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Diquark condensation in dense SU(2) matter

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We report on a lattice study of two-color QCD with adjoint staggered fermions at high density. We find that the model has no early onset and we report on results for diquark condensation, from simulations with and without a diquark source term.

1. INTRODUCTION

Numerous model calculations suggest that the ground state of QCD at high baryonic density is characterized by diquark condensation which spontaneously breaks gauge and/or baryon number symmetry (see [1,2] for recent reviews). In [3,4] we presented evidence that two-color QCD with one flavor of adjoint staggered fermions has interesting QCD-like features, in particular the absence of baryonic Goldstone modes in the spectrum. Here we extend the study of this model to include possible diquark condensates. We refer to [3] for a detailed description of the model and its properties, while we summarize here only the main features: N flavors of staggered fermion, in the adjoint representation of the SU(2) gauge group, have a U($2N$) chiral-flavor symmetry (at zero mass m and zero chemical potential μ). This is explicitly (spontaneously) broken by the mass (the chiral condensate) to Sp($2N$). SSB gives rise to $2N^2-1$ Goldstone modes, of which $N(N-1)$ are baryonically charged. When $N > 1$ and $\mu > m_\pi/2$ the vacuum starts rotating from the chiral condensate direction to that of a diquark condensate [5], which in the case of $N = 2$ is (modulo a baryonic rotation):

$$qq_3 = \frac{i}{2}\left[\chi^{p\,tr}(x)\varepsilon^{pq}\chi^q(x) + \bar{\chi}^p(x)\varepsilon^{pq}\bar{\chi}^{q\,tr}(x)\right],$$

where ε^{pq}, $p,q = 0,1$ is the completely antisymmetric tensor in flavor space. If $N = 1$ no baryonic Goldstone mode is expected, and no local, gauge invariant, Lorentz scalar diquark condensate is possible. Therefore no early onset ($\mu \sim m_\pi$) transition is expected. One interesting possibility is a gauge non singlet — hence color superconducting — diquark condensate:

$$qq^i_{sc} = \frac{1}{2}\left[\chi^{tr}(x)t^i\chi(x) + \bar{\chi}(x)t^i\bar{\chi}^{tr}(x)\right],$$

where t^i, $i = 1,3$ are color generators. Note however that the fermionic determinant of the $N = 1$ model (although real) does not have a definite sign. The indication in [3,4] was that the effect of the sign is indeed important. If we ignore the sign, the model is well described by Chiral Perturbation Theory (χPT) [5], including the presence of baryonic Goldstone modes. The latter are suppressed only by the inclusion of the sign. Here we want to complement the previous analysis with the study of diquark condensation [6].

2. ALGORITHMS

We studied the $N = 1$ model by means of both HMC [7] and TSMB [8] algorithms. Because of lack of ergodicity, with HMC we could explore only the positive determinant phase. TSMB is defined by a set of parameters (degrees of polynomia n_i, the number of heatbath and overrelaxation iterations for the bosonic fields I_h and I_o, the number of metropolis iterations for the gauge fields I_M and the number of noisy correc-

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Table 1
Runs at $\beta = 2.0$, $m = 0.1$, on a $4^3 \times 8$ lattice.

μ	Alg.	n_1	λ/ϵ	N_{sw}	N_{int}^{plaq}
0.0	TSMB	24	$2 \cdot 10^3$	$1 \cdot 10^5$	$2.1 \cdot 10^5$
0.0	TSMB	16	$2 \cdot 10^3$	$2 \cdot 10^5$	$1.7 \cdot 10^5$
0.0	HMC	—	$2 \cdot 10^3$	$3 \cdot 10^4$	$1.9 \cdot 10^5$
0.30	TSMB	48	$8 \cdot 10^4$	$1 \cdot 10^5$	$9.2 \cdot 10^6$
0.37	TSMB	80	$4 \cdot 10^5$	$1 \cdot 10^5$	$2.5 \cdot 10^7$

N_{int}^{plaq} is the autocorrelation length (expressed in number of matrix multiplications) for the plaquette, evaluated from a run of N_{sw} sweeps. $n_2 = 90$ at $\mu = 0$, otherwise $n_2 \sim 10 n_1$.

Table 2
Runs at $\beta = 2.0$, $m = 0.1$, on a $4^3 \times 8$ lattice.

μ	n_1	n_2	N_g	p_-	$\langle r \rangle$
0.00	16	120	380	0.00	1.00
0.30	48	500	216	0.00	0.9982(3)
0.36	64	700	140	0.14	0.476(14)
0.37	80	800	275	0.22	0.45(2)
0.38	100	1000	440	0.33	0.30(4)
0.40	100	1000	265	0.44	0.085(24)

N_g is the effective number of *independent* configurations. p_- is the fraction with a negative determinant and $\langle r \rangle$ the average reweighting factor including the sign.

tions I_c). One has to tune such parameters in order to minimize the autocorrelation. The latter must be expressed in term of matrix multiplications. The number of matrix multiplications per sweep is roughly given by:

$$N_{mult} \simeq \frac{7}{2} n_1 (I_h + I_o + I_M) + (n_2 + n_3) I_c .$$

In general when the chemical potential increases the lowest eigenvalues (ϵ) approach zero and the simulations become difficult. In Table 1 we show how the condition number (λ/ϵ, λ being the largest eigenvalue) as well as the plaquette autocorrelation change by increasing μ. The autocorrelation of fermionic observables is typically about 10 times shorter. From Table 2 we see that until the point $\mu = 0.38$ the average sign is still not too small, while points above $\mu = 0.4$ are already exceedingly difficult.

3. DIQUARK CONDENSATES

Our model has very interesting features, which distinguish it from other SU(2) models. It also has a sign problem. It is natural and interesting to study the effect of the inclusion of the sign. We distinguish two models: the one where only positive (determinant) configurations are sampled and the one where both positive and negative configurations are included with their sign. The positive sector turns out to be in very good agreement with χPT predictions; i.e., the positive model behaves effectively like an $N = 2$ model.

In order to study possible diquark condensations we introduced a source j for qq_3. This is done in a partially quenched way: by introducing the source only in the measurements. χPT predicts also the dependence on j of the chiral and the diquark condensate [5]. In particular the sum

$$\langle \bar{\psi}\psi \rangle_0^2 = \langle \bar{\psi}\psi \rangle^2 + \langle qq_3 \rangle^2 \qquad (1)$$

must, to lowest order in χPT, be a constant independent of both on μ and j (for NLO see [9]). In fact, as one can see on fig. 1, this relation is very well satisfied in the positive sector. The validity of the relation (1) also allows an extrapolation of $\langle qq_3 \rangle$ to zero j, which would otherwise be quite difficult.

Since the condensate qq_{sc} is not gauge invariant we studied its susceptibility χ_{sc}

$$\chi_{sc} = \frac{1}{3} \langle \bar{\chi}(x) t^i \bar{\chi}^{tr}(x) \chi^{tr}(x) t^i \chi(x) \rangle .$$

This has a finite value even without a source term. In the low density regime χ_{sc} shows little variation with μ. Once we reach the χPT transition point, χ_{sc} drops rapidly in the positive sector. We interprete this as an effect of Pauli blocking or phase space suppression: as the ground state is filled up by fermions there is less phase space left to accommodate the fermion loops that contribute to χ_{sc}. When the sign is properly taken into account this effect disappears and χ_{sc} remains stable through the transition (fig. 2).

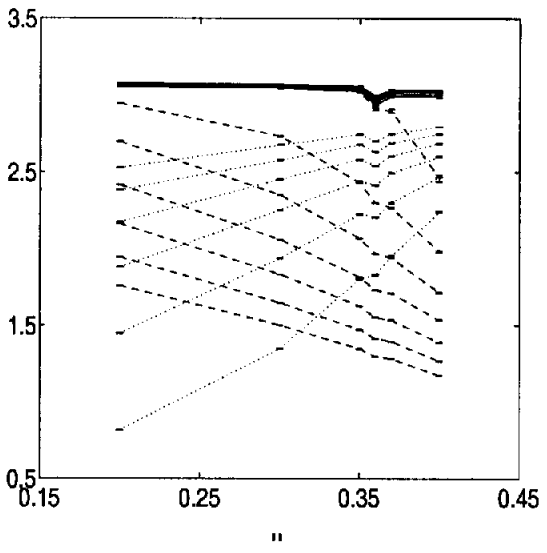

Figure 1. Dotted (increasing) lines are qq_3, dashed (decreasing) lines are $\langle\bar{\psi}\psi\rangle$ (at different values of the source between 0 and 0.5). Solid lines are the sum in (1). The error bars do not take into account the autocorrelation.

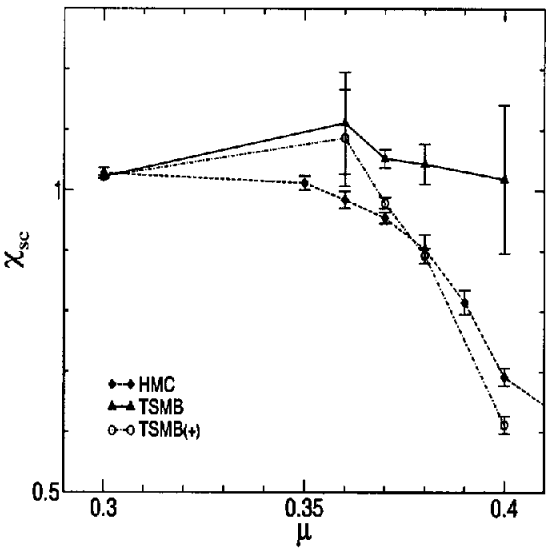

Figure 2. Superconducting susceptibility as a function of μ. The effect of including the sign is compared with the positive sector.

4. CONCLUSIONS

The interesting lesson one can learn from this model is that the early onset transition, typical of all other SU(2) models, is indeed delayed as expected from the analysis of possible diquark condensates. Numerically this is obtained by a delicate balance between configurations with positive and negative determinant. This makes simulations prohibitively expensive in the region where one would expect a true onset transition at high density.

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