Mécanique statistiques des équations de Saint Venant et dissipation d'énergie par les modes agéostrophiques

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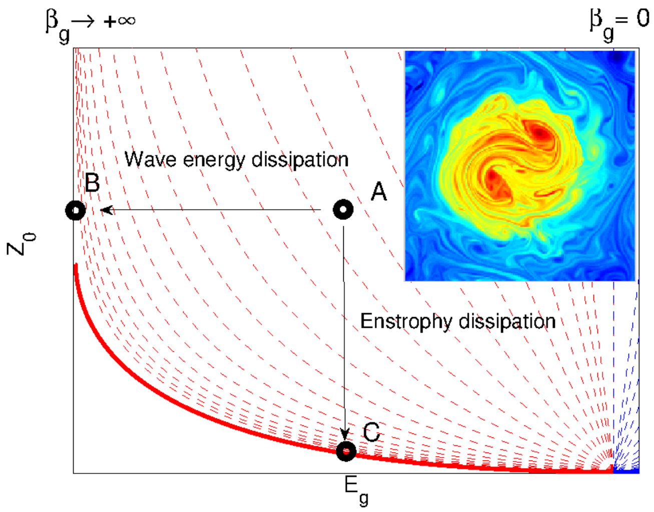

Figure 1: The small insert is a snapshot of the potential vorticity field showing the tendency of a freely quasi-geostrophic flow to self-organize into a large scale anticyclonic vortex above a topographic bump, which is predicted by equilibriums statistical mechanics. This flow is characterized by its microscopic enstrophy Z0 and its energy Eg. We show that this self-organization phenomenon is robust for shallow water equations, but that some of the energy of the large scale flow is lost into inertial-gravity waves (energy Ew). For an initial entrophy Z0 and a an initial total energy E, we predict the ratio Eg/Ew. The two key parameters are the enstrophy Zb of the flow at rest and the total energy E. The inverse temperature associated with the quasi-gesotrophic flow is related to the wwaves through βg=1/Ew. At high temperature, the quasi-geotrophic potential vorticity if fully homogenized. At low temperature, we recover Fofonoff flow that follows topography contours.

Geophysical flows are highly turbulent, yet embody large-scale coherent structures, such as ocean rings, jets, and large-scale circulations. Understanding how these structures appear and predicting their shape are major theoretical challenges. The statistical mechanics approach to geophysical flows is a powerful complement to more conventional theoretical and numerical methods. In the inertial limit, it allows to describe, with only a few thermodynamical parameters, the long-time behavior of the largest scales of the flow. Recent studies in quasi-geostrophic models provide encouraging results: a model of the Great Red Spot of Jupiter, an explanation of the drift properties of ocean rings, the inertial structure of mid-basin eastward jets, bistability phenomena in complex turbulent flows, and so on. Generalization to more comprehensive hydrodynamical models, which include gravity wave dynamics and allow for the possibility of energy transfer through wave motion, would be extremely interesting. Namely, both are essential in understanding the geophysical flow energy balance. However, due to difficulties in essential theoretical parts of the statistical mechanics approach, previous methods describing statistical equilibria were up to now limited to the use of quasi-geostrophic models. The current study fills this gap. The new theory we propose describes geophysical phenomena using statistical mechanics applied to the shallow water model, and is easily generalizable to the primitive equations. Invariant measures of the shallow water model are built based on the Hamiltonian structure and the Liouville theorem. The theory predicts a balanced large scale flow that contains only a fraction of the initial energy. The excess of energy is stored among all accessible gravity wave degrees of freedom. The theory thus predicts the fraction of energy dissipated through interaction between the vortical part of the dynamics and wave motions.

1. F. Bouchet, and A. Venaille, 2012, Statistical mechanics of two-dimensional and geophysical flows, Physics Reports, 515, 5, June 2012, 227–295