Deterministic Leader Election in Anonymous Sensor Networks Without Common Coordinated System

Yoann Dieudonne, Franck Petit

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Yoann Dieudonné[1] and Franck Petit[1]

[1] *MIS, Université de Picardie Jules Verne Amiens, France*

Dans ce papier, nous nous focalisons sur le problème qui consiste à élire un leader dans un réseau de *n* capteurs anonymes ne partageant aucun système commun de coordonnées. En supposant que lorsque les robots disposent de la propriété de latéralité, nous donnons une caractérisation complète sur les positions des capteurs permettant de distinguer un leader, et ce quelque soit *n*. Lorsqu'ils ne disposent pas de la propriété de latéralité, nous montrons que cette caractérisation reste vraie si et seulement si *n* est impair. Ces résultats sont vrais même si les capteurs possèdent une mémoire et une visibilité infinie, sont mobiles et peuvent communiquer entre eux.

Keywords: Election de leader distribué, Sense d'orientation, Latéralité, Réseaux de capteurs.

1 Introduction

In distributed settings, many problems that are hard to solve otherwise become easier to solve with a *leader* to coordinate the system. Due to its importance, the problem of electing a leader is covered in depth in many books related to distributed systems, *e.g.,* [San07]. The *Leader Election* (LE) Problem consists in moving the system from an initial configuration were all entities are in the same state into a final configuration were all entities are in the same state, except one, the leader.

In this paper, we address the leader election problem in *sensor networks* under very weak assumptions *e.g., uniformity* (or, *homogeneity* — all the sensors follow the same program —, *anonymity* — the sensors are *a priori* indistinguishable —, *disorientation* — the sensors share no kind of coordinate system nor common sense of direction nor unit measure. In weak distributed environments, many tasks have no solution. In particular, in uniform anonymous general networks, the impossibility of breaking a possibly symmetry in the initial configuration makes the leader election unsolvable deterministically [Ang80]. We come up with the following question : "*Given a set of such weak sensors scattered on the plane, what are the (minimal) geometric conditions to be able to deterministically agree on a single sensor ?*"

2 Preliminaries

In this section, we define the distributed system considered in this paper. Next, we review some formal definitions and basic results on words and Lyndon words.

2.1 Model

Consider a set of *n sensors* (or *agents*, *robots*) arbitrarily scattered on the plane such that no two sensors are located at the same position. The sensors are *uniform* and *anonymous*, i.e, they all execute the same program using no local parameter (such that an identity) allowing to differentiate any of them. However, we assume that each sensor is a computational unit having the ability to determine the positions of the *n* sensors within an infinite decimal precision. We assume no kind of communication medium. Each sensor has its own local *x*-*y* Cartesian coordinate system defined by two coordinate axes (*x* and *y*), together with their *orientations*, identified as the positive and negative sides of the axes.

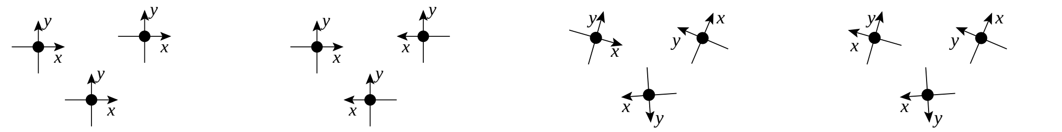
In this paper, we discuss the influence of *Sense of Direction* and *Chirality* in a sensor network.

Definition 2.1 (Sense of Direction) *A set of n sensors has sense of direction if the n sensors agree on a common direction of one axis (x or y) and its orientation. The sense of direction is said to be* partial *if the agreement relates to the direction only* —i.e., *they are not required to agree on the orientation.*

In Figure 1, the sensors have sense of direction in the cases (a) and (b), whereas they have no sense of direction in the cases (c) and (d).

Given an x-y Cartesian coordinate system, the *handedness* is the way in which the orientation of the y axis (respectively, the x axis) is inferred according to the orientation of the x axis (resp., the y axis).

Definition 2.2 (Chirality) *A set of n sensors has chirality if the n sensors share the same handedness.*



(a) Sense of Direction and Chirality | (b) Sense of Direction and No Chirality | (c) No Sense of Direction and Chirality | (d) No Sense of Direction and No Chirality

FIG. 1: Four examples showing the relationship between Sense of Direction and Chirality

In Figure 1, the sensors have chirality in the cases (a) and (c), whereas they have no chirality in the cases (b) and (d).

2.2 Lyndon Words

Let k and j be two positive integers. The k^{th} *power* of a word w is the word denoted s^k such that $s^0 = \varepsilon$, and $s^k = s^{k-1}s$. A word u is said to be *primitive* if and only if $u = v^k \Rightarrow k = 1$. Otherwise ($u = v^k$ and $k > 1$), u is said to be *strictly periodic*. The j^{th} *rotation* of a word w, notation $R_j(w)$, is defined by :

$$R_j(w) \stackrel{\text{def}}{=} \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ a_j,\ldots,a_l,a_1,\ldots,a_{j-1} & \text{otherwise } (w = a_1,\ldots,a_l,\ l \geq 1) \end{cases}$$

Note that $R_1(w) = w$.

A word w is said to be *minimal* if and only if $\forall j \in 1,\ldots,l$, $w \preceq R_j(w)$.

Definition 2.3 (Lyndon Word) *A word w ($|w| > 0$) is a Lyndon word if and only if w is nonempty, primitive and minimal, i.e., $w \neq \varepsilon$ and $\forall j \in 2,\ldots,|w|,\ w \prec R_j(w)$.*

3 Leader Election

The *leader election* problem considered in this paper is stated as follows : Given the positions of n sensors in the plane, the n sensors are able to deterministically agree on the same position L called the leader.

3.1 Leader Election with Chirality

In this subsection, we assume a sensor networks having the property of chirality. A *configuration* π of the sensor network is a set of positions $p_1,\ldots,p_n$ ($n > 1$) occupied by the sensors. Given a configuration π, *SEC* denotes the *smallest enclosing circle* of the positions in π. Note that *SEC* is unique and can be computed in linear time [Wel91]. The center of *SEC* is denoted O. *SEC* passes either through two of the positions that are on the same diameter (opposite positions), or through at least three of the postions in π. Note that if $n = 2$, then *SEC* passes both sensors and no sensor can be located inside *SEC*, in particular at O. Since the sensors have the ability of chirality, they are able to agree on a common orientation of *SEC*, denoted $\circlearrowright$.

Given a smallest enclosing circle *SEC*, we can associate a word $W(r)$ for each radius r with at least one robot on it and not at the center. Let $\mathcal{R}$ be the finite set of radii such that at least one sensor is located on r

but O. Let $p_1,\dots,p_k$ be the respective positions of k robots ($k \geq 1$) located on the same radius $r \in \mathcal{R}$. Let w_r be the word such that

$$w_r \stackrel{\text{def}}{=} \begin{cases} 0 \text{ if there exists one sensor at } O \\ a_1,\dots,a_k \text{ with } a_1 = d(O,p_1) \text{ and } \forall i \in [2,k], a_i = d(p_{i-1},p_i) & \text{, otherwise} \end{cases}$$

Let r be a radius in $\mathcal{R}$. The successor of r, denoted by $Succ(r,\circlearrowright)$, is the next radius in $\mathcal{R}$, according to $\circlearrowright$. The i^{th} successor of r, denoted by $Succ_i(r,\circlearrowright)$, is the radius such that $Succ_0(r,\circlearrowright) = r$, and $Succ_i(r,\circlearrowright) = Succ(Succ_{i-1}(r,\circlearrowright),\circlearrowright)$. Given r and its successor $r' = Succ(r,\circlearrowright)$, $\sphericalangle(rOr')$ denotes the angle between r and r'. Given an orientation $\circlearrowright$, let $CW^{\circlearrowright}$ be the set of configuration words, computed by any sensor s, build over $\mathcal{R}$ such that for each radius $r \in \mathcal{R}$, the associated configuration word $W(r)$ is equal to $(0,0)$ if $w_r = 0$, otherwise $W(r)$ is equal to the word $a_1,\dots,a_k$ such that $k = \sharp\mathcal{R}$ and $\forall i \in [1,k]$, $a_i = (Succ_{i-1}(r,\circlearrowright), \sphericalangle(Succ_{i-1}(r,\circlearrowright) O Succ_i(r,\circlearrowright)))$.

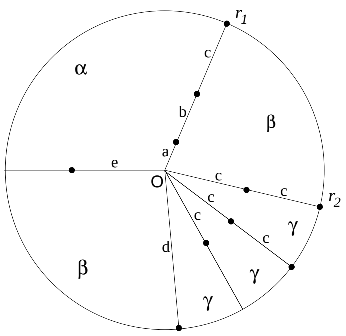


FIG. 2: Computation of Configuration words — the sensors are the black bullets.

In Figure 2, if $\circlearrowright$ is the clockwise orientation, then : $W(r_1) = (abc,\beta)(c^2,\gamma)^2(c,\gamma)(d,\beta)(e,\alpha)$ and $W(r_2) = (c^2,\gamma)^2(c,\gamma)(d,\beta)(e,\alpha)(abc,\beta)$. If $\circlearrowright$ is the counterclockwise orientation, then : $W(r_1) = (abc,\alpha)(e,\beta)(d,\gamma)(c,\gamma)(c^2,\gamma)(c^2,\beta)$ and $W(r_2) = (c^2,\beta)(abc,\alpha)(e,\beta)(d,\gamma)(c,\gamma)(c^2,\gamma)$.

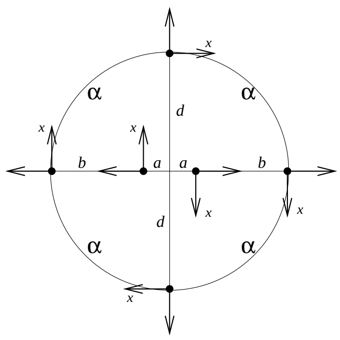


FIG. 3: A counter example illustrating Theorem 3.1.

Let $A_{CW^{\circlearrowright}}$ be the set of letters over $CW^{\circlearrowright}$. Let (u,x) and (v,y) be any two letters in $A_{CW^{\circlearrowright}}$. Define the order $\lessdot$ over $A_{CW^{\circlearrowright}}$ as follows : $(u,x) \lessdot (v,y) \Leftrightarrow (u \lneqq v$ or $(u = v$ and $x < y))$. The lexicographic $\preceq$ order over $CW^{\circlearrowright}$ is naturally built over $\lessdot$.

Theorem 3.1 *Given a configuration π of any number $n \geq 2$ sensors with chirality scattered on the plane, the n sensors are able to deterministically agree on the same sensor L if and only if there exists a radius $r \in \mathcal{R}$ such that $W(r)$ is a Lyndon Word.*

Sketch of proof. If there exists a radius r such that $W(r)$ is a Lyndon Word then r is unique. So, the leader is the robot on r which is the nearest from the center of *SEC*. If there exists no radius $r \in \mathcal{R}$ such that $W(r)$ is a Lyndon Word, then for each r, $W(r)$ is strictly periodic. So, for each sensor there exists at least another sensor which can have the same view of the world (see Figure 3). In that case, we cannot determiniscally distinguish an unique sensor L. □

3.2 Leader Election without Chirality

Without chirality, the sensors are not able to agree on a common orientation of *SEC*. No chirality implies that for each radius r there are two possible word according to the clockwise or counterclockwise orientation. The main difficult is, with respect to their handedness, some of the n sensors choose to orient *SEC* according to ↻, whereas some other to ↺.

In spite of this constraint, we can build the same set A_{CW} as for the case assuming chirality in both direction of *SEC*. Over this set, assuming that n is odd, we can provide a deterministic algorithm following a similar method as in the previous section. So, the statement of Theorem 3.1 also holds assuming no chirality if n is odd. However, the equivalence does not work with an even number of sensors. A counter example is shown in Figure 4. For any orientation in $\{↻, ↺\}$, there exists one Lyndon word equal to $(d,\alpha)(d,\beta)(d,\gamma)(d,\beta)$. However, the symetry of the configuration does not allow to choose any sensor as a leader.

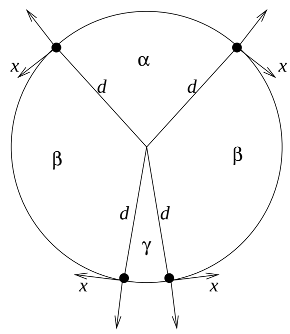


FIG. 4: A counter example showing that the statement of Theorem 3.2 does not hold if n is even.

Theorem 3.2 *Given a configuration π of any number $n \geq 2$ sensors without chirality scattered on the plane, the n sensors are able to determiniscally agree on the same sensor L if and only if n is odd and there exists a radius r such that $W(r)$ is a Lyndon Word.*

4 Conclusion

We studied the leader election problem in networks of anonymous sensors sharing no kind of common coordinate system. Assuming anonymous sensors with chirality, we gave a complete characterization on the sensors positions to deterministically elect a leader for any number $n > 1$ of sensors. We also showed that our characterization still holds with sensors without chirality if and only if the number of sensors is odd.

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