REPRESENTATION OF ALGEBRAIC CONVEX GEOMETRIES

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The classical result of P.H. Edelman and R.E. Jamison [3] states that every finite convex geometry can be represented by a join of the chain-convex geometries defined on the same base set through the special (compatible) linear orderings of the elements of the base set. A chain-convex geometry, on the linearly ordered base set (X,<), is the collection of all initial segments of (X,<). The join of chain-convex geometries is computed by taking all possible intersections of members of those geometries. In the current work we find the generalization of this statement for the case of (infinite) algebraic convex geometry.

Introduction. Convex geometry is a set system generated by the closure operator with the anti-exchange axiom. These systems model the concept of convexity in various settings. They are also closely connected to anti-matroids, which are set systems with the property of accessibility. In particular, the latter were used in modelling the states of human learners and found practical applications in designing the automatic tutoring systems. In current work we develop the theoretical foundations of infinite convex geometries in case their closure operator satisfies the finitary property: closure of any subset is a union of closures of its finite subsets. In such case, the convex geometry is called algebraic.

Results and discussion. We prove that the result of [3] can be extended to algebraic case. We propose a new type of join that we need to apply for the chain-convex geometries in the representation: it needs to be the minimal algebraic subset generated by all members of the joinands. This will require to add the joins of all up-directed families of subsets, after computing all the intersections of chain-convex geometries in the representation. Earlier work of N. Wahl [4] is discussed, and some new conclusions and properties of algebraic convex geometries are presented. The first draft of this work is available in arxiv publication [1].

Conclusions. The work gives new incentive for the development of theory of infinite convex geometries that was initiated in K. Adaricheva, V. Gorbunov, V. Tumanov [2].

Acknowledgments. We are indebted to J. B. Nation, who is a collaborator on chapter on convex geometries for the third volume of monograph on Lattice Theory, G. Gratzer and F. Wehrung eds. We thank M. Nakamura and J.P. Doignon for their pointers and discussion of paper [4].

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