Kerr BH is described by Equations (13)–(15) with

$$\Delta < 0. \tag{46}$$

Geodesics arranged in the equatorial plane, satisfying Equations (16)–(19), under condition (46), would yield the following (see Equation (22)):

$$p(1) \cdot p(2) = m^2 \frac{1}{r^2 \Delta} \left(\{ \pm A_1 A_2 - B_1 B_2 \Delta \} - \sqrt{C_1} \sqrt{C_2} \right). \tag{47}$$

Let us consider when, in these circumstances, on-horizon collisions would yield diverging energy outcomes. For the same reasons as in the case of spherically symmetric BHs, the inner horizon r_- , but not the event horizon r_+ , could be regarded as a source of this kind of event.

One may notice that the inner horizon could be reached by all kinds of geodesics. Indeed, in this case,

$$C_i = A_i^2 + (B_i^2 + r^2) |\Delta| > 0; \tag{48}$$

Hence, a critical geodesic, $A_2(r_-) = 0$, would reach the horizon. Consequently, Equation (47) is regarded as robust. Then, in the vicinity of the horizon, $|\Delta| \ll 1$, one finds two kinds of possible ultrahigh-energy (*infinite*) collisions.

- (a) The case of colliding particles with positive and negative A , i.e., with a minus sign in the first term on the rhs of Equation (47)

$$- A_1 A_2. \tag{49}$$

Then, one obtains

$$p(1) \cdot p(2) \cong m^2 \frac{2|A_1 A_2|}{r_-^2 |\Delta|} \sim \frac{1}{|\Delta|} \xrightarrow{r \rightarrow r_-} \infty. \tag{50}$$

- (b) The case of a critical geodesic of particle (2)

$$A_2(r_-) = (r_-^2 + a^2) e_2 + a l_2 = 0 \tag{51}$$

Then, one formally finds

$$p(1) \cdot p(2) \cong m^2 \frac{1}{2} \left\{ \frac{A_2}{A_1} + \frac{A_1}{A_2} + \frac{1}{r_+^2} \left(B_1 \sqrt{\frac{A_2}{A_1}} - B_2 \sqrt{\frac{A_1}{A_2}} \right)^2 \right\} \xrightarrow{r \rightarrow r_-} \infty. \tag{52}$$

However, closer inspection indicates that

$$C_2 = A_2^2 + (B_2^2 + r^2) |\Delta| \cong (B_2^2 + r_-^2) |\Delta| \tag{53}$$

in the vicinity of the (inner) horizon; hence, instead of Equation (52), one finds the following asymptotic behavior:

$$p(1) \cdot p(2) \cong m^2 \frac{A_1 \sqrt{B_2^2 + r_-^2}}{r_-^2 \sqrt{|\Delta|}} \sim \frac{1}{\sqrt{|\Delta|}} \xrightarrow{r \rightarrow r_-} \infty. \tag{54}$$

4. Discussion

It is rather well known that high-energy collisions due to strong gravitational fields may occur in the vicinity of the event horizon of Kerr BHs. Such collisions would be unbounded in the case of extreme Kerr BHs and a critically tuned geodesic for one of the colliding particles. Recently, it was indicated that the interior of both spherically and axially symmetric BHs could be an arena for hypothetical ultrahigh-energy collisions if one of the colliding particles, the one with “zero” or “negative” energy/momentum, resides inside the horizon.

The main outcome of this paper is the conclusion that there exists a coherent picture for high-energy collisions in the vicinity of horizon(s) of spherically and axially symmetric black holes. This picture is also coherent for the exterior and interior of black holes. The meaning of this coherence is explained below. There are two possible scenarios of ultrahigh-energy collisions occurring in the vicinity of a BH, both outside and inside. The existence of these two kinds of outcomes is, rather surprisingly, founded on the same grounds for the BH exterior and interior. Even more surprising is the fact that the explanation of such a duality can be formulated originating from the interior but not the exterior of the black hole.

In the first of these two scenarios of ultrahigh-energy collisions, a particle freely falling toward the horizon and crossing it collides inside the horizon with a ‘zero-energy’ particle; this kind of particle may be referred to as “critical” (see also [13]). The collision energy would then diverge as $|\delta|^{-1/2}$, where $\delta = \frac{f}{\Delta}$ denotes the dimensionless distance from the horizon, $\delta(r_h) = 0$. Due to the reasons presented above, such collisions would be effective in the vicinity of the inner horizon. In fact, in this scenario, the ‘zero-energy’ particle is the one at rest and is following its geodesic. Therefore, the other particle may, in principle, follow an arbitrary geodesic, originating outside or inside the horizon, thus being of “positive” or “negative” energy, i.e., moving along a “+” or “−” t -direction respectively.

Outside the horizon of spherically symmetric BHs, a conserved energy parameter e_i is simply a constant number, and there is no strict analogy to a “zero” energy geodesic. However, as the parameter $e_2 \sim \delta^{1/2}$ might be arbitrarily small, corresponding to the initial position of such a particle located arbitrarily close to the horizon, the collision with a freely falling particle would result in an energy output greater than any fixed amount. However, of course it cannot be infinite, and that is the realization of the weakly divergent case $\delta^{-1/2}$ (see [13]).

In the case of a Kerr BH, there is an analogy of a ‘zero-energy’ geodesic. The generalized energy parameter, $A_i(r)$, is r -dependent, and the critical geodesic, $A_2(r_h) = 0$, plays the role of a ‘zero-energy’ geodesic. However, the analogy is limited. In the case of nonextreme Kerr BHs, the critical geodesic cannot reach the event horizon; in the case of extreme Kerr BHs, the time of travel to the horizon along the critical geodesic turns out to be infinite. Thus, the meaning of “weakly diverging” is revealed also in this case; an arbitrarily large value of collision energy is achievable, but it cannot be infinite, even in principle.

In the second scenario, a freely falling particle crosses the horizon of a spherically symmetric S or RN BH and collides with a ‘negative-energy’ particle, i.e., with a particle moving along the “− t ” direction. In such a case, the collision energy would diverge as $|\delta|^{-1}$. Here, the point is that the particles colliding on the (inner) horizon are going in opposite directions. Hence, the geodesic of one of them, the one going along the “− t ” direction, has to originate inside the BH, but the other geodesics may originate both inside and outside the BH. In the case of the interior of a Kerr BH, the interpretation is different as the momenta scalar product is expressed as

$$\frac{1}{\delta} \left(\{ \pm A_1 A_2 - B_1 B_2 \delta \} - \sqrt{A_2^2 + (B_2^2 + r^2) |\delta|} \sqrt{A_2^2 + (B_2^2 + r^2) |\delta|} \right) \quad (55)$$

for the sign “−” in the first term in Equation (46); the second scenario taking place—collision energy in the vicinity of the event horizon—is of the order $|\delta|^{-1}$.

Outside the horizon of spherically symmetric and axially symmetric BHs, the arguments concerning the weakly diverging singularity of energy collisions are similar. In both cases, the expressions for the momenta scalar product are similar

$$\frac{1}{\delta} \left(e_1 e_2 - \delta \frac{l_1 l_2}{r^2} \pm \sqrt{e_1^2 - \delta \left(1 + \frac{l_1^2}{r^2} \right)} \sqrt{e_2^2 - \delta \left(1 + \frac{l_2^2}{r^2} \right)} \right), \tag{56}$$

$$\frac{1}{r^2 \delta} \left(\{A_1 A_2 - B_1 B_2 \delta\} \pm \sqrt{A_2^2 + (B_2^2 + r^2) \delta} \sqrt{A_1^2 + (B_1^2 + r^2) \delta} \right), \tag{57}$$

being large for the “+” sign in Equations (56) and (57). However, the meaning of “+” is such that the colliding particles are going in opposite directions: “in” and “out” going to and from the BH, respectively. There is no natural source of the particles outgoing from the BHs (see the next section), and that is the reason why these kinds of collisions may lead to an energy outcome that is arbitrarily large but not infinite (even in principle) and, thus, “weakly diverging”.

Let us point out another well-known perspective of infinite-energy collisions. A two-particle collision viewed through the momenta scalar product is an invariant quantity that might be regarded as the speed of particle (1) as measured by particle (2),

$$p(1) \cdot p(2) = m^2 \frac{1}{\sqrt{1 - v^2}}. \tag{58}$$

Then, the diverging energy collision is equivalent to meaning that the speed of particle (1) tends to the speed of light (as measured by particle (2)). The critical geodesic tangential to the horizon of the extremal Kerr BH appeared to be an example of measuring the speed of light of a particle crossing the horizon. Although finally excluded as being ineffective (see below), this hypothetical possibility enforces the revision of the widely accepted point of view of the speed of the particle crossing the BH’s horizon (see, e.g., [18,19]).

One can consider a particular collision in the vicinity of the Schwarzschild horizon. Let particle (1) follow its radial geodesic (this is not a necessary but simplifying condition) and collide near the horizon, at $r = \bar{r}$, with particle (2) that is undergoing uniform circular, i.e., non-geodesic, motion. The velocity vector of particle (2) is a two-component one.

$$u(2) = u^t \partial_t + u^\varphi \partial_\varphi. \tag{59}$$

Therefore, the speed of particle (1) as measured by particle (2) according to Equation (7),

$$u(1) \cdot u(2) = e_1 u^t = \frac{1}{\sqrt{1 - v^2}}, \tag{60}$$

turns out to be

$$v^2 = \frac{(e_1 u^t)^2 - 1}{(e_1 u^t)^2}. \tag{61}$$

Due to the normalization condition,

$$u(2)^2 = 1,$$

one finds that

$$(u^t)^2 = \frac{1}{f(\bar{r})} \left(1 + \frac{1}{r^2} (u^\varphi)^2 \right). \tag{62}$$

Therefore, the speed tends to the value of the speed of light,

$$v^2 \cong \frac{e_1^2 \left(1 + \frac{1}{r^2} (u^\varphi)^2\right) - f}{e_1^2 \left(1 + \frac{1}{r^2} (u^\varphi)^2\right)} \xrightarrow{\bar{r} \rightarrow r_h} 1, \quad (63)$$

and the collision energy,

$$p(1) \cdot p(2) \sim \frac{1}{\sqrt{f(\bar{r})}} \xrightarrow{\bar{r} \rightarrow r_h} \infty, \quad (64)$$

tends to infinity when the circle approaches the horizon. This is an argument in favor of the revision of the problem to the on-horizon speed of a particle approaching and crossing the BH horizon; it is beyond the scope of this presentation to discuss that question, which is left open at the moment.

5. Final Remarks

Therefore, as indicated above, out of the variety of ultrahigh-energy collisions, those occurring in the exterior of black holes, although unbounded, would not be singular, leading to an infinite-energy outcome. In the terminology proposed by Zaslavskii [13], they would be weakly diverging. On the other hand, collisions inside black holes in the vicinity of the inner (but not the outer) horizon seem to open the way to unbounded energy collisions, associated, in principle, with the infinite-energy outcome (in Zaslavskii's terminology, strongly diverging). They may be diverging within two possible scenarios due to the existence of 'zero-energy' particles and 'negative-energy' particles.

However, those potentially infinite-energy collisions could also not be realized.

The perspective proposed in this paper, regarding the interior of the black hole as a dynamic medium, provides two levels of arguments with regard to this fundamental issue. The first level is of a qualitative character: ultrahigh-energy, diverging, on-(inner)horizon collisions in RN black holes are impossible. The inner RN BH horizon is the endpoint of contraction of cylinder-like spacetime. The cylinder length contraction with a sphere of radius r_-^{RN} is an exit-point of the most violent process one can ever imagine: an infinitely long cylinder shrinking rapidly. During this violent contraction of the cylinder, it is impossible to arrange initial conditions for the two particles to 'meet', i.e., collide at the exit instant r_-^{RN} .

The second level is quantitative confirmation of the above argument. In the case of a $\delta^{-1/2}$ scenario, it is as described below. One can expect that particle (1), radially falling,

$$p(1) = \left\{ \frac{e_1}{f}, -\sqrt{e_1^2 - f}, 0, 0 \right\}, \quad (65)$$

and critical particle (2), the 'zero-energy' one,

$$p(2) = \left\{ 0 - \sqrt{-f}, 0, 0 \right\}, \quad (66)$$

would collide on the inner horizon, r_-^{RN} . The "zero"-energy particle occupies a fixed position on the t -axis, say t_2 . One can assume that the geodesic of particle (1) originates at the instant r_0 and position t_X in the interior of an RN black hole. Then, the condition for an on-horizon collision at t_2 ,

$$t_2 - t_X = \int_{t_X}^{t_2} dt = - \int_{r_0}^{r_-^{RN}} \frac{e_1}{f \sqrt{e_1^2 - f}} dr \rightarrow \infty, \quad (67)$$

should provide the value of t_X . However, the right-hand side turns out to be divergent; thus, one cannot determine t_X !

The same argument may be used in the second scenario, ultrahigh-energy collision, diverging as $|\delta|^{-1}$. Hence, these cannot be implemented either.

A similar way of reasoning might be given for the case of axially symmetric black holes. We avoid presenting here the rather lengthy algebra related to this point.

The impossibility of the strong version of on-horizon collisions due to the dynamic character of the interior of BHs appears to provide a nice interpretation of the arguments presented by Zaslavskii [13] in terms of Carter–Penrose diagrams. Specifically, in the interior of RN BHs, particles traveling along $\pm t$ directions are carried away to the appropriate infinities, $\pm\infty$, at the final stage of contraction. In the Carter–Penrose diagram, those trajectories tend to the left, $t \rightarrow +\infty, e > 0$, or right, $t \rightarrow -\infty, e < 0$, null infinities. A bifurcation point at this diagram represents the endpoint of the geodesics of all of the particles, $e = 0$, resting at their fixed positions on the t -axis.

Therefore, the final conclusion is that nature prevents the possibility of on-horizon collisions with an accompanying infinite-energy outcome, even on the inner black hole horizon. Nevertheless, there are two universal scenarios, in the sense defined above, of ultrahigh-energy collisions with an unbounded, i.e., greater than any fixed value, energy outcome, robust both outside and inside black holes.

Despite the sense of this conclusion, in the context of our discussion, there is still an open interesting question posted by a variety of authors (e.g., [8]) but first discussed by Bejger et al. [3]. Specifically, what would be the effect of high-energy collisions observed at *spatial infinity*? One can identify three distinct regimes: (a) collisions inside BHs, (b) collisions outside axisymmetric BHs, and (c) collisions outside Schwarzschild BHs. Case (a) is irrelevant; phenomena taking place inside BHs are not observed. Case (b) was discussed by Bejger et al. [3]. The analysis provided in [3] shows that high-energy collisions observed far from the Kerr BH’s horizon lead to modest energy gain. Due to the energy extraction from Kerr BHs, the maximal energy gain may be fixed with a coefficient of 1.3. In the case of the Schwarzschild BHs, there is no energy gain; hence, the consequences of a high-energy collision as observed at infinity are more or less obvious. Indeed, the energy of a particle freely falling from infinity recorded at r_0 by a static observer is as follows (see Equation (12)):

$$E(r_0) = \frac{e_\infty}{\sqrt{f(r_0)}}. \quad (68)$$

This energy, if converted into the energy of the photon sent back, would be recorded at infinity as the particle’s initial energy e_∞ . This simplified example illustrates the meaning of “no energy gain in this case”.

This is a more general outcome applicable to astrophysical black holes. The BSW effect and its current variations seem not to lead to exciting effects observed on the Earth (i.e., at “infinity”).

Let us make a final comment.

It relates to the problem of (the lack of) “natural sources” of “outgoing” geodesics near the horizon exterior to BHs. There are no such outgoing geodesics, in the sense of a classical, i.e., nonquantum approach. Taking into account the quantum aspect of the picture, in particular consideration of Hawking radiation outgoing from black holes, one may expect ultrahigh-energy collisions but occurring beyond the horizon, as Hawking radiation comes out of black holes due to the tunneling effect. Although unbounded, such collisions could not lead to infinite-energy outcomes.

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