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A Model Description

Jussi Lintunen, Jani Laturi and Jussi Uusivuori

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## Abstract

Jussi Lintunen, Jani Laturi and Jussi Uusivuori

Natural Resources Institute Finland (Luke), Economics and society, Jokiniemenkuja 1, 01370 Vantaa, first-name.lastname@luke.fi

Finnish Forest and Energy Policy model (FinFEP) is a partial equilibrium model of the Finnish forest and energy sectors. The processing technologies of the two sectors are based on plant-level data. The approach falls into the bottom-up modeling tradition. In addition to the processing sectors, the model consists of a detailed description of forest resources on forestry land available for wood production in Finland. The model generates harvesting behavior based on optimized management rules and the state of the forest resources. In line with the usual CGE modeling convention, we formulate the partial equilibrium of the two sectors as a mixed complementarity problem (MCP). In this report, we describe the modeling approach in detail level, and give examples of applications that the model can be used for.

**Keywords:** Forest sector, Energy sector, Modeling, Age-structured forest, Partial equilibrium, Dynamic optimization, Bottom-up approach, Mixed complementarity problem

## Foreword

This report documents the modeling work for FinFEP-version 1.0. This version is the outcome of several years' work by the team. The idea of building a new model goes back to 2005 by which time it had become clear that an economic policy model would be needed in Finland to bridge the gap between timber supply from forest resource owners and timber demand by forest and energy industries, as well as to combine the forest and energy sectors into one single model.

Over the years, several individuals have contributed to the completion of this first version of FinFEP. We thank Hanna-Liisa Kangas for her contribution in the early stages of the development of the modeling framework for the processing sectors, Matti Mäkelä for his work on data collection, Sini Niinistö for her contribution to the forest-stand growth modeling, and Johanna Pohjola for her calibration and model testing efforts.

We are also thankful for the assistance and hints received from Luke's Motti forest growth simulator and National Forest Inventory (NFI) researchers Jari Hynynen, Antti Ihalainen, Kari Korhonen, Mika Lehtonen, Risto Ojansuu and Olli Salminen. Early support by Lauri Hetemäki and Risto Seppälä for the idea of building a new model is gratefully acknowledged. We also thank Jari Kuuluvainen for his detailed and insightful comments on an earlier version of this report and Aapo Rautiainen for checking the consistency and clarity of the report.

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Vantaa, 14 October 2015

Jussi Uusivuori  
Professor  
Natural Resources Institute Finland (Luke)

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# 1. Introduction

## 1.1. Background and motivation

In recent years forest management and the forest sector have become increasingly open to influences from energy, climate, environmental and economic policies. Both national and international policy measures need to be taken into account when analyzing the behavior of agents operating within the sector. A wide angle on the economic analysis of policies concerning forest sector is needed because of close linkages between various sectors. Acts and initiatives taken primarily within other sectors may have profound consequences in the forest sector. For example, EU-level and national targets for biofuels, CO<sub>2</sub> reductions and renewable energy shares, will have profound impacts in the forest sector. Similarly, biodiversity protection policies in many cases target forest environments. The converse is true also: policies meant to affect forest management can have far-reaching impacts in other sectors. For example, forest taxation or subsidies for forest residue collection for energy purposes affect timber supply and have impacts on the entire energy sector.

Here, we present a partial equilibrium model of the forest and energy sector in Finland (FinFEP model, Finnish Forest and Energy Policy). The model allows studying the effectiveness of various policy instruments to achieve preset objectives, and analyzing the impacts that changes in external conditions have on the Finnish forest and energy sectors. In individual parts, the model has been applied to investigate the willingness of pulp and paper industries to invest in biorefineries, the willingness of sawmill industries to invest in pellet production, and the cofiring problem of power plants (Kangas et al. 2009, Lintunen & Kangas 2010, Mäkelä et al. 2011, Kangas et al. 2011).

A fundamental question in forest economics is what is the best way of dividing forest resources between different uses. This allocation problem can take the form of dividing a piece of forestland between age-classes, types of forest management or between conservation and timber production. But it can also take the form of dividing forest biomass resources between different end-uses, e.g., between energy wood, wood for industrial materials, woody biomass for soil fertilization, and wood in standing trees. In FinFEP-model the use of forest resources, as well as the allocation of energy sources between forest-based and other energy sources are optimized. Market behavior and industrial investment behavior are described in each period. The optimization is based on applying economic theory and detailed data consisting of information on the market, policy conditions, technology and natural resources.

The modeling initiative described here has three main objectives:

- 1 Linking forest and energy sectors
- 2 Linking forest industry and accurate nonindustrial private forest owners' behavioral descriptions
- 3 Enable improved integration of policy tools in the model to see their impacts.

The constraints with respect to which profits or utility are maximized are related to economic, ecological, social and cultural conditions. In modeling work, the economic conditions typically take the form of budget constraints and the ecological conditions are often formulated as laws of motions related to biological growth, or as relations describing management reactions. Social and economic conditions can take the form of legal restrictions, as in the case of limiting timber harvesting rights or imposing a replanting requirement.

The model whose conceptual premises are demonstrated in this work, can be applied to study the impacts of policy measures on the use of forest resources. More specifically, the FinFEP-model can be used to assess:

- (i) The cost effectiveness of various policy measures and the optimality of policies
- (ii) The market impacts of policies: short-run and long-run
- (iii) Impacts of industrial or natural resource investments

- (iv) Climate change impacts
- (v) Impacts of global trends in consumer behavior
- (vi) Impacts of changes in land-owner behavior

Information is especially needed on the cost efficiency of policy instruments and their impacts on the industrial and energy systems. To calculate policy costs, different policy instruments can be compared under various scenarios concerning external conditions such as oil and CO<sub>2</sub> prices, or demand for Finnish forest industry products. The policy impacts measured include those on energy and industrial wood demand and supply, other fuel types, wood prices, forest industry profitability, new investments, and the age-class structure and carbon content of forest resources.

Policy instruments that can be incorporated into the model include:

- (a) Energy subsidies for small-sized wood
- (b) Support for electricity production from wood chips (feed-in tariff)
- (c) Feed-in tariff for small CHP plants
- (d) Carbon rentals for landowners
- (e) Forest taxation

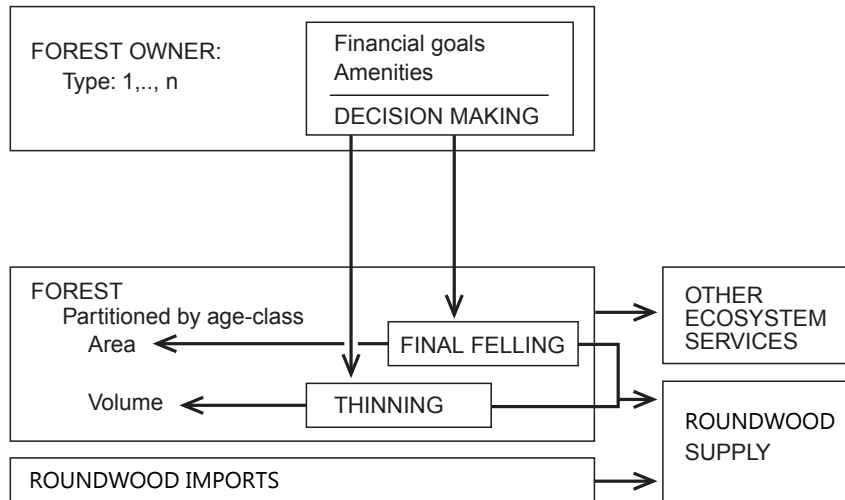
## 1.2. Previous modeling work

Toppinen and Kuuluvainen (2010) and Latta et al. (2013b) reviewed recent developments and applications of partial equilibrium models within the forest sector. According to Latta et al. (2013b) most forest sector models that are in use today have connections to models developed in the 1980s, such as: TAMM – the Timber Assessment Market Model (Adams & Haynes 1980) describing North American solid wood products markets, POPYRUS (Gilles & Buongiorno 1987) describing the North American pulp and paper markets, IIASA GTM, the International Institute for Applied Systems Analysis Global Trade Model (Kallio et al. 1987, Daigneault et al. 2012) for global forest products and trade, and TSM, The Timber Supply Model (Lyon & Sedjo 1983). Of these, the three first ones are referred to as applying a recursive dynamic framework based on the net social surplus period-by-period optimization, while TSM is said to utilize intertemporal optimization over the entire time horizon simultaneously.

Of forest sector models developed since the 1990s Latta et al. (2013b) list the following from the category of recursive models: EFI-GTM (Kallio et al. 2004), NTM-II (Bolkesjø et al. 2006), The French Forest Sector Model (FFSM) (Cauria et al. 2009), and the Subregional Timber Supply Model (SRTS) of the United States South (Abt et al. 2000). As intertemporal optimization models they list e.g. the The Forest and Agriculture Optimization Model (FASOM) (Adams et al. 1996, Alig et al. 1997) for the United States, its European counterpart The European Forest and Agriculture Sector Optimization Model (EUFASOM) (Schneider et al. 2008, Lauri et al. 2012), and the Norwegian Forest Sector Model (NorFor) (Sjølie et al. 2011). The FASOM-model was also used by McCarl et al. (2000), and by Latta et al. (2013a).

Toppinen and Kuuluvainen (2010) classify forest sector models for Europe into three categories: (1) those studying the effects on the future development of the forest sector of assumptions concerning GDP growth, forest growth and technological changes, such as Trømborg et al. (2000) who used Global Forest Products Model GFPM by Zhang et al. (1997) to study the effects of economic growth, timber supply and technological changes on the production, consumption and trade of forest products in the world; (2) those studying the effects of forest conservation, such as Kallio et al. (2006) who used the EFI-GTM model to study the forest sector impacts of increased biodiversity conservation in European forests; (3) and those that are concerned with forest bio-energy, such as Bolkesjø et al. (2006) and Trømborg et al. (2007) who used the NMT-II model to study the possible development in the use of forest bio-energy in Norway. GFPM has since been used also by Raunikaar et al. (2010), and by Buongiorno et al. (2011), while EFI-GTM was used by Moiseyev et al. (2011).





**Figure 1.** A diagrammatic representation of timber supply in the FinFEP-model.

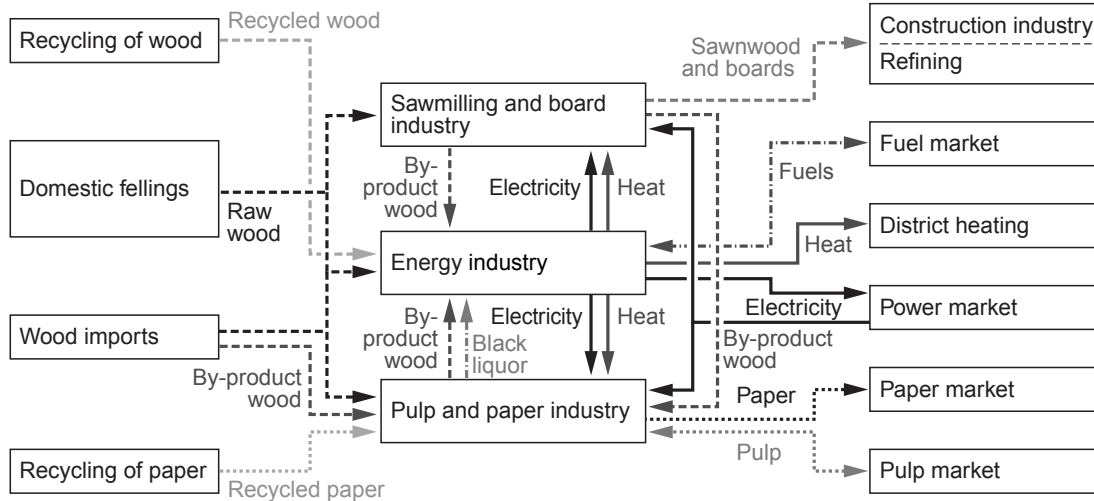
United States Forest Products Module (USFPM) operates recursively, and has been used by Ince et al. (2011) and Nepal et al. (2013). SF-GTM is a spatial partial equilibrium model simulating the Finnish forest sector (Ronnala 1995, see also Kallio et al. 2013).

### 1.3. Our approach

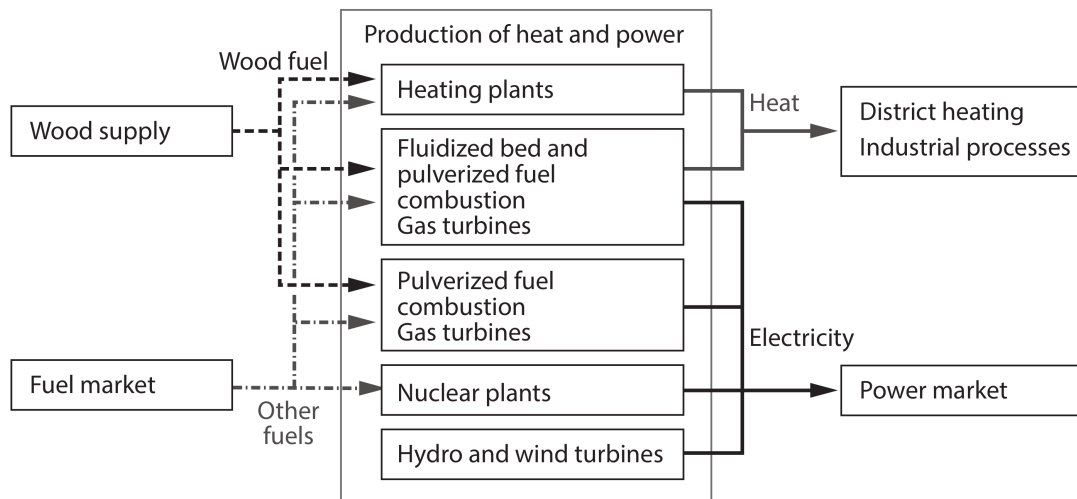
Recursively dynamic models are dynamic because decisions have an effect on state variables in the future periods. However, the control decisions such as investments are based on current economic conditions only, i.e. the decision makers are myopic. Among models with perfect foresight, an opposite type of model is one where the decision makers know all the decisions made by themselves and others in the current and all the future periods. In FinFEP, the dynamic optimization is made under bounded rationality. First, the forest owners base their decision on an assumed stochastic price process that only approximates the price series generated by the model equilibrium. Second, the capacity investments are made with perfect foresight on the next period but the more distant future is assessed imperfectly.

We construct an economic equilibrium model that combines the profit maximizing firms of Finnish forest and energy sectors with the data on the Finnish forest resources, managed by forest owners maximizing their objective function. The endogenous wood supply in FinFEP is based on a stand-level optimization of forest management. Both commercial thinning and regeneration harvests are included. The objective function in the optimization problem is determined by forest-owner preferences. Optimization is performed with a separate sub-model, shown as the forest-owner decision making box in Figure 1, which outlines the structure of the forest resource module of the FinFEP-model. The stand level optimization is performed under exogenously developing stochastic timber prices. The price process is such that it approximates the eventual statistical properties of the timber prices in the equilibrium model. Since the price process is stationary, the harvest policies (harvest rules) obtained from the optimization are stationary. After the optimization step the stationary harvest policies are aggregated to represent the regional-level harvest behavior. The aggregation is performed through statistical modeling of the policies that gives parametrized approximations of the actual policies. These approximations are inserted into the equilibrium model and together with the forest resource data they form the timber supply curves of the FinFEP. As shown in Figure 1, the harvest rules determine both the final felling and thinning management in the model. Final fellings determine how the forest area and age-structure of the forest develop, whereas thinning controls the development of the stand-level properties such as volume. They both contribute to the supply of roundwood. Finally, wood imports are added to the aggregate supply of timber. In addition, the growing stock of timber generates ecosystem services other than roundwood and their development is linked to the development of the regional forest resources.

The processing module of the FinFEP-model consists of input use decisions made by profit maximiz-



**Figure 2.** A diagrammatic representation of product and energy flows in the FinFEP-model.



**Figure 3.** A diagrammatic representation of energy industry in the FinFEP-model.

ing forest and energy sector firms. The firms face product demand that is partly endogenous and partly determined through exogenous demand functions. The supply of inputs is mostly endogenous, through forest owner decisions and production decisions of the intermediate good producers. In addition, there is an exogenous supply function for imports such as fossil fuels. The markets are assumed to be competitive and, therefore, all the firms are modeled as price takers. The processing facilities are aggregated at regional level. To obtain an accurate enough description of the input-output flows, the production technologies are based on input-output data. However, we do not optimize all the inputs of products endogenously. Instead, we optimize a set of variables of interest and set the use of the rest of the inputs of production at their cost minimizing level.<sup>1</sup> Thus, our approach combines the profit maximization and cost minimization frameworks of the theory of the firm.

In the equilibrium, the supply and demand of every good in the model are in balance and equilibrium prices are obtained for each good. A simplified illustration of product flows in the model are presented in Figure 2. The processing technologies of the industries are modeled regionally. Thus, for example in energy generation, the different combustion technologies are modeled individually. In addition to the conventional thermal power and heat generation, the model includes domestic wind, hydro and nuclear power generation. The energy industry is further elaborated in Figure 3.

<sup>1</sup>Profit maximizing firms automatically use their inputs at cost minimizing way.

The paper is structured as follows. In Section 2, we formulate the timber supply step by step. In Section 3 we present the optimization problems of the firm operating in the forest and energy sectors. The partial equilibrium and its implementation as a mixed complementarity problem are presented in Section 4. In Section 5 we illustrate the possibilities of the model in policy analysis.

## 2. Forest resource module

### 2.1. Stand level description

The set of trees in a one-hectare forest stand of age  $a_t$  is described through three state variables: the number of trees,  $n_t$ , the average volume of individual trees,  $v_t$ , and a measure of the relative width of the volume distribution,  $s_t$ . We assume that the volume distribution is uniform at all times, i.e.  $v_t \sim U(\underline{v}_t, \bar{v}_t)$ . The measure  $s_t$  is one half of the width of the distribution's support relative to the mean value. Thus, the smallest tree of the stand has the volume  $\underline{v}_t = (1 - s_t)v_t$  and the largest  $\bar{v}_t = (1 + s_t)v_t$ . As the volume distribution is symmetric, the volume of a stand is directly obtained as  $q_t = n_t v_t$ . The volume of a tree determines the shares of different timber grades,  $w$ , obtainable from the stem of the tree. We model three grades based on diameter: small-diameter tree, pulpwood and logs, i.e.  $w \in W := \{small, pulp, logs\}$ . The volume of individual trees determines the shares of the timber grades. We model these shares through continuous age-dependent function  $\sigma_{wa}(v_t)$ . Given these functions, the volume,  $v_w$ , of a timber grade  $w$  in a tree of volume  $v$  in a stand of age-class  $a$  is

$$v_w = \sigma_{wa}(v)v, \quad (1)$$

where  $\sum_{w \in W} \sigma_{wa}(v) = 1$ . For brevity, we use a short-hand notation  $z_t := (n_t, v_t, s_t)$  to denote the state of the trees on a given stand.

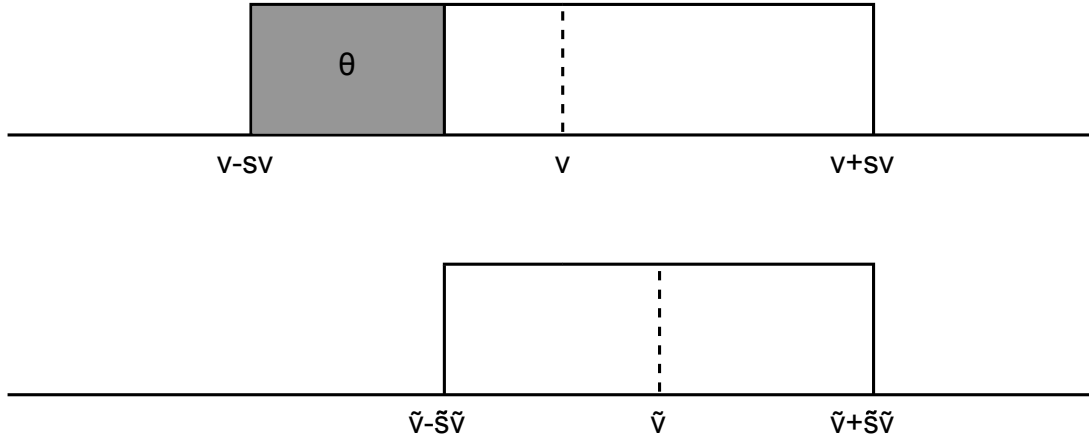
Harvest occurs at the beginning of a period and for the rest of the period, the stand continues to grow. We allow for two kinds of harvests: regeneration and thinning harvests,  $\gamma_t$  and  $\theta_t$ , respectively. Regeneration harvest is a binary decision, i.e.  $\gamma_t \in \{0, 1\}$ , where unity denotes the decision to clear-cut and regenerate the stand, whereas, zero indicates the decision not to regenerate the stand. Thinning management is denoted as a share of trees that are removed. We allow for both thinning from below and above. The thinning decision is denoted as  $\theta_t \in [-\bar{\theta}, \bar{\theta}]$ . If  $\theta_t > 0$ , thinning is performed from below, i.e. felling of trees starts from smallest trees and proceeds towards larger ones. If  $\theta_t < 0$  indicates that thinning is from above. Absolute value of  $\theta_t$  indicates the share of removed trees. Harvest yield from a stand of age  $a_t$  and state  $z_t$  by a clear-cut is simply

$$q_w^{clear}(n_t, v_t, a_t) := \sigma_{wa_t}(v_t)n_t v_t, \quad (2)$$

for each timber grade  $w$ . Thus, we assume that the average volume is sufficient in determining the average shares of timber grades. For harvest yield from thinning management, however, we take into account the volume distribution more accurately. With thinning, we first calculate the timber grade shares at mean volume,  $v_t$  and at both limits  $\underline{v}_t$  and  $\bar{v}_t$ . We proceed by approximating the volume dependence of grade shares in the support of volume distribution by a linear relation. Using definitions  $\underline{\sigma}_{wt} := \sigma_{wa}(\underline{v}_t)$ ,  $\bar{\sigma}_{wt} := \sigma_{wa}(\bar{v}_t)$ ,  $\underline{\Delta}_{wt} := \sigma_{wa}(v_t) - \sigma_{wa}(\underline{v}_t)$  and  $\bar{\Delta}_{wt} := \sigma_{wa}(\bar{v}_t) - \sigma_{wa}(v_t)$ , we arrive in the following thinning yield function

$$q_w^{thin}(\theta_t, z_t, a_t) := \begin{cases} \left[ (1 - s_t)\underline{\sigma}_{wt} + (s_t\underline{\sigma}_{wt} + (1 - s_t)\underline{\Delta}_{wt})\theta_t + \frac{4}{3}s_t\underline{\Delta}_{wt}\theta_t^2 \right] n_t v_t \theta_t & \text{if } \theta_t \geq 0, \\ \left[ (1 + s_t)\bar{\sigma}_{wt} + (s_t\bar{\sigma}_{wt} + (1 + s_t)\bar{\Delta}_{wt})\theta_t + \frac{4}{3}s_t\bar{\Delta}_{wt}\theta_t^2 \right] n_t v_t |\theta_t| & \text{if } \theta_t < 0, \end{cases} \quad (3)$$

where  $|\theta_t| \leq 1/2$ . The resulting harvest yield function is continuous and differentiable everywhere except at  $\theta_t = 0$ . The thinning from below yields initially low volumes and timber grade shares are those of small trees. As thinning intensity grows, the unit volumes increase and timber shares change accordingly. For the thinning from above the opposite happens for the unit volumes. See details in Appendix A.



**Figure 4.** Uniform distribution and the development of volume distribution.

The state of the stand after thinning is given by  $\tilde{z}_t := (\tilde{n}_t, \tilde{v}_t, \tilde{s}_t)$ . Under the assumption of uniform volume distribution, the state of the stand after thinning management is

$$\tilde{n}_t = (1 - |\theta_t|) n_t, \quad (4)$$

$$\tilde{v}_t = (1 + s_t \theta_t) v_t \quad (5)$$

and

$$\tilde{s}_t = \frac{1 - |\theta_t|}{1 + s_t \theta_t} s_t. \quad (6)$$

It proves useful to collect these relations into function  $\tilde{z}_t = \tilde{z}(\theta_t, z_t)$ . The effect of thinning from below on the volume distribution is illustrated in Figure 4. A thinning removes the share  $\theta$  of trees. As the thinning is made from below, the after-harvest average volume increases. Analogously, the average volume of trees decreases, when the stand is thinned from above.

The development of the stand is described through equations of motion for the three state variables. The after-thinning state determines the development of the stand during the current period. We specify age-dependent survivability, gross growth rate of average tree volume and gross growth of the width of the volume distribution function,  $N_a(\tilde{z})$ ,  $V_a(\tilde{z})$  and  $S_a(\tilde{z})$ , respectively. However, if the stand is clear-cut and regenerated, the development is very different as the trees are replaced with a new cohort. The new cohort has fixed properties  $(n_0, v_0, s_0)$ . The growth model is Markovian and can be summarized as

$$z_{t+1} = G_{a_t}(z_t, \gamma_t) := \begin{cases} (n_1, v_1, s_1), & \text{if } \gamma_t = 1, \\ (N_{a_t}(\tilde{z}_t)\tilde{n}_t, V_{a_t}(\tilde{z}_t)\tilde{v}_t, S_{a_t}(\tilde{z}_t)\tilde{s}_t), & \text{if } \gamma_t = 0. \end{cases} \quad (7)$$

In addition, the stand ages according to the function  $a^+(a, \gamma)$ :

$$a_{t+1} = a^+(a_t, \gamma_t) := \gamma_t + (1 - \gamma_t) \min\{a_t + 1, A\}, \quad (8)$$

where  $A$  is the oldest age-class to be included in the model.

## 2.2. Stand-level harvest decision

Since our aim is to model timber supply, we chose not to rely on a constant price Faustmann model. Instead, we use a model with changing prices. In forest economics, it is a well-known fact that a variable price complicates the forest owners' problem considerably. A full equilibrium model is out of question because of the mere size of the modeled system. Thus, we approximate the equilibrium behavior by studying the forest owner behavior under an exogenous stochastic price process. We assume that the

price process is slowly mean-reverting. Thus, the forest owner reacts to price changes logically: Exceptionally high prices induce harvests and low prices make waiting motives stronger. In practice, we define a AR(1) process for pulpwood and log timber grades:<sup>2</sup>

$$p_{w,t+1} = \mu_w - \eta_w(p_{wt} - \mu_w) + \varepsilon_{wt}, \quad (9)$$

where  $\varepsilon_{wt} \sim N(0, \sigma_w^2)$  is the IID innovation of the process. Parameters  $\mu_w$  and  $\eta_w \in [0, 1]$  are the expected price and the persistence of the price process for timber grade  $w$ , respectively.

The average forest owner behavior in Finland has not typically followed that of NPV maximizing, i.e. a Faustmannian, forest owner. Therefore, we consider three different forest owner types and apply amenity values of Hartmanian type with differing relative weights between monetary profits and amenity values (Hartman 1976). The periodic payoff,  $R_f$ , of the forest owner of type  $f \in F$  is defined as

$$R_f(a_t, z_t, \mathbf{p}_t, \theta_t, \gamma_t) := (1 - \tau) \left[ R^H(a_t, z_t, \mathbf{p}_t, \theta_t, \gamma_t) + S(a_t, \tilde{z}_t) \right] + \frac{\alpha_f}{1 - \alpha_f} R^A(\gamma_t, \tilde{z}_t), \quad (10)$$

where  $\alpha_f \in [0, 1]$  is the weight of the amenity payoffs in the forest owner preferences and a profit tax is denoted by  $\tau$ . The function  $S(\cdot)$  denotes the possible policy benefits,  $R^A(\cdot)$  the amenity benefits<sup>3</sup> and  $R^H(\cdot)$  gives the harvest revenues

$$R^H(a_t, z_t, \mathbf{p}_t, \theta_t, \gamma_t) := \sum_w p_{wt} \left[ \gamma_t q_w^{clear}(n_t, v_t, a_t) + (1 - \gamma_t) q_w^{thin}(\theta_t, z_t, a_t) \right] - C(\theta_t, \gamma_t, z_t), \quad (11)$$

where the harvest yield functions  $q_w^{clear}(\cdot)$  and  $q_w^{thin}(\cdot)$  are determined by equations (2) and (3), respectively. The function  $C(\cdot)$  represents the harvesting and regeneration costs. See Appendix D.3 for details.

In each period, the forest owner chooses the thinning and harvest management,  $\theta_t$  and  $\gamma_t$ , respectively. It is assumed that the forest owner maximizes the expected net present value of the stream of payoff over an infinite horizon, i.e.

$$W_f(a_0, z_0, \mathbf{p}_0) = \max_{\{\theta_t, \gamma_t\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t R_f(a_t, z_t, \mathbf{p}_t, \theta_t, \gamma_t), \quad (12)$$

subject to (1)–(9). We formulate the dynamic optimization problem as a stationary dynamic program with Bellman equation

$$W_f(a_t, z_t, \mathbf{p}_t) = \max \left\{ W_f^{thin}(a_t, z_t, \mathbf{p}_t), W_f^{clear}(a_t, z_t, \mathbf{p}_t) \right\}, \quad (13)$$

where the value of thinning management is given by

$$W_f^{thin}(a_t, z_t, \mathbf{p}_t) := \max_{\theta_t} \left\{ R_f(a_t, z_t, \mathbf{p}_t, \theta_t, 0) + \beta \mathbb{E}_t W_f(\min\{a_t + 1, A\}, G_{at}(\tilde{z}(\theta_t, z_t), 0), \mathbf{p}_{t+1}) \right\} \quad (14)$$

and the value of clear cutting the stand by

$$W_f^{clear}(a_t, z_t, \mathbf{p}_t) := R_f(a_t, z_t, \mathbf{p}_t, 0, 1) + \mathbb{E}_t \beta W_f(1, z_1, \mathbf{p}_{t+1}). \quad (15)$$

Equation (14) gives the value of waiting, if  $\theta_t = 0$ . In both equations, the development of stand age and other stand characteristics follows directly from equations (8) and (7), respectively. The exogenous development of prices is determined by the price process (9). The resulting dynamic program has six state variables: the four endogenous state variables describing the state of the stand ( $a_t$ ,  $n_t$ ,  $v_t$  and  $s_t$ ) and the two exogenously developing timber prices. For numerical solution we use a piece-wise linear

<sup>2</sup>For small sized roundwood and harvest residues, the prices are assumed to be time invariant.

<sup>3</sup>The exact formulation of the policy payoff may vary depending on policies studied or calibration used and, therefore, we do not specify their functional forms here.

approximation for the value function. In addition, we discretize the harvest actions. A solution is found using value function iteration.

As the growth conditions vary across forest site-classes,  $s \in S$ , and regions of Finland,  $r \in R$ , the optimization needs to be performed for each growth condition  $(r, s)$ . The solutions of the optimization problems include a pair of stationary optimal harvesting rules, i.e. policies,

$$\theta_t = \Theta_{rsfat}(\mathbf{z}_t, \mathbf{p}_t) \quad (16)$$

and

$$\gamma_t = \Gamma_{rsfat}(\mathbf{z}_t, \mathbf{p}_t), \quad (17)$$

that give the optimal intensity of thinning management and clearing activity for each contingency. Since the harvest policies are stationary, the rules are valid throughout the relevant time horizon.

### 2.3. Aggregation of harvest rules

In order to be useful in FinFEP, the harvest rules (or policies) are transformed into a parametrized formulation. The first step is to simulate a time-series of stochastic prices and create the resulting time-series of harvest decisions using the optimal rules (16) and (17). The simulation results in realizations  $(\theta_{rsfat}, \gamma_{rsfat})$  of the harvest decisions for each realization of the states of the stand  $(a, \mathbf{z}_t)$  and prices  $\mathbf{p}_t$ . A parametrized formulation of rules (16) and (17) is obtained through an approximation by a statistical fit

$$\theta_{rsfat} = \hat{\Theta}_{rsfa}(\mathbf{z}_t, \mathbf{p}_t) + \varepsilon_{rsfat}^\theta \quad (18)$$

and

$$\gamma_{rsfat} = \hat{\Gamma}_{rsfa}(\mathbf{z}_t, \mathbf{p}_t) + \varepsilon_{rsfat}^\gamma, \quad (19)$$

where error terms  $\varepsilon^i$  have expectation value of 0 and positive variance. The simulated time-series of harvest decisions is used as data in the estimation step. For thinning management (18) it is natural to apply regression models developed for censored variables and probability models for binary clearing decision.<sup>4</sup>

Conceptually, this transformation aggregates the individual forest owner decisions into one aggregate harvest rule for a set of forest owners. In the case of thinning, the aggregation means that all the forest owners of given type and with a stand of given site-type will thin their stands identically. This kind of aggregation is necessary as deviating thinning management would make stand properties  $\mathbf{z}_t$  different and the model size would grow each time period. For clearing, the aggregation transforms binary decisions into continuous ones, i.e.  $\{0, 1\} \rightarrow [0, 1]$ . Thus we allow for deviation in clear-cut behavior in a given group of otherwise identical forest owners. These differences can be motivated, for example, through differences in site-specific conditions that are not included in the model or through non-modeled variations in forest owner preferences.

### 2.4. Timber supply and age-class dynamics

The aggregated harvest decision rules are the behavioral basis of the timber supply in FinFEP. Since we are not able to use the exact harvest policies obtained from the dynamic program, we approximate them with statistical fit described above. This means that for the timber supply functions we use the following relations

$$\theta_{rsfat} = \hat{\Theta}_{rsfa}(\mathbf{z}_t, \mathbf{p}_t) \quad (20)$$

and

$$\gamma_{rsfat} = \hat{\Gamma}_{rsfa}(\mathbf{z}_t, \mathbf{p}_t), \quad (21)$$

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<sup>4</sup>See implementation details in Appendix D.4.

obtained as estimates from regression models (18) and (19), respectively. To obtain the timber supply function for the timber grade  $w$ , we need to combine these behavioral rules with the forest resource data.

In practice, what is needed is the information on site-class, age and forest owner structure of the forest land,  $x_{rsfat}$ , in all the regions. In addition, the stand properties,  $z_t$ , are needed in order to apply the harvest rules above and to obtain correct yield levels. When this information is combined, we arrive at the following timber supply

$$y_{wrt} = \sum_{s,f,a} \left[ q_w^{clear}(n_t, v_t, a_t) \gamma_{rsfat} + q_w^{thin}(\theta_{rsfat}, z_t, a_t) (1 - \gamma_{rsfat}) \right] x_{rsfat}, \quad (22)$$

where the per hectare harvest yields  $q_w^{clear}$  and  $q_w^{thin}$  are defined in equations (2) and (3), respectively. The thinning and harvest decisions,  $\theta_{rsfat}$  and  $\gamma_{rsfat}$ , are obtained from equations (20) and (21), respectively.

The stand level equations (1) – (7) are all included in FinFEP.<sup>5</sup> The final piece in the forest resource module is the dynamics of the age-classes. For this purpose, the equations of motion are based on clear-cut decisions  $\gamma_{rsfat}$ :

$$\begin{aligned} x_{rsf1,t+1} &= \sum_{a=1}^A \gamma_{rsfat} x_{rsfat}, \\ x_{rsf,a+1,t+1} &= (1 - \gamma_{rsfat}) x_{rsfat}, \quad \text{for } a \in \{1, \dots, A-2\}, \\ x_{rsfA,t+1} &= \sum_{a=A-1}^A (1 - \gamma_{rsfat}) x_{rsfat}. \end{aligned} \quad (23)$$

Thus, the oldest age-class does not grow any older as indicated by stand-level equation of motion for age (8). Note that the clear-cut decisions,  $\gamma_{rsfat}$ , are obtained from equation (21).

## 3. Processing module

### 3.1. Technology

The modeled production processes combine a large number of inputs to produce one or more outputs. For example, a pulp mill uses several inputs such as wood, chemicals, labor and energy (in the form of steam and electricity). The products include, in addition to the pulp itself, bark, black liquor and other chemicals. As another example, a CHP-plant produces both heat and electricity by utilizing labor and a technology-dependent variety of fuels. All the production processes are constrained by the machinery in place, which is summarized by the capital input. We model the machinery as technology and each technology can be utilized in a number of processes. The capacity constraints are technology specific, whereas the optimization of inputs is performed for each process.

We divide the factors of production into three categories of variables: free, predetermined and exogenous. The free variables are the ones that are directly optimized to maximize profit. The predetermined variables are endogenous, but their level is determined by the optimal use of the free variable in a given production process.<sup>6</sup> The level of exogenous variables and, thus, their effect on production costs is consistently linked to the level of free input use.

We assume, that outputs are produced using processes that are described through Leontief technology where the inputs are perfect complements. The outcomes of different processes are identical

<sup>5</sup>The timber grade share function  $\sigma_{wa}(v)$  is determined through a regression on the stand-level data.

<sup>6</sup>Predetermination follows from assumptions of Leontief production function and cost minimizing production process. As the inputs are perfect complements, if the costs are minimized, the inputs are used in fixed ratios determined by the production function.

in their properties and, therefore, they are perfect substitutes. In order to describe this kind of input-output transformation, we use a nested perfect substitutes – perfect complements -function

$$y_i = f_i(\mathbf{x}, \hat{\mathbf{x}}) := \sum_j \min \left\{ a_{ij}x_j, \frac{\hat{x}_{j1}}{b_{ij1}}, \frac{\hat{x}_{j2}}{b_{ij2}}, \dots, \frac{\hat{x}_{jn}}{b_{ijn}} \right\}, \quad (24)$$

where  $y_i$  is the output of product  $i$ ,  $\mathbf{x}$  are the free variables used in the production and  $\hat{x}_j$  are the predetermined variables used together with free input in the production process  $j$ . The parameter  $a_{ij}$  denotes the output  $y_i$  from the input use  $x_j$  in process  $j$ , whereas  $b_{ijk}$  indicates the need of predetermined variable  $k$  when output  $i$  is produced by the process  $j$ .

The costs of production are based on endogenous prices of the variables in the model. Therefore, free and predetermined inputs have endogenous unit costs. The exogenous variables in the model, e.g. labor, transport fuels, chemicals of the pulp and paper industry, do not have endogenous prices but costs related to their use are based on reported cost data and calibration. The costs of using exogenous variables are determined by the use of free variables and they are denoted by

$$c(\mathbf{x}). \quad (25)$$

The costs are increasing and convex ( $c' > 0$  and  $c'' \geq 0$ ). In practice, the costs can be based on level of production but also on individual input use, such as in the case of transportation costs.

The use of inputs in each of the technologies is constrained by a maximum production capacity. The capacity constraint is

$$K_t \geq g(\mathbf{x}_t) \quad (26)$$

for static input optimization in each period. Depending on the technology, the capacity may constraint input use or output produced. The function  $g(\cdot)$  maps the input use to the relevant constraint type. The initial capacity is based on initial plant level data. In later periods the capacity develops endogenously through optimized investments and exogenous depreciation. Thus, the time development of production capacity follows the equation of motion

$$K_{t+\tau} = \delta_\tau K_t + \sum_{s=1}^{\tau} (1 - \delta_{s-1}) I_{t+\tau-s}, \quad (27)$$

where  $I_t$  denotes the amount of production capacity increment through investment at period  $t$ ,  $\delta_s$  is the share of production capacity that has depreciated in  $s$  periods from its installment and  $\tau \geq 1$ . The investments are costly and are represented by a cost function

$$c_I(I), \quad (28)$$

The cost function is increasing and convex:  $c'_I(I) \geq 0$  and  $c''_I(I) \geq 0$ . The investments on production capacity present the capital costs of production.

### 3.2. Production

Firms optimize input use in a technology and the level of investments to the production capacity of that technology. The production generates profits  $\pi(\mathbf{x}_{t,dt})$ . The optimization is constrained by capacity constraint (26) and investing in production capacity is costly. The investments have long lasting effects on the capacity (27) and, therefore, investment decisions are made optimizing over time. The Lagrangian function of joint input use and investment optimization is

$$L(\mathbf{x}_{t,dt}, I_t, \lambda_{t,dt}) = \sum_{t=0}^{\infty} \beta^t \left( \sum_{dt} w_{dt} \{ \pi(\mathbf{x}_{t,dt}) + \lambda_{t,dt} [K_t - g(\mathbf{x}_{t,dt})] \} - c_I(I_t) \right), \quad (29)$$

with a given level of initial production capacity  $K_0$ . The current capacity  $K_t$  is obtained using (27):

$$K_t = \delta_t K_0 + \sum_{s=1}^t (1 - \delta_{s-1}) I_{t-s}. \quad (30)$$



The use of some of the inputs is optimized for each subperiod  $dt$ . As a result, some of the inputs and outputs have different prices in subperiod level. Therefore, the optimization problem is a weighted average of subperiod profits and the capacity constraint may bind on some subperiods and not on others. The parameter  $w_{dt}$  denotes the weights for subperiods  $dt \in \{1, 2, \dots, n\}$  with  $\sum_{dt} w_{dt} = 1$ .<sup>7</sup> The optimization problem is formally identical for all the firms producing all the output goods of the model.

To illustrate the nature of the problem further, the optimization problem (29) can be formally divided into two subproblems. First, the input use is based on a static profit maximization where free variables,  $\mathbf{x}$ , are optimized in each period and subperiod when needed and the input use is constrained by a capacity constraint (26). Thus, the Lagrangian of the constrained static maximization problem is

$$L_x(\mathbf{x}_{t,dt}, \lambda_{t,dt}) = \sum_{dt} w_{dt} \{ \pi(\mathbf{x}_{t,dt}) + \lambda_{t,dt} [K_t - g(\mathbf{x}_{t,dt})] \}. \quad (31)$$

Lagrange multiplier  $\lambda_{t,dt}$  is the usual shadow price of a constraint, i.e. it denotes the profit increase from a marginal capacity increase. Given the transformation function (24), the profits to be maximized can be represented as

$$\pi(\mathbf{x}) := \sum_i p_i \sum_j a_{ij} x_j - \sum_j p_j x_j - \sum_k p_k \sum_{i,j} b_{ijk} a_{ij} x_j - c(\mathbf{x}). \quad (32)$$

The levels of predetermined variables have been replaced by levels of free variables by using cost minimization together with Leontief production technology. Because profit function depends linearly on input price (32), equation (31) ensures that, in an annually optimized process, the annual mean price of a subperiodically priced good is used as a unit cost. Analogously, a subperiodically priced product built in an annually optimized process obtains the annual mean price of the good as its unit price.

The second part of the optimization problem of the firms is the optimization of investments. While the input use optimization is a static problem, the investment decision is dynamic. Firms make technology specific investment decisions each period. As indicated by equation (27), these investments replace depreciating production possibilities but also can be used to increase the production capacity.<sup>8</sup> The marginal benefits of capital increments due to the investments are valued through the Lagrange multiplier,  $\lambda$ , of the capacity constraint equation (26). Since the installed capacity is long-lived, the value of investment is accumulated over several periods. The periodic investment decision can be isolated from the optimization problem (29) and the Lagrangian function can be formally represented as

$$L_I(I_t) = I_t \sum_{s=1}^{\infty} \beta^s (1 - \delta_{s-1}) \sum_{dt} w_{dt} \lambda_{t+s,dt} - c_I(I_t). \quad (33)$$

The marginal benefits of the investment,  $\lambda_{t+s,dt}$ , are endogenous (see equations (29) and (31)) and are scaled down by depreciation and discounting as the benefits are obtained in the future.<sup>9</sup>

### 3.3. Inter-regional trade

The consumption or processing of a good can occur in a different region from where the production of the good takes place, i.e. we allow for inter-regional trade of goods. We assume that there is an unspecified number of competitive traders who transport goods between regions with a linear technology and that they maximize their profits. Thus, the profit maximization problem of a trader transporting goods from  $r_0$  to  $r_1$  is simply

$$\max_{t_{r_0 r_1}} (p_{r_1} - p_{r_0} - c_{r_0 r_1}) t_{r_0 r_1} \quad (34)$$

<sup>7</sup>If input decisions are made at annual level, then  $n = 1$ .

<sup>8</sup>It may prove useful to implement dual investment option, where the other can only be used in decreasing the depreciation and the other is new plant investment with high costs. The latter could be made in new regions whereas the former would be restricted to regions with that technology already existing.

<sup>9</sup>The setup described here indicates that the firms would have a perfect foresight on future periods. However, the way of implementation we follow causes the foresight to become imperfect. The implementation is discussed in Section 4.2 and in Appendix B.

subject to  $t_{r_0r_1} \geq 0$ . Here  $t_{r_0r_1}$  is the amount of good traded and  $c_{r_0r_1}$  is the fixed unit cost of transportation between regions  $r_0$  and  $r_1$ . The optimization problem is static and is performed for both direction and for all the regions adjacent to each other. Given the optimization problem, it is rather obvious that there is transport of goods between regions if the price difference is greater or equal to the unit transport cost. It is equally clear that with linear technology, the equilibrium can sustain only those price differences that are less than or equal to the unit cost of transportation.

## 4. Equilibrium and implementation

### 4.1. Partial equilibrium

FinFEP-model describes a competitive equilibrium where firms in the forest and energy sectors optimize their input use, investments and inter-regional trade (Section 3). The timber supply is based on optimized harvest rules of the forest owners of each type (Section 2.4).<sup>10</sup> In addition to endogenous supply and demand, there are exogenous supply  $S_t(p_t)$  and demand  $D_t(p_t)$  functions, representing the contributions of the external sectors and foreign countries. Accordingly, the modeled equilibrium is a partial equilibrium.

In an economic equilibrium, every agent optimizes its behavior. The optimality conditions determine the equilibrium. Given the presented model structure, the optimality conditions for input use are

$$\frac{\partial L(\mathbf{x}_{t,d}, I_t, \lambda_{t,d})}{\partial x_{jt,d}} \leq 0, \quad x_{jt,d} \geq 0 \quad \text{and} \quad x_{jt,d} \frac{\partial L(\mathbf{x}_{t,d}, I_t, \lambda_{t,d})}{\partial x_{jt,d}} = 0. \quad (35)$$

jointly with an explicit un-equality constraint

$$K_t - g(\mathbf{x}_t) \geq 0, \quad \lambda_t \geq 0 \quad \text{and} \quad (K_t - g(\mathbf{x}_t))\lambda_t = 0, \quad (36)$$

and for the investments the optimality conditions are

$$\frac{\partial L(\mathbf{x}_{t,d}, I_t, \lambda_{t,d})}{\partial I_t} \leq 0, \quad I_t \geq 0 \quad \text{and} \quad I_t \frac{\partial L(\mathbf{x}_{t,d}, I_t, \lambda_{t,d})}{\partial I_t} = 0. \quad (37)$$

The Lagrangian function is given by equation (29). For the inter-regional exports (Section 3.3), the conditions are

$$p_{r_1} - p_{r_0} - c_{r_0r_1} \leq 0, \quad t_{r_0r_1} \geq 0 \quad \text{and} \quad (p_{r_1} - p_{r_0} - c_{r_0r_1})t_{r_0r_1} = 0. \quad (38)$$

The dynamics of the capacity is given by the equality constraint (27). The development of forest resources is given by equations (1)–(7) and (23).

The equilibrium consists of prices,  $p_t$ , for each good included in the model under which all the endogenous agents optimize their actions and market clearing conditions

$$S_t + S(p_t) \geq D_t + D(p_t), \quad p_t \geq 0 \quad \text{and} \quad (S_t + S(p_t) - D_t - D(p_t))p_t = 0 \quad (39)$$

are satisfied for every good. Here  $S_t$  and  $D_t$  denote the endogenous supply and demand levels, respectively. The endogenous supply and demand include both the output produced and inputs used as well as inter-regional imports and exports, respectively. The timber supply is presented by equation (22). If for a good the market clearing condition holds as an equation, the price of a given good is positive. In the case of supply exceeding the demand, the price is zero.

<sup>10</sup>The harvest rules are optimized in an separate sub-model and, therefore, the harvest rules are exogenous to the solved equilibrium.

## 4.2. GAMS implementation

In economics, the general equilibrium model of a Walrasian economy can be presented as a mixed complementarity problem (MCP) (e.g. Mathiesen 1985, 1987, Ferris & Pang 1997).<sup>11</sup> Similarly, a MCP formulation can be used in a partial equilibrium setting. In a strictly concave maximization problem the solution to MCP is unique as the Karush-Kuhn-Tucker conditions uniquely determine the solution. The equilibrium conditions presented in the previous section (equations (35)–(39)) as well as all the equality constraints mentioned above are transformed into MCP formulation.

In the optimization problem for the investments into capacity (33), the time horizon of benefit streams is set to be infinite. Naturally, an infinite horizon is infeasible for the numerical solution methods. Thus, we need to truncate the horizon in the model.<sup>12</sup> Since the model has a large number of variables and equations, the model horizon is kept quite short (e.g. 2–3 periods). In practice, the model is solved recursively using a rolling window approach, where the starting period of the model horizon is shifted by one period in each recursive step. As an outcome of this feature the model exhibits imperfect foresight.

We have implemented the MCP describing the partial equilibrium of the FinFEP into General Algebraic Modeling System (GAMS) and we solve the MCP using the PATH solver (Dirkse & Ferris 1995). Rutherford (1995) and Ferris and Munson (2000) give in-depth information on using PATH solver in GAMS environment.

## 4.3. Data and calibration

The data for processing module (Section 3) is collected from various public sources. For example, the process parameters and production capacities were based on engineering-level data on individual plants presented in e.g. environmental reports of the companies. In this manner a database of plant level process data was constructed for Finland. The cost parameters for exogenous variables and investments were more difficult to obtain. Although, a reasonable range is easily obtainable from different studies and assessment reports, some uncertainty regarding their exact level remains.

The data describing stand development were simulated using the MOTTI forest simulator (e.g. Hynynen et al. 2002), which yielded growth predictions for different harvesting regimes. MOTTI contains up-to-date representation of the tree growth dynamics in Finnish growing conditions. MOTTI is built on deterministic growth models based on extensive measurements at permanent and temporary inventory plots and field experiments of the Natural Resources Institute Finland (Luke). The simulation results were used as data in the estimation of functions  $N_a$ ,  $V_a$ ,  $S_a$  and  $\sigma_{wa}$  (Section 2.1). Data on the current state of the forest resource was obtained from the National Forest Inventory (NFI) (e.g. Tomppo 2006). The utilized data were rather disaggregated as we used five year age-classes, in 18 regions of Finland, for three tree species and five site classes. To smooth out some of the sampling noise, a joint method of simulation and regression analysis was used for determining the applied stand parameters.

The parameters of the model, for which reliable data sources were not available, were initially based on expert assessment. Their values were later calibrated when the model was finalized. In the calibration process the equilibrium solution was made to match the observed data. The result of the calibration is not unique as all the model variables do not have direct statistical counterparts that could have been used as a calibration reference. The final assessment of the parameters was based on expert opinion.

## 5. Policy modeling

The FinFEP model can be used in analyzing of the effects of various policy instruments. Since the agents' behavior is determined through explicit optimization problems, the model is particularly well suited for

<sup>11</sup>Mathematical background of MCP is presented in Appendix C

<sup>12</sup>Truncation does not affect the static optimization problems of input use and inter-regional trade. However, it has an effect on dynamic optimization of investments. We describe in Appendix B how the implications of truncation are handled.

policies that directly give economic incentives to behavioral changes. These policies include Pigouvian taxes, feed-in tariffs and premia, production and investment subsidies etc. Naturally, the pure fiscal taxes are included in the model and, therefore, side effects of their changes can be analyzed too. For the environmental policies, we can study the effects, given the level of an instrument, or we can determine the needed level of an instrument, given a target level for an effect.

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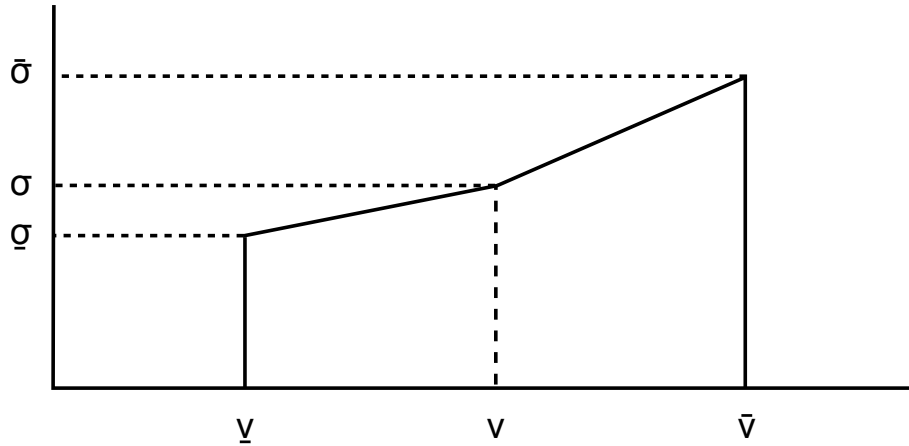
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## A. Thinning yield

Here we derive the thinning yield functions (3) in detail. To avoid clutter, we keep the notation at minimum. We therefore drop all the unnecessary indices and argument from the symbols and consider them as parameters. The basis of our approach is a piecewise linear approximation of the relation between timber grade share  $\sigma$  and the volume of an average tree  $v$ , presented by function  $\sigma(v)$  (see Figure 5).<sup>13</sup> We calculate the three nodes of the function approximation  $\underline{\sigma} := \sigma(\underline{v})$ ,  $\sigma := \sigma(v)$  and  $\bar{\sigma} := \sigma(\bar{v})$ , where  $\underline{v} = v - sv$  and  $\bar{v} = v + sv$  are the minimum and maximum volumes of the volume distribution.



**Figure 5.** A piecewise linear approximation of the  $\sigma(v)$  function used in harvest yield calculation.

To keep the calculations simple, we restrict ourselves to the case in which  $|\theta| \leq 1/2$ . Under this restriction, the timber grades follow an approximation

$$\sigma(\theta) = \underline{\sigma} + 2\Delta\theta, \quad (40)$$

when thinning is made from below, i.e.  $\theta \geq 0$ . Here we use shorthand notation  $\Delta := \sigma - \underline{\sigma}$ . If the thinning is made from above,  $\theta < 0$ , the relation is analogously

$$\sigma(\theta) = \bar{\sigma} - 2\bar{\Delta}|\theta|, \quad (41)$$

where  $\bar{\Delta} := \bar{\sigma} - \sigma$ . Depending on the sign of  $\Delta$ , the increasing thinning from above either increases or decreases the share of given timber grade. Similar observation can be made for thinning from above but the sign is opposite, as the harvesting proceeds from large trees towards smaller ones. The other component of the harvest yield is the volume of harvested trees. Their dependence on thinning intensity is directly observed to be

$$v(\theta) = \underline{v} + 2sv\theta, \quad (42)$$

for thinning made from below, and

$$v(\theta) = \bar{v} - 2sv|\theta|, \quad (43)$$

for thinning from above. It is obvious that harvest from below proceeds towards larger and thinning from above towards smaller trees.

The harvest yield,  $q(\theta)$ , is defined as an integral over a product of grade share and volume of trees times the number of trees,  $n$ , i.e.

$$q(\theta) := n \int_0^\theta \sigma(\tilde{\theta})v(\tilde{\theta})d\tilde{\theta}, \quad (44)$$

where, with the piecewise linear approximation introduced above, the product is a simple quadratic function of  $\theta$ . Thus, the integration is straightforward and results in the harvest yield function presented in the text (equation (3)).<sup>14</sup>

<sup>13</sup>The sum of grade shares over all the share is equal to unity for all values of  $v$ . Thus, the linear approximation is consistent.

<sup>14</sup>For thinning from above the absolute value operator may be confusing, but by using an auxiliary variable e.g.  $y := |\theta|$ , the integration proceeds as with the thinning from below case, except the cross term of the product has a negative sign.



## B. Investments and the truncation of horizon

We truncate the formally infinite optimization horizon of the firm into  $T$  periods. We assume that new capacity appears in the next period after the investment. We allow investments only for periods  $t < T$ . The truncated version of the investment optimization problem

$$\max_{I_t} I_t \sum_{s=1}^{\infty} \beta^s (1 - \delta_{s-1}) \sum_{dt} w_{dt} \lambda_{t+s,dt} - c_I(I_t) \quad (45)$$

is

$$\max_{I_t} I_t \sum_{s=1}^{T-t} \beta^s (1 - \delta_{s-1}) \sum_{dt} w_{dt} \lambda_{t+s,dt} - \tilde{c}_{I,T-t}(I_t). \quad (46)$$

It is apparent, that since the horizon  $T - t$  depends on the period of investment decision  $t$ , the sum of revenues depends on the period. Thus, the investment costs have to be time dependent for the truncated investment model to be consistent. To avoid this impractical data problem, we first derive a *periodic investment cost*,  $\hat{c}_I$ , by dividing the original investment cost by the sum  $\sum_{s=1}^{\infty} \beta^s (1 - \delta_{s-1})$ . This results in a reformulated optimization problem

$$\max_{I_t} I_t \sum_{s=1}^{\infty} \omega_s \sum_{dt} w_{dt} \lambda_{t+s,dt} - \hat{c}_I(I_t), \quad (47)$$

where

$$\omega_s := \frac{\beta^s (1 - \delta_{s-1})}{\sum_{i=1}^{\infty} \beta^i (1 - \delta_{i-1})} \quad (48)$$

is the weight of a lag  $s$ , i.e.  $\omega_s \geq 0$  and  $\sum_s \omega_s = 1$ . In the scaled version of the investment problem, the scaled investment costs are compared with a weighted average of marginal benefits from the investment. The weights  $\omega_s$  are largest for  $s = 1$  and decrease monotonically, if either  $\beta < 1$  or  $\delta_s > 0$ , for all  $s$ .

This scaled version of the infinite problem (47) proves useful in the truncated model. Truncating the problem leads to

$$\max_{I_t} I_t \sum_{s=1}^{T-t} \hat{\omega}_{s,T-t} \sum_{dt} w_{dt} \lambda_{t+s,dt} - \hat{c}_I(I_t), \quad (49)$$

where the investment cost functions are equal for all the periods  $t$ . The cost function is the same as in the scaled version of the infinite horizon problem (47). Comparing equations (47) and eq:invScaledTrunc shows that only thing that changes is the weighting parameter  $\omega_s$ , which has become time dependent  $\hat{\omega}_{s,T-t}$ . The lag weights in the case of truncated horizon are obtained analogously to the infinite horizon case as

$$\hat{\omega}_{s,T-t} := \frac{\beta^s (1 - \delta_{s-1})}{\sum_{i=1}^{T-t} \beta^i (1 - \delta_{i-1})}, \quad (50)$$

for all  $s \leq T - t$ . To illustrate the procedure, let us first examine the investment problem at period  $t = T - 1$ . Then the investment is based only on the period  $T$  revenues, i.e.  $s = 1$ , indicating  $\omega_{11} = 1$ . In period  $t = T - 2$ , the investment is based on revenues from periods  $T - 1$  and  $T$ . Now  $s \in \{1, 2\}$  and corresponding weights are  $\omega_{12} = \beta/S$  and  $\omega_{22} = \beta^2(1 - \delta_1)/S$ , where the denominator is defined as  $S := \beta + \beta^2(1 - \delta_1)$ . Here we have used the assumption  $\delta_0 = 0$ . If  $\beta < 1$  and  $\delta_1 > 0$  it is clear that the first period after the investment obtains a larger weight than the second. This is because in the second period the revenues are obtained only later and the capacity depreciates. Similarly, the weights are obtained for all  $t < T - 2$ .

The main benefit of the approach is that the same investment cost function can be used for all the periods. The time dependent weight parameter is more practical to work with in the model implementation. It is also worth noting that the scaling does not alter the marginal revenues obtained from the investment  $\lambda_{t+s,dt}$ . This is essential for the applicability of our approach. Namely, the marginal revenues are determined by the input use and the capacity constraint equations and, therefore, should not be affected by a scaling of some other equations in the MCP.

## C. Mixed complementarity problem (MCP)

The MCP formulation allows for constraints on individual variables as well as on the whole optimization problem. MCP formulation is constructed from equation-variable pairs that are complementary to each other. Complementarity is understood here in the usual sense, i.e. the values of functions,  $f(x)$ , and variables,  $x$  are orthogonal,  $x'f(x) = 0$ . In MCP framework the orthogonality is a bit more complicated due to the constraints: The MCP is defined through a function  $f : \Omega \rightarrow \mathbb{R}^n$  and a box  $B = [l, u]$ , where  $l_i \leq u_i$  (with equality, the variable is constant and can be removed from the problem),  $l_i \in \mathbb{R}$  and  $u_i \in \mathbb{R}$  for all  $i$  ( $B \subset \Omega$ ). The problem is to find a  $x \in B$  subject to conditions

$$\begin{aligned} (x - l)' f_+(x) &= 0 \\ (u - x)' f_-(x) &= 0 \end{aligned} \tag{51}$$

The elements  $i$  of the vectors  $f_+(x)$  and  $f_-(x)$  are defined as  $f_{+i}(x) := \max\{0, f_i(x)\}$  and  $f_{-i}(x) := -\min\{0, f_i(x)\}$ , respectively.<sup>15</sup> In MCP framework the notation  $x \perp f(x)$  denotes that vectors satisfy the condition (51). Dirkse (1994) has shown that MCP can equivalently be stated as: find a  $x \in B$  such that  $(y - x)' f(x) \geq 0$  for all  $y \in B$ . Thus, the MCP is a variational inequality on the parallelepiped  $B$ . The solution for the problem is a vector,  $x$ , each element in which satisfies one of the following three conditions

$$\begin{aligned} x_i = l_i &\quad \wedge \quad f_i(x) \geq 0 \\ x_i \in (l_i, u_i) &\quad \wedge \quad f_i(x) = 0 \\ x_i = u_i &\quad \wedge \quad f_i(x) \leq 0. \end{aligned}$$

Without upper boundary we end up with nonlinear complementarity problem (NCP) and without both boundaries the problem is a system of nonlinear equations. In the mixed complementarity setting lower and upper bounds exist variably. Thus the name “mixed”.

## D. FinFEP Version 1.0

### D.1. Data

The model includes a description of those forests in Finland that are available for wood production (puuntuotannon metsämaa). The forest resource data used in FinFEP are based on the 10th National forest inventory of Finland (NFI10), field data having been collected between years 2004 - 2008. The forest area has been partitioned to almost 500 forest sites with 30 ages-classes each (see Table 1 ). In the model the total area and growing stock volumes from NFI10 are used as initial states for the forests.

Forest area left out of FinFEP consist of Ahvenanmaa and north-Lapland regions, as well as treeless sites and the least growing sites with spruce as dominant species. This reduces the total forest area and volume in the model by about 5 % and 2 %, respectively, as presented in Table 2. FinFEP model uses Nuts 3 category (regions) as the level of regional aggregation. Table 3 shows the division of the regional forestry units between the provincial regions as applied in FinFEP.

**Table 1.** NFI data partition of forest resources in the FinFEP model

Site partitions	Description	Number of options
Area	The Finnish Forest Centre, Regional Units	14 (2 with 2 sub-regions)
Species	By dominant trees (pine, spruce , broadleaf trees)	3
Site-class	Tax-class (Land capability classification used in NFI)	5
Age	Five year age-classes (last age-class: older than 145 years)	30
Site type	Mineral soil and peatland	2

<sup>15</sup>Note that  $f(x) = f_+(x) - f_-(x)$ .

Table 4 presents the locations which have been used in the MOTTI stand-level forest simulations and the corresponding growth parameters are based on these simulations. Growth parameters based on simulations for these locations are used also to describe other regions and site-classes. The growth description used for each region, tree species and tax-class in the model is presented in Table 5.

**Table 2.** NFI10 and FinFEP model's initial forest land area and timber volume

	Area 1000 ha	Volume milj m <sup>3</sup>
NFI10	1815	1947
Ahvenanmaa, Ylä-Lappi, Spruce (tax-class 4), treeless sites	86	44
FinFep V1.0	1729	1904

**Table 3.** Correspondence between forest centers and provincial regions as applied in the FinFEP model.

FinFEP Region (Nuts 3)	NFI The Finnish Forest Centre, Regional Units	Share
Uusimaa	Rannikko-Etela	1
	Häme-Uusimaa	0.2
Varsinais-Suomi	Lounais-Suomi	0.5
Kanta-Häme	Häme-Uusimaa	0.4
Paijät-Häme	Häme-Uusimaa	0.4
Kymenlaakso	Kaakkois-Suomi	0.5
Etela-Karjala	Kaakkois-Suomi	0.5
Satakunta	Lounais-Suomi	0.5
Pirkanmaa	Pirkanmaa	1
Keski-Suomi	Keski-Suomi	1
Etela-Pohjanmaa	Etelä- ja Keski-Pohjanmaa	0.8
Pohjanmaa	Rannikko-Pohjanmaa	1
Etelä-Savo	Etelä-Savo	1
Pohjois-Savo	Pohjois-Savo	1
Pohjois-Karjala	Pohjois-Karjala	1
Kainuu	Kainuu	1
Keski-Pohjanmaa	Etelä- ja Keski-Pohjanmaa	0.2
Pohjois-Pohjanmaa	Pohjois-Pohjanmaa	1
Lappi	Etelä-Lappi	1
	Ylä-Lappi	0
Ahvenanmaa	Ahvenanmaa	0

**Table 4.** Locations used in MOTTI simulations

Location	Code	Temperature sum	Altitude
Porvoo	P	1331	27
Hollola	H	1227	136
Tampere	T	1190	128
Lapua	L	1118	57
Juva	J	1185	108
Siilinjärvi	S	1126	107
Utajärvi	U	993	112
Kemijärvi north	K	771	242

**Table 5.** Growth description used for each region, by tree species and tax-class. The subscript indicates the tax-class, if it differs from the tax-class of the modeled stand. The asterisk denotes the growth description for peatland. The spruce stands with tax-class 4 were omitted from the modeled forest area.

Region	Pine					Spruce					Birch				
	Tax-class														
	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
Uusimaa	H	P	P	S	S <sub>3</sub>	P	P	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Varsinais-Suomi	H	P	P	S	S <sub>3</sub>	P	P	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Kanta-Häme	H	H	H	S	S <sub>3</sub>	H	H	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Päijät-Häme	H	H	H	S	S <sub>3</sub>	H	H	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Kymenlaakso	H	P	P	S	S <sub>3</sub>	P	P	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Etelä-Karjala	H	H	H	S	S <sub>3</sub>	H	H	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Satakunta	S	L	L	S	S <sub>3</sub>	L	L	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Pirkanmaa	H	T	T	S	S <sub>3</sub>	T	T	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Keski-Suomi	S	S	S	S	S <sub>3</sub>	S	S	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Etelä-Pohjanmaa	S	L	L	S	S <sub>3</sub>	L	L	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Pohjanmaa	S	L	L	S	S <sub>3</sub>	L	L	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Etelä-Savo	S	J	J	S	S <sub>3</sub>	J	J	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Pohjois-Savo	S	S	S	S	S <sub>3</sub>	S	S	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Pohjois-Karjala	S	S	S	S	S <sub>3</sub>	S	S	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Kainuu	S	U	U <sub>1</sub>	U <sub>2</sub>	S <sub>3</sub>	U	U	U	S*	–	S	S	S	S*	S <sub>3</sub> *
Keski-Pohjanmaa	S	L	L	S	S <sub>3</sub>	L	L	S	S*	–	S	S	S	S*	S <sub>3</sub> *
Pohjois-Pohjanmaa	S	U	U <sub>1</sub>	U <sub>1</sub>	U <sub>2</sub>	U	U	U	S*	–	S	S	S	S*	S <sub>3</sub> *
Lappi	S	U	K <sub>1</sub>	K <sub>1</sub>	K <sub>1</sub>	S <sub>3</sub> *	S <sub>3</sub> *	K <sub>0</sub>	K <sub>1</sub>	–	S	K	K	K	S <sub>3</sub> *

## D.2. Stand dynamics

The development of the stand is determined through volume growth, natural mortality and harvests. The volume growth was estimated from the simulated MOTTI-data. The gross volume growth  $G_i := v_{a+1}/v_a$  was specified to follow a log-linear specification with the number of trees,  $n_i$ , and the average volume of trees,  $v_i$ , as explanatory variables:

$$\log G_i = \alpha_0 + \alpha_1 \log n_i + \alpha_2 v_i + \varepsilon_i^g.$$

The survivability is measured as  $S_i = n_{a+1}/n_a$  and we denote its expected value as  $\mathbb{E}S_i = \mu_i$ . The explanatory variables used are the number of trees and the average volume of a tree and their interaction. As the survivability is constrained between zero and one, we used a GLM-approach for the estimation. For consistent estimation we applied a quasi-likelihood approach (e.g McCullagh & Nelder 1989; chap. 9.1), with the logit link function

$$\log \frac{\mu_i}{1 - \mu_i} = \beta_0 + \beta_1 \log n_i + \beta_2 v_i + \beta_3 \log(n_i v_i) + \varepsilon_i^s.$$

and variance function

$$V(\mu_i) = \mu_i(1 - \mu_i).$$

Both the growth and survivability function were estimated for each age-class. The data used in estimation were selected to consist of the target age-class and the adjacent age-classes. For the first and last age-class only the data of two age-classes were used. The estimated relations were used in determining the functions  $N_a(\cdot)$  and  $V_a(\cdot)$ , in equation (7). Function  $S_a(\cdot)$  was assumed to be unity, i.e.  $S_a(\cdot) = 1$ .

For the estimation of the shares of different timber grades,  $\sigma_{va}$ , we followed similar quasi-likelihood approach as we did with the survivability. However, for the timber grade shares we used only the average

**Table 6.** Parameters of price processes

Grade	$\mu_w$	$\eta_w$	$\sigma_w^2$
small	25	0	0
pulpwood	40	0.8	4
logs	55–60	0.8	9

volume of a tree as an explanatory variable. Separate models were estimated for small-diameter stems and logs. The share of pulpwood was obtained as a residual. Separate functions were estimated for each age-class.

### D.3. Forest owner's problem

The parameters of the price processes are given in Table 6. The figures should be interpreted as roadside prices. The expected price  $\mu_w$  for logs depends on the tree species, being highest for softwood and lowest for hardwood. The amenity payoff is obtained from the standing stock with an emphasis on larger stems, i.e.

$$R^A(\gamma_t, \tilde{z}) := (1 - \gamma_t)\tilde{n}_t\tilde{v}_t^2$$

As usual, the tilde refers to the after-thinning state of the stand. The harvest costs are given by the following function

$$C(\theta_t, \gamma_t, z_t) := (1 - \gamma_t)[F_\theta + c_\theta(v_t)n_tv_t\theta_t] + \gamma_t[F_\gamma + c_\gamma(v_t)n_tv_t]$$

where the unit costs are given by

$$c_i(v) := \frac{c_h}{e^{a_{ih}v}b_{ih}} + \frac{c_t}{e^{a_{it}v}b_{it}}.$$

The subscripts  $h$  and  $t$  indicate harvester and tractor, respectively. Fixed cost  $F$  and parameters  $a$  and  $b$  are separate for thinning and clear-cutting. The parameters of the harvest cost function were based on machinery time use data (Väätäinen et al. 2008).

### D.4. Aggregation

The thinning policy is estimated using a Tobit model where the thinning share  $\theta_i$  is censored both from below and above. Thus, the data generating process consists of a linear model for the underlying latent variable  $\theta_i^*$

$$\theta_i^* = \mathbf{x}_i'\boldsymbol{\beta} + \varepsilon_i^\theta$$

together with censoring of observable variable  $\theta_i$

$$\theta_i = \begin{cases} 0 & \text{if } \theta_i^* \leq 0 \\ \theta_i^* & \text{if } \theta_i^* \in (0, \bar{\theta}) \\ \bar{\theta} & \text{if } \theta_i^* \geq \bar{\theta}. \end{cases}$$

The upper limit used is  $\bar{\theta} = 0.4$ . The explanatory variable used are prices of pulpwood and logs, mean harvest revenue,  $(1-s_i)p_in_iv_i$ , and per hectare volume,  $n_iv_i$ , and volume squared,  $(n_iv_i)^2$ . The average price of harvested timber,  $p_i$ , is calculated using the observed timber grade shares and their prices.

For the estimation of the clear-cut policy we use a linear probability model with the same explanatory variables as in the case of thinning. When inserted into FinFEP, the values obtained from the estimated clear-cut function are truncated into interval  $[0, 1]$ .

**Table 7.** Goods of the model and their species classification.

Good	Symbol	Species	Good	Symbol	Species
Newspaper	News	none	Electricity	el	none
Coated fine	Fine_co	none	Heat	heat	none
Uncoated fine	Fine_uc	none	Logging residues	res	none
Coated magazine	Magaz_co	none	Stumps	stumps	none
Uncoated magazine	Magaz_uc	none	Small-sized trees	small	spruce, pine, birch
Containerboard	Board_con	none	Pulpwood	pulp	spruce, pine, birch
Cartonboard	Board_car	none	Logs	logs	spruce, pine, birch
Tissue	Tissue	none	Industrial chips	chips	spruce, pine, birch
Chem. pulp: hardwood	hardchem	none	Sawdust	dust	none
Chem. pulp: softwood	softchem	none	Dry sawdust	drydust	none
Mechanical pulp	mech	none	Bark	bark	none
Recycled pulp	recycled	none	Recycled paper	rec_paper	none
Sawn wood: pine	PineSawn	none	Fischer-Tropsch gas	FTgas	none
Sawn wood: spruce	SpruceSawn	none	Black liquor	bl	none
Sawn wood: birch	BirchSawn	none	Peat	peat	none
Spruce boards	SpruceBoard	none	Coal	coal	none
Birch boards	BirchBoard	none	Natural gas	gas	none
Biofuel	biofuel	none	Fuel oil	oil	none
Pellet high quality	pelh	none	Wind	wind	none
Pellet low quality	pell	none			

### D.5. Processing and markets

The costs of production (25) consist of non-linear costs such as intra-regional transport costs and co-firing costs of the power plants. Both of these costs follow a similar specification as described by Kangas et al. (2009) and Lintunen & Kangas (2010). We specify a quadratic production costs that include e.g. intra-regional transport cost. The case of co-firing costs is more complicated. The exact specification is

$$C^{CF}(\mathbf{x}) := \frac{c_0}{2}(s_w - \sigma)^2 X^2,$$

where the share of wood fuels is  $s_w := X_w/X$  and the cost minimizing wood fuel share is  $\sigma \in [0, 1]$ . The sums of fuel use are denoted by capital letters, i.e.  $X = \sum_i \rho_i x_i$  and  $X_w = \sum_i 1_W(i) \rho_i x_i$ , where  $\rho_i$  the energy density of a fuel  $i$ . The indicator function has value zero or one depending on whether  $i \in W$  or not:  $1_W(i) = 1$  if  $i \in W$  and elsewhere  $1_W(i) = 0$ , where  $W$  is the set of wood fuels. This results in the following marginal cost

$$\frac{\partial C^{CF}(\mathbf{x})}{\partial x_i} = c_0 \rho_i (1_W(i) - \sigma)(s_w - \sigma)X.$$

The  $s_w - \sigma$  term is positive if wood fuels are used more than their optimal level. Thus, in that case an increase of wood fuel use leads to an increase in costs, whereas an increase in non-wood fuel use leads to a cost decrease. The opposite holds, if the wood use is below the cost optimum.

Table 7 presents the set of goods included in the model. Currently, there are 46 endogenous variables in the model.<sup>16</sup> The number of goods is bound to increase as new technologies and corresponding goods are introduced into the model. In the current model version logging residues, stumps and bark of different tree species are totaled into an aggregate without species classification. If needed, this constraint can be removed. The final goods (paper, sawn wood and boards) have pure exogenous demand functions. The intermediate goods (e.g. pulpwood, electricity, heat, pulp, pellets) have endogenous demand augmented with exogenous demand functions. The other wood inputs and fossil fuels have only endogenous demand in the model. The price elasticities of exogenous demand functions are based on qualitative estimates. Table 8 presents the elasticities used. On the supply side, the supply of electric-

<sup>16</sup>Wind power is formally presented as an input for modeling technical reasons, but as an exogenous natural phenomena, it is not counted in to the number of goods.

**Table 8.** Price elasticities of demand. Parentheses denote goods for which exogenous demand constitutes only a small share of total demand.

Good	Elasticity
News, Fine_co, Fine_uc, Magaz_co, Magaz_uc, Board_con, Board_car, Tissue, softchem, (hardchem), (pelh)	5
PineSawn, SpruceSawn, BirchSawn, SpruceBoard, BirchBoard	3
(Pulpwood)	0.5
Electricity	0.2
Heat	0.1

**Table 9.** Products from the production processes. Bark is produced only when using roundwood as a free input and, therefore, it is presented in parenthesis.

Process	Products
News 1 & 2	News
Fine_co	Fine_co
Fine_uc	Fine_uc
Magaz_co	Magaz_co
Magaz_uc 1 & 2	Magaz_uc
Board_con	Board_con
Board_car	Board_car
Tissue	Tissue
softchem	softchem, bl, (bark)
hardchem	hardchem, bl, (bark)
mech	mech, heat, (bark)
recycled pulp	recycled pulp
PineSawn	PineSawn, chips, dust, drydust, bark
SpruceSawn	SpruceSawn, chips, dust, drydust, bark
BirchSawn	BirchSawn, chips, dust, drydust, bark
SpruceBoard	SpruceBoard, chips, dust, drydust, bark
BirchBoard	BirchBoard, chips, dust, drydust, bark
biofuel	biofuel, Fgas, heat
pelh	pelh, (bark)
pell	pell

ity from domestic hydro and nuclear plants is set price-invariant, but the levels are calibrated for each intra-annual sub-period. The electricity, roundwood and chips imports have price elasticity 0.5.

The production described in the model consists of processing and energy generation. The products and inputs of the modeled processes are presented in Tables 9 and 10, respectively. Table 11 gives the products and inputs of the modeled power plant technologies. From these tables it is directly observed that the sawmills play an important role in the model as they generate a large variety products and by-products (e.g. Mäkelä et al. 2011). Similarly, the pulp industry as well as the biofuel and pellet producers and the wood-firing power plants have several possible free inputs from which to choose. Therefore, this part of the model is responsible for a substantial share of computational capacity needed to run the model.

**Table 10.** Inputs used in production processes. Free inputs can be chosen from the set available for a given technology, whereas all the predetermined inputs are needed in production with all free inputs.

Process	Free inputs	Predetermined inputs
News 1	mech	softchem, electricity, heat
News 2	mech	softchem, recycled pulp, electricity, heat
Fine_co	hardchem	softchem, electricity, heat
Fine_uc	hardchem	softchem, electricity, heat
Magaz_co	mech	softchem, electricity, heat
Magaz_uc 1	mech	softchem, electricity, heat
Magaz_uc 2	mech	softchem, recycled pulp, electricity, heat
Board_con	hardchem	softchem, electricity, heat
Board_car	mech	softchem, hardchem, electricity, heat
Tissue	hardchem	electricity, heat
mech	chips, pulp, logs	electricity
softchem	chips, pulp, logs	electricity, heat
hardchem	chips, pulp, logs	electricity, heat
recycled pulp	recycled paper	electricity, heat
PineSawn	pulp, logs	electricity, heat
SpruceSawn	pulp, logs	electricity, heat
BirchSawn	logs	electricity, heat
SpruceBoard	logs	electricity, heat
BirchBoard	logs	electricity, heat
biofuel	bark, chips, drydust, dust, pulp, logs, res, small, stumps, pelh, pell	electricity
pelh	chips, drydust, dust, pulp, logs	electricity, heat
pell	bark, pulp, logs, res, small	electricity, heat

**Table 11.** Power plant types and their products and fuels.

Power plant type	Symbol	Product	Fuel
Gas turbine	gt_oil	el	gas, oil
Gas turbine (CHP)	gt	el, heat	gas, oil, FTgas
Gas turbine combined cycle	gtcc	el, heat	gas, FTgas
Pulverized fuel	pf	el	pelh, pell, coal
Pulverized fuel (peat fired)	pf_peat	el, heat	pelh, pell, peat
Pulverized fuel (CHP)	pf_heat	el, heat	pelh, pell, coal
Fluidized bed	fb	el, heat	res, small, pulp, logs, stumps, chips, dust, drydust, bark, pelh, pell, peat, coal
Fluidized bed (high wood share)	chp_wood	el, heat	res, small, pulp, logs, stumps, chips, dust, drydust, bark, pelh, pell, peat, coal
Heating plant (wood fired)	heat_wood	heat	res, small, pulp, logs, chips, dust, bark, pelh, pell, oil
Oil boiler	ob	heat	gas, oil
Recovery boiler	rec	el, heat	bl
Wind turbine	wind_turb	el	wind





luke.fi

Natural Resources Institute  
Finland  
Viikinkaari 4  
FI-00790 Helsinki, Finland  
tel. +358 29 532 6000