OPTIMUM SHAPES OF A PERIODIC ARRAY OF ISOTHERMAL PIPES EMBEDDED IN A SLAB SUBJECTED TO UNIFORM CONVECTION

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Objective. The project's main objective is to obtain optimum shapes of a periodic array of isothermal pipes that maximize heat transfer. The pipes are embedded in a two-dimensional slab whose surfaces are subjected to uniform convection.

Introduction. The area of Shape Optimization has fascinated scientists and philosophers since antiquity. However, the computation of an optimum geometry presents a formidable task, both theoretically and numerically. For example, the first mathematically rigorous proof of the Isoperimetric Theorem, which states that "Among all planar regions with a given perimeter, the circle encloses the greatest area", was obtained only in the 19th century.

Method of solution. The problem is addressed by first transforming the "unknown" shape to a fixed shape using the generalized Schwarz-Christoffel transformation [1]. Subsequently, we use the boundary element method [2] to obtain the total transport rate. The shape is parameterized with the parameters of the Schwarz-Christoffel transformation which are the variables of the optimization procedure and the transport rate is the objective function.

Results and Conclusion. There is excellent agreement between numerical and asymptotic results for the heat transport rate. Subsequently, we pose the Shape Optimization problem as a nonlinear programming problem (constrained nonlinear optimization), i.e. find the constrained extremum of a scalar function of several variables, where the variables are the parameters of the generalized Schwarz-Christoffel transformation and the objective function is the heat transport rate. The Shape Optimization problem under consideration is solved numerically using a special implementation of a sequential quadratic programming (SQP) method [3]. If the thickness of the slab is relatively large, the surfaces of the slab are approximately isothermal, hence the optimum shape is independent of the Biot number. If, in addition, the distance between the two pipes is also large, the optimum shape is a circle. When, on the other hand, the slab is relatively thin, the optimum shape tends to become flatter as the Biot number is decreased. In general, a smaller Biot number results in a flatter optimum shape.

References.

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