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ANAC'S degenerate mathematical model: a sensitivity analysis

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ABSTRACT

The Brazilian Civil Aviation Agency's (ANAC) surveillance missions have great relevance for the effectiveness of its performance. Hence, several studies have already aimed to optimize this process, and mathematical models are conceived for this purpose. However, some of these linear programming models have a degenerate structure, which compromises their sensitivity analysis related to the dual model and further analysis of the model's scenarios. Thus, the objective of this work is to present a case study consisting of ways to perform sensitivity analysis in ANAC's mathematical models with degenerate solutions. To that end, the method of sensitivity analysis proposed by Koltai and Tatay (2011) is applied in a mathematical model elaborated to assist in the designation of inspectors for surveillance missions in ANAC's Operational Standards Superintendence (SPO), as proposed by Pinheiro (2018). Finally, the objective was achieved and this article contributes to the Academy and the market by presenting a reference of how to perform a more assertive sensitivity analysis in a degenerate case.

***Keywords**: Linear Programming, Sensitivity Analysis, Degenerate, ANAC*

1. INTRODUCTION

The Brazilian Civil Aviation Agency's (ANAC) surveillance missions consist of the displacement of inspectors to carry out activities in locations in the various federative units of Brazil. Each mission has a specific demand, which requires qualified professionals and includes airfare costs and daily expenses. The country's recession scenario has been affecting the execution of these missions, as well as ANAC being subject to the budgetary contingencies imposed to most federal agencies. Budgetary contingency in this case consists of delays or even the non-execution of a part of the expenditure planning provided for in the Annual Budget Law (LOA) due to insufficient credit.

ANAC’s Fiscal Year Management Report 2016 to 2019 present the budgetary constraints, tax restrictions and LOA credits arrest as clear difficulties for the proper operational implementation of the Agency's budget. This situation generated restrictions in the processes of certification and inspection since they depend on that budget for inspector’s airfare and daily expenses in order to perform on site activities.

ANAC's Strategic Plan from 2020 to 2026 aims to promote the allocation of resources in a strategic and effective manner (OE13) due to the context of increasingly scarce resources. In addition, OE2 brings the importance of certification and inspection to ensure the perpetual maintenance of air transport security and OE12 says that it is necessary to manage people effectively. Thus, efforts are made to improve the mechanisms and procedures for managing resources, including human resources, in order to guarantee the effectiveness of ANAC's activities.

Another problem regarding ANAC's certification and inspection missions is the designation of inspectors. Some studies have investigated ways to optimize this process. The first of these, from Nascimento (2016), focused on improving the process for optimal and efficient distribution of ANAC’s inspections, performing the mapping AS IS (current situation) and TO BE (future situation that one aims to achieve) of this process. The following papers from Freitas Júnior (2017), Celestino, Reis and Júnior (2018), Reis and Celestino (2018), Pinheiro (2018) and Silva (2018), developed mathematical models to support the decision to allocate inspectors to ANAC´s surveillance missions.

In these last studies, when performing the sensitivity analysis of the model, it was verified that the results obtained were degenerates. When this happens, the results of sensitivity analysis obtained by the Dual process become uncertain and it is necessary to perform more calculations in order to obtain the correct information to support the decision-making. Therefore, in view of the importance and limitation of resources in the process of inspector designation and missions, this work aims to to exemplify how sensitivity analysis in ANAC’s previously developed mathematical models can be created in order to obtain valid results, given its degenerate characteristic.

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This research's choice of theme is justified by its relevance to the scientific world, since there are few studies that address sensitive analysis, especially in models with degeneration. This indicates that there is still much to be explored on the subject. Thus, this research seeks to contribute with a demonstration of the application of the Koltai and Tatay method (2011) of sensitivity analysis to the model formulated by Pinheiro (2018) in order to support the decision of a real problem at ANAC. Considering that this is a model with a degenerate solution, the application will provide a more accurate interpretation of the solutions and could be used in the future as a reference for decision makers who want to apply the method.

2. THEORETICAL BACKGROUND

2.1. Linear Programming and Duality

Linear Programming (LP) is a technique that uses mathematical models to deal with problems of allocation of limited resources between activities that compete with each other, as well as other problems that have a similar mathematical formulation (Hillier & Lieberman, 2013). LP models are formed, in general, by decision variables, parameters, objective function and constraints.

Decision making process can control decision variables, so when testing these variables' values one can find the solution of the problem. Parameters are model variables that cannot be controlled by the decision-maker, so it refers to fixed values that should be considered for the solution of the problem. The objective function represents the measure of performance to be maximized or minimized in the model. Constraints, on the other hand, represent the limitations or requirements in the set of possible decisions (Colin, 2013).

Another important feature of LP problems is their duality, that is, one can express every LP problem in two ways. The original problem is called primal and the problem associated with it, dual. "The properties of the primal are closely linked to those of the dual, and the optimal value of the objective function is the same for both forms" (Caixeta-Filho, 2011, p. 54). Thus, primal and dual share several associations. Table 1 presents some of the associations that one should consider when transforming primal into dual, or vice versa.

Table 1: Relations between primal and dual problems

Primal problem	Dual problem
Objective function of maximizing	Objective function of minimizing
Coefficients of the objective function	Constraint constants
Constraint constants	Coefficients of the objective function
Number of variables	Number of constraints
Number of constraints	Number of variables

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Constraint type ≤	Constraint type ≥
Constraint type ≥	Variable ≤ 0
Constraint type =	Variable without signal restriction

Source: Prepared by the author

According to Lachtermacher (2018), there are two reasons for studying dual problems. The first relates to the amount of constraints. When the primal has too many constraints it is easier to solve the problem by its dual, since the amount of constraints will be equal to the number of variables of the primal's objective function.

Therefore, there will probably be a smaller number of constraints, which makes it easier to find the optimal solution. The second refers to the economic interpretations obtained from the dual problem decision variables' values (also called shadow price or dual price). According to the Complementary Slack Theorem, the dual decision variables are associated with the primal slack/excess variables.

In general, they represent the value by which the objective function would change into if the resource quantity were to be reduced/increased one unit. To analyze this type of change, there is a procedure performed in the LP models that is called sensitivity analysis.

2.2. Sensitivity Analysis in Linear Programming

When solving LP problems, one assumes that all model parameters (independent terms, coefficients of the objective function and of constraint) are constant and known with certainty (Fávero & Belfiore, 2013). However, the application of the solution in the real world can generate changes in some parameters, causing uncertainty about the quality of the optimal solution. Thus, to minimize the inaccuracy regarding these changes, one performs an analysis to verify the possible up and down variations of the model's parameters values that do not cause alteration in the optimal solution (Lachtermacher, 2018). Such a study is called sensitivity analysis.

The main objectives of sensitivity analysis are: (1) to identify the sensitive parameters that change the optimal solution; and (2) determine the possible intervals of values for non-sensitive parameters along which the optimal solution will remain unchanged (Hillier & Lieberman, 2013). In this fashion, sensitivity analysis helps decision-makers evaluate how changes in the model and in the real world can affect the solution (Colin, 2013), in addition to identifying how much the solution is dependent on a particular constant or coefficient (Lachtermacher, 2018).

According to Lachtermacher (2018) and Fávero and Belfiore (2013), one can use sensitivity analysis for two distinct cases. The first, called sensitivity analysis, can be characterized by evaluating the possibilities of variations and influences in the optimal solution of a problem when there is only one change at a time; while the second, called post-optimization sensitivity analysis, evaluates when more than one change occurs simultaneously.

In addition, Colin (2013) presents that from the theoretical point of view sensitivity analysis can happen in: changes in the values of the objective function's coefficients; changes in the right sides of the constraints; changes in the constraints' coefficients (left sides) and introduction and removal of variables and constraints. In each of these cases, one performs sensitivity analysis in a certain way and uses indicators that present the possible changes in each of these factors, without a change in the optimal solution. These indicators are the following: reduced cost, shadow price, allowed increase and allowed reduction.

The reduced cost represents the cost of including a variable in the optimal solution in order that a variable's value ceases to equal zero in the optimal solution. Thus, it is observed that the reduced cost is a measure that can be calculated only for the non-basic variables of the model, that is, those that assume null values in the optimal solution. Therefore, for the basic variables, the reduced costs will always be zero (Fávero & Belfiore, 2013).

The shadow price, also called dual price, represents the equivalent unit value of a resource. This concept is observed mainly in sensitivity analyses based on changes in the value of one of the constants on the right side of the constraint, that is, when there are variations in the availability of resources. Therefore, the shadow price demonstrates the increase or decrease in the value of the objective function if one adds or withdraws a unit in the current amount of resources available in the constraint (Fávero & Belfiore, 2013).

Finally, sensitivity reports also show the lower and upper limits of decision variables, objective function coefficients and constraint constants. One can refer these limits as allowable decrease and allowable increase. They represent the lowest and largest values that decision variables, coefficients of the objective function and constraint constants can assume (considering that all other variables remain constant) as long as no constraint fails to be fulfilled causing the solution to become unfeasible (Lachtermacher, 2018).

The allowable decrease and allowable increase values can eventually equal zero. This fact can indicate two different situations. First, when it occurs for constraints, it means that the

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optimal solution is a degenerate, that is, the solution of the model has one or more basic variables with the value equal to zero. Secondly, when it occurs for the objective function's coefficients (there are no degenerate solutions) it means multiple optimal solutions exist, hence different values for the decision variables reach the same optimal value in the objective function.

In the presence of multiple optimal solutions, it is possible to calculate workarounds using sensitivity report information. However, the presence of optimal degenerate solutions impacts the interpretation of the sensitivity report in several ways ergo one should perform the analysis with ultimate care.

2.3. Degenerate Solution

Degeneration occurs when one or more basic variables equal zero. Thus, if the optimal solution for a LP has less than **m** positive variables, it is called a degenerate solution (Moore & Weatherford, 2005, TEA-4), **m** being the number of constraints. Normally, one can identify a degenerate solution sensitivity report when the allowable increase or allowable decrease for a shadow price is zero.

According to Taha (2008), from a practical point of view, a degenerate solution reveals that the model has at least one redundant constraint. From the theoretical point of view, degeneration has two implications, which are:

a) The phenomenon of cycling or cyclic return: solutions enter a sequence of changes that never improve the value of the objective function and never satisfy the optimality condition;

b) Interactions with different categorization of their variables as basic and non-basic result in identical values for the value of the objective function.

2.4. Interpretation of Sensitivity Analysis of a Degenerate Solution

When an optimal solution is a degenerate, the results obtained in sensitivity analysis are no longer reliable. Therefore, one needs to observe certain points at the time of their interpretation. According to Moore and Weatherford (2005), Ragsdale (2001, apud Lachtermacher, 2018), Ragsdale and Lachtermacher (2009, apud Belfiore & Fàvero, 2013) the following conclusions can be drawn in this situation:

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a) Reduced costs and/or shadow prices (and their intervals) are still valid, but may not be unique. Thus, it is possible that two different solutions are generated, presenting the same optimal values for the decision variables and values of the objective function, but with some or all of the reduced costs and/or shadow prices (and intervals) different;

b) The variation intervals (allowed increase and allowed decrease) of the coefficients of the objective function are still valid, but the coefficient can assume values outside this interval and still not change the optimal solution;

c) When a variation's interval (allowable increase and allowable decrease) of the coefficient in one of the variables of the objective function is also zero, the multiple optimal solutions' statement of occurrence is not reliable.

In general, one can observe that the results of sensitivity analysis become uncertain in the presence of optimal degenerate solutions. Thus, more calculations are needed to obtain the information necessary for management decision making (Jansen el al, 1997 apud Koltai & Tatay, 2011).

3. RESEARCH METHODOLOGY

3.1. Study Design

According to Fontelles, Simões, Farias and Fontelles (2009) this research is classified as applied, observational, quantitative, exploratory, documentary, transversal and retrospective.

3.2. Characterization of organization, population and sample

The object of this research was the Brazilian Civil Aviation Agency (ANAC), a Brazilian federal regulatory agency, responsible for the regulation and supervision of civil aviation and airport infrastructure activities.

Regarding its organizational structure, article 2 of the Internal Rules of ANAC (2016) states that the agency is composed of a Board of Directors and its Assistance Bodies (advisory, office, ombudsmen, internal affairs, attorney and internal audit), Specific Bodies (superintendence) and Collegiate Bodies (advisory council and plenary).

That said, the problem this research addresses will focus on the Superintendence of Operational Standards (SPO). In addition, the research's target relates to the macro-inspection process, particularly when applied within the superintendence. The superintendence, in its turn, according to Article 31 of the Internal Rules of ANAC (2016), is responsible for the following:

> "IX - carry out supervisory actions with regard to continued surveillance, which involves permanent monitoring of the activities of the regulated entities in order to guide them, maintain the risk of operations within an acceptable level of civil aviation safety and improve the provision of services to the passenger;" (ANAC, 2016, p. 20, independent translation)

Thus, this research studies the surveillance missions in which SPO is responsible for certifying and supervising the operational scope in order to ensure minimum standards of safety and efficiency regarding operational safety.

The population subject to this research is composed of federal public servants stationed in ANAC's Superintendence of Operational Standards (SPO). The sample consists of the servants that have attributions related to the SPO surveillance missions such as superintendence inspectors. This is a non-probabilistic sample for data's availability convenience, obtained from the records of ANAC's surveillance missions.

3.3. Research Instruments

The instruments used in this research will be the Superintendence of Operational Standards' mathematical model (SPO) proposed by Pinheiro (2018) and LP's additional problems method proposed by Koltai and Tatay (2011). Thus, the latter will be applied in Pinheiro's model in order to find the correct sensitivity analysis.

3.4. Mathematical Model of the Operational Standards Superintendence (SPO)

The mathematical model proposed by Pinheiro (2018) aims to optimize SPO's designation of inspectors for surveillance missions. It considers offering the origins and demands of the destinations to allocate inspectors in order to meet mission requirements and minimize air transport costs.

Therefore, the model's objective function (OF) aims to minimize the sum of the costs of displacement of inspectors for inspection missions, being subject to 16 sets of supply constraints (SC), 7 sets of demand constraints (DC) and 1 constraint of the designated inspector (DesigC), which represents the final allocated sum. Supply constraints indicate that the number of inspectors allocated must be less than or equal to the number of available inspectors per specific skill group (S) in each source. In addition, demand constraints indicate that the number

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of inspectors allocated must be equal to the required number of inspectors for each specialty group (G) at each destination.

The model's implementation included 16 origins, 69 targets, 7 specialty groups (G), 46 specific skill sets of inspectors (H) and 16 specific auxiliary skill sets (S). Thus, the resolution of SPO's mathematical model involved 68,448 variables and 18,404 constraints, resulting in the optimal designation of inspectors. However, when analyzing the sensitivity reports, it was observed that some supply and demand constraints had allowable increase intervals or allowable decrease equal to zero, which characterizes a degenerate solution.

3.5. Sensitivity Analysis Method through Additional LP Problems

The authors Koltai and Tatay (2011) present that the standard form of a primal linear programming problem, which is dual, is:

$$\text{Primal: Max } (c^T x) \qquad Ax \leq b \qquad c \geq 0 \qquad [1]$$

$$\text{Dual: Min } (b^T y) \qquad A^T y \geq c \qquad y \leq 0 \qquad [2]$$

Some additional LP problems need to be resolved to get actual values of shadow prices and sensitivity intervals. Figure 1 presents these problems.

	Maximal decrease	Maximal increase
Sensitivity analysis of objective function coefficients (OFC)	$\mathbf{A}^T\underline{\mathbf{y}} \geq \underline{\mathbf{c}} - \gamma_i \underline{\mathbf{e}}_i$ $\underline{\mathbf{b}}^T\underline{\mathbf{y}} = OF^* - \gamma_i x_i^*$ $\gamma_i \geq 0$ Max (γ_i); (3) Optimal solution: γ_i^-	$\mathbf{A}^T\underline{\mathbf{y}} \geq \underline{\mathbf{c}} + \gamma_i \underline{\mathbf{e}}_i$ $\underline{\mathbf{b}}^T\underline{\mathbf{y}} = OF^* + \gamma_i x_i^*$ $\gamma_i \geq 0$ Max (γ_i); (4) Optimal solution: γ_i^+
Sensitivity analysis of the left shadow price $(\delta < 0)$ (y_i^-)	$\mathbf{A}\underline{\mathbf{x}} \leq \underline{\mathbf{b}} + \delta \underline{\mathbf{e}}_j - \xi_j \underline{\mathbf{e}}_j$ $\underline{\mathbf{c}}^T\underline{\mathbf{x}} = OF^* - \xi_j y_j^*$ $\xi_j \geq 0$ Max (ξ_j) (5) Optimal solution: $n\xi_j^-$	$\mathbf{A}\underline{\mathbf{x}} \leq \underline{\mathbf{b}} + \delta \underline{\mathbf{e}}_j + \xi_j \underline{\mathbf{e}}_j$ $\underline{\mathbf{c}}^T\underline{\mathbf{x}} = OF^* + \xi_j y_j^*$ $\xi_j \geq 0$ Max (ξ_j) (6) Optimal solution: $n\xi_j^+$
Sensitivity analysis of the right shadow price $(\delta > 0)$ (y_j^+)	$\mathbf{A}\underline{\mathbf{x}} \leq \underline{\mathbf{b}} + \delta \underline{\mathbf{e}}_j - \xi_j \underline{\mathbf{e}}_j$ $\underline{\mathbf{c}}^T\underline{\mathbf{x}} = OF^* - \xi_j y_j^*$ $\xi_j \geq 0$ Max (ξ_j) (7) Optimal solution: $p\xi_j^-$	$\mathbf{A}\underline{\mathbf{x}} \leq \underline{\mathbf{b}} + \delta \underline{\mathbf{e}}_j + \xi_j \underline{\mathbf{e}}_j$ $\underline{\mathbf{c}}^T\underline{\mathbf{x}} = OF^* + \xi_j y_j^*$ $\xi_j \geq 0$ Max (ξ_j) (8) Optimal solution: $p\xi_j^+$

Figure 1 Additional LP problems
Source: Koltai and Tatay, (2011, p. 394.)

These additional LP problems find a parameter's maximum reduction value (how much must be subtracted from a parameter to obtain the lowest value of the validity interval) and

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maximum increase (when it must be added to the parameter to obtain the highest value of the validity interval). For each coefficient of the objective function, one must calculate (3) and (4) and for each constraint, one must calculate (5), (6), (7) and (8) to obtain the values of maximum reduction, maximum increase and shadow prices. Hence, for a LP problem with Variable I and J constraints, 2I+6J one should calculate additional LP problems to obtain the interval information for each objective function coefficient (OFC) and right-hand side (RHS) element of the original problem.

Summary of notations:

A	Matrix of coefficients with elements a_{ji} (j=1, ..., J; i=1, ..., I)
B	Right-hand side (RHS) vector with elements b_j (j=1, ..., J)
C	Objective function coefficient vector with elements c_i (i=1, ..., I)
x	Primal problem variable with elements x_i (i=1, ..., I)
x*	Primal problem optimal solution with elements x_i^* (i=1, ..., I)
y	Dual problem variable with elements y_j (j=1, ..., J)
y*	Dual problem optimal solution with elements y_j^* (j=1, ..., J)
OF*	Optimal value of the objective function
e_i	Unit vector with elements I and with $e_i = 1$ e $e_k = 0$ for all $k \neq i$.
e_j	Unit vector with elements J and with $e_j = 1$ e $e_k = 0$ for all $k \neq j$.
δ	Disturbance of a parameter on the right side (RHS)
y_j^-	Left shadow price of an element on the right side (RHS) b_j ($\delta < 0$)
y_j^+	Right shadow price of an element on the right side (RHS) b_j ($\delta > 0$)
γ_i	Change in the coefficient of the objective function c_i
γ_i^-	Viable reduction of the coefficient of the objective function c_i
γ_i^+	Viable increase in the coefficient of the objective function c_i
ξ_j	Right side element change (RHS) b_j
n ξ_j^-	Viable decrease in b_j belonging to the left shadow price

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n ξ_j^+ Viable increase in b_j belonging to the left shadow price

p ξ_j^- Viable decrease in b_j belonging to the right shadow price

p ξ_j^- Viable increase in b_j belonging to the right shadow price

4. RESULTS

4.1. Adaptation of the SPO Mathematical Model

As previously stated, according to authors Koltai and Tatay (2011), for a LP problem with I variables and J constraints, 2I+6J additional LP problems should be calculated to obtain the sensitivity interval information for each OFC and RHS element of the original problem, in addition to the shadow prices. According to the results presented by Pinheiro (2018), it would take a total of 247320 (2*68448 + 6*18404) additional LP problems to find the correct information of the sensitivity analysis model.

Due to the large number of additional problems one has to solve, an alternative to decrease the number of problems would be to follow the suggestions put forward by authors Koltai and Tatay (2011) in order to exclude problems that do not need to be resolved. The first suggestion refers to mathematical analysis, which says that when right and left shadow prices are equal, there are only two additional problems one needs to solve (for maximum decrease and increase).

Therefore, to find the constraints with this feature, one should observe the validity interval of the shadow price in the sensitivity reports available to LP solvers. If the maximum decrease and maximum increase are other than zero the shadow prices on the right and left will be identical. The second suggestion refers to management analysis, in which the choice of additional LP problems to be calculated depends on the variables and constraints that the manager wants to better analyze before making a decision.

Therefrom, in order to exemplify the method of sensitivity analysis through additional LP problems, one solved them by considering only two origins, two destinations, a specialty group, a specific skill group and a specific skill auxiliary group. Some data from Scenario 1 used by Pinheiro (2018) were applied for validation testing of the model, which are:

a) Origins (**i**): Sao Paulo SBSP Airport (SP) and Confins SBCF Airport (MG);

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b) Destinations (**j**): SBFL Airport of Florianopolis (SC) and SBUL airport of Uberlandia (MG);

c) Specialty group (**g**): A;

d) Specific skill group(**h**): 1A;

e) Specific skill auxiliary group (**s**): 1.

In addition, Tables 2, 3 and 4 show the matrix of flight ticket costs between origins and destinations, the supply of inspectors in each origin and the demand of inspectors in each destination, respectively. Regarding the supply, one considered that only 30% of the inspectors of each origin would be made available to meet the demands of these missions in order to make the model degenerate.

Table 2: Airline ticket cost matrix

Origin / Destination	SBFL	SBUL
SBSP	R$ 231	R$ 197
SBCF	R$ 345	R$ 483

Source: Prepared by the author

Table 3 Annual supply of inspectors

Airport	Total Supply - Group 1	Available Supply - Group 1
SBSP	20	6
SBCF	20	6

Source: Prepared by the author

Table 4 Annual demand for planned inspections

Airport	Total Demand - Group A
SBFL	10
SBUL	2

Source: Prepared by the author

Considering that data, one can express Pinheiro's (2018) model adaptation by the following equations:

Objective Function:

$$MIN\ 231\ Allocated_{SPSP,SPFL,1A} + 197\ Allocated_{SPSP,SPUL,1A} \quad [3]$$

$$+345\ Allocated_{SPCF,SPFL,1A} + 483\ Allocated_{SPCF,SPUL,1A}$$

Supply Constraints:

$$Allocated_{SPSP,SPFL,1A} + Allocated_{SPSP,SPUL,1A} \leq Offer_{SPSP,1} \quad [4]$$

$$Allocated_{SPCF,SPFL,1A} + Allocated_{SPCF,SPUL,1A} \leq Offer_{SPCF,1} \quad [5]$$

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Demand Constraints:

$$Allocated_{SPSP,SPFL,1A} + Allocated_{SPCF,SPFL,1A} \geq Demand_{SPFL,A} \quad [6]$$

$$Allocated_{SPSP,SPUL,1A} + Allocated_{SPCF,SPUL,1A} \geq Demand_{SPUL,A} \quad [7]$$

4.2. Calculation of Additional LP Problems for the Adapted SPO Model

To calculate the additional LP problems of Pinheiro's adapted model, one first had to solve the primal and dual problems of the model. Consequently, both primal and dual problems, the optimal value of the objective function (OF*) and the optimal values of the decision variables were obtained (x_i* e y_j*):

$$\text{OF*} = 3.388 \quad [8]$$

$$x_1^* = 4, x_2^* = 2, x_3^* = 6, x_4^* = 0 \quad [9]$$

$$y_1^* = -114, y_2^* = 0, y_3^* = 345, y_4^* = 311 \quad [10]$$

According to Moore and Weatherford (2005), when the number of positive variables in the optimal solution is lower than the number of constraints, the solution is degenerate. Accordingly, since SPO's adapted model has four constraints it should also have four basic variables (i.e., different from zero). However, it only has three, which characterizes it as degenerate. For this reason, presented below are the calculations to obtain the correct values of the sensitivity analysis for the coefficients of the objective function (OFC) and for the constraints (RHS).

4.3. Additional LP Issues for OFCs

After resolving the primal and dual, with the matrices values ($A^T y, c, b^T y$) of the dual problem, the value of OF* and of x_i^*, the additional LP problems were calculated for the OFC of Pinheiro's adapted model, as shown in Figure 2.

	Maximal decrease	Maximal increase
Sensitivity analysis of objective function coefficients (OFC)	$\mathbf{A}^T\underline{\mathbf{y}} \geq \underline{\mathbf{c}} - \gamma_i \underline{\mathbf{e}}_i$ $\underline{\mathbf{b}}^T\underline{\mathbf{y}} = OF^* - \gamma_i x_i^*$ $\gamma_i \geq 0$ Max (γ_i); (3) Optimal solution: γ_i^-	$\mathbf{A}^T\underline{\mathbf{y}} \geq \underline{\mathbf{c}} + \gamma_i \underline{\mathbf{e}}_i$ $\underline{\mathbf{b}}^T\underline{\mathbf{y}} = OF^* + \gamma_i x_i^*$ $\gamma_i \geq 0$ Max (γ_i); (4) Optimal solution: γ_i^+

Figure 2 Additional LP problems for OFCs
Source: Koltai and Tatay (2011)

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It can be seen in Figure 2 that the problems consider an e_i. This e_i representes a unit vector that takes the value of 1 when "i" is the same as the one used for the calculation and 0 when it is different. For instance, in the model adapted from the SPO, i = (1, 2, 3, 4). When calculating the intervals (maximal decrease and maximal increase) for i = 1, $e_1 = 1, e_2 = 0, e_3 = 0$ and $e_4 = 0$. It is also observed that the objective function of the problems is to maximize the value of γ_i. The γ_i represents the most one can change the parameter so that the optimal solution of the variable remains the same. In this way, the maximal decrease and increase were calculated for each OFC of the model adapted from the SPO, resulting in a total of eight (2*4) additional problems. The sensitivity results obtained by Koltai and Tatay method (2011) are presented in Table 5.

Table 5 OFC sensitivity results

Koltai and Tatay Method (2011)				
i	**Variable (x_i)**	**Coefficient (OFC)**	**Allowable Decrease (γ_i^-)**	**Allowable Increase (γ_i^+)**
1	SPSP, SBFL, 1ª	231	172	**0**
2	SPSP, SPUL, 1ª	197	**0**	**172**
3	SPCF, SBFL, 1ª	345	**0**	**172**
4	SBCF, SBUL, 1ª	483	**172**	**0**

Source: Prepared by the author

In Table 5, the highlights in gray values refer to those in which the model does not present a solution or presents infinite solutions, according to the results found with Excel Solver. For allowable decreases in x_2 e x_3 and the allowable increases of x_1 e x_4, Solver results showed that it was not possible to find a viable solution for the set of constraints presented. For the allowable decrease of x_4 and the allowable increases of x_2 e x_3, Solver results showed that the 'Define cell' values did not converge, that is, with each iteration it found a different value that improved the objective function, so it found himself far from a final value. Because of these errors, these results are not reliable.

One could solve the primal problem due to the errors. Additionally, in order to test the validity of the intervals obtained by the Koltai and Tatay method (2011) one could solve it by reducing and increasing the OFCs based on the intervals found, with the intention of finding the real limits. The results obtained are shown in Table 6.

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Table 6 Final sensitivity results of OFC

Actual limits obtained after testing the intervals				
i	**Variable (x_i)**	**Coefficient (OFC)**	**Allowable Decrease (γ_i^-)**	**Allowable Increase (γ_i^+)**
1	SPSP, SBFL, 1ª	231	172	∞
2	SPSP, SPUL, 1A	197	∞	172
3	SPCF, SBFL, 1A	345	∞	172
4	SBCF, SBUL, 1A	483	**171**	∞

Source: Prepared by the author

Table 6 shows the actual sensitivity limits of Pinheiro's adapted model coefficients, that is, even if each OFC vary within the interval presented the optimal solution of the variable will not be changed. However, this is only true if one OFC is changed and the others remain constant. When comparing the results shown in Tables 5 and 6, one observes that the allowable decrease $\gamma_2^-, \gamma_3^-\ e\ \gamma_4^-$ and the allowable increase $\gamma_1^+\ e\ \gamma_4^+$ have changed (highlighted in gray). These values are the same as those that presented an error in to the results found with Excel Solver. That said, Koltai and Tatay method (2011) proved effective in finding the correct sensitivity intervals of OFCs.

4.4. Additional LP Issues for RHS

After calculating the sensitivity intervals of the OFCs, the additional LP problems were calculated for Pinheiro's adapted model RHS, according to Figure 3. For that, the matrix values $(Ax, b, c^T x)$ of the primal problem, the value of OF* and the y_i^* were used.

	Maximal decrease	Maximal increase
Sensitivity analysis of the left shadow price ($\delta < 0$) (y_i^-)	$\underline{\mathbf{A}}\underline{\mathbf{x}} \le \underline{\mathbf{b}} + \delta\underline{\mathbf{e}}_j - \xi_j\underline{\mathbf{e}}_j$ $\underline{\mathbf{c}}^T\underline{\mathbf{x}} = OF^* - \xi_j y_j^*$ $\xi_j \ge 0$ Max (ξ_j) (5) Optimal solution: $n\xi_j^-$	$\underline{\mathbf{A}}\underline{\mathbf{x}} \le \underline{\mathbf{b}} + \delta\underline{\mathbf{e}}_j + \xi_j\underline{\mathbf{e}}_j$ $\underline{\mathbf{c}}^T\underline{\mathbf{x}} = OF^* + \xi_j y_j^*$ $\xi_j \ge 0$ Max (ξ_j) (6) Optimal solution: $n\xi_j^+$
Sensitivity analysis of the right shadow price ($\delta > 0$) (y_j^+)	$\underline{\mathbf{A}}\underline{\mathbf{x}} \le \underline{\mathbf{b}} + \delta\underline{\mathbf{e}}_j - \xi_j\underline{\mathbf{e}}_j$ $\underline{\mathbf{c}}^T\underline{\mathbf{x}} = OF^* - \xi_j y_j^*$ $\xi_j \ge 0$ Max (ξ_j) (7) Optimal solution: $p\xi_j^-$	$\underline{\mathbf{A}}\underline{\mathbf{x}} \le \underline{\mathbf{b}} + \delta\underline{\mathbf{e}}_j + \xi_j\underline{\mathbf{e}}_j$ $\underline{\mathbf{c}}^T\underline{\mathbf{x}} = OF^* + \xi_j y_j^*$ $\xi_j \ge 0$ Max (ξ_j) (8) Optimal solution: $p\xi_j^+$

Figure 3 Additional PL problems for RHS
Source: Koltai and Tatay, (2011)

One notices that the problems presented in Figure 3 consider a disturbance $\delta < 0\ e\ \delta > 0$ in order to observe how the model behaves if its parameters suffer a small reduction or a small increase. Thus, for Pinheiro's adapted model the disturbances -1 and 1 were considered.

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However, before calculating the sensitivity intervals of RHS, the dual problem was used to recalculate each shadow price, considering these very disturbances in order to find the right shadow prices (y_j^-) and left (y_j^+). Hence, for each y_j the dual was solved in consideration of $b_j - 1\ e\ b_j + 1$, performing a total of eight (2*4) calculations, with the following values of y_j^- e y_j^+:

$$y_1^- = y_1^+ = -114 \quad [11]$$

$$y_2^- = y_2^+ = 0 \quad [12]$$

$$y_3^- = y_3^+ = 345 \quad [13]$$

$$y_4^- = y_4^+ = 311 \quad [14]$$

It is observed that the results obtained for all y_j^- e y_j^+ were equal to the optimal values that had already been found in the first resolution of the dual. Thus, in the case of Pinheiro's adapted model the shadow prices for both decrease and increase are the same, so one needs to solve only two additional LP problems to find the sensitivity intervals, since the results will be the same. If the shadow prices were different, four additional LP problems would have to be calculated, and problems (5) and (6) would consider the value of y_j^- and the disturbance $\delta < 0$, and problems (7) and (8) would consider the value of y_j^+ and the disturbance $\delta > 0$.

Therefrom, according to Figure 3, the problems consider a e_j, which is a unit vector equal to that used in OFC problems. Additionally, the objective function is to maximize the value of ε_j, which represents the most the parameter can be changed so that the shadow price related to that constraint remains the same. Thereon, the maximum decrease and increase values were calculated for each RHS, and the total number of 8 (2*4) additional problems. The sensitivity results obtained by Koltai and Tatay method (2011) are presented in Table 7.

Table 7 RHS sensitivity results

Koltai and Tatay Method (2011)					
j	**Variable (y_j)**	**Constraint (RHS)**	**Shadow price (y_j^-)**	**Allowable Decrease ($n\varepsilon_j^-$)**	**Allowable Increase ($n\varepsilon_j^+$)**
1	y_1	6	-114	0	6
2	y_2	6	0	0	**0**
3	y_3	10	345	6	0
4	y_4	12	311	2	0

Source: Prepared by the author

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In Table 7, in light of the allowable increase value y_2, highlighted in gray, Solver result showed that the 'Define cell' values did not converge. Thus, as seen in some OFC sensitivity interval values, one cannot rely on these results.

To solve this error using Koltai and Tatay (2011) method, one could reduce and increase the OFCs (which are the RHS of the primal problem, according to the relationship between the primal and the dual) to solve the dual problem based on the intervals found, with the intention of finding the real limits. The results obtained are shown in Table 8.

Table 8 Final sensitivity results of the RHS

Actual limits obtained after testing the intervals					
j	**Variable (y_j)**	**Constraint (RHS)**	**Shadow price (y_j^-)**	**Allowable Decrease ($n\varepsilon_j^-$)**	**Allowable Increase ($n\varepsilon_j^+$)**
1	y_1	6	-114	0	**5**
2	y_2	6	0	0	∞
3	y_3	10	345	**5**	0
4	y_4	12	311	2	0

Source: Prepared by the author

Table 8 shows the constraints' real sensitivity limits of Pinheiro's adapted model. These results show the interval in which each RHS can vary without changing the shadow price related to it. However, this is only true if one RHS changes and the others remain constant.

In comparison to the results shown in Tables 7 and 8, one can observe that only the allowable increases $n\varepsilon_1^+$ e $n\varepsilon_2^+$ and the allowable decrease $n\varepsilon_3^-$ changed. Since $n\varepsilon_2^+$ presented an error in Solver's solution, it was possibly to find its true value by testing it. On the other hand, the Koltai and Tatay method (2011) found the value of six for both $n\varepsilon_1^+$ and $n\varepsilon_3^-$, but when one added or deducted the same amount (6) iny$_1$ and y_3, respectively, different shadow prices were found.

Regarding y_1, by increasing its OFC by six and solving the problem more than once, Solver returned the optimal value of y_1 as -114 and 0, with the same optimal solution of R$ 2704. Similarly, reducing the OFC of y_3 by six and solving the problem more than once, Solver returned the optimal value of y_3 as 345 and 231, with the same optimal solution of R$ 1318. This indicates that with these changes the additional problems of y_1 e y_3 presented multiple optimal solutions, i.e., different values for the decision variables reached the same optimal value in the objective function.

Because of the multiple solutions, it was more reliable to change the sensitivity limits $n\varepsilon_1^+$ and $n\varepsilon_3^-$ to five, as it guarantees the shadow price will remain the same with the parameter

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change, though the result of the Koltai and Tatay (2011) method was not incorrect. Thus, the Koltai and Tatay method (2011) was also effective in finding the valid sensitivity intervals of RHS.

5. DISCUSSION

The analysis of the results made it possible to observe some of the ways to identify degenerate solutions presented in the Theoretical Background, such as:

a) The optimal solution had a smaller number of positive variables than the number of restrictions;

b) The permitted reduction and permitted increase values for some restrictions and shadow prices were equal to zero.

These characteristics confirm the fact that the Pinheiro's adapted model presents degenerate solutions. Therefore, the correct sensitivity analysis of the model was found through the application of the Koltai and Tatay method (2011) along with small corrections. Hence, the final sensitivity results of the OFC and RHS obtained in this study and the results presented by the Excel Solver sensitivity report is compared in Tables 9 and 10 below, in order to identify the differences and possible problems that could occur if the manager did not pay attention to this.

Table 9 Comparison of OFC sensitivity results

i	Variable (x_i)	Coefficient (OFC)	Actual limits obtained after testing the intervals		Excel Solver	
			Allowable decrease(γ_i^-)	Allowable increase (γ_i^+)	Allowable decrease	Allowable increase
1	SPSP, SBFL, 1ª	231	172	∞	172	**114**
2	SPSP, SPUL, 1A	197	∞	172	∞	172
3	SPCF, SBFL, 1A	345	∞	172	**114**	172
4	SBCF, SBUL, 1A	483	**171**	∞	**172**	∞

Source: Prepared by the author

Table 9 values highlighted in gray represent those that presented differences. Additionally, Solver presented a report with three incorrect interval values for three different variables. Regarding x_1 e x_3, when using the values of +114 or –114, the optimal solution is not affected, for the actual value of the limit is infinite. However, the decisions would be limited to these values where it could have considered higher values, which would also maintain the optimal solution. On the other hand, as to x_4, if the Manager used the value –172, the optimal

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solution would be affected. In that event, the Manager could make a non-optimal decision by not noticing the problem.

Table 10 Comparison of RHS sensitivity results

j	Variable (y_j)	Constraint (RHS)	Actual limits obtained after testing the intervals			Solver do Excel		
			Shadow price (y_j^-)	Allowable decrease ($n\varepsilon_j^-$)	Allowable increase ($n\varepsilon_j^+$)	Shadow price	Allowable decrease	Allowable increase
1	y_1	6	-114	0	**5**	-114	0	**6**
2	y_2	6	0	0	∞	0	0	∞
3	y_3	10	345	**5**	0	345	**6**	0
4	y_4	12	311	2	0	311	2	0

Source: Prepared by the author

Table 10 values highlighted in gray represent those that presented differences and only the values of $n\varepsilon_1^+$ e $n\varepsilon_3^-$ changed. These values were equivalent to 6 in the result obtained by the Koltai and Tatay method (2011 and they have been set to 5 since the result generated multiple solutions. Consequently, the results would be the same should the change be disregarded and the sensitivity report made available by Solver for the constraints did not present errors in this case. Therefore, if the Manager had used it, he would have had no problems.

Finally, one could realize that 24 calculations were performed to obtain the correct sensitivity analysis values for Pinheiro's adapted. This value is lower than predicted, as Koltai and Tatay (2011) stated that for a problem of LP with I variables and J constraints, 2I+6J additional problems should be calculated. Accordingly, for the case of SPO that has 4 variables and 4 constraints, 32 (2*4+6*4) additional problems should have been calculated. The reason for this difference was due to left and right shadow prices values being equal. However, contrarily to what presented the authors, this was only observed later on, in detriment of it being detected in Solver's sensitivity report as to the shadow prices validity intervals.

6. CONCLUSION AND RECOMMENDATION

This research sought to exemplify how sensitivity analysis can be performed in the mathematical models of ANAC, given its degenerate characteristic. One can conclude that the objective was achieved by applying the Koltai and Tatay method (2011) in the ANAC model proposed by Pinheiro (2018).

This case study allowed the application of technical knowledge to a real problem in a public organization, making it possible to evidence errors in sensitivity analysis that could cause problems in an inattentive decision-making process. In addition, by applying the method, it is possible to verify its technical and practical feasibility, confirming its usage in other models that suffer from degeneration, due to its generic formulation. Thereupon, it contributes both to the Academy and to the market by gathering studies and applications on the subject, serving as a reference for future studies or for professionals who are dealing with degenerate problems and need to perform a more assertive sensitivity analysis.

However, one difficulty refers to the amount of additional calculations needed to validate sensitivity analysis and its non-automated nature. One can observe that though the model used in the study was small it still took more than twenty additional problems to determine the sensitivity intervals. Therefore, in a larger manually performed model the application of the same process would be very long and more exposed to errors. Thus, it would be ideal that the Software provides all types of sensitivity analysis. Since commercial packages still do not implement it, the main suggestion is to include ways to automate the sensitivity analysis in future works.

Finally, for similar studies, there should be a focus on sensitivity analysis under degeneration in which more than one parameter changes simultaneously. This is because the analyzed method considers one change at a time for decision makers deal with changes in several variables at the same time in real situations.

REFERENCES

ANAC (2020). **Plano estratégico – 2020-2026** [Strategic plan – 2020-2026], ANAC, Brasília.

ANAC (2017). **Relatório de Gestão do Exercício de 2016** [2016 Fiscal Year Management Report], ANAC, Brasília. Available: https://www.anac.gov.br/acesso-a-informacao/auditorias/processos-de-contas-anuais-exercicio-2016. Acess:21/04/2021.

ANAC (2018). **Relatório de Gestão do Exercício de 2017** [2017 Fiscal Year Management Report], ANAC, Brasília. Available: https://www.anac.gov.br/acesso-a-informacao/auditorias/processos-de-contas-anuais-exercicio-2017. Acess:21/04/2021.

ANAC (2019). **Relatório de Gestão do Exercício de 2018** [2018 Fiscal Year Management Report], ANAC, Brasília. Available: https://www.anac.gov.br/acesso-a-informacao/auditorias/processos-de-contas-anuais-exercicio-2018. Acess:21/04/2021.

ANAC (2020). **Relatório de Gestão do Exercício de 2019** [2019 Fiscal Year Management Report], ANAC, Brasília. Available: https://www.anac.gov.br/acesso-a-informacao/auditorias/processos-de-contas-anuais-exercicio-2019. Acess:21/04/2021.

Caixeta-Filho, J. V. (2011). **Pesquisa Operacional**: Técnicas de Otimização Aplicadas a Sistemas Agroindustriais [Operational Research: Optimization Techniques Applied to Agroindustrial Systems], 2 ed. São Paulo: Atlas.

Celestino, V. R. R., Reis, S. A. D., & Júnior, L. S. P. D. F. (2018). **Modelo Matemático para Alocação de Inspetores**: Um Estudo de Caso na Agência Nacional de Aviação Civil [Mathematical Model for Allocation of Inspectors: A Case Study at the National Civil Aviation Agency]. In: CONGRESSO DE GESTÃO DE OPERAÇÕES EM ORGANIZAÇÕES PÚBLICAS, Brasília, 21-22 May. Brasília: GOOP,2018.

Colin, E. C. (2013). **Pesquisa Operacional**: 170 aplicações em estratégia, finanças, logística, produção, marketing e vendas [Operational Research: 170 applications in strategy, finance, logistics, production, marketing and sales]. Rio de Janeiro: Ltda.

Fávero, L. P., & Belfiore, P. (2013). **Pesquisa operacional para concursos de engenharia** [Operational research for engineering contests]. Rio de Janeiro: CIP-Brasil.

Fontelles, M. J., Simões, M. G., Farias, S. H., & Fontelles, R. G. S. (2009). **Metodologia da Pesquisa Científica**: Diretrizes para a Elaboração de um Protocolo de Pesquisa [Scientific Research Methodology: Guidelines For Elaboration Of A Research Protocol]. Work carried out at the Center for Biostatistics Applied to research at the University of the Amazon - UNAMA. Belém: University of the Amazon.

Freitas Júnior, S. P. (2017). **Modelo de transbordo para problemas de designação de inspetores**: Um estudo de caso na Agência Nacional de Aviação Civil [Transhipment model for inspector assignment problems: A case study at the National Civil Aviation Agency]. Dissertation (Graduation in Administration). Brasília: University of Brasilia, Available: https://bdm.unb.br/handle/10483/18936. Acess:21/04/2021.

Hillier, F. S., & Lieberman, G. J. (2013). **Introdução à Pesquisa Operacional** [Introduction to Operational Research], 9 ed. Porto Alegre: AMGH.

Koltai, T., & Tatay, V. (2011). A practical approach to sensitivity analysis in linear programming under degeneracy for management decision making. **International Journal of Production Economics**, 131(1), 392-398. DOI: 10.1016/j.ijpe.2010.04.037.

Lachtermacher, G. (2018). **Pesquisa Operacional na Tomada de Decisões** [Operational Research in Decision Making], 5 ed. Rio de Janeiro: CIP-BRASIL.

Moore, J. H., & Weatherford, L. R (2005). **Tomada de Decisão em Administração com Planilhas Eletrônicas** [Decision Making in Administration with Spreadsheets], 6 ed. Porto Alegre: Bookman.

Nascimento, V. P. (2016). **Projeto de Aplicação de Gestão de Processos – Otimização na ANAC/SAR na Certificação e Supervisão em Aviação Civil**: do Pedido à Certificação, Pesquisa e Resultado, Diretrizes para Parâmetros [Process Management Application Project - Optimization at ANAC/SAR in Civil Aviation Certification and Supervision: from Order to Certification, Research and Results, Guidelines for Parameters]. Dissertation (Specialization). Niterói: Getúlio Vargas Foundation.

Pinheiro, R. S. (2018). **Modelo matemático para apoio a decisão no processo de designação de inspetores em missões de fiscalização na ANAC** [Mathematical model to support the decision in the process of appointing inspectors on inspection missions at ANAC]. Dissertation (Graduation in Administration). Brasília: University of Brasilia, Available: https://bdm.unb.br/handle/10483/24560. Acess: 21/04/2021.

Reis, S. A., & Celestino, V. R. R. (2018). **Um modelo matemático para alocação de pessoas na gestão da capacidade em serviços públicos** [A mathematical model for allocating people to capacity management in public services]. In: ANAIS DO SIMPÓSIO BRASILEIRO DE

http://www.ijmp.jor.br v. 13, n. 5, May - July 2022
ISSN: 2236-269X
DOI: 10.14807/ijmp.v13i5.1701

ENGENHARIA DE PRODUÇÃO, 15, Bauru, 7-9 November. Bauru: SIMPEP, 2018.

Silva, J. V. N. (2018). **Modelo de designação para administração pública**: um caso sobre a Superintendência de Aeronavegabilidade da Agência Nacional de Aviação Civil [Designation model for public administration: a case about the Airworthiness Superintendence of the National Civil Aviation Agency]. Dissertation (Graduation in Administration). Brasília: University of Brasilia.

Taha, H. A. (2008). **Pesquisa Operacional**: Uma visão Geral [Operational Research: An Overview], 8 ed. São Paulo: Prentice Hall.