OPERATOR ON TYPES OF NUMBERINGS

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Introduction. A numbering is a family (A_n) of sets which can be enumerated or approximated uniformly in n. Numberings are ordered by many-one reduction; here a numbering (A_n) is many-one reducible to (B_n) iff there is a recursive function f such that $(A_n = \mathbf{B}/(_n))$ for all n. Rogers initiated the study of the semilattice of numberings under many-one reduction and Ershov [1-3] transferred it in particular to the study of the k-r.e. and, more generally, a-r.e. sets where the sets (A_n) are approximated uniformly by enumerating and deenumerating elements a limited number of times; this number can either be a constant k or an ordinal which is counted down each time the number is enumerated or deenumerated. An important question investigated is how the numberings on various levels of the hierarchy differ and which types of semilattices can be realized as the semilattice of all a-r.e. numberings with the same range as a givena-r.e. numbering (A_n). Friedberg[4] showed that the family of all r.e. sets has a numbering in which set occurs only once; on the other hand, Vyugin [5] showed that there are numberings of r.e. sets whose Rogers semilattice does not contain a minimal numbering. Goncharov[7] contributed substantially to the field of r.e. numberings by constructing a numbering which has (up to many-one equivalence) exactly two Friedberg numberings.

Recently, the study of numberings has shifted much to the Ershov hierarchy and looked on how the result about r.e. numberings can be replicated on the various levels of this hierarchy [7-8].

Results and discussion. We established some reductions between various types of numberings:

If a (a+k)-r.e. numbering can realize a certain type of Rogers semilattice, so can a (a +k +1)-r.e. numberings;

Every type of Rogers semilattice realized by an r.e. numbering is also realized by an a-r.e. for every recursive ordinal a which is not a power of w and which is not 0 while if a is a power of w then there is no a-r.e. numbering without minimal numberings in the Rogers semilattice;

Conclusions. The reductions constructed permit to give various alternative proofs to known results about d.r.e. and, in general,a-r.e. numberings. For example, one can show that for every recursive ordinal a which is not an w-power and every Rogers semilattice of r.e. sets, the corresponding semilattice is also realized within the a-r.e. numberings.

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