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The Gini Test for Survival Data in Presence of Small and Unbalanced Groups

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Abstract

The aim of this note is to study the performance of the Gini concentration test for survival data in presence of unbalanced and small samples. We compared the performance of the asymptotic test with an alternative permutation distribution test, illustrating by simulation that if groups are very small the latter test should be used. Also, we show how the definition of the length of time considered in the construction of the test statistic can be chosen to improve the performance of the test.

Keywords: Gini index; Nonparametric test; Permutation test; Right censored survival data

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1 INTRODUCTION

The Gini index is one of the most common statistical indices employed in social sciences for measuring concentration in the distribution of a positive random variable (typically, income).

This concentration index may be useful in detecting differences in heterogeneity also between the survival distributions of two groups of individuals to whom different treatments have been administered, thus providing a different assessment of such difference compared to what is provided by traditional tests.

In [1] a nonparametric test has been proposed based on the Gini index for testing the equality of two survival distributions from the point of view of concentration. The authors derived the asymptotic distribution of the test statistics and performed a simulation analysis, in which they compared the Gini test with other tests for the difference between two survival distributions, such as the log-rank, Wilcoxon, and Gray-Tsiatis tests (see, among others, [2], [3] and [4]). The simulation study in that paper was based on using two balanced and mid-sized groups ($n_1 = n_2 = 50$).

However, in many situations the scenario may be quite different since treatment groups may be smaller and not equally sized. Some issues of instability were originally reported for such cases in [5].

Here we analyze the behavior of the Gini test for the cases of (i) small-sized and (ii) unbalanced groups. In particular, we want to study whether the asymptotic distribution of the Gini test (in particular, the asymptotic variance estimate of the test statistic) are reliable and stable under different scenarios.

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2 MATERIALS AND METHODS

Consider a nonnegative random variable X, a survival time, with survival function $S(x)$ and finite expected value μ. Let $X_1, \ldots, X_n$ be an i.i.d. sample from X observed only partially, in particular after random right censoring (independent of X). In other words, we observe the right censored sample $(\widetilde{X}_i, D_i)$, $i = 1, \ldots, n$, where $\widetilde{X}_i = X_i \wedge U_i$ and $D_i = 1(\widetilde{X}_i = X_i)$ for some *i.i.d.* censoring times $U_1, \ldots, U_n$, distributed independently of the survival times.

We apply the Gini index to lifetime data in which individuals have finite follow-up time for survival; in particular, we consider the *restricted* Gini index

$$G_t = 1 - \frac{\int_0^t S^2(u)du}{\int_0^t S(u)du}, \tag{1}$$

rather than the traditional unrestricted Gini index G (whose integrals in (1) would run from zero to infinity); see [6]. The time t indicates the longest follow-up time in the data.

The estimator of the restricted Gini index for right censored data is defined as

$$\widehat{G}_t = 1 - \frac{\int_0^t \widehat{S}^2(u)du}{\int_0^t \widehat{S}(u)du},$$

where $\widehat{S}(u)$ is the Kaplan-Meier estimator of $S(u)$; see [7].

2.1 The Gini asymptotic test

The asymptotic distribution of $\widehat{G}_t$ has been obtained in [1]; in particular, under some regularity conditions, as $n \to \infty$,

$$\sqrt{n}\left(\widehat{G}_t - G_t\right) \xrightarrow{\mathrm{d}} \mathcal{N}(0, \tau_t)$$

where

$$\tau_t = \int_0^t \left[4\frac{[\overline{\nu}_t(v)]^2}{W_t^2} + \frac{[\overline{\mu}_t(v)]^2 V_t^2}{W_t^4} - 4\frac{\overline{\mu}_t(v)\overline{\nu}_t(v)V_t}{W_t^3}\right] d\sigma^2(v), \tag{2}$$

with $\overline{\mu}_t(v) = \mu_t - \mu_v = \int_v^t S(u)du$, $\overline{\nu}_t(v) = \nu_t - \nu_v = \int_v^t S^2(u)du$, $W_t = \int_0^t S(u)du$, and $V_t = \int_0^t S^2(u)du$.

[1] constructed a test for comparing two survival functions estimated from two independent samples observed in two different treatment groups. The Gini test statistic is

$$T := \frac{\left(\widehat{G}_{1,t} - \widehat{G}_{2,t}\right)^2}{\widehat{Var}(\widehat{G}_{1,t}) + \widehat{Var}(\widehat{G}_{2,t})}, \tag{3}$$

where $\widehat{G}_{j,t}$ is the estimator of the restricted Gini index for censored data referred to the treatment group j and $\widehat{Var}(\widehat{G}_{j,t})$ is the estimator of the sampling variance of $\widehat{G}_{j,t}$, for group j, $j = 1, 2$.

Under the null hypothesis of equality of the two survival distributions, the statistic T has an approximate chi-squared distribution with 1 degree of freedom, while under any alternative to the null hypothesis T is distributed as an approximate noncentral chi-squared distribution with noncentrality parameter $\eta = [(G_{1,t} - G_{2,t})^2]/(\tau_{1,t}/n_1 + \tau_{2,t}/n_2)$, where $\tau_{j,t}$ indicates the asymptotic variance of $\sqrt{n_i}\widehat{G}_{i.t}$ and n_i is the sample size of group j, $j = 1, 2$.

2.2 The Gini permutation test

In presence of small treatment groups the large sample normal distribution of the restricted Gini statistic $\widehat{G}_t$ may not be reliable, and its exact distribution is in general not known; therefore, we suggest to consider permutation tests as a nonparametric alternative to the Gini asymptotic test when the group sizes are small (see, e.g., [8]). The Gini permutation test is a permutation test procedure applied to the Gini statistic $\widehat{G}_t$, and it is obtained as follows:

(i) Compute the Gini statistic $\widehat{G}_t$ for the original data.

(ii) Repeat the following M times (with index $m = 1, ..., M$):

- sample a permutation π^m from all permutations of the $(n_1 + n_2)$ group labels;
- compute the Gini statistic value $g_t^{(m)}$ from the original data, but with the permuted group labels π^m.

(iii) Estimate the permutation distribution of the Gini statistic with the empirical cumulative distribution function

$$\widehat{F}_{\widehat{G}_t}(g) = \widehat{P}(\widehat{G}_t \leq g) = \frac{1}{M}\sum_{m=1}^{M} 1(g_t^{(m)} \leq g).$$

(iv) Obtain the permutation p-value p_0 corresponding to the value $\widehat{G}_t$ observed on the original data in (i) from the empirical distribution $\widehat{F}_{\widehat{G}_t}(g)$ in (iii). If $p_0 \leq \alpha$ for the given significance level α, we reject the null hypothesis of equality of the two survival distributions.

3 A SIMULATION STUDY

We focus on cure rate models, i.e. survival models that study diseases such that patients have a positive probability of being cured. They assume that the patient population can be split into two groups: the noncured patients, who experience the event of interest (relapse, death, etc.) before a given finite length of time, and the cured patients, who do not appear to be experiencing recurrence or worsening even after prolonged follow-up.

More specifically, cure rate models set a *cure time* t^* so that anyone who experiences the event of interest before t^* is considered to be a noncured patient, and anyone who survives after t^* is a cured patient. The fraction of cured patients is indicated with θ, $\theta \in [0, 1]$. The portion $(1 - \theta)$ of noncured patients experience the event of interest according to the conditional distribution function $F^*(u) = 1 - S^*(u)$, while the failure time of the cured patients is degenerate at infinity, i.e. their distribution function is equal to 0 (w.p. 1). Overall, the survival function of the patient population is given by the mixture model

$$S(u) = \theta + (1 - \theta)S^*(u). \tag{4}$$

We performed a simulation study in which the population is characterized by a fixed cure time $t^* = 3$ after which the risk of failure is zero, and such that anyone who survives past t^* is considered to be cured. The conditional survival distribution of the noncured patients, $S^*(u)$, was taken as being exponential with hazard rate λ and truncated at $t^* = 3$, so that $S^*(3) = 0$. We considered two different censoring scenarios: (i) we assumed the censoring variable U to follow a uniform distribution on the interval $(0, 3.37)$ and, alternatively, (ii) we considered the censoring

variable U to be degenerate at the value 10, thus imposing no censoring for the noncured patients. We used $M = 100$ permuted samples.

We compared the Gini asymptotic test and the Gini permutation test for different combinations of hazard rates (λ_1, λ_2), cure rates (θ_1, θ_2), censoring, sample size (n) and group sizes (n_1, n_2). We estimated the probability of rejecting the null hypothesis for each test, both under the null hypothesis and under some alternatives, using a level of significance of 5%.

In the applications discussed in [1], the time t characterizing the restricted Gini statistic G_t was taken as being fixed, and in that article G_t has been estimated up to the largest observed or censored survival time. As these times (say, $\tilde{t}_1$ and $\tilde{t}_2$) are typically different between the two treatment groups, the test statistic in (3) was constructed from $\widehat{G}_{1,\tilde{t}_1}$ and $\widehat{G}_{2,\tilde{t}_2}$.

Differently here, we construct the test statistic in (3) in such a way that the group-specific statistics are integrated up to the same survival time $Tmax$, which is equal to the maximum observation of the most populated group. This choice was observed to guarantee a higher stability in presence of unbalanced groups.

We also allowed for two different scenarios: $Tmax$ could be either (i) the largest survival time (regardless of its censoring status), as in [1] (henceforth, *last observation*); or (ii) the largest uncensored observed survival time (henceforth, *last event*).

4 RESULTS

From the simulation study shown in Tables 1 - 4 emerges that, in case of no censoring for the noncured patients (that is $U \equiv 10$) and same noncured distributions ($\lambda_1 = \lambda_2$), for small and balanced groups (such as $n_1 = n_2 = 15$, $n_1 = n_2 = 25$) the asymptotic test has a similar performance to that of the permutation test (except for $\theta = 0.1$ and $Tmax$ equal to the last observation).

When the two survival curves differ with respect to the noncured distribution but have the same cure rate (i.e. $\theta_1 = \theta_2$ and $\lambda_1 \neq \lambda_2$), then the Gini test is more powerful when one integrates until the last event rather than until the last observation. In both cases, the power decreases as θ increases, since the number of events reduces. This holds for both balanced and unbalanced groups.

Vice versa, when the two survival curves differ with respect to the cure rate but have equal noncured distributions (i.e. $\theta_1 \neq \theta_2$ and $\lambda_1 = \lambda_2$), then the Gini test is more powerful if one integrates until the last observation rather than until the last event.

Finally, in case of strongly unbalanced groups (for example with $n_1 = 10$ and $n_2 = 100$) the simulations suggest that the Gini asymptotic test is not reliable both if $Tmax$ is the last observation and if $Tmax$ corresponds to the last event. In this type of situations we strongly recommend using the permutation test.

Summing up, we have shown that the Gini asymptotic test does not perform well in case of strongly unbalanced groups, if we integrate the Gini statistic until the last observation. Defining $Tmax$ as the last event improves significantly the Gini asymptotic test's power, but the best results are obtained if one also replaces the asymptotic version of the test with the permutation test. In case of censoring, the simulations' results based on the asymptotic test when $Tmax$ corresponds to the last observation are very similar to the results in [1]. In presence of censoring, the permutation test still shows better results than the asymptotic test when the two groups are small and strongly unbalanced.

5 CONCLUSIONS

We have studied the performance of the Gini concentration test for survival data introduced in [1] in the case of unbalanced and small groups. Introducing a slight modification in the original test we observed an increase in the power of the test and in the ability to correctly recover the type I error probability under such settings. We have also compared the Gini asymptotic test with a permutation test, and have detected the kinds of scenarios under which the permutation test should be preferred to the asymptotic test.

Note that, when possible, the choice between balanced and unbalanced treatment groups can be made in order to optimize the Gini test's performance; indeed, it is easy to see that for a fixed sample size n, the relative group size maximizing the Gini test's power is given by $n_1/n = \sqrt{\tau_{1,t}}/\sqrt{\tau_{1,t} + \tau_{2,t}}$. (This result can be easily proved by differentiating the non-centrality parameter η with respect to n_1 for a fixed n.) Therefore, if one knows *a priori* that the Gini statistic's variances of the two treatment groups ($\tau_{1,t}$ and $\tau_{2,t}$) are similar, then a balanced design seems optimal from an inferential viewpoint; otherwise, if some previous pilot studies show different degrees of variability in the groups, then an unbalanced design would be better to detect differences between the two distributions.
An R function is available that implements these methods and it can be obtained from the authors (a Stata command is currently being developed).

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Table 1: Estimated powers for Gini test, with balanced groups

						$U \equiv 10$				$U \sim Unif(0, 3.37)$			
						$Tmax = LO$		$Tmax = LE$		$Tmax = LO$		$Tmax = LE$	
n_1	n_2	λ_1	λ_2	θ_1	θ_2	A	P	A	P	A	P	A	P
15	15	1	1	0.10	0.10	0.13	0.05	0.06	0.04	0.07	0.05	0.06	0.04
				0.20	0.20	0.05	0.05	0.07	0.05	0.06	0.05	0.07	0.05
				0.25	0.25	0.06	0.06	0.09	0.06	0.08	0.06	0.08	0.06
				0.10	0.20	0.06	0.06	0.07	0.06	0.09	0.06	0.10	0.07
				0.20	0.25	0.05	0.04	0.08	0.05	0.09	0.06	0.07	0.05
		3	1	0.10	0.10	0.46	0.29	0.47	0.43	0.45	0.39	0.48	0.44
				0.20	0.20	0.22	0.22	0.34	0.29	0.39	0.37	0.41	0.36
				0.25	0.25	0.19	0.19	0.32	0.28	0.38	0.33	0.38	0.32
				0.10	0.20	0.46	0.42	0.40	0.42	0.54	0.51	0.50	0.45
		1	3	0.10	0.10	0.41	0.23	0.25	0.21	0.31	0.24	0.33	0.27
				0.20	0.20	0.21	0.19	0.30	0.24	0.31	0.24	0.35	0.32
				0.10	0.20	0.19	0.17	0.27	0.24	0.29	0.24	0.26	0.21
25	25	1	1	0.10	0.10	0.07	0.03	0.07	0.05	0.05	0.04	0.05	0.04
				0.20	0.20	0.05	0.04	0.07	0.06	0.08	0.06	0.07	0.06
				0.30	0.30	0.05	0.06	0.05	0.04	0.06	0.05	0.08	0.06
				0.10	0.30	0.11	0.19	0.11	0.10	0.13	0.12	0.15	0.13
				0.20	0.40	0.25	0.26	0.19	0.15	0.18	0.15	0.17	0.14
		3	1	0.10	0.10	0.55	0.50	0.67	0.68	0.61	0.60	0.62	0.60
				0.20	0.20	0.36	0.43	0.66	0.64	0.57	0.53	0.56	0.52
				0.30	0.30	0.16	0.15	0.52	0.47	0.38	0.36	0.49	0.44
				0.10	0.30	0.66	0.82	0.83	0.85	0.77	0.80	0.81	0.81
				0.20	0.30	0.46	0.44	0.74	0.72	0.67	0.64	0.66	0.63
		1	3	0.10	0.10	0.46	0.41	0.49	0.48	0.48	0.47	0.50	0.46
				0.20	0.20	0.31	0.37	0.56	0.55	0.48	0.49	0.54	0.51
				0.30	0.30	0.20	0.17	0.47	0.41	0.40	0.35	0.48	0.43
				0.10	0.30	0.09	0.21	0.26	0.26	0.25	0.23	0.25	0.22
				0.20	0.30	0.11	0.12	0.33	0.28	0.29	0.27	0.30	0.25
50	50	1	1	0.10	0.10	0.06	0.06	0.06	0.06	0.05	0.07	0.04	0.03
				0.20	0.20	0.03	0.03	0.04	0.05	0.05	0.04	0.06	0.05
				0.50	0.50	0.06	0.04	0.07	0.05	0.04	0.03	0.06	0.05
				0.10	0.30	0.12	0.25	0.16	0.16	0.17	0.17	0.14	0.13
				0.20	0.40	0.49	0.50	0.31	0.28	0.23	0.23	0.25	0.23
		3	1	0.10	0.10	0.77	0.82	0.88	0.89	0.81	0.84	0.90	0.89
				0.20	0.20	0.47	0.50	0.89	0.87	0.83	0.84	0.90	0.88
				0.50	0.50	0.07	0.05	0.50	0.42	0.21	0.19	0.49	0.44
				0.10	0.30	0.93	0.97	0.99	0.99	0.96	0.97	0.98	0.99
				0.20	0.40	0.88	0.86	0.98	0.98	0.97	0.97	0.98	0.97
		1	3	0.10	0.10	0.79	0.77	0.81	0.83	0.74	0.77	0.79	0.80
				0.20	0.20	0.50	0.51	0.82	0.81	0.83	0.82	0.83	0.82
				0.50	0.50	0.07	0.06	0.40	0.34	0.26	0.22	0.45	0.39
				0.10	0.30	0.09	0.22	0.41	0.42	0.39	0.44	0.41	0.39
				0.20	0.40	0.17	0.15	0.16	0.14	0.09	0.11	0.16	0.14

Note: $Tmax = LO$ means that $Tmax$ is equal to the *last observation*, $Tmax = LE$ means that $Tmax$ is equal to the *last event*, A refers to the Gini asymptotic test, P refers to the Gini permutation test.

Table 2: Estimated powers for Gini test, with slightly unbalanced groups

						$U \equiv 10$				$U \sim Unif(0, 3.37)$			
						$Tmax = LO$		$Tmax = LE$		$Tmax = LO$		$Tmax = LE$	
n_1	n_2	λ_1	λ_2	θ_1	θ_2	A	P	A	P	A	P	A	P
15	30	1	1	0.10	0.10	0.17	0.06	0.09	0.06	0.08	0.04	0.11	0.08
				0.20	0.20	0.06	0.06	0.08	0.05	0.06	0.03	0.08	0.05
				0.30	0.30	0.04	0.04	0.06	0.03	0.07	0.04	0.06	0.04
				0.10	0.30	0.14	0.12	0.07	0.05	0.10	0.07	0.10	0.05
				0.20	0.30	0.06	0.03	0.06	0.05	0.06	0.03	0.08	0.05
		3	1	0.10	0.10	0.46	0.07	0.39	0.34	0.34	0.20	0.35	0.26
				0.20	0.20	0.26	0.14	0.42	0.37	0.38	0.25	0.41	0.34
				0.30	0.30	0.19	0.15	0.38	0.29	0.31	0.23	0.36	0.28
				0.10	0.30	0.56	0.45	0.55	0.48	0.48	0.40	0.54	0.44
				0.20	0.30	0.36	0.25	0.55	0.47	0.44	0.35	0.53	0.46
		1	3	0.10	0.10	0.53	0.25	0.57	0.54	0.53	0.44	0.59	0.54
				0.20	0.20	0.33	0.25	0.58	0.57	0.52	0.46	0.55	0.53
				0.30	0.30	0.12	0.11	0.50	0.45	0.44	0.36	0.47	0.42
				0.10	0.30	0.21	0.21	0.30	0.25	0.30	0.26	0.25	0.22
				0.20	0.30	0.11	0.08	0.36	0.31	0.34	0.29	0.36	0.30
25	50	1	1	0.10	0.10	0.10	0.04	0.03	0.03	0.08	0.05	0.05	0.03
				0.20	0.20	0.04	0.05	0.06	0.04	0.05	0.04	0.04	0.04
				0.50	0.50	0.09	0.07	0.08	0.06	0.08	0.05	0.09	0.05
				0.10	0.30	0.12	0.12	0.11	0.09	0.09	0.06	0.11	0.10
				0.20	0.40	0.33	0.20	0.20	0.14	0.14	0.11	0.19	0.14
		3	1	0.10	0.10	0.59	0.34	0.63	0.60	0.57	0.48	0.57	0.50
				0.20	0.20	0.38	0.38	0.67	0.64	0.57	0.51	0.65	0.61
				0.50	0.50	0.06	0.04	0.33	0.28	0.19	0.15	0.35	0.30
				0.10	0.30	0.73	0.75	0.84	0.81	0.70	0.67	0.81	0.74
				0.20	0.30	0.55	0.50	0.81	0.76	0.72	0.68	0.80	0.74
		1	3	0.10	0.10	0.71	0.53	0.76	0.75	0.70	0.63	0.76	0.75
				0.20	0.20	0.42	0.39	0.79	0.79	0.72	0.71	0.76	0.75
				0.30	0.30	0.16	0.16	0.66	0.60	0.52	0.48	0.67	0.63
				0.10	0.30	0.10	0.15	0.38	0.36	0.35	0.33	0.42	0.37
				0.20	0.40	0.11	0.04	0.28	0.20	0.10	0.08	0.29	0.21
50	100	1	1	0.10	0.10	0.06	0.06	0.05	0.04	0.05	0.04	0.06	0.06
				0.20	0.20	0.03	0.04	0.05	0.05	0.06	0.07	0.06	0.06
				0.50	0.50	0.05	0.04	0.05	0.04	0.05	0.04	0.05	0.04
				0.10	0.30	0.18	0.20	0.22	0.18	0.17	0.14	0.18	0.16
				0.20	0.40	0.68	0.55	0.35	0.26	0.31	0.25	0.28	0.23
		3	1	0.10	0.10	0.82	0.80	0.93	0.93	0.80	0.79	0.86	0.82
				0.20	0.20	0.56	0.62	0.91	0.92	0.89	0.89	0.90	0.90
				0.50	0.50	0.08	0.08	0.40	0.36	0.28	0.27	0.44	0.40
				0.10	0.30	0.93	0.98	0.99	0.99	0.92	0.93	0.95	0.95
				0.20	0.40	0.93	0.92	0.99	0.99	0.99	0.99	1.00	0.99
		1	3	0.10	0.10	0.94	0.91	0.95	0.95	0.91	0.88	0.93	0.93
				0.20	0.20	0.63	0.59	0.95	0.95	0.95	0.95	0.96	0.96
				0.50	0.50	0.07	0.06	0.62	0.51	0.31	0.24	0.59	0.51
				0.10	0.30	0.08	0.15	0.64	0.57	0.49	0.48	0.60	0.53
				0.20	0.40	0.24	0.10	0.38	0.28	0.13	0.08	0.42	0.34

Note: see Note of Table 1.

Table 3: Estimated powers for Gini test, with fairly unbalanced groups

						$U \equiv 10$				$U \sim Unif(0, 3.37)$			
						$Tmax = LO$		$Tmax = LE$		$Tmax = LO$		$Tmax = LE$	
n_1	n_2	λ_1	λ_2	θ_1	θ_2	A	P	A	P	A	P	A	P
15	75	1	1	0.10	0.10	0.23	0.06	0.08	0.05	0.11	0.04	0.10	0.05
				0.20	0.20	0.08	0.06	0.08	0.06	0.09	0.06	0.07	0.05
				0.30	0.30	0.06	0.05	0.08	0.04	0.09	0.05	0.07	0.04
				0.10	0.30	0.17	0.09	0.08	0.03	0.09	0.04	0.08	0.04
				0.20	0.30	0.10	0.04	0.06	0.04	0.08	0.01	0.06	0.03
		3	1	0.10	0.10	0.54	0.05	0.49	0.38	0.41	0.18	0.35	0.23
				0.20	0.20	0.32	0.11	0.44	0.37	0.39	0.20	0.43	0.31
				0.30	0.30	0.19	0.11	0.40	0.34	0.38	0.23	0.40	0.31
				0.10	0.30	0.65	0.43	0.63	0.53	0.58	0.35	0.62	0.46
				0.20	0.30	0.43	0.24	0.64	0.52	0.56	0.35	0.60	0.45
		1	3	0.10	0.10	0.68	0.14	0.71	0.59	0.65	0.35	0.64	0.54
				0.20	0.20	0.37	0.17	0.69	0.68	0.65	0.48	0.66	0.61
				0.30	0.30	0.13	0.11	0.56	0.49	0.44	0.35	0.53	0.50
				0.10	0.30	0.19	0.12	0.40	0.27	0.33	0.21	0.40	0.30
				0.20	0.30	0.11	0.07	0.48	0.38	0.37	0.27	0.46	0.38
25	125	1	1	0.10	0.10	0.10	0.05	0.07	0.04	0.09	0.05	0.07	0.03
				0.20	0.20	0.04	0.05	0.07	0.06	0.08	0.06	0.06	0.04
				0.30	0.30	0.06	0.05	0.06	0.05	0.06	0.05	0.08	0.07
				0.10	0.30	0.20	0.09	0.09	0.05	0.11	0.04	0.12	0.05
				0.20	0.30	0.11	0.04	0.08	0.06	0.06	0.03	0.08	0.05
		3	1	0.10	0.10	0.65	0.17	0.69	0.65	0.64	0.51	0.61	0.51
				0.20	0.20	0.41	0.38	0.72	0.72	0.65	0.53	0.68	0.62
				0.50	0.50	0.08	0.07	0.34	0.31	0.21	0.20	0.38	0.32
				0.10	0.30	0.79	0.79	0.87	0.81	0.76	0.73	0.84	0.79
				0.20	0.40	0.83	0.79	0.97	0.94	0.92	0.86	0.95	0.89
		1	3	0.10	0.10	0.84	0.40	0.86	0.80	0.77	0.56	0.85	0.80
				0.20	0.20	0.51	0.37	0.83	0.84	0.81	0.77	0.82	0.81
				0.50	0.50	0.07	0.05	0.46	0.33	0.23	0.14	0.49	0.35
				0.10	0.30	0.14	0.13	0.50	0.38	0.45	0.34	0.50	0.41
				0.20	0.40	0.17	0.01	0.33	0.20	0.16	0.08	0.31	0.22
50	250	1	1	0.10	0.10	0.06	0.05	0.06	0.05	0.08	0.05	0.09	0.08
				0.20	0.20	0.03	0.04	0.05	0.05	0.06	0.04	0.04	0.03
				0.50	0.50	0.07	0.06	0.05	0.05	0.06	0.04	0.07	0.04
				0.10	0.30	0.26	0.12	0.23	0.17	0.17	0.11	0.17	0.13
				0.20	0.40	0.80	0.56	0.46	0.34	0.39	0.25	0.34	0.23
		3	1	0.10	0.10	0.88	0.80	0.94	0.93	0.88	0.86	0.90	0.87
				0.20	0.20	0.61	0.72	0.97	0.96	0.93	0.93	0.95	0.94
				0.50	0.50	0.12	0.12	0.49	0.45	0.29	0.29	0.49	0.43
				0.10	0.30	0.94	0.97	0.98	0.97	0.94	0.94	0.99	0.98
				0.20	0.40	0.97	0.96	1.00	1.00	1.00	0.99	1.00	0.99
		1	3	0.10	0.10	1.00	0.94	0.98	0.97	0.98	0.93	0.97	0.96
				0.20	0.20	0.80	0.66	0.99	0.99	0.98	0.98	0.98	0.98
				0.50	0.50	0.09	0.06	0.66	0.54	0.33	0.24	0.71	0.63
				0.10	0.30	0.05	0.07	0.77	0.65	0.64	0.52	0.71	0.63
				0.20	0.40	0.40	0.05	0.45	0.24	0.13	0.06	0.43	0.28

Note: see Note of Table 1.

Table 4: Estimated powers for Gini test, with strongly unbalanced groups

						$U \equiv 10$				$U \sim Unif(0, 3.37)$			
						$Tmax = LO$		$Tmax = LE$		$Tmax = LO$		$Tmax = LE$	
n_1	n_2	λ_1	λ_2	θ_1	θ_2	A	P	A	P	A	P	A	P
10	100	1	1	0.10	0.10	0.36	0.06	0.11	0.03	0.14	0.04	0.16	0.05
				0.20	0.20	0.13	0.04	0.10	0.05	0.14	0.04	0.12	0.05
				0.10	0.20	0.32	0.10	0.08	0.03	0.15	0.04	0.11	0.03
				0.20	0.25	0.10	0.04	0.10	0.04	0.13	0.05	0.14	0.05
		3	1	0.10	0.10	0.53	0.03	0.25	0.19	0.32	0.08	0.26	0.10
				0.20	0.20	0.26	0.05	0.32	0.23	0.27	0.05	0.28	0.15
				0.10	0.20	0.53	0.11	0.32	0.21	0.35	0.10	0.30	0.12
				0.20	0.25	0.35	0.08	0.36	0.26	0.33	0.11	0.34	0.16
		1	3	0.10	0.10	0.65	0.11	0.59	0.34	0.63	0.26	0.57	0.37
				0.20	0.20	0.40	0.09	0.58	0.46	0.58	0.29	0.57	0.44
				0.10	0.20	0.50	0.17	0.57	0.38	0.59	0.27	0.50	0.37
				0.20	0.25	0.26	0.10	0.52	0.41	0.55	0.29	0.50	0.38

Note: see Note of Table 1.