Compensation and responsibility

Marc Fleurbaey, François Maniquet

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March 1999; revised August 2005

1 Introduction

Many distributive issues involve situations of unequal endowments that call for compensating transfers. When the endowments themselves are transferable, things are relatively simple, at least in the first best context, because transfers can remove the initial inequality. But in other situations endowments are not transferable, or are only transferable at a prohibitive cost, because of the available technology or because of prevailing moral constraints. For instance, between individuals, features of human capital like education, social background, genetic endowment, health or bodily characteristics are seldom considered as being transferable. Between regions or countries features like natural environment, landscape, population characteristics, even industry, are, in many decision contexts, not modifiable. In such cases, transfers will not remove all the inequalities but will compensate endowment inequality by a countervailing resource inequality. Sometimes things are still simple because there is an obvious index of performance which measures how much of compensation is achieved. For instance, if equality of weight between racers is sought, it is easy to put artificial weight on lighter racers so as to achieve the desired equality, even though transferring weight is in itself impossible. If equality of welfare between individuals is the social objective, and there is an accepted measure of welfare with interpersonal comparability, then it is relatively easy to devise transfers of resources in the direction of such equality, and, if full inequality is not possible because initial inequalities are too great (or because of second-best impediments), resorting to the maximin criterion seems reasonable.

In many contexts such an undisputed index of compensation is not available. There may be for instance different evaluations of the unequal endowments, leading to a preference aggregation problem. Moreover, it is often the case that among initial characteristics

*We thank K. Suzumura for helpful advice in the preparation of this chapter and detailed comments on a first draft, and L. Kranich for helpful suggestions. We are also grateful to R. Arneson, R. Gary-Bobo, F. Gaspart, M. Le Breton, H. Moulin, E. Ooghe, J. Roemer, E. Schokkaert, Y. Sprumont, K. Tadenuma, W. Thomson, A. Trannoy, B. Tungodden, P. Vallentyne, D. Van de gaer and N. Yoshihara for comments or discussions, and to participants at a Summer School in Namur.

†CATT, IDEP, Université de Pau et des Pays de l'Adour, France. Email: marc.fleurbaey@univ-pau.fr.
‡FNRS, GREBE, FUNDP Namur, and CORE. Email: francois.maniquet@fundp.ac.be.

that make individuals (or groups, regions, countries, etc.) unequal, some do not call for compensation, in view of the prevailing moral sentiment. For instance, a non-negligible part of welfare inequalities comes from bodily beauty and attractiveness, although it is seldom argued that such inequalities should induce transfers. Whatever the reason (e.g. organizing transfers on this basis might be damaging to self-respect), this entails that individuals bear the consequences of such inequalities. Similarly, the exercise of responsibility is often thought to generate legitimate inequalities. The classical example of expensive tastes (Arrow 1973, Dworkin 1981a) is a case in point. An expensive taste for rare food or extravagant activities is a kind of handicap in view of achieving happiness, although it may be argued that such a handicap is less of an urgency than bodily disability, because individuals must somehow assume responsibility for their goals. When the disputed characteristic is fully chosen by individuals, it is also often argued that they should bear the consequences, even disregarding the obvious underlying moral hazard issue. Similar considerations arise in the case of other entities like regions or countries. It may sometimes be defended that federal or national subsidies to a region should not cater to inequalities due to consumption patterns, pollution behavior, religious practices, etc. In all such cases, the scale of compensation for illegitimate inequalities is difficult to assess because the individual index of performance typically mixes the effects of the various characteristics, those that justify compensation and those that do not.

The literature under review in this chapter deals with this problem of compensation. The origins of this literature can be traced back to four different sources. First, the utilitarian approach to this issue has been studied by Arrow (1971) and Sen (1973), who pointed out the paradox that a handicapped individual is likely to have a lower marginal utility and therefore, by application of the utilitarian criterion, to receive less resources than a better-off individual. As an antidote to this unappealing consequence, Sen proposed the "*Weak Equity Axiom*", requiring that when an individual has a lower level of utility than another at all levels of income, the optimal allocation must not give less income to him than to the other.

Second, the theory of social choice, as early as in Arrow's seminal monograph (Arrow 1963), has called attention on the issue of inequalities due to personal characteristics, and the notion of extended sympathy was coined to make it possible to evaluate such inequalities. But it did not clearly identify tools to deal with the separation between characteristics that call for compensation and other characteristics, and that was indeed a difficult exercise in the purely abstract framework of social choice, in which transfers of resources are not even explicitly described.

The third origin of the literature presented here is the theory of equity, which was formulated in economic models and therefore was equipped with more precise concepts to evaluate inequalities. In Kolm (1972), a general approach to the assessment of equity based on partial sets of individual characteristics was already proposed, with a full range of criteria going from the most restrictive, equity as No-Envy on resources, to the most comprehensive, justice as equal utility. But this first step was largely ignored, although the related issue of compensating for productivity handicaps was raised by Pazner and Schmeidler (1974) in the model of production with unequal skills, and their negative result about the general existence of efficient and envy-free allocations triggered some research effort, but the resulting literature simply tried to define weaker notions of equity, without relating this difficulty to the general compensation problem.

The fourth source of inspiration was the Rawlsian tradition in political philosophy, initiated by Rawls (1971) and developed by Dworkin (1981a,b), Sen (1985), Arneson (1989), Cohen (1989) and van Parijs (1995), and also commented upon by Roemer (1985a, 1986). This philosophical literature gave a strong endorsement to egalitarian goals, but made the point that it may be reasonable to deny compensation for some welfare deficits, laying the stress on the importance of compensating individuals only for handicaps for which they should not be deemed responsible.

We now briefly summarize the main insights obtained in the more recent theory of compensation, and introduce some basic concepts. One first contribution of the literature reviewed here is a distinction of several relevant ethical issues in the problem of compensation. At first glance, it may appear that the goal of compensating for handicaps, or equalizing opportunities, is a simple goal. But consider the following example.

Example 1. Suppose that individual welfare is determined by the formula

$$u = (x + y)\, z,$$

where x, y and z are real numbers. The quantity x denotes an external resource allocated by redistributive policies, y is social background, and z is a measure of personal effort and merit. Assume that y and z are independently distributed, and that each of these two variables takes on the values 1 and 3 in equal proportions. The population is then partitioned into four subgroups of equal size, as depicted in the following table:

$y \setminus z$	1	3
1	the "undeserving poor"	the "deserving poor"
3	the "undeserving rich"	the "deserving rich"

We assume that the *per capita* amount of available external resource is 4, and that it can be freely allocated among the four subgroups. Let us consider that the goal of redistributive policy is to compensate for inequalities in social background y, whereas inequalities due to personal merit z are deemed acceptable. What kind of policy is adequate in this perspective? A natural candidate is to seek equality of $x + y$ across all individuals. This is achieved by policy A described in the following table:

Policy A

$y \setminus z$	1	3
1	$x = 5$ $u = 6$	$x = 5$ $u = 18$
3	$x = 3$ $u = 6$	$x = 3$ $u = 18$

Assuming that individuals are in control of variable z, one can consider in this case that equality of opportunity is perfectly achieved, since every individual, after transfer, has a welfare function equal to $u = 6z$.

But the simplicity of this solution hides several different underlying ethical values. In order to see this, consider an alternative policy:

Policy B

$y \setminus z$	1	3
1	$x = 2$ $u = 3$	$x = 8$ $u = 27$
3	$x = 0$ $u = 3$	$x = 6$ $u = 27$

This alternative policy no longer equalizes $x + y$ across individuals, and displays a strong bias in favor of the "deserving" subpopulation. But one can still argue that it compensates for social background y, and even that it equalizes opportunities. Indeed, in any subpopulation with equal z, social background is not a source of welfare inequality under policy B, and every individual, after transfer, faces the same welfare function: $u = 3$ if $z = 1$; $u = 27$ if $z = 3$.

These examples of policies show that compensating for inequalities in variable y does not require equalizing $x + y$ over the whole population, but only within the "deserving" subpopulation and within the "undeserving" subpopulation, separately. This, by itself, is generally insufficient to fully determine what redistributive policy is the best. Additional ethical principles are needed to determine how to allocate resources between the "deserving" and the "undeserving" subpopulations. These additional principles will define the appropriate relationship between merit (z) and welfare (u) over the population. In brief, they will decide how merit should be rewarded. The two policies proposed in this example illustrate two different "reward principles". Policy A illustrates an approach which is prominent in the philosophical literature as well as in the literature surveyed in the beginning of this chapter. It expresses a desire of neutrality by avoiding any redistribution on the basis of merit. If, as expressed above, inequalities due to personal merit z are deemed acceptable, it may seem quite reasonable not to interfere with the consequences of such inequalities. The population is then left to bear the consequences of differential merit, without any bias in favor of a subpopulation. Policy B, in contrast, does display a bias in favor of the deserving subpopulation, and such a bias may be justified by a sum-utilitarian view of social welfare. Indeed, policy B maximizes the sum of individual utilities, under the constraint that social background inequalities be fully neutralized. The deserving subpopulation is favored under this policy because, by its own effort and merit, it is more able to transform resources into welfare. Measuring social welfare by the sum is usually considered appropriate when one has no preference for equality in the distribution of welfare, and since inequalities due to differential merit are deemed acceptable it may seem reasonable to adopt a utilitarian approach toward such inequalities.

This example highlights the main ethical principles which will be relevant in this chapter. First, there is a "*principle of compensation*" (Barry 1991, Fleurbaey 1995a), whose limited and separate scope is to neutralize the differential influence over agents' outcomes of the characteristics which elicit compensation.[1] Informally, it is expressed in terms of making sure that such characteristics do not entail by themselves inequalities in final outcomes, or simply that opportunities are made equal. This may be translated into the requirement that any pair of agents who differ only in characteristics to be compensated should obtain equal outcomes. It is also present in the idea that if all unequal characteristics were to be compensated, full equality of outcomes should be the goal. And it will also be shown below to underlie the idea that changes in the population profile of characteristics to be compensated should benefit or harm all individuals, in solidarity.

Then, there are various independent reward principles, and two of them are significant in the literature. The first principle, illustrated by policy A in Example 1, is to neutrally respect inequalities entailed by other characteristics. It is usually introduced through the argument that differences in such characteristics should not entail any compensatory redistribution, or that agents who differ only in such characteristics should be submitted to equal treatment in terms of resource allocation or transfers. It is also behind the idea that if all agents were identical in the characteristics which elicit compensation, there would be no reason to make any transfer between the agents. It may also be shown to underlie various requirements that the redistributive policy should be independent in some way of the population profile of characteristics which do not elicit compensation. This reward principle was called the "*principle of natural reward*" in Fleurbaey (1995a), in reference to the fact that by taking a neutral attitude one does not advocate any "artificial" reward favoring the agents who exercise their responsibility in a particular way.[2] This principle receives a strong backing in the philosophical literature. In particular, Rawls's and Dworkin's theories of equality of resources[3] are built around the core principle that once resources are properly equalized, just institutions may let individual welfare be freely determined by the exercise of individual responsibility in the formation of one's view of life, personal ambitions and goals. No favor should be granted to any subpopulation in virtue of how it exercises its responsibility in this respect. Although Arneson and Cohen disagree with Rawls's and Dworkin's definition of the sphere of individual responsibility,[4] they also endorse a similar reward principle.[5]

[1] This is to be distinguished from the Kaldor-Hicks "Compensation Principle", which has to do with the gains of a subpopulation compensating the losses of the rest of the population.

[2] Barry (1991) proposed calling it "the principle of responsibility", and a similar terminology is retained in part of the literature reviewed here, e.g. Fleurbaey and Maniquet (1998), Maniquet (2002). This raises the issue of deciding whether "artificial" rewards really impede the exercise of responsibility. Imagine that transfers are made so as to make the reward scheme steeper than it would naturally be (effort is rewarded more than naturally). This is exemplified by policy B in Example 1. It is dubious that this diminishes the degree of responsibility that agents can exercise, although this violates the principle of natural reward.

[3] See in particular Rawls (1982) and Dworkin (1981b, 2000).

[4] Instead of considering that individuals should be responsible for their ambitions and use of resources, they argue that individuals should be deemed responsible only for their genuine choices (ambitions and preferences may be influenced in various ways, in which case, according to Arneson and Cohen, individuals should not be held responsible for such characteristics).

[5] 'We should ... compensate only for those welfare deficits which are not in some way traceable to the individual's choices' (Cohen 1989, p. 914). 'Distributive justice does not recommend any intervention by

The alternative, *utilitarian*, reward principle (illustrated by policy B in Example 1) is adopted by a few authors in the literature on equality of opportunities.[6] The idea is that, since inequalities due to characteristics of responsibility are acceptable, the social objective should simply be to maximize the sum of individual outcomes, once the undue influence of characteristics calling for compensation has properly been taken into account. There are different ways in which this undue influence can be reckoned, and therefore different *modified utilitarian social welfare functions* in this branch of the literature, to be presented below. Interestingly, the thesis that it would be legitimate to make transfers not only toward those with characteristics to be compensated, but also in favor of those who exercise responsibility in a "good" way (that is, in such a way as to increase their marginal utility), seems never to have been supported explicitly in the relevant philosophical literature. The contrast between the natural reward principle and the utilitarian reward principle reflects a deep opposition, in welfare economics and political philosophy, between two views of what it means to be indifferent to inequalities. One view, which inspires the natural reward principle, is based on the libertarian principle that the state should then remain neutral and refrain from tampering with the distribution. The alternative view, based on the utilitarian doctrine, is that the state may intervene and possibly alter the distribution, in order to promote the global good.

The literature to be reviewed here has clarified the distinctions and logical independence between the principle of compensation and the reward principles. In addition, it has also revealed that there exist tensions between the principle of compensation and the reward principles. In particular, as explained below in various contexts, one often finds logical incompatibility between axioms representing the principle of compensation and axioms representing the natural reward principle. Roughly, the reason is that transfers designed to fully compensate for undue inequalities may sometimes automatically alter the distribution in a way that is clearly non-neutral. This happens when the influence of characteristics which elicit compensation is not separable from the influence of the

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society to correct inequalities that arise through the voluntary choice or fault of those who end up with less, so long as it is proper to hold the individuals responsible for the voluntary choice or faulty behavior that gives rise to the inequalities.' (Arneson 1990, p. 176 —see however Arneson 2000, p. 344 for a more nuanced view) These authors do not seem to consider the possibility of a policy like policy B in Example 1, which actually *widens* inequalities due to responsibility characteristics. Consider a modified version of Example 1, with $u = 20 - 100/(x+y)z$. In this modified example, a low utility now implies a high marginal utility. Then policy A, in which only inequalities due to y are corrected, leads to $u = 3.33$ for $z = 1$ and $u = 14.44$ for $z = 3$. In contrast, a utilitarian planner maximizing the sum of utilities under the constraint that a poor receives two units more of x than a rich (compensation for y) would choose the policy B':

Policy B'

$y \setminus z$	1	3
1	$x = 6.61$ $u = 6.86$	$x = 3.39$ $u = 12.41$
3	$x = 4.61$ $u = 6.86$	$x = 1.39$ $u = 12.41$

partially correcting inequalities due to z as well because the undeserving have a high marginal utility.

[6] See in particular Roemer (1993, 1998), Van de gaer (1993), Vallentyne (2002).

other individual characteristics. This raises an interesting ethical dilemma, because when the incompatibility arises the social allocation of resources has to reflect one of the two ethical principles (compensation, natural reward) more than the other one. One might think that the principle of compensation, largely endorsed in the literature, is obviously the one which should be given priority in such a conflict. But this is far from obvious, as illustrated in the following example.

Example 2. This example retains the main data of Example 1, but the welfare function is now modified into

$$u = xz + y.$$

One interpretation of this function is that personal effort z makes one more sensitive to external resources x. For instance, the deserving poor ($y = 1$, $z = 3$) respond very well to transfers of resources ($u = 3x + 1$), whereas such transfers are rather ineffective for undeserving poor (for whom $y = z = 1$, so that $u = x + 1$).

Here is a policy which fully achieves compensation, so that inequalities in y do not create welfare differences, and every individual faces the same ex post welfare function: $u = 6$ if $z = 1$; $u = 14$ if $z = 3$.

Policy A1

$y \setminus z$	1	3
1	$x = 5$ $u = 6$	$x = 4.333$ $u = 14$
3	$x = 3$ $u = 6$	$x = 3.666$ $u = 14$

This policy is not strongly biased in favor of the deserving or the undeserving, since both subpopulations receive half of the total resources. Nonetheless, within the poor subpopulation ($y = 1$), a bias seems to be expressed in favor of the undeserving (who receive a larger x than the deserving), whereas the reverse occurs within the rich subpopulation. Moreover, one may object to fully compensating the undeserving poor. Indeed, if such people are responsible for the low $z = 1$, which makes their welfare little sensitive to transfers, one may question compensating them not only for a low y, but also, indirectly, for a low z. Similarly, one may object to full compensation among the deserving, and consider that the remarkable effort made by the deserving poor is not a reason to give them a smaller differential of resources (the differential of x between poor and rich is 2 among the undeserving, and only 2/3 among the deserving).

A similar objection could be made under a different interpretation of the variables. Suppose that the welfare function $xz+y$ reflects subjective preferences about combinations of external and internal resources (x, y). Under this new interpretation, the undeserving are simply y-lovers, whereas the deserving are x-lovers. The undeserving poor consider that they have a deep handicap, whereas the deserving poor consider that their handicap is mild. A full compensation policy such as A1 tries to cater to both opinions at the same time, giving a strong transfer to poor y-lovers, and a small one to poor x-lovers. One may object that there is no reason to accept two different evaluations of the handicap of having a low y, and that society should define its transfer policy on the basis of a consistent evaluation of handicaps.[7]

Whatever the interpretation of the example, such objections to full compensation may lead to an alternative policy like the following one, which is more in line with the natural reward principle.

Policy A2

$y \setminus z$	1	3
1	$x = 4.5$ $u = 5.5$	$x = 4.5$ $u = 14.5$
3	$x = 3.5$ $u = 6.5$	$x = 3.5$ $u = 13.5$

In this policy, the poor receive the same differential amount of x, independently of their merit (or preference, under the alternative interpretation). This particular policy performs compensation under the assumption that everyone could have $z = 2$. Indeed with this value of z, a poor would get $u = 2 \times 4.5 + 1 = 10$, just like a rich, for whom $u = 2 \times 3.5 + 3 = 10$. As a consequence, of course, compensation is no longer achieved within each subpopulation. The undeserving poor are not fully compensated, the deserving poor are more than compensated.

Notice that policy A1 is not the only one to achieve compensation, and similarly, policy A2 is not the only one to comply with the idea of natural reward. As explained below, one may narrow the set of admissible policies by combining one of the two principles with axioms expressing mild versions of the other principle.

There appears to be also a dilemma between the principle of compensation and the utilitarian reward principle, since the modified utilitarian social welfare functions often fail to select allocations which achieve full compensation. This is explained below.

Finally, the contribution of the theory of compensation is not only a clarification of the concepts and an analysis of ethical dilemmas and conflicts of values. It has produced a variety of solutions, allocation rules and social objectives, which embody the relevant ethical principles and determine precise ways of defining appropriate compensatory policies. Many results take the form of axiomatic characterizations, which are helpful in showing the logical links between some basic ethical principles, expressed in various axioms, and possible redistributive solutions. Some of these solutions are actually observed in various institutions and policies, and this literature contributes to enlarging the set of options available to policy-makers confronted with compensation issues.

This chapter is organized as follows. The next section (Sect. 2) reviews the results obtained in the basic model describing the case when there is no production, and only one good (such as money) is transferable in order to compensate for handicaps (as in Examples 1 and 2 above). Section 3 is devoted to the Mirrlees-Pazner-Schmeidler model of cooperative production in which the individual characteristics to be compensated are unequal productive skills. Section 4 is devoted to the literature which adopts the utili-

[7] For such a criticism of the principle of compensation, see van Parijs (1997).

tarian kind of reward or, more generally, deals with an abstract social choice framework. Section 5 reviews other related parts of the literature.[8]

The main mathematical conventions and notations, in this chapter, are as follows. The set of real (resp. non-negative, positive) numbers is $\mathbb{R}$ (resp. $\mathbb{R}_+, \mathbb{R}_{++}$). Vector inequalities are denoted $\geq, >, \gg$, and set inclusion is denoted $\subseteq$, $\subset$. The symbol $\geq_{\text{lex}}$denotes the leximin (lexicographic maximin) criterion applied to vectors of real numbers. Namely, $x \geq_{\text{lex}} x'$ if the smallest component of x is greater than the smallest component of x', or they are equal and the second smallest component of x is greater than the second smallest component of x', and so on. A function $f : A^n \to A$ is said to be idempotent if for all $a \in A$, $f(a, ..., a) = a$. For any set N, let Π_N denote the set of permutations over N (i.e., bijections from N to N). For any function $f : A \to B$, let $f(A) \subseteq B$ denote the range of f. An ordering is a binary relation that is reflexive and transitive. For any set A, $\#A$ denotes its cardinal.

2 Fair monetary compensation

This section deals with the case when the transferable resource by which compensation of handicaps is performed is not produced and is one-dimensional.[9] This is relevant to various practical problems of compensation, especially the allocation of a central budget to several agencies or administrative units with some autonomy in management. It may be applied to social assistance toward disabled individuals when the impact of redistribution on earning incentives is null (e.g. the population under consideration does not work) or may be ignored (fixed labor supply). Such impact on incentives is taken into account in the model studied in the next section.

This section starts with a description of the model. We then list the various solutions which have been proposed for the compensation problem, and review the axiomatic analysis of the relevant ethical principles. The last subsection focuses on the particular case of quasi-linear utilities, which has attracted much attention in the literature.

2.1 The model

There are several ways of modelling this problem, which are almost equivalent. One possibility is to have an ordering $\succeq$ defined over triples (x, y, z), where x is an amount of transferable resource, y is a characteristic which elicits compensation, and z a characteristic which does not elicit compensation. This ordering describes the agents' performances, and

$$(x_i, y_i, z_i) \succeq (x_j, y_j, z_j)$$

[8] In Fleurbaey (1998) one can find another survey which emphasizes the link between the economic and the philosophical literatures. See also Peragine (1999), for a survey focused on the notion of opportunity, and Suzumura and Yoshihara (2000).

[9] The literature reviewed in this section includes Bossert (1995), Bossert and Fleurbaey (1996), Bossert, Fleurbaey and Van de gaer (1999), Cappelen and Tungodden (2002, 2003b), Fleurbaey (1994, 1995c,d), Iturbe (1997), Iturbe and Nieto (1996), Maniquet (2002), Moulin (1994), Sprumont (1997) and Tungodden (2005).

means that agent i is at least as well-off as agent j. One may also use a function representing this ordering: $f(x_i, y_i, z_i)$ is then agent i's outcome or performance (well-being, income, etc.). One may also simply write this function as $u_i(x_i, y_i)$, where the mapping $u_i(.,.)$ itself incorporates the influence of parameter z_i.

Another approach, adopted by Fleurbaey (1994, 1995d) and Iturbe and Nieto (1996), simply endows every agent with preferences over pairs (x, y). This approach is more parsimonious in terms of information, because no interpersonal comparison of outcome is made in this case. Notice, however, that preferences over (x, y) contain more than the usual consumer preferences over commodities, since y is a non-transferable personal characteristic.

It turns out that, whatever the approach, the main solutions to this problem, anyway, do not use more information than these personal, ordinal non-comparable, preferences over pairs (x, y). Since this can be viewed as an interesting ethical conclusion in itself, we adopt here the richer framework with fully interpersonally comparable utilities $u_i(x_i, y_i)$ so as to highlight the way in which interpersonal comparisons of well-being are eventually ruled out, somewhat paradoxically, in the compensation problem.

An *economy* is denoted $e = (y_N, u_N, \Omega)$, where $N = \{1, ..., n\}$ is a finite population of size n, $y_N = (y_1, ..., y_n)$ is the profile of characteristics to be compensated (hereafter called "*talents*"), $u_N = (u_1, ..., u_n)$ is the profile of *utility functions*, and $\Omega \in \mathbb{R}_{++}$ is the amount of *resource* to be allocated among the population. An agent's utility function u_i, for $i \in N$, is defined over pairs (x, y), where x is a quantity of resource and y a personal talent, and, since utilities are assumed to be fully interpersonally comparable, we can, for instance, say that agent i is at least as well-off as agent j if

$$u_i(x_i, y_i) \geq u_j(x_j, y_j).$$

A pair (x_i, y_i) will be called hereafter a bundle of "extended resources".

An *allocation* is a vector $x_N = (x_1, ..., x_n) \in \mathbb{R}^n_+$. It is *feasible* if $\sum_{i \in N} x_i = \Omega$. The set of feasible allocations for the economy e is denoted $F(e)$. Notice that, when utility functions are increasing in x, all feasible allocations are Pareto-optimal in this setting (since we did not assume free disposal in the definition of feasibility).

An *allocation rule* is a correspondence S such that for every e in a given domain, $S(e)$ is a subset of feasible allocations in e. Although the literature reviewed in this section has almost exclusively dealt with allocation rules,[10] we will here make allusions to social ordering functions as well. A *social ordering function* is a mapping R such that for every e in a given domain, $R(e)$ is a complete ordering over the set of allocations $\mathbb{R}^n_+$. We write $x_N R(e) x'_N$ (resp., $x_N P(e) x'_N$, $x_N I(e) x'_N$) to mean that x_N is weakly better than x'_N (resp., strictly better than, indifferent to). The social ordering function R is said to *rationalize* the allocation rule S on some domain if for every e in this domain,

$$S(e) = \{x_N \in F(e) \mid \forall x'_N \in F(e),\ x_N R(e) x'_N\}.$$

The main domain of economies considered here, denoted $\mathcal{E}$, is the set of economies such that: $n \geq 2$; $y_N \in Y^n$, where Y is a given set with at least two elements; for all i,

[10] The exceptions are Bossert, Fleurbaey and Van de gaer (1999) and Maniquet (2002).

$u_i : \mathbb{R}_+ \times Y \rightarrow \mathbb{R}$ is continuous and strictly increasing in the first argument. Let $\mathcal{U}$ denote the set of such utility functions.

This model bears a strong similarity with the model of fair allocation of indivisible goods with money, studied by Svensson (1983) and others. The current model studies what would happen in that model if indivisible goods were already allocated arbitrarily, and only money could still be transferred. The current model also bears some similarity with the bankruptcy model (O'Neill 1982, Aumann and Maschler 1985, Young 1987), in which some money must be allocated on the basis of claims. The current model represents the claims not as simple and fixed demands of money, but as expressions of needs and preferences. On these two related models, see Thomson (this volume).

2.2 Fairness as No-Envy, and related solutions

A central notion in the literature on fair allocations[11] is No-Envy, suggested by Foley (1967) and analyzed by Kolm (1972), Pazner and Schmeidler (1974) and Varian (1974). In brief, No-Envy is obtained when no agent would rather have another's bundle of resources. This notion appears to be very relevant to the problem of compensation, but some precautions must be taken in its application. Indeed, in the current model one can apply the No-Envy requirement in two ways, either on external resources only:

$$\forall i, j,\ u_i(x_i, y_i) \geq u_i(x_j, y_i),$$

or on extended bundles:[12]

$$\forall i, j,\ u_i(x_i, y_i) \geq u_i(x_j, y_j).$$

The first option is inappropriate, since in this model it is equivalent to

$$\forall i, j,\ x_i \geq x_j,$$

and therefore entails $x_i = \Omega/n$ for all i, which prevents any compensation of inequalities in y. In contrast, the second option is quite reasonable as an expression of the principle of compensation as well as of the principle of natural reward.[13] In line with compensation, two agents i, j with the same utility function $u_i = u_j$ will end up with equal utility levels:

$$\left.\begin{array}{l} u_i(x_i, y_i) \geq u_i(x_j, y_j) \\ u_j(x_j, y_j) \geq u_j(x_i, y_i) \end{array}\right\} \Rightarrow u_i(x_i, y_i) = u_j(x_j, y_j),$$

[11] This literature is surveyed in Thomson (this volume).

[12] Under the alternative formulation $u_i = f(x_i, y_i, z_i)$, one can think of a third kind of application:

$$\forall i, j,\ f(x_i, y_i, z_i) \geq f(x_j, y_j, z_j),$$

which directly implies full equality of utilities over the population.

[13] Apart from Kolm (1972), in which No-Envy is considered for any combination of external resources and personal characteristics, early attempts to apply the notion of No-Envy to the compensation of handicaps were problematic. Champsaur and Laroque (1981) limited the No-Envy test to external resources and concluded negatively but unsurprisingly the impossibility of making income transfers in favor of handicapped agents. Roemer (1985b) applied the No-Envy test to extended resources but failed to allow people to feel envy for others' personal characteristics, therefore concluding also negatively about the compensation performed through application of the No-Envy test (see Fleurbaey (1994) for a discussion).

implying full compensation between them. And in line with natural reward, two agents with the same talent $y_i = y_j$ will receive the same resource:

$$\left.\begin{array}{c} u_i(x_i, y_i) \geq u_i(x_j, y_j) \\ u_j(x_j, y_j) \geq u_j(x_i, y_i) \end{array}\right\} \Rightarrow \left.\begin{array}{c} x_i \geq x_j \\ x_j \geq x_i \end{array}\right\} \Rightarrow x_i = x_j,$$

which prevents any biased transfer between agents who differ only in their utility function. This can be illustrated with Example 1.

Example 1 (pursued). When agent i has a utility function $u_i(x, y) = (x + y)\, z_i$, the No-Envy condition applied to extended resources implies

$$\forall i, j,\ x_i + y_i \geq x_j + y_j,$$

which entails equality of $x + y$ over the whole population, as performed by policy A. This policy is the only one to comply with the No-Envy condition.

In summary, No-Envy is a test of equality of resources, and can be used as a test of compensation provided it bears on extended resources.

No-Envy Allocation Rule (S_{NE}, Foley 1967, Roemer 1985b, Fleurbaey 1994):
$\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$x_N \in S_{NE}(e) \Leftrightarrow \forall i, j \in N,\ u_i(x_i, y_i) \geq u_i(x_j, y_j).$$

It is immediate to see on simple examples that this allocation rule is likely to be empty in many non-pathological economies. Assume for instance that for any given x, agent i prefers agent j's talent to her own, and conversely. Then any transfer will just increase the envy of one of them. With Example 2 one has a less extreme illustration of this difficulty.

Example 2 (pursued). When agent i has a utility function $u_i(x, y) = xz_i + y$, the No-Envy condition applied to extended resources entails

$$\forall i, j,\ x_i z_i + y_i \geq x_j z_i + y_j,$$

For instance, take i among the undeserving poor, and j among the deserving rich. One then must have

$$\left\{\begin{array}{l} x_i + 1 \geq x_j + 3 \\ 3x_j + 3 \geq 3x_i + 1 \end{array}\right. \Leftrightarrow x_j + 2 \leq x_i \leq x_j + 2/3,$$

which is impossible. Agent i requires a large transfer not to be envious, which renders agent j envious.

This problem is similar to that encountered by Pazner and Schmeidler (1974) in the model of production with unequal skills (see next section), and a line of research is to look for nice weakenings of the No-Envy condition that reduce the non-existence problem.

Here are three examples of allocation rules derived from this idea. The first one combines two ideas separately proposed by Feldman and Kirman (1974), who suggested to choose allocations with the minimal number of occurrences of envy, and by Daniel (1975), who suggested to choose allocations where the number of agents envying any given agent is equal to the number of agents this agent envies. For any economy e, let $B(e)$ be the set of such "balanced" allocations: $x_N \in B(e)$ if and only if $x_N \in F(e)$ and for all $i \in N$,

$$\#\{j \in N \mid u_j(x_i, y_i) > u_j(x_j, y_j)\} = \#\{j \in N \mid u_i(x_i, y_i) < u_i(x_j, y_j)\}.$$

and let $E(e, x_N)$ denote the number of envy occurrences in allocation x_N :

$$E(e, x_N) = \#\{(i, j) \in N^2 \mid u_i(x_i, y_i) < u_i(x_j, y_j)\}.$$

Balanced and Minimal Envy (S_{BME}, Fleurbaey 1994):
$\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$x_N \in S_{BME}(e) \Leftrightarrow x_N \in B(e) \text{ and } \forall x'_N \in B(e),\ E(e, x'_N) \geq E(e, x_N).$$

Fleurbaey (1994) shows that a sufficient condition for $S_{BME}(e)$ to be non empty is:[14]

$$\begin{aligned} \exists \delta &> 0, \forall x_N \in F(e), \exists i \in N, x_i > 0, \\ \#\{j &\in N \mid u_i(x_i, y_i) < u_i(x_j, y_j)\} \leq \#\{j \in N \mid u_j(x_i, y_i) \geq u_j(x_j + \delta, y_j)\}. \end{aligned}$$

The second solution was proposed in a more general context by Diamantaras and Thomson (1990). It tries to minimize the intensity of envy, this intensity being measured for every agent by the resource needed to make this agent non-envious. An advantage of this rule is that it is single-valued.

Minimax Envy Intensity (S_{MEI}, Diamantaras and Thomson 1990, Fleurbaey 1994):
$\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$x_N \in S_{MEI}(e) \Leftrightarrow \forall x'_N \in F(e),\ \max_{i \in N} IE_i(e, x'_N) \geq \max_{i \in N} IE_i(e, x_N),$$

where $IE_i(e, x_N) = \min\{\delta \in \mathbb{R} \mid \forall j \in N \setminus \{i\}\,,\ u_i(x_i + \delta, y_i) \geq u_i(x_j, y_j)\}$.[15]

The third solution makes use of all agents' opinions about the relative well-being of two agents. It tries to minimize the size of subsets of agents thinking that one agent is worse-off than another agent. It takes inspiration from van Parijs' scheme of "undominated diversity" (van Parijs 1990), which seeks to avoid situations in which one agent is deemed unanimously worse-off than another one, and from the family of solutions put forth by

[14] As shown in Fleurbaey (1994), this condition is logically weaker than the conditions given in Daniel (1975).

[15] This rule is well-defined only in economies where $IE_i(e, x_N)$ is bounded from above, namely, in economies where

$$\forall i, j \in N, \exists x \in \mathbb{R}_+,\ u_i(x, y_i) \geq u_i(\Omega, y_j).$$

Assuming $\min \emptyset = +\infty$, one can extend the definition, but then the solution is not well-behaved toward agents who are always envious.

Iturbe and Nieto (1996), which generalizes van Parijs' idea and seeks to avoid such a unanimity among a subgroup of a given size and containing the worse-off agent. Let

$$I_i^m = \{G \subset N \mid \#G = m,\ i \in G\}.$$

Minimal Unanimous Domination (S_{MUD}, Fleurbaey 1994, Iturbe and Nieto 1996): $\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$x_N \in S_{MUD}(e) \Leftrightarrow \exists m \in \{1, ..., n\},$$
$$\begin{cases} \text{(i) } \forall i, j \in N, \forall G \in I_i^m, \exists k \in G,\ u_k(x_i, y_i) \geq u_k(x_j, y_j), \\ \text{(ii) } \forall p < m, \forall x'_N \in F(e), \exists i, j \in N, \exists G \in I_i^p, \\ \qquad \forall k \in G,\ u_k(x'_i, y_i) < u_k(x'_j, y_j). \end{cases}$$

Fleurbaey (1994) states that $S_{MUD}(e)$ is non empty if

$$\forall i, j \in N, \exists k \in N,\ u_k\left(\frac{\Omega}{n-1}, y_i\right) \geq u_k(0, y_j),$$

and Iturbe and Nieto (1996) give an alternative sufficient condition:

$$\forall x_N \in F(e), \exists i, k \in N, x_i > 0, \forall j \in N,\ u_k(x_i, y_i) \geq u_k(x_j, y_j).$$

This rule makes a very indirect use of the No-Envy test, and can be viewed as aggregating the opinions of the population over the transfers of resources to be made between two given agents. Along this line another kind of allocation rule has been proposed, which makes use of one preference relation at a time in order to look for equality of extended resources. Aggregation of opinions can then be made in two ways. Either the preference relation used in the computation of the allocation can be based on the profile of preferences in the population, as in an Arrovian social choice problem. Or aggregation can be made at the level of allocations, for instance by averaging the allocations obtained by taking every agent's preferences in turn as the reference. This suggests two different sorts of allocation rules. Let Φ be a mapping from $\bigcup_{n\geq 1} \mathcal{U}^n$ to $\mathcal{U}$.

Φ-Conditional Equality ($S_{\Phi CE}$, Fleurbaey 1995d): $\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$x_N \in S_{\Phi CE}(e) \Leftrightarrow \forall i, j \in N,\ \tilde{u}(x_i, y_i) = \tilde{u}(x_j, y_j) \text{ or}$$
$$[\tilde{u}(x_j, y_j) > \tilde{u}(x_i, y_i) \text{ and } x_j = 0].$$

where $\tilde{u} = \Phi(u_1, ..., u_n)$.

Notice that equality of extended resources is performed here on the basis of the maximin criterion, because it may be impossible to fully compensate some inequalities in personal characteristics with the available resources. In that case some inequalities in well-being persist and the better-off agents are left with no external resource, while only disadvantaged agents receive positive resources (and all of them have the same well-being according to the reference preferences).

Average Conditional Equality (S_{ACE}, Fleurbaey 1995d):
$\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$x_N \in S_{ACE}(e) \Leftrightarrow x_N = \frac{1}{n}\sum_{k=1}^{n} x^k$$

where

$$\forall i,j,k \in N,\ u_k(x_i^k, y_i) = u_k(x_j^k, y_j) \text{ or } \left[u_k(x_i^k, y_i) < u_k(x_j^k, y_j) \text{ and } x_j^k = 0\right].$$

The Average Conditional Equality rule can be generalized by allowing the weights between the n proposed allocations x^k to differ. This can be done without violating anonymity. For instance, one could argue that allocations based on minority preferences should be overweighted, or, on the contrary, underweighted, or that certain talents should entail a greater weight.

By construction, these two allocation rules are non empty on the whole domain $\mathcal{E}$ of economies considered here, and they are single-valued. Moreover, the Conditional Equality rule can be immediately interpreted as derived from a social ordering function which rationalizes it,[16] and is defined as follows.

Φ-Conditional Equality ($R_{\Phi CE}$, Roemer 1993, Bossert, Fleurbaey and Van de gaer 1999):
$\forall e \in \mathcal{E}$, $\forall x_N, x'_N \in \mathbb{R}_+^n$,

$$x_N R_{\Phi CE}(e) x'_N \Leftrightarrow (\tilde{u}(x_i, y_i))_{i\in N} \geq_{\text{lex}} (\tilde{u}(x'_i, y_i))_{i\in N},$$

where $\tilde{u} = \Phi(u_1, ..., u_n)$.

A rather different approach is inspired by the Egalitarian Equivalence criterion proposed by Pazner and Schmeidler (1978a), which can be easily adapted to the current framework. The idea is to render all agents indifferent between their current bundle of extended resources (x_i, y_i) and a reference bundle which is the same for all. Again this suggests two (families of) allocation rules, depending on whether some unique reference talent $\tilde{y}$ is used to compute Egalitarian Equivalent allocations, or the reference $\tilde{y}$ varies and the average of the resulting allocations is retained. Let Ψ be a mapping from $\bigcup_{n\geq 1} Y^n$ to Y.

Ψ-Egalitarian Equivalence ($S_{\Psi EE}$, Pazner and Schmeidler 1978a, Fleurbaey 1995d):
$\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$\begin{aligned} x_N &\in S_{\Psi EE}(e) \Leftrightarrow \exists \tilde{x} \in \mathbb{R}_+, \\ \forall i,j &\in N, u_i(x_i, y_i) = u_i(\tilde{x}, \tilde{y}) \text{ or } [u_i(x_i, y_i) > u_i(\tilde{x}, \tilde{y}) \text{ and } x_i = 0]. \end{aligned}$$

[16] The Minimax Envy Intensity allocation rule is also rationalized by a social ordering function, defined as follows.

Minimax Envy Intensity (R_{MEI})
$\forall e \in \mathcal{E}$, $\forall x_N, x'_N \in \mathbb{R}_+^n$,

$$x_N R_{MEI}(e) x'_N \Leftrightarrow (-IE_i(e, x_N))_{i\in N} \geq_{\text{lex}} (-IE_i(e, x'_N))_{i\in N}.$$

But this social ordering function does not satisfy the Pareto criterion, since one may have $x_N R_{MEI}(e) x'_N$ and $x'_N \gg x_N$. In particular, it would no longer rationalize S_{MEI} under a free disposal assumption.

where $\tilde{y} = \Psi(y_1, ..., y_n)$.

Average Egalitarian Equivalence (S_{AEE}, Moulin 1994):
$\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$x_N \in S_{AEE}(e) \Leftrightarrow x_N = \frac{1}{n}\sum_{k=1}^{n} x^k$$

where

$$\begin{aligned} \forall k &\in N, \exists \tilde{x} \in \mathbb{R}_+, \forall i \in N, \\ u_i(x_i^k, y_i) &= u_i(\tilde{x}, y_k) \text{ or } \left[u_i(x_i^k, y_i) > u_i(\tilde{x}, y_k) \text{ and } x_i^k = 0\right]. \end{aligned}$$

These allocation rules are single-valued but may be empty over the domain $\mathcal{E}$. For convenience we will consider here the subdomain $\mathcal{E}'$ of economies with utility functions such that

$$\forall y \in Y,\ u(0, y) = 0$$

and

$$\forall y, y' \in Y, \forall x \in \mathbb{R}_+, \exists x' \in \mathbb{R}_+,\ u(x', y') \geq u(x, y).$$

On this subdomain these two allocation rules are non-empty.[17] The Egalitarian Equivalent allocation rule is rationalized on $\mathcal{E}'$ by the following social ordering function. This social ordering function applies the leximin criterion to the individual levels of resource x^* which would make the agents accept the reference talent $\tilde{y}$ instead of their current situation (x_i, y_i). This gives priority to the agents with a low x^*, i.e. agents who either have a low x_i or dislike their talent y_i.

Ψ-Egalitarian Equivalence ($R_{\Psi EE}$, Pazner and Schmeidler 1978a, Bossert, Fleurbaey and Van de gaer 1999):
$\forall e \in \mathcal{E}'$, $\forall x_N, x'_N \in \mathbb{R}^n_+$,

$$x_N R_{\Psi EE}(e) x'_N \Leftrightarrow (\hat{x}(x_i, y_i, u_i, \tilde{y}))_{i \in N} \geq_{\text{lex}} (\hat{x}(x'_i, y_i, u_i, \tilde{y}))_{i \in N}$$

where $\tilde{y} = \Psi(y_1, ..., y_n)$, and $\hat{x}(x_i, y_i, u_i, \tilde{y})$ is defined as the solution x^* of

$$u_i(x_i, y_i) = u_i(x^*, \tilde{y}).$$

It is worth stressing that all these allocation rules (and social ordering functions) avoid interpersonal comparisons of utilities. The only information they need is the profile of preferences over (x, y) represented by u_N. This can be intuitively understood as resulting from the fact that what is sought here is equality of extended bundles (x_i, y_i), not of utilities. Therefore utilities are essentially irrelevant. This idea will be made more precise below.[18]

[17] See Fleurbaey (1995d) for an exact definition of the largest domain over which Egalitarian Equivalent allocations exist.

[18] This does not imply that this theory takes sides in the philosophical debate about whether individuals should be held responsible for their utilities and preferences (Dworkin 2000, Cohen 1989). Indeed, we have assumed nothing about the concrete meaning of the variable y, so that y may contain any trait related to subjective satisfaction. The concrete content of the separation between y and u (or z) has to be decided outside the model.

In conclusion, it seems relatively easy to define reasonable allocation rules (or social ordering functions) for the current problem. Is it the case that all of the above solutions are equally appealing? The axiomatic analysis of the model has shown that the answer is no. In the next subsection, we review the main axioms proposed in the literature.

2.3 Axioms and ethical principles

Following the literature, we focus on allocation rules and all axioms presented in this subsection bear on allocation rules defined over some domain $\mathcal{D}$ (either $\mathcal{E}$ or $\mathcal{E}'$).

A basic axiom, satisfied by all reasonable allocation rules, is that names of agents are irrelevant.

Anonymity:
$\forall e = (y_N, u_N, \Omega) \in \mathcal{D}$, $\forall x_N \in S(e)$, $\forall \pi \in \Pi_N$,

$$x_{\pi(N)} \in S(y_{\pi(N)}, u_{\pi(N)}, \Omega).$$

A related axiom, which is implied by ***Anonymity*** when the allocation rule is single-valued, is the following horizontal equity requirement.

Equal Treatment of Equals:
$\forall e \in \mathcal{D}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$[y_i = y_j \text{ and } u_i = u_j] \Rightarrow x_i = x_j.$$

The No-Envy condition can be used to define not only an allocation rule, but also an axiom bearing on allocation rules.

No-Envy Axiom (NE):
$\forall e \in \mathcal{D}$, $S(e) \subset S_{NE}(e)$.

Since $S_{NE}(e)$ is empty for some e in $\mathcal{E}$, this axiom cannot be satisfied. One must therefore weaken the requirement. As explained above, this axiom embodies the principle of compensation as well as the principle of natural reward, because it implies a substantial degree of equality in extended bundles. The axiomatic analysis reviewed here has studied how to weaken this axiom in the dimension of either compensation or natural reward. Most of the axioms presented below are actually sufficiently weak so as to express only one of these two ethical principles.

We begin by listing axioms which express the *principle of compensation*, namely, the goal of neutralizing the impact of unequal characteristics over utilities. The first axiom is inspired by the intuitive requirement that agents with the same utility function, who therefore differ only in characteristics to be compensated, should obtain equal utility after transfer (unless the better-off has a zero resource). Full compensation among such agents can be obtained only when their difference in y does not entail any inequality in utility. The fact that agents with identical utility functions are considered eliminates any problem of separating the influence of y from the influence of $u(.,.)$ in the production of inequalities in utility levels.

Equal Utility for Equal Function (**EUEF**, Fleurbaey 1994):
$\forall e \in \mathcal{D}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$u_i = u_j \Rightarrow u_i(x_i, y_i) = u_j(x_j, y_j) \text{ or } [u_i(x_i, y_i) < u_j(x_j, y_j) \text{ and } x_j = 0].$$

Notice that since $u_i = u_j$, this axiom is a direct weakening of ***No-Envy*** and says that no envy should occur among agents with identical utility functions, except when the envied agent has a zero resource. One also sees that it is only when all agents with identical utility function obtain the same utility level that every agent faces one and the same set of opportunities, defining an opportunity as a pair (*utility function, utility level*). Any agent who adopts a particular utility function is then assured of getting the same utility level as the others who adopted the same function. On the other hand, recall that, as explained in Example 2, this axiom is not totally uncontroversial and may be rejected as displaying too much faith in the agents i and j's own (identical but possibly idiosyncratic) evaluation of the impact of y.

The next axiom makes the same requirement, but only when all agents have identical utility functions, which can be interpreted as the case when all characteristics which differ among agents are to be compensated.

Equal Utility for Uniform Function (**EUUF**, Fleurbaey 1994, Bossert 1995):
$\forall e \in \mathcal{D}$, $\forall x_N \in S(e)$,

$$\begin{aligned} [\forall i, j \in N,\ u_i = u_j] &\Rightarrow \forall i, j \in N, \\ u_i(x_i, y_i) &= u_j(x_j, y_j) \text{ or } [u_i(x_i, y_i) < u_j(x_j, y_j) \text{ and } x_j = 0]. \end{aligned}$$

The next axiom is even weaker, because it requires equality of utilities (or No-Envy) only when utility functions are not only identical, but also belong to some specified subset which may be arbitrarily small.

Equal Utility for Reference Function (**EURF**, Fleurbaey 1995d):
$\exists \tilde{u} \in \mathcal{U}, \forall e \in \mathcal{D}$, $\forall x_N \in S(e)$, if $\forall i \in N,\ u_i = \tilde{u}$, then

$$\forall i, j \in N,\ u_i(x_i, y_i) = u_j(x_j, y_j) \text{ or } [u_i(x_i, y_i) < u_j(x_j, y_j) \text{ and } x_j = 0].$$

The next axiom is somewhat different and deals with changes in the profile of characteristics. It says that a change in this profile should affect all agents' final utilities in the same way. The rationale for this condition is that since those characteristics elicit compensation, there is no reason to make some agents benefit while others would lose. This may be related to Rawls's idea 'to regard the distribution of natural talents as a common asset and to share in the benefits of this distribution whatever it turns out to be' (Rawls 1971, p. 101). In the current model it makes sense to apply this solidarity requirement only to agents who receive positive resources, because agents who do not receive resources will have their utility level depend only on the change in their own talent, independently of changes in the whole profile.

Solidarity (**S**, Fleurbaey and Maniquet 1999):
$\forall e = (y_N, u_N, \Omega),\ e' = (y'_N, u_N, \Omega) \in \mathcal{D}$, $\forall x_N \in S(e)$, $\forall x'_N \in S(e')$,

$$\begin{aligned} \forall i &\in N \text{ such that } x'_i > 0,\ u_i(x_i, y_i) \geq u_i(x'_i, y'_i) \text{ or} \\ \forall i &\in N \text{ such that } x_i > 0,\ u_i(x_i, y_i) \leq u_i(x'_i, y'_i). \end{aligned}$$

It is easy to illustrate why these axioms express the principle of compensation and not at all the principle of natural reward, by observing that they are all satisfied by the following welfare egalitarian allocation rule (based on the maximin criterion applied to utilities):

Maximin Utility (S_{MU}):
$\forall e \in \mathcal{E}$, $\forall x_N \in F(e)$,

$$x_N \in S_{MU}(e) \Leftrightarrow \forall x'_N \in F(e),\ \min_{i \in N} u_i(x_i, y_i) \geq \min_{i \in N} u_i(x'_i, y_i).$$

This allocation rule does not satisfy the principle of natural reward in any sense because it does not seek equality of extended bundles.

We now turn to axioms reflecting the *principle of natural reward*, namely, the goal of compensating only talents and not other characteristics like utility functions. The first axiom says that an agent unanimously considered as more talented than another one should not receive more resources.

Protection of Handicapped (**PH**, Fleurbaey 1994):
$\forall e \in \mathcal{D}$, $\forall i, j \in N$,

$$[\forall x \in \mathbb{R}_+, \forall k \in N,\ u_k(x, y_i) \leq u_k(x, y_j)] \Rightarrow [\forall x_N \in S(e),\ x_i \geq x_j].$$

This axiom is similar to Sen's *Weak Equity Axiom*,[19] and boils down to it when all utility functions are identical. When utility functions are heterogeneous, this axiom only applies when unanimity is obtained, which means that it is quite weak in this respect. However, most of the axioms of a similar vein studied here are actually even weaker than it.

It is tempting to view this axiom as expressing the principle of compensation rather than the principle of natural reward. But this would be a mistake. The protection granted to handicapped agents is only minimal, and the allocation rule which always divides Ω equally ($x_i = \Omega/n$ for all i) does satisfy this axiom, without performing any kind of compensation. This axiom never requires any kind of compensation, even when all agents have identical utility functions. On the other hand, it does requires giving equal resources to agents having identical talents y, and therefore prevents any biased transfer in favor of some agents just because they have "good" utility functions. This is why this axiom directly, and rather strongly, expresses the principle of natural reward.

The second, weaker axiom is, precisely, formulating the requirement that agents with equal talents should receive the same treatment in terms of resources.

[19] As recalled in the introduction, this axiom says that when an individual has a lower level of utility than another at all levels of income, the optimal allocation must not give him less income.

Equal Resource for Equal Talent (**ERET**, Fleurbaey 1994):
$\forall e \in \mathcal{D}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$y_i = y_j \Rightarrow x_i = x_j.$$

The motivation for this requirement has been discussed in Examples 1 and 2, in the introduction. It guarantees a neutral treatment of different utility functions.

On the other hand, it may be criticized for failing to take account of the fact that different values of talent y may alter the opportunity set and then require a sensitivity of transfers to utility functions. This criticism is illustrated in the following example.

Example 3. This example retains the main data of Examples 1 and 2, except that the utility function is now defined as

$$u = x + yz.$$

With this function, a greater talent y makes one more able to benefit from one's effort. One possible policy respecting the condition formulated in the ***E****qual* ***R****esource for* ***E****qual* ***T****alent* axiom is the following. It is based on the idea that any agent (rich or poor) who would have $z = 2$ would obtain $u = 8 = 6 + 2 \times 1 = 2 + 2 \times 3$.

Policy AA2

$y \setminus z$	1	3
1	$x = 6$ $u = 7$	$x = 6$ $u = 9$
3	$x = 2$ $u = 5$	$x = 2$ $u = 11$

This policy, like policy A2 in Example 2, fails to compensate in one subpopulation (here, the deserving) and overcompensates in the other (here, the undeserving). Here, one may complain that giving the same transfer to all poor fails to take account of the fact that by exerting effort they are less able to improve their lot than the rich, and since this is due to a low endowment in y they should not suffer from this. Compensation should then take account not only of the fact that they have a lower talent, but also of the fact that this low talent makes them less able to benefit from their own effort. As a consequence, x should depend on z as well as on y.[20]

In essence, this criticism is pointing at a conflict between this ***E****qual* ***R****esource for* ***E****qual* ***T****alent* axiom and the principle of compensation (in particular the ***E****qual* ***U****tility for* ***E****qual* ***F****unction* axiom). This conflict will be formally delineated below.

[20] An argument along these lines is made in Tungodden (2005). A defender of natural reward may reply that agents being responsible for their effort, they cannot complain if by choosing an especially low or high effort they obtain more or less than others. This kind of defense is even more convincing when, in an alternative interpretation, $u = x + yz$ simply represents preferences about bundles (x, y). The deserving poor are then just y-lovers who happen to have a low endowment in y, so that one may object to accepting their view that this is a great handicap. See Fleurbaey (1995b, 2001), Vallentyne (2002) and Vandenbroucke (2001) for critical discussions of natural reward, and Cappelen and Tungodden (2004a) for an in-depth study of reward schemes.

A weaker requirement than ***E**qual **R**esource for **E**qual **T**alent*, in the natural reward line of inspiration, is that the relative position of x_i with respect to the mean resource Ω/n should depend only on talent.

Fair Relative Resource for Equal Talent (**FRRET**, Sprumont 1997):
$\forall e \in \mathcal{D}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$y_i = y_j \Rightarrow (x_i - \frac{\Omega}{n})(x_j - \frac{\Omega}{n}) \geq 0.$$

Notice that this formulation is relatively weak since no constraint bears on x_j when $x_i = \Omega/n$.

The next axiom weakens this again by applying the requirement only to economies with a uniform talent.

Equal Resource for Uniform Talent (**ERUT**, Fleurbaey 1994, Bossert 1995):
$\forall e \in \mathcal{D}$, $\forall x_N \in S(e)$,

$$\forall i, j \in N,\ y_i = y_j \Rightarrow \forall i \in N,\ x_i = \frac{\Omega}{n}.$$

And a further weakening[21] goes by applying this only to economies with a uniform talent in a certain, arbitrarily small, subset.

Equal Resource for Reference Talent (**ERRT**, Fleurbaey 1995d):
$\exists \tilde{y} \in Y$, $\forall e \in \mathcal{D}$, $\forall x_N \in S(e)$, if $\forall i \in N,\ y_i = \tilde{y}$, then

$$\forall i \in N,\ x_i = \frac{\Omega}{n}.$$

Another kind of condition relies on the idea that if utilities as such are not to elicit any compensation, the allocation rule should be essentially independent of utilities.

Independence of Utilities (**IU**, Bossert 1995):
$\forall e = (y_N, u_N, \Omega),\ e' = (y_N, u'_N, \Omega) \in \mathcal{D}$,

$$S(e) = S(e').$$

This axiom is very strong because it forbids any use of the agents' preferences in the measurement of talent differentials and in determining the scale of compensation. A weaker requirement is ordinalism, which is satisfied by all allocation rules presented in the previous subsection.

[21] Another weak condition of natural reward has been proposed in Boadway et alii (2002). In the current model, it would say that every subpopulation with a given utility function should receive its per capita share of Ω. In other words, there should be no transfers among subpopulations of different utility functions. In the subdomain of economies where talents and utility functions are independently distributed, this condition is implied by ***E**qual **R**esource for **E**qual **T**alent*, and (under ***E**qual **T**reatment of **E**quals*) implies ***E**qual **R**esource for **U**niform **T**alent*. It is logically independent of ***F**air **R**elative **R**esource for **E**qual **T**alent*. Outside this subdomain it is not reasonable (e.g. if there are more poor among the deserving, it is acceptable to give more resources to the deserving subpopulation than its per capita share).

Ordinalism (O):
$\forall e = (y_N, u_N, \Omega),\ e' = (y_N, u'_N, \Omega) \in \mathcal{D}$,

$$R_N = R'_N \Rightarrow S(e) = S(e'),$$

where R_N (resp. R'_N) is the profile of preferences represented by u_N (resp. u'_N).

Finally, we turn to another ancient notion of fairness, namely, that equal division should be a minimum right guaranteed to every agent (Steinhaus 1948). Moulin (1991) has shown how to extend this notion by devising lower bounds and upper bounds suitable for division problems. In the current context of compensation, Moulin (1994) suggests to define a bound based on what an agent would obtain if others shared his responsibility characteristics (utility function here). Let $EUUF(y_N, \Omega, u)$ denote the allocation resulting from the application of ***E**qual **U**tility for **U**niform **F**unction* (maximin of utilities) to the economy where all agents have the same utility function u. This defines a bound that cannot operate on all economies as a lower bound, and Moulin suggests to use it as an upper bound when this happens.

Egalitarian Bound (EB, Moulin 1994):
$\forall e = (y_N, u_N, \Omega) \in \mathcal{D}$, $\forall x_N \in S(e)$,

$$\forall i \in N,\ x_i \geq EUUF_i(y_N, \Omega, u_i) \text{ or } \forall i \in N,\ x_i \leq EUUF_i(y_N, \Omega, u_i).$$

Although this axiom clearly has a flavor of compensation (it implies ***E**qual **U**tility for **U**niform **F**unction*), one can argue that it also contains a pint of natural reward, because it forbids excessive compensation as performed for instance by the welfare egalitarian allocation rule S_{MU}. In fact it also implies ***E**qual **R**esource for **U**niform **T**alent*, as indicated in the following proposition.

Proposition 1 *The following table describes the logical implications between axioms (for $\mathcal{D} = \mathcal{E}$ or $\mathcal{E}'$).*

Compensation		*Natural*	*Reward*
S		O$\Leftarrow$	IU
$\Downarrow^{(1)}$			$\Downarrow^{(2)}$
EUEF	$\Leftarrow$ NE$^{(3)}$ $\Rightarrow$	PH$\Rightarrow$	ERET
			$\Downarrow$
$\Downarrow$	$\Downarrow$		FRRET
			$\Downarrow$
EUUF	$\Leftarrow$ EB $\Rightarrow$		ERUT
$\Downarrow$			$\Downarrow$
EURF			ERRT

$^{(1)}$*Assuming that S satisfies* **A**nonymity.
$^{(2)}$*Assuming that S satisfies* **E**qual **T**reatment of **E**quals.
$^{(3)}$*Considered on the subdomain where it can be satisfied.*

Proof. We focus only on the non obvious implications.

S⇒EUEF, under Anonymity. Consider $e = (y_N, u_N, \Omega)$ with $u_i = u_j = u$. Let $e' = (y'_N, u_N, \Omega)$ be such that $y'_i = y_j$, $y'_j = y_i$, and $y'_k = y_k$ for all $k \neq i, j$. By Anonymity, if $x_N \in S(e)$, then $x'_N \in S(e')$, with $x'_i = x_j$, $x'_j = x_i$, and $x'_k = x_k$ for all $k \neq i, j$. First case: if $x_i = x_j = 0$, then EUEF is satisfied. Second case: $x_i > 0 = x_j$. By Solidarity, either $u(x_j, y_j) \geq u(x'_j, y'_j)$ or $u(x_i, y_i) \leq u(x'_i, y'_i)$, which is the same, and this implies either $u(x_i, y_i) = u(x_j, y_j)$ or $[u(x_i, y_i) < u(x_j, y_j)$ and $x_j = 0]$, as EUEF requires. Third case: $x_i x_j > 0$. By Solidarity, either $u(x_j, y_j) \geq u(x'_j, y'_j)$ and $u(x_i, y_i) \geq u(x'_i, y'_i)$, or $u(x_j, y_j) \leq u(x'_j, y'_j)$ and $u(x_i, y_i) \leq u(x'_i, y'_i)$. Both mean $u(x_i, y_i) = u(x_j, y_j)$, and EUEF is satisfied.

NE⇒EB. Consider an allocation x_N such that there exists i such that $x_i < EUUF_i(y_N, \Omega, u_i)$. This implies that $EUUF_i(y_N, \Omega, u_i) > 0$ and therefore, for all $j \in N$,

$$u_i(EUUF_i(y_N, \Omega, u_i), y_i) \leq u_i(EUUF_j(y_N, \Omega, u_i), y_j).$$

Since $\sum_{j \in N} EUUF_j(y_N, \Omega, u_i) = \sum_{j \in N} x_j = \Omega$, necessarily there is j such that $x_j > EUUF_j(y_N, \Omega, u_i)$. This implies that $u_i(x_i, y_i) < u_i(x_j, y_j)$, and thus x_N is not envy-free. As a consequence, any envy-free allocation is such that for all $i \in N$, $x_i \geq EUUF_i(y_N, \Omega, u_i)$. Therefore EB is satisfied in such an allocation. ■

The pattern of incompatibilities displayed in the next table graphically shows that there is a tension between the principle of compensation and the principle of natural reward. It is impossible to fully satisfy both principles at the same time.

Proposition 2 *The following table describes the incompatibilities (marked by $\bigotimes$) between the main axioms of compensation and natural reward (for $\mathcal{D} = \mathcal{E}$ or $\mathcal{E}'$), and shows what axioms are satisfied by the various solutions (on their respective domain).*

	IU	ERET	FRRET	ERUT	ERRT
S	$\bigotimes$	$\bigotimes$	$\bigotimes$	$\bigotimes$	$S_{\Psi EE}$ (Ψ *const.*)
EUEF	$\bigotimes$	$\bigotimes$	$\bigotimes$	$S_{\Psi EE}$ (Ψ *idempot.*)	
EUUF	$\bigotimes$	$S_{\Phi CE}, S_{MUD}, S_{ACE}$ (Φ *idempotent*)		$S_{AEE}, S_{BME}, S_{MEI}$	
EURF	$S_{\Phi CE}$ (Φ *const.*)				

Proof. We omit the easy proof of the incompatibility between S and ERUT, and between EUUF and IU, and focus on the proof that EUEF and FRRET are incompatible on $\mathcal{E}'$ (and therefore on $\mathcal{E}$ as well). Assume the utility function is defined by $u = x\,(yz + \hat{y}\hat{z})$, where $(z, \hat{z})$ are preference parameters. Consider an economy with $\Omega = 4$, and four agents, described in the following table:

i	y_i	$\hat{y}_i$	z_i	$\hat{z}_i$
1	1	2	1	2
2	1	2	2	1
3	2	1	1	2
4	2	1	2	1

FRRET requires

$$\begin{cases} (x_1 - 1)(x_2 - 1) \geq 0 \\ (x_3 - 1)(x_4 - 1) \geq 0, \end{cases}$$

and EUEF, in this particular case, simply requires

$$\begin{cases} 5x_1 = 4x_3 \\ 4x_2 = 5x_4. \end{cases}$$

First possibility: $x_1 > 1$. Then $x_2 \geq 1$ by FRRET and $x_3 > 1$ by EUEF, the latter implying $x_4 \geq 1$ by FRRET. All this is incompatible with $x_1 + x_2 + x_3 + x_4 = 4$.

Second possibility: $x_1 < 1$. Then $x_2 \leq 1$ by FRRET, which implies $x_4 < 1$ by EUEF, implying in turn $x_3 \leq 1$ by FRRET. This is again impossible.

Third possibility: $x_1 = 1$. Then $x_3 = 5/4$ by EUEF, implying $x_4 \geq 1$ by FRRET. The latter entails $x_2 > 1$ by EUEF. And again it is impossible to have $x_1 + x_2 + x_3 + x_4 = 4$. ■

The axiom ***P**rotection of **H**andicapped* is satisfied by S_{ACE}, by $S_{\Phi CE}$ for some Φ, and by S_{MUD}. The axiom ***E**galitarian **B**ound* is satisfied by none of the above solutions.[22]

The literature provides few characterizations of allocation rules for this model. It is first necessary to provide the definition of an ancillary axiom, capturing the idea that the reallocation problem over a subpopulation should be correctly dealt with by the allocation rule applied to the whole population. ***C**onsistency* says that a suballocation of a selected allocation must also be selected in the subeconomy defined by the corresponding subgroup of agents and the total amount of money they receive in this suballocation.

Consistency (**C**, Thomson 1988):
$\forall e = (y_N, u_N, \Omega) \in \mathcal{D}$, $\forall G \subset N$, $\forall x_N \in S(e)$,

$$x_G \in S(y_G, u_G, \sum_{i \in G} x_i).$$

Proposition 3 *(Fleurbaey 1995d, Bossert and Fleurbaey 1996) An allocation rule S defined on $\mathcal{E}$ is single-valued and either satisfies* **I**ndependence of **U**tilities *and* **E**qual **U**tility

[22] Here is an example of a solution satisfying ***E**galitarian **B**ound* as well as ***E**qual **U**tility for **E**qual **F**unction* and ***O**rdinalism.* In $e = (y_N, u_N, \Omega)$, it chooses some $\tilde{y}$ and it selects allocations x_N such that: $\exists \tilde{x} \in \mathbb{R}$, $\forall i \in N$,

$$u_i(x_i, y_i) = u_i(\tilde{x} + \hat{x}_i, \tilde{y}) \text{ or } [u_i(x_i, y_i) > u_i(\tilde{x} + \hat{x}_i, \tilde{y}) \text{ and } x_i = 0].$$

where $\hat{x}_i$ is defined by $u_i(\hat{x}_i, \tilde{y}) = u_i(EUUF_i(y_N, \Omega, u_i), y_i)$.

for **R**eference **F**unction*, or satisfies* **C**onsistency*,* **E**qual **R**esource for **U**niform **T**alent *and* **E**qual **U**tility for **R**eference **F**unction *if and only if* $S = S_{\Phi CE}$ *for some constant function* Φ.

Proof. Assume S satisfies IU and EURF. Take $\tilde{u} \in \mathcal{U}$, as posited in EURF. Consider an economy such that for all i, $u_i = \tilde{u}$. In this economy, EURF requires selecting the only allocation such that

$$\forall i, j \in N, \quad \tilde{u}(x_i, y_i) = \tilde{u}(x_j, y_j) \text{ or } [\tilde{u}(x_i, y_i) < \tilde{u}(x_j, y_j) \text{ and } x_j = 0].$$

By IU, this allocation must be selected in any economy with the same profile y_N, whatever u_N. Therefore $S = S_{\Phi CE}$ with $\Phi(u_N) \equiv \tilde{u}$.

Now, one easily checks that C and ERUT imply IU for a single-valued allocation rule. Hence the second result.[23] ■

Proposition 4 *(Fleurbaey 1995d) An allocation rule* S *defined on* $\mathcal{E}'$ *is single-valued and satisfies* **C**onsistency*,* **E**qual **R**esource for **R**eference **T**alent *and* **E**qual **U**tility for **U**niform **F**unction *if and only if* $S = S_{\Psi EE}$ *for some constant function* Ψ.

Proof. On $\mathcal{E}'$, Egalitarian Equivalent allocations are such that for all i, $u_i(x_i, y_i) = u_i(\tilde{x}, \tilde{y})$, with $\tilde{x} > 0$ whenever $\Omega > 0$. Assume S satisfies C, ERRT and EUUF. Take some $\tilde{y} \in Y$, as posited in ERRT, and some $e = (y_N, u_N, \Omega) \in \mathcal{E}'$. Let x_N be an Egalitarian Equivalent allocation in e, such that $u_i(x_i, y_i) = u_i(\tilde{x}, \tilde{y})$ for all i. Construct the $2n$-agent economy $e' = ((y_N, \tilde{y}, ..., \tilde{y}), (u_N, u_N), \Omega + n\tilde{x})$. By C and EUUF, one must have for all $i \leq n$, $u_i(S_i(e'), y_i) = u_i(S_{i+n}(e'), \tilde{y})$. By C and ERRT, one must have for all $i, j > n$, $S_i(e') = S_j(e')$. These two conditions imply that for all $i > n$, $S_i(e') = \tilde{x}$, and that for all $i \leq n$, $u_i(S_i(e'), y_i) = u_i(\tilde{x}, \tilde{y})$. By C, therefore, $S(e) = x_N$. This implies that $S = S_{\Psi EE}$ with $\Psi(y_N) \equiv \tilde{y}$. ■

A noticeable feature of these results is that the characterized allocation rules satisfy ***O**rdinalism* while the axioms ***E**qual **R**esource for **U**niform **T**alent* and ***E**qual **R**esource for **R**eference **T**alent* do not by themselves imply it. It is an interesting feature of this literature that it gives an ethical justification to ordinalism, in addition to the traditional positivist justification underlying New Welfare Economics.[24]

There is no axiomatic study of social ordering functions for this model, with the exception of Maniquet (2004), who studies some weak axioms of compensation and natural reward and their consequences about how to measure individual welfare. For instance, the Φ-Conditional Equality ordering function $R_{\Phi CE}$ defined above relies on the leximin

[23] Fleurbaey (1995d) also characterizes S_{ACE} on the basis of EUUF and an axiom expressing that the allocation rule aggregates opinions about the individual talents. The result in Fleurbaey (1995d) is actually incorrect as stated, because under the guise of ***A**nonymity* the proof makes an implicit use of a third axiom saying that permutations of preferences only do not affect the selected allocation. This third axiom is independent of the others.

[24] In a similar analysis focused on social ordering functions, Maniquet (2004) shows how natural reward axioms entail ordinalism, under a consistency requirement.

criterion applied to individual indices measured by $\tilde{u}(x_i, y_i)$. Maniquet shows that, when combined with a consistency property, compensation and natural reward requirements imply that the social ordering function has the structure of a classical "welfarist" criterion applied to vectors of individual indices of well-being.

2.4 The quasi-linear case

The case when utility functions are quasi-linear is particularly simple and has been the topic of many papers.[25] It is assumed that utility functions are as follows:

$$u_i(x_i, y_i) = x_i + v_i(y_i).$$

As usual in the quasi-linear case, negative quantities of consumption x are allowed, and for simplification the total amount to be distributed is $\Omega = 0$. The quasi-linear case is particularly relevant to applications where u_i measures a monetary outcome. This version of the model is due to Bossert (1995), who described $v_i(y_i)$ as individual pre-tax income, x_i as an income transfer, and $u_i(x_i, y_i)$ as final income. This model is also relevant to other applications, for instance when agents are administrative units (local administrations, local branches of a national organization, etc.) and $v_i(y_i)$ is their initial budget balance, to be corrected by transfers x_i between units. See Subsection 4.7 for examples of applications.

Bossert (1995), and the subsequent literature, actually adopted a parameterized description of the utility functions:

$$u_i(x_i, y_i) = x_i + v(y_i, z_i).$$

This formulation is mathematically convenient, and also graphical in order to understand that the ethical goal is to neutralize inequalities due to y (principle of compensation) and to preserve inequalities due to z (principle of natural reward). We retain it in the sequel. It is then convenient to describe an economy by the pair of profiles $e = (y_N, z_N)$.

The domain of definition of allocation rules studied here is the set of economies with $n \geq 2$, $y_N \in Y^n$, $z_N \in Z^n$, where Y and Z are subsets of euclidean spaces, and v is a mapping from $Y \times Z$ to $\mathbb{R}$. Let $\mathcal{E}_{ql}$ denote this domain, $\mathcal{E}_{ql}^Y$ the same domain when Y is an interval of $\mathbb{R}$ and v is continuous and increasing in y, and $\mathcal{E}_{ql}^Z$ the same domain when Z is an interval of $\mathbb{R}$ and v is continuous and increasing in z. Let $\mathcal{E}_{ql}^{YZc}$ denote the subdomain of $\mathcal{E}_{ql}^Y \cap \mathcal{E}_{ql}^Z$ such that y and z are complementary, i.e. such that $v(y, z) - v(y', z)$ is non-decreasing in z for any given $y > y'$.[26]

The quasi-linear case provides a simpler framework for the formulation of many axioms and solutions. It also sheds more light on the trade-off between compensation and natural

[25] See Bossert (1995), Bossert and Fleurbaey (1996), Bossert, Fleurbaey and Van de gaer (1999), Cappelen and Tungodden (2002, 2003b), Iturbe (1997), Moulin (1994), Sprumont (1997) and Tungodden (2005).

[26] This assumption corresponds to the idea that the productivity of talent increases with effort, or equivalently that the productivity of effort increases with talent.

reward, and in particular reveals that the root of the problem, in this case, is the non-separability of v in y and z. When v is additively separable in y and z, then all the main solutions coincide and the tension between compensation and natural reward disappears. This is explained below.

2.4.1 Allocation rules

There are a few facts and notions specific to this particular domain.

First, the subdomain in which the No-Envy allocation rule is non empty now has a precise definition.

Proposition 5 *(Svensson 1983) For all $e \in \mathcal{E}_{ql}$, $S_{NE}(e)$ is non empty if and only if*

$$\forall \pi \in \Pi_N, \sum_{i \in N} v(y_i, z_i) \geq \sum_{i \in N} v(y_{\pi(i)}, z_i).$$

Second, the definitions of the allocation rules can be simplified because zero is no longer a lower bound to resources. We only provide here a sample of these more explicit definitions.

Φ-Conditional Equality ($S_{\Phi CE}$):
$\forall e \in \mathcal{E}_{ql}$,

$$(S_{\Phi CE})_i (e) = -v(y_i, \tilde{z}) + \frac{1}{n} \sum_{j=1}^{n} v(y_j, \tilde{z}).$$

where $\tilde{z} = \Phi(z_1, ..., z_n)$.

Ψ-Egalitarian Equivalent ($S_{\Psi EE}$):
$\forall e \in \mathcal{E}_{ql}$,

$$(S_{\Psi EE})_i (e) = -v(y_i, z_i) + v(\tilde{y}, z_i) + \frac{1}{n} \sum_{j=1}^{n} \left(v(y_j, z_j) - v(\tilde{y}, z_j)\right).$$

where $\tilde{y} = \Psi(y_1, ..., y_n)$.

Third, new allocation rules can be defined. The next one is similar to $S_{\Psi EE}$ in that it refers to a benchmark level of pre-transfer utility, $v(\tilde{y}, z_i)$, but instead of giving this level of utility to agent i plus an increment, it applies a proportional adjustment so as to meet the resource constraint. Notice that the idea of egalitarian-equivalence is lost in this operation.

Ψ-Proportionally Adjusted Equivalent ($S_{\Psi PAE}$, Iturbe 1997):
$\forall e \in \mathcal{E}_{ql}$,

$$(S_{\Psi PAE})_i (e) = -v(y_i, z_i) + \frac{\sum_{j=1}^{n} v(y_j, z_j)}{\sum_{j=1}^{n} v(\tilde{y}, z_j)} v(\tilde{y}, z_i),$$

where $\tilde{y} = \Psi(y_1, ..., y_n)$.

Bossert (1995) proposes an average version of this allocation rule. The *Average Proportionally Adjusted Equivalent* (S_{APAE}) allocation rule is constructed by computing the average of $S_{\Psi PAE}$ allocations with $\tilde{y} = y_j$ for every j, successively.

Finally, when the variable y or z is one-dimensional, it is possible to define the following allocation rules.

Balanced Egalitarian (S_{BE}, Sprumont 1997):
$\forall e \in \mathcal{E}_{ql}^{Y}$,

$$(S_{BE})_i (e) = -v(y_i, z_i) + v(\hat{y}, z_i),$$

where $\hat{y}$ is defined as the solution to

$$\sum_{j=1}^{n} v(y_j, z_j) = \sum_{j=1}^{n} v(\hat{y}, z_j).$$

Notice that this solution would belong to the family of $S_{\Psi EE}$ if the function Ψ could have (y_N, z_N) as its argument. The next solution is dual to this one, and similarly, it would belong to the family of $S_{\Phi CE}$ if the function Φ could have (y_N, z_N) as its argument.

Balanced Conditionally Egalitarian (S_{BCE}, Sprumont 1997):
$\forall e \in \mathcal{E}_{ql}^{Z}$,

$$(S_{BE})_i (e) = -v(y_i, \hat{z}) + \frac{1}{n} \sum_{j=1}^{n} v(y_j, z_j),$$

where $\hat{z}$ is defined as the solution to

$$\sum_{j=1}^{n} v(y_j, z_j) = \sum_{j=1}^{n} v(y_j, \hat{z}).$$

Cappelen and Tungodden (2002, 2003b) propose an allocation rule which splits the proceeds among agents with similar values of z. First rank the agents in such a way that $z_1 \leq ... \leq z_n$. The first agent then receives an equal split of the total v which would be obtained if all agents had the same z_1. The second agent also receives this plus an equal split of the additional v which would be obtained if all agents $i = 2, ..., n$ had the same z_2. And so on.

Serially Egalitarian (S_{SE}, Cappelen and Tungodden 2002):[27]
$\forall e \in \mathcal{E}_{ql}^{Z}$,

$$(S_{BE})_i (e) = -v(y_i, z_i) + \frac{1}{n} \sum_{j=1}^{n} v(y_j, z_1) + \sum_{j=2}^{i} \frac{1}{n-j+1} \sum_{k=j}^{n} \left[v(y_k, z_j) - v(y_k, z_{j-1}) \right].$$

All of these allocation rules are based on complex computations of $v(y, z)$ for combinations of characteristics which are not necessarily observed in the population. One can

[27] Cappelen and Tungodden (2003b) also introduce the symmetric rule which starts from z_n. There is also a dual to S_{SE}, which they do not consider, and which taxes agents in proportion to the v that would be obtained if individuals adopted $y_1 \leq ... \leq y_i$ in sequence.

define adapted versions of some of these allocation rules so as to rely only on the observed $v_i = v(y_i, z_i)$. Define the sets $N_y = \{i \in N \mid y_i = y\}$ and $N_z = \{i \in N \mid z_i = z\}$.

Observable Average Conditional Egalitarian (S_{OACE}, Bossert, Fleurbaey and Van de gaer 1999):
$\forall e \in \mathcal{E}_{ql}$,

$$(S_{OACE})_i(e) = -\frac{1}{\#N_{y_i}} \sum_{j \in N_{y_i}} v_j + \frac{1}{n} \sum_{j=1}^{n} v_j.$$

Observable Average Egalitarian Equivalent (S_{OAEE}, Bossert, Fleurbaey and Van de gaer 1999):
$\forall e \in \mathcal{E}_{ql}$,

$$(S_{OAEE})_i(e) = -v_i + \frac{1}{\#N_{z_i}} \sum_{j \in N_{z_i}} v_j.$$

2.4.2 Axioms

The definitions of some of the axioms can be simplified since zero is no longer a lower bound to resources. Moreover, quasi-linearity entails a property which is not generally true in the previous model (it served to define $\mathcal{E}'$ in the previous subsection):

$$\forall y, y' \in Y, \forall u \in U, \forall x \in \mathbb{R}, \exists x' \in \mathbb{R},\ u(x', y') \geq u(x, y),$$

and these two features guarantee that full equality of utilities is always possible. This simplifies the axioms of compensation. For instance, ***E**qual **U**tility for **E**qual **F**unction* now reads as follows:

Equal Utility for Equal Function (EUEF):
$\forall e \in \mathcal{E}_{ql}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$z_i = z_j \Rightarrow x_i + v(y_i, z_i) = x_j + v(y_j, z_j).$$

A similar simplification applies to ***E**qual **U**tility for **U**niform **F**unction* and ***E**qual **U**tility for **R**eference **F**unction*. The ***S**olidarity* axiom is also simplified somewhat.

Solidarity (S):
$\forall e = (y_N, z_N),\ e' = (y'_N, z_N) \in \mathcal{E}_{ql}$, $\forall x_N \in S(e)$, $\forall x'_N \in S(e')$,

$$\begin{aligned} \forall i &\in N,\ x_i + v(y_i, z_i) \geq x'_i + v(y'_i, z_i) \text{ or} \\ \forall i &\in N,\ x_i + v(y_i, z_i) \leq x'_i + v(y'_i, z_i). \end{aligned}$$

The literature has also introduced new, specific axioms. One of these, which is the dual counterpart of ***F**air **R**elative **R**esource for **E**qual **T**alent*, refers to the mean utility defined as: $\bar{v}(e) = (1/n) \sum_{i \in N} v(y_i, z_i)$, and requires that two agents with the same utility function should be similarly ranked with respect to the mean utility.

Fair Ranking for Equal Function (FREF, Sprumont 1997):
$\forall e \in \mathcal{E}_{ql}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$z_i = z_j \Rightarrow (x_i + v(y_i, z_i) - \bar{v}(e))\,(x_j + v(y_j, z_j) - \bar{v}(e)) \geq 0.$$

The next axiom is a strengthening of ***S****olidarity*, based on the argument that there is no reason to make some agents benefit unequally from variations in the profile, in particular, it would be undesirable to let an agent whose characteristics are improved to benefit more than other agents.

Additive Solidarity (**AS**, Bossert 1995):[28]
$\forall e = (y_N, z_N),\ e' = (y'_N, z_N) \in \mathcal{E}_{ql},\ \forall x_N \in S(e),\ \forall x'_N \in S(e'),$

$$\forall i, j \in N,\ x_i + v(y_i, z_i) - (x'_i + v(y'_i, z_i)) = x_j + v(y_j, z_j) - (x'_j + v(y'_j, z_j)).$$

At the opposite, a weaker version of the ***S****olidarity* axiom applies only when mean utility is unchanged.

Weak Solidarity (**WS**, Iturbe 1997):
$\forall e = (y_N, z_N),\ e' = (y'_N, z_N) \in \mathcal{E}_{ql},\ \forall x_N \in S(e),\ \forall x'_N \in S(e'),$

$$\bar{v}(e) = \bar{v}(e') \Rightarrow \forall i \in N,\ x_i + v(y_i, z_i) = x'_i + v(y'_i, z_i).$$

Another kind of solidarity requires proportional moves of the agents' final utilities:

Multiplicative Solidarity (**MS**, Iturbe 1997):
$\forall e = (y_N, z_N),\ e' = (y'_N, z_N) \in \mathcal{E}_{ql},\ \forall x_N \in S(e),\ \forall x'_N \in S(e'),$

$$\forall i, j \in N,\ [x_i + v(y_i, z_i)] \left[x'_j + v(y'_j, z_j)\right] = [x'_i + v(y'_i, z_i)] [x_j + v(y_j, z_j)] .^{29}$$

The axioms of natural reward defined in the previous model do not need any adaptation here. Notice that Ordinalism is built in the model since we make use only of the quasi-linear representation of the quasi-linear preferences.

The axiom of ***E****galitarian* ***B****ound* can be given a more explicit definition.

Egalitarian Bound (**EB**):
$\forall e = (y_N, z_N) \in \mathcal{E}_{ql},\ \forall x_N \in S(e),$

$$\forall i \in N,\ x_i \geq EUUF_i(y_N, z_i) \text{ or } \forall i \in N,\ x_i \leq EUUF_i(y_N, z_i),$$

where

$$EUUF_i(y_N, z_i) = -v(y_i, z_i) + \frac{1}{n}\sum_{j=1}^{n} v(y_j, z_i).$$

We now study the modified relationships between the axioms. One sees that the quasi-linear case displays a fuller duality between the axioms and solutions related to the principles of compensation on the one hand, natural reward on the other hand.

[28] An equivalent formulation of this axiom, adopted by Bossert (1995), refers to the change of only one agent's characteristics.

[29] When this product is different from zero, one then obtains that the agents' utilities change in the same proportion:

$$\frac{x_i + v(y_i, z_i)}{x'_i + v(y'_i, z_i)} = \frac{x_j + v(y_j, z_j)}{x'_j + v(y'_j, z_j)}.$$

Proposition 6 *The following table describes the logical implications between the axioms.*

Compensation		*Natural*	*Reward*
AS⇒S⇐MS			IU
⇓			
WS			⇓$^{(2)}$
⇓$^{(1)}$			
EUEF	⇐ NE$^{(3)}$ ⇒	PH⇒	ERET
⇓			⇓
FREF	⇓		FRRET
⇓			⇓
EUUF	⇐ EB ⇒		ERUT
⇓			⇓
EURF			ERRT

$^{(1)}$*Assuming that* S *satisfies* **A**nonymity.

$^{(2)}$*Assuming that* S *satisfies* **E**qual **T**reatment of **E**quals.

$^{(3)}$*Considered on the subdomain where it can be satisfied.*

The proof is similar to that of Prop. 1.[30]

Proposition 7 *The following table describes the incompatibilities (marked by* ⊗*) between the main axioms of compensation and natural reward, and shows what axioms are satisfied by the various solutions. The pairs of axioms* AS *and* ERUT*,* EUEF *and* ERET*,*

[30] Notice that a weak version of ***I**ndependence of **U**tilities*, limited to changes of z_N which do not change $\bar{v}(e)$, could play a dual role to ***W**eak **S**olidarity* in the table.

EUUF *and* IU *are compatible if and only if* v *is additively separable in* y *and* z.[31]

	IU	ERET	FRRET	ERUT	ERRT
AS	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$S_{\Psi EE}$ (Ψ *const.*)
MS	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$S_{\Psi PAE}$ (Ψ *const.*)
S,WS	$\otimes$	$\otimes$	$\otimes$ (S_{BE} *on* E^Y_{ql})	$\otimes$	
EUEF	$\otimes$	$\otimes$	$\otimes$	S_{AEE}, S_{APAE} S_{OAEE}, S_{SE} $S_{\Psi EE}$(Ψ *idemp.*)	
FREF	$\otimes$	$\otimes$ (S_{BCE} *on* $\mathcal{E}^Z_{ql}$)	$\otimes$		
EUUF	$\otimes$	S_{ACE}, S_{OACE} $S_{MUD}, S_{\Phi CE}$ (Φ *idempotent*)			
EURF	$S_{\Phi CE}$ (Φ *const.*)				

Proof. If v is not additively separable, there exist y, y', z, z' such that

$$v(y', z) - v(y, z) \neq v(y', z') - v(y, z').$$

These values will be used in various examples below.

AS and ERUT are incompatible if v is not additively separable. Consider an economy e with two agents 1 and 2 with profile, respectively, $(y, z), (y, z')$. By ERUT, one must have $x_1 = x_2 = 0$. Consider another economy e' with two agents and a new profile $(y', z), (y', z')$. By ERUT again, one must have $x'_1 = x'_2 = 0$. And by AS, one must have

$$x'_1 + v(y', z) - x_1 - v(y, z) = x'_2 + v(y', z') - x_2 - v(y, z'),$$

which is impossible.

EUUF and IU are incompatible if v is not additively separable. Consider an economy e with two agents 1 and 2 with profile, respectively, $(y, z), (y', z)$. By EUUF, one must have $x_1 + v(y, z) = x_2 + v(y', z)$. Consider another economy e' with two agents and a new profile $(y, z'), (y', z')$. By EUUF again, one must have $x'_1 + v(y, z') = x'_2 + v(y', z')$. And by IU, one must have $x'_1 = x_1$ and $x'_2 = x_2$,which is impossible.

EUEF and ERET are incompatible if v is not additively separable. Consider an economy with four agents 1 through 4 with profile, respectively, (y, z), (y, z'), (y', z) and (y', z'). By EUEF, one must have

$$\begin{aligned} x_1 + v(y, z) &= x_3 + v(y', z), \\ x_2 + v(y, z') &= x_4 + v(y', z'). \end{aligned}$$

[31] See also Cappelen and Tungodden (2004b) for another perspective on the conflict between compensation and natural reward.

By ERET, one must have $x_1 = x_2$ and $x_3 = x_4$. This is again impossible.

FREF and FRRET are incompatible (unlike the previous pairs of axioms, they may be compatible in some cases of non-separable v). Take an economy with four agents, a function $v((y,\hat{y}),(z,\hat{z})) = yz + \hat{y}.\hat{z}$. The profile of the parameters $y, \hat{y}, z, \hat{z}$ is as in the proof of Prop. 2. One computes $\bar{v}(e) = 4.5$. FREF then requires

$$\begin{cases} (x_1 + 0.5)(x_3 - 0.5) \geq 0 \\ (x_2 - 0.5)(x_4 + 0.5) \geq 0, \end{cases}$$

while FRRET requires

$$\begin{cases} x_1 x_2 \geq 0 \\ x_3 x_4 \geq 0. \end{cases}$$

Try $x_1 \geq 0$. This implies $x_3 \geq 0.5$, and $x_4 \geq 0$ (from $x_3 x_4 \geq 0$), and therefore $x_2 \geq 0.5$. This makes it impossible to achieve $x_1 + x_2 + x_3 + x_4 = 0$.

Try $x_1 < 0$. This implies $x_2 \leq 0$, and therefore $x_4 \leq -0.5$, and therefore $x_3 \leq 0$. Same contradiction. ■

In the domain $\mathcal{E}_{ql}$, the axiom ***E**galitarian **B**ound* is now satisfied by S_{AEE}.

Characterizations specific to this model have been provided. In Bossert and Fleurbaey (1996) it is shown that S_{ACE} is the only single-valued anonymous allocation rule defined on $\mathcal{E}_{ql}$ and satisfying ***E**qual **R**esource for **E**qual **T**alent*, ***E**qual **U**tility for **U**niform **F**unction* and an additional axiom stipulating that, when one agent k's characteristic z_k changes, the change in x_i registered by any i should not depend on the value of z_j for $j \neq i, k$. One motivation for such an axiom, which is logically weaker than ***I**ndependence of **U**tilities*, is that resource transfers for any agent should not be sensitive to how this agent's characteristic z is compared to the rest of the population. Similarly, S_{AEE} is the only single-valued anonymous allocation rule defined on $\mathcal{E}_{ql}$ and satisfying ***E**qual **R**esource for **U**niform **T**alent*, ***E**qual **U**tility for **E**qual **F**unction* and an additional axiom in the vein of ***S**olidarity* (and implied by it), stipulating that, when one agent k's characteristic y_k changes, the change in u_i registered by any i should not depend on the value of y_j for $j \neq i, k$.

It is also easy to check that an allocation rule satisfies ***A**dditive **S**olidarity* (resp. ***M**ultiplicative **S**olidarity*) and ***E**qual **R**esource for **R**eference **T**alent* if and only if it is a $S_{\Psi EE}$ (resp. $S_{\Psi PAE}$) with a constant Ψ (Bossert and Fleurbaey 1996, resp. Iturbe 1997), and that S_{BE} is the only allocation rule defined on $\mathcal{E}_{ql}^{Y}$ and satisfying ***W**eak **S**olidarity* and ***E**qual **R**esource for **U**niform **T**alent* (Iturbe 1997).

Bossert (1995) characterizes S_{APAE} as the only allocation rule satisfying ***E**qual **R**esource for **U**niform **T**alent* and an axiom saying that, when one agent k's characteristic y_k changes to y'_k, the change in u_i registered by any i should be equal to the difference $v(y'_k, z_i) - v(y_k, z_i)$, up to a multiplicative term depending on the profile and incorporating the feasibility constraint. The difference $v(y'_k, z_i) - v(y_k, z_i)$ represents the change in pre-tax income which would be obtained by i if this agent was submitted to the same change of characteristic as agent k. This axiom is logically stronger than ***E**qual **U**tility for **U**niform **F**unction* and may be viewed as expressing an idea of solidarity, although it is not compatible with ***W**eak **S**olidarity*.

Characterizations of S_{OACE} and S_{OAEE} are provided in Bossert, Fleurbaey and Van de gaer (1999).

In a model with a continuum of agents, Sprumont (1997) characterizes S_{BE} as the only single-valued allocation rule defined on a domain similar to $\mathcal{E}_{ql}^{Y}$ and satisfying ***Equal Utility for Equal Function*** and ***Fair Relative Resource for Equal Talent***, and he dually characterizes S_{BCE} as the only single-valued allocation rule defined on a domain like $\mathcal{E}_{ql}^{Z}$ and satisfying ***Equal Resource for Equal Talent*** and ***Fair Ranking for Equal Function***.

Tungodden (2005) and Cappelen and Tungodden (2002, 2003b) study weak variants of ***Independence of Utilities***, focusing on the issue of how other agents may be affected when an agent j changes his z_j. One variant says that ***Independence of Utilities*** applies only when the agent j changing z_j has $y_j = \widetilde{y}$. Combined with ***Equal Utility for Equal Function***, this immediately characterizes $S_{\Psi EE}$ with $\Psi \equiv \widetilde{y}$ on a subdomain with fixed y_N (see Tungodden 2005 and Cappelen and Tungodden 2003b). Another variant says that when an agent j changes z_j, all other agents i have their transfer x_i changed by the same amount (so that their differential outcomes do not change). This can be justified as an application of a solidarity principle, the other agents not being responsible for the change in z_j.[32] Yet another variant says that when z_j increases, then x_i does not decrease for any $i \neq j$, the idea being that an increase in effort should not hurt others. Among the $S_{\Psi EE}$ allocation rules, and on the domain $\mathcal{E}_{ql}^{YZc}$, this axiom is satisfied only by $S_{\Psi EE}$ with $\Psi(y_N) = \min_i y_i$, and Cappelen and Tungodden (2002) characterize it with this axiom, the previous one, ***Equal Utility for Equal Function*** and an ancillary condition restricting the outcome gap between any pair of agents to lie between the maximal and minimal productivity.[33] They also characterize $S_{\Psi EE}$ with $\Psi(y_N) = \max_i y_i$ with the symmetric axiom saying that when z_j increases, then x_i does not increase for any $i \neq j$. Finally, on the domain $\mathcal{E}_{ql}^{YZc}$ they characterize the serial rule S_{SE} with ***Equal Utility for Equal Function*** and an axiom saying that an increase in an agent's z_j does not affect the agents with $z_i \leq z_j$. Interestingly, these various axioms are all logically weaker than ***Independence of Utilities***, so that they are satisfied by $S_{\Phi CE}$ for constant Φ, but turn out to be also compatible with a high degree of compensation, as shown in the quoted results.

If v is additively separable so that $v(y,z) = v_1(y)+v_2(z)$, then ***Independence of Utilities*** and ***Additive Solidarity*** (and all weaker axioms) are compatible. They are satisfied by

[32] With this axiom and ***Equal Utility for Equal Function***, Cappelen and Tungodden (2004a) characterize a generalized version of egalitarian-equivalence, on a subdomain of $\mathcal{E}_{ql}^{Z}$ with fixed y_N:

$$(S_{GEE})_i(e) = -v(y_i, z_i) + r(z_i) + \frac{1}{n}\sum_{j=1}^{n}\left(v(y_j, z_j) - r(z_j)\right),$$

where $r : Z \to \mathbb{R}$ is an arbitrary function.

[33] This condition is: if $z_i > z_j$, then

$$v(\min y, z_i) - v(\min y, z_j) \leq S_i(e) + v(y_i, z_i) - S_j(e) - v(y_j, z_j) \leq v(\max y, z_i) - v(\max y, z_j).$$

the "canonical" allocation rule defined by

$$x_i = -v_1(y_i) + \frac{1}{n}\sum_{j=1}^{n} v_1(y_j),$$

which then coincides with $S_{\Phi CE}$, S_{ACE}, $S_{\Psi EE}$, S_{AEE}, S_{APAE}, S_{BE}, and S_{BCE}.[34] Notice that the characterizations mentioned in this subsection and the previous one are still valid in this particular context of additive separability.

3 Unequal productive skills

The second environment where properties of compensation and responsibility have been studied is the production environment.[35] It is defined with respect to a group of agents sharing a technology transforming one input (typically their labor) into one output. Agents may differ by their production skill and by their preferences towards labor-time-consumption bundles.[36]

The common ethical premiss of the literature surveyed in this section is that the principle of compensation applies to skills (e.g. skills are due to innate or inherited physical or intellectual abilities), whereas preferences are under the agents' responsibility.[37] In addition, it is also considered that different preferences do not justify any differential treatment, which is in line with the principle of natural reward.

We begin by defining the production model. Then, we define the basic compensation and natural reward axioms. As in the pure distribution problem studied above, incompatibilities between axioms reflecting the two principles arise. The remaining of the section is devoted to analyzing several ways out of these negative results. Compared to the previous section, substantially different concepts and results are presented, due to the particular structure of the production problem.

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[34] It coincides with $S_{\Psi PAE}$ for

$$\Psi(y_1, ..., y_n) = v_1^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} v_1(y_i)\right).$$

[35] This model has been introduced by Mirrlees (1971) and Pazner and Schmeidler (1974). The problem of compensation as such has been studied in this model by Bossert, Fleurbaey and Van de gaer (1999), Fleurbaey and Maniquet (1996a, 1998, 1999, 2001, 2002, 2005), Gaspart (1996, 1998), Kolm (1996a, 2004a,b), and Maniquet (1998).

[36] In a similar model, Moulin and Roemer (1989) study the very different problem of sharing a technology without compensation for any individual characteristic (self-ownership), but their result is akin to results in Fleurbaey and Maniquet (1999) because one of their axiom actually implies some compensation (see Fleurbaey and Maniquet (1999) for details).

[37] Contrary to the previous model which was more abstract, this model is unambiguously in line with Rawls' and Dworkin's view that individuals should assume responsibility for their preferences, even when such preferences are not under their control.

3.1 The model

There are two goods, an input contribution (labor time) ℓ and a consumption good c. An *economy* is a list $e = (s_N, u_N, f)$, where $N = \{1, ..., n\}$ is a finite population, s_i denotes agent i's production skill, u_i denotes agent i's utility function defined on bundles $x = (\ell, c)$, and $f : \mathbb{R}_+ \to \mathbb{R}_+$ is a one-input-one-output production function yielding a total production equal to $f\left(\sum_{i \in N} s_i \ell_i\right)$. The agents' consumption set is $X = [0, \overline{\ell}] \times \mathbb{R}_+$, where $\overline{\ell}$ is the maximal labor time which an agent can provide.

The relevant domain, denoted $\mathcal{E}$, consists of economies $e = (s_N, u_N, f)$ such that $n \geq 2$; for all $i \in N$, $s_i \geq 0$, and u_i is a continuous and quasi-concave function on X, non-increasing in ℓ and increasing in c; f is increasing and concave. Let $\mathcal{U}$ denote the set of utility functions satisfying the above conditions.

In an economy $e = (s_N, u_N, f) \in \mathcal{E}$, an *allocation* is a vector of bundles $x_N = (x_1, ..., x_n) \in X^n$. It is *feasible* for e if and only if

$$\sum_{i \in N} c_i \leq f\left(\sum_{i \in N} s_i \ell_i\right).$$

We denote by $F(e)$ the set of feasible allocations for $e \in \mathcal{E}$, and by $F_i(e)$ the projection of $F(e)$ on agent i's consumption set, for every $i \in N$. The literature has concentrated initially on allocation rules. As in the model studied in the previous section, however, most allocation rules obtained here can be rationalized by social ordering functions. Moreover, axioms directly bearing on social ordering functions have also been proposed, in the special case where the production function is linear. Let $\mathcal{E}^L \subset \mathcal{E}$ denote the subdomain of economies such that $f(q) = q$ for all $q \in \mathbb{R}_+$ (since skills s_N can always be renormalized this covers the more general case of linear production functions).

An *allocation rule* is a correspondence S which associates to every $e \in \mathcal{E}$ a non-empty subset of its feasible allocations $S(e) \subset F(e)$. A *social ordering function* is a mapping R associating every $e \in \mathcal{E}^L$ with a complete ordering $R(e)$ over the set of allocations X^n (and $P(e)$, resp. $I(e)$, denotes the related strict preference, resp. indifference, relation).

At this point, we may introduce some basic requirements which will be used repeatedly in the sequel. First, we have the usual (strong) Pareto efficiency requirement.

Pareto-Efficiency (PE):
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall x'_N \in F(e)$

$$[\forall i \in N,\ u_i(x'_i) \geq u_i(x_i)] \Rightarrow [\forall i \in N,\ u_i(x'_i) = u_i(x_i)].$$

For all $e \in \mathcal{E}$, let $PE(e)$ denote the set of Pareto-efficient allocations for e. Let us define the budget set $B(s, w, x) \subset X$ by

$$B(s, w, (\ell, c)) = \{(\ell', c') \in X \mid c' - ws\ell' \leq c - ws\ell\}.$$

This is the budget of an agent with skill s, who is just able to get bundle $x = (\ell, c)$ when w is the relative price of efficient labor. We will say that w is a supporting price for a

Pareto-efficient allocation $x_N \in PE(e)$ when:

$$\begin{cases} \forall i \in N,\ x_i \in \arg\max_{x \in B(s_i,w,x_i)} u_i(x), \\ \left(\sum_{i\in N} c_i, \sum_{i\in N} s_i \ell_i\right) \in \arg\max_{(Y,L):Y\leq f(L)} Y - wL. \end{cases}$$

Let $W(x_N)$ denote the set of supporting prices for x_N.

A second basic requirement is that the replica of a selected allocation be a selected allocation for the replicated economy.

Replication Invariance:
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall \nu$ positive integer,

$$\nu x_N \in S(\nu s_N, \nu u_N, f^\nu),$$

where νx_N means that x_N is replicated ν times (and similarly for νs_N and νu_N), and $f^\nu \in \mathcal{F}$ is defined by $f^\nu(q) = \nu f\left(\frac{q}{\nu}\right)$.

Third, we retain the simple horizontal equity requirement that two identical agents always reach the same welfare level.

Equal Treatment of Equals:
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$[s_i = s_j \text{ and } u_i = u_j] \Rightarrow [u_i(x_i) = u_j(x_j)].$$

Fourth, there is the requirement that a selected allocation remains selected after a contraction in the production set which leaves this allocation feasible.

Contraction Independence (**CI**, Moulin 1990):
$\forall e = (s_N, u_N, f), e' = (s_N, u_N, g) \in \mathcal{E}$, $\forall x_N \in S(e)$,

$$[\forall\, q \in \mathbb{R}_+,\ g(q) \leq f(q) \text{ and } x_N \in F(e')] \Rightarrow [x_N \in S(e')].$$

Finally, we have the requirement that if an allocation is selected, then all the allocations which are Pareto-indifferent to this allocation are also selected.

No Discrimination (**ND**, Thomson 1983):
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall x'_N \in F(e)$,

$$[\forall i \in N,\ u_i(x'_i) = u_i(x_i)] \Rightarrow [x'_N \in S(e)].$$

Like most of the literature on fair allocation, the literature reviewed here has focused on a restricted informational basis with ordinal non-comparable preferences. As in Section 2, we depart from it and retain utility functions here, in order to make it more transparent that this informational limitation is not arbitrary and is a consequence of the ethical requirements posited in this context, in particular those pertaining to the natural reward principle. All allocation rules presented below do satisfy the ***Ordinalism*** axiom (defined similarly as in Section 2).

3.2 Fairness in compensation and reward

In the model just defined, the skill parameter, for which agents should be compensated, is not an argument of preferences. But this is actually not a fundamental difference with the model of the previous section. Indeed, the current model may equivalently be written in terms of efficient labor $\hat{\ell}_i = s_i \ell_i$. The feasibility constraint is then simply $\sum_{i \in N} c_i \leq f\left(\sum_{i \in N} \hat{\ell}_i\right)$. More importantly, individual utility is then computed as

$$u_i(\ell_i, c_i) = u_i(\hat{\ell}_i / s_i, c_i),$$

where one sees that the parameter s_i does enter the utility function. The most relevant differences between the current model and the previous one are, actually, the following. First, the parameter s_i enters preferences in a special way (as a denominator to $\hat{\ell}_i$), and in particular, the fact that s_i is a real number, and that utility is always non-decreasing in it (in the domain $\mathcal{E}$), excludes any disagreement problem about how to rank individual talents. Second, transferable resources $(\hat{\ell}_i, c_i)$ are two-dimensional, which entails that Pareto-efficiency is no longer trivially satisfied. Third, resources can be transformed by production. The presence of production gives this model a particular structure with specific moral issues. For instance, several notions described in this section are based on the idea that consumption should be somehow proportional to labor, or minimally, that nobody should work for nothing.[38]

As a guide to the definition of proper conditions reflecting the principles of compensation and of natural reward, it is then convenient to apply the No-Envy condition to extended bundles $(\hat{\ell}_i, c_i, s_i)$. This yields:

$$\forall i, j \in N,\ u_i(\hat{\ell}_i / s_i, c_i) \geq u_i(\hat{\ell}_j / s_j, c_j).$$

Since $u_i(\hat{\ell}_j / s_j, c_j) = u_i(\ell_j, c_j)$, one obtains the ordinary No-Envy condition as it was applied by Pazner and Schmeidler (1974) in this context:

No-Envy (NE):
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall i, j \in N$, $u_i(x_i) \geq u_i(x_j)$.

Pazner and Schmeidler (1974) showed that in some economies of the domain $\mathcal{E}$, there does not exist any Pareto-efficient allocation satisfying the No-Envy condition. In other words, no allocation rule S satisfies the above ***No-Envy*** axiom and the ***Pareto-Efficiency*** axiom.[39] Again, it appears that this negative result is just a consequence of the tension between the principle of compensation and the principle of natural reward, which are both embodied in the No-Envy condition.

First, let us list basic axioms which weaken the ***No-Envy*** requirement and focus on only one of the two conflicting principles. In line with the compensation principle, one

[38] Another difference with the model of Section 2 is that one resource $(\hat{\ell}_i)$ is a bad. Besides, the consumption range for efficient labor differs between agents: $[0, s_i \bar{\ell}]$. This reduces the transferability of $\hat{\ell}$.

[39] In contrast with the model of the previous section, the ***No-Envy*** axiom is not empty. This is due to the fact that s_i enters the utility function $u_i(\hat{\ell}_i / s_i, y_i)$ in a special way. For instance, No-Envy is satisfied by giving the bundle $(\ell, c) = (0, 0)$ to all agents.

would like to compensate for differences in skills, so that two agents having the same utility functions reach the same welfare level. The axioms of ***E****qual* ***U****tility for* ***E****qual* ***F****unction*, ***E****qual* ***U****tility for* ***U****niform* ***F****unction* and ***E****qual* ***U****tility for* ***R****eference* ***F****unction* can be immediately adapted from the previous setting. We present one of them, in order to avoid any ambiguity.

Equal Utility for Equal Function (**EUEF**):[40]
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$[u_i = u_j] \Rightarrow [u_i(x_i) = u_j(x_j)].$$

Since external resources are multi-dimensional in this model, the natural reward principle can no longer be formulated in terms of a simple equality of resources between equally talented agents. But the No-Envy condition does express an idea of equality of multi-dimensional resources.[41] Therefore, weakening No-Envy by applying it only to equally skilled agents seems a good way of adapting the natural reward principle to the current setting.

No-Envy among Equally Skilled (**NEES**, Fleurbaey and Maniquet 1996a):
$\forall\, e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$[s_i = s_j] \Rightarrow [u_i(x_i) \geq u_i(x_j)].$$

In the same vein, one can then define an axiom of ***N****o-****E****nvy among* ***U****niformly* ***S****killed* (NEUS).

Interestingly, however, there is a problem with defining an axiom based on some arbitrary reference skill. For any economy with a uniform strictly positive skill s, indeed, it is possible to rescale the production function in such a way that the set of feasible bundles is the same as in another economy with any other uniform skill s'.[42] Presumably,

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[40]Working only with ordinal non comparable preferences, Fleurbaey and Maniquet (1996a, 1999) actually used the following axiom, which says that agents with identical preferences should have bundles on the same indifference curve (R_i denotes agent i's preference relation, I_i denotes indifference):
Equal Welfare for Equal Preferences:
$\forall e \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall i, j \in N$,

$$[R_i = R_j] \Rightarrow [x_i\, I_i\, x_j].$$

The "Equal Welfare" label is a little misleading since it suggests that interpersonal comparisons of utilities are smuggled in the analysis. Actually, this axiom is a direct consequence of ***E****qual* ***U****tility for* ***E****qual* ***F****unction* and of the ***O****rdinalism* axiom. Indeed, two agents with identical preferences could have the same utility function, in which case ***E****qual* ***U****tility for* ***E****qual* ***F****unction* would require giving them equal utilities, that is, in this case, giving them bundles on the same indifference curve. Under ***O****rdinalism*, this latter consequence must still hold when the agents have identical preferences but different utility functions. Therefore, this axiom of ***E****qual* ***W****elfare for* ***E****qual* ***P****references* is the correct translation of ***E****qual* ***U****tility for* ***E****qual* ***F****unction* to a setting with ordinal non-comparable preferences.

[41]In addition, as noticed by Kolm (1972, 1996b), requiring No-Envy among two agents is equivalent to requiring that there must be some common opportunity set over which these agents could choose their preferred bundles.

[42]Define $g(q) = f(qs/s')$. Then $g(\sum_i s'\ell_i) = f(\sum_i s\ell_i)$ for all ℓ_N. An allocation x_N is feasible in $e = ((s, ..., s), u_N, f)$ if and only if it is feasible in $e' = ((s', ..., s'), u_N, g)$.

we are interested only in allocation rules which are neutral to such rescaling. Under such a scale invariance constraint, an axiom of No-Envy among agents with a reference skill, similar to ***E**qual **R**esource for **R**eference **T**alent* of the previous section, would actually be equivalent to ***N**o-**E**nvy among **U**niformly **S**killed*, for any strictly positive reference skill. It has not been noted in the literature, however, that if the reference skill is equal to zero, the requirement is independent of any rescaling of skills and production function.[43] In addition, one may argue that the case when all agents have zero skill is particularly telling. If no agent is productive, then we have a pure distribution economy as in the previous model, and it seems clear that an equal sharing of the unproduced resource $f(0)$ is the only reasonable allocation, as recommended by ***N**o-**E**nvy*, when none of them works as will be the case in Pareto-efficient allocations (when labor has some disutility). It would be very strange to discriminate between agents on the basis of their preferences over labor, when they do not work.

No-Envy among Zero Skilled (**NEZS**):
$\forall\, e = (s_N, R_N, f) \in \mathcal{E},\, x_N \in S(e)$,

$$[\forall i \in N,\ s_i = 0] \Rightarrow [\forall i, j \in N,\ x_i\, R_i\, x_j].$$

One may also define stronger compensation and natural reward properties. The ***S**olidarity* axiom of the previous section can be applied here rather directly. ***S**kill **S**olidarity* is consistent with the idea of a collective sharing in the benefits of skills. It requires that all the agents be affected in the same direction if the profile of personal skills changes.

Skill Solidarity (**SS**, Fleurbaey and Maniquet 1999):
$\forall\, e = (s_N, u_N, f), e' = (s'_N, u_N, f) \in \mathcal{E},\, \forall x_N \in S(e),\, \forall x'_N \in S(e')$,

$$\forall i \in N,\ u_i(x_i) \geq u_i(x'_i) \text{ or } \forall i \in N,\ u_i(x'_i) \geq u_i(x_i).$$

Strong natural reward axioms like ***I**ndependence of **U**tilities* cannot be directly transposed to the current model, because with multi-dimensional external resources such axioms would conflict with ***P**areto-**E**fficiency*. But some independence of changes in the profile of utility functions is achievable under ***P**areto-**E**fficiency*. This is done in particular by the ***M**onotonicity* axiom. This axiom, introduced in Maskin (1977), requires that a selected allocation remain in the selection after a change in one agent's preferences whenever this change enlarges her lower contour set at her assigned bundle. ***M**onotonicity* has played an important role in the implementation literature. It is quite interesting to find a connection between incentive compatibility conditions, which require the allocation rule not to be sensitive to personal characteristics which the agents can easily conceal or misrepresent, and natural reward conditions, which justify a disregard of personal characteristics for which the agents are responsible.

[43] An alternative solution to this difficulty is to limit application of the No-Envy condition to agents with a reference value of "wage rate" ws_i, where w is a supporting price. This is particularly natural in the subdomain $\mathcal{E}^L$, where, in any given economy, the supporting price is the same for all Pareto-Efficient allocations. See Fleurbaey and Maniquet (2005), and the definition of $S_{\Psi EE}$ below.

Monotonicity (**M**, Maskin 1977):
$\forall\, e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall i \in N$, $\forall u_i' \in \mathcal{U}$,

$$[\forall x \in F_i(e),\ u_i(x_i) \geq u_i(x) \Rightarrow u_i'(x_i) \geq u_i'(x)] \Rightarrow x_N \in S\left(s_N, (u_{N\setminus\{i\}}, u_i'), f\right).$$

When the allocation rule satisfies ***E**qual **T**reatment of **E**quals*, ***M**onotonicity* implies ***N**o-**E**nvy among **E**qually **S**killed*.[44] ***M**onotonicity* also implies ***O**rdinalism*, as shown by Maskin.

Several properties have been studied in the literature which are even stronger than ***M**onotonicity* (see e.g. Nagahisa (1991)). The following property, introduced in Gaspart (1996), requires that a selected allocation remain in the selection after a change in the agents' utility functions and in the skill profile whenever the allocation is Pareto-efficient in the new economy. This axiom implies ***M**onotonicity* for Pareto-efficient allocation rules. Interestingly, this axiom also has a flavor of compensation, since the selection is also required to be independent, in some cases, of changes in the skill profile.[45]

Pareto Preserving Independence (**PPI**, Gaspart 1996, 1998):
$\forall\, e = (s_N, u_N, f)$, $e' = (s_N', u_N', f) \in \mathcal{E}$, $\forall x_N \in S(e)$,

$$[x_N \in PE(e')] \Rightarrow [x_N \in S(e')].$$

We may now present the general structure of this set of axioms.

[44] This is a consequence of a result in Fleurbaey and Maniquet (1997). The intuition for the proof is as follows. Let $e = (s_N, u_N, f)$. Suppose $s_i = s_j$ and $x_N \in S(e)$ is such that $u_i(x_i) < u_i(x_j)$, i.e. i envies j. Then one can find a function $u^* \in \mathcal{U}$ such that $u^*(x_i) < u^*(x_j)$ and:

$$\begin{aligned} \forall x &\in X,\ u_i(x_i) = u_i(x) \Rightarrow u^*(x_i) = u^*(x), \\ \forall x &\in X,\ u_j(x_j) \geq u_j(x) \Rightarrow u^*(x_j) \geq u^*(x). \end{aligned}$$

By Monotonicity, $x_N \in S(s_N, (u_{N\setminus\{i,j\}}, u^*, u^*), f)$. Since $u^*(x_i) \neq u^*(x_j)$, this violates ***E**qual **T**reatment of **E**quals*. By a similar argument, one can show that ***M**onotonicity* and ***E**qual **U**tility for **E**qual **F**unction* imply ***N**o-**E**nvy*.

[45] An axiom of independence of skill levels is used in Yoshihara (2003). This axiom says that $S(e) = S(e')$ whenever $F(e) = F(e')$ and e, e' differ only in the skill profile. The condition $F(e) = F(e')$ is never obtained when f is increasing and the axiom is used by Yoshihara only for cases of constant f. Closer to Gaspart's axiom, Yamada and Yoshihara (2004) introduce an axiom which says that a selected allocation remains selected whenever the skills of non-working agents change without altering the Pareto efficiency of the allocation.

Proposition 8 *The following table describes the logical implications between axioms:*

Compensation		*Natural Reward*
		PPI
		$\Downarrow^{(3)}$
SS		M $\Rightarrow$O
$\Downarrow^{(1)}$		$\Downarrow^{(2)}$
EUEF	$\Leftarrow$ NE $\Rightarrow$	NEES
$\Downarrow$		$\Downarrow$
EUUF		NEUS
$\Downarrow$		$\Downarrow$
EURF		NEZS

$^{(1)}$*Assuming that S satisfies* **A**nonymity.
$^{(2)}$*Assuming that S satisfies* **E**qual **T**reatment of **E**quals.
$^{(3)}$*Assuming that S satisfies* **P**areto-**E**fficiency.

The following allocation rules, studied in Fleurbaey and Maniquet (1996a, 1999) and Gaspart (1996, 1998), are important when studying the possible combinations of compensation and natural reward properties. First, the Egalitarian Equivalent rule selects Pareto-efficient allocations having the property that there is some consumption level c_0 such that all agents are indifferent between their assigned bundle and consuming c_0 without working at all.

Egalitarian Equivalence (S_{EE}, Kolm 1968,[46] Pazner and Schmeidler 1978a):
$\forall e = (s_N, u_N, f) \in \mathcal{E}$,

$$S_{EE}(e) = \{x_N \in PE(e) \mid \forall i \in N, \exists\, c_0 \in \mathbb{R}_+,\ u_i(x_i) = u_i(0, c_0)\}$$

This rule is just a member of more general family, defined as follows. An allocation is selected if it is Pareto-efficient and every agent's utility is equal to her indirect utility over a reference budget (the same for all agents).

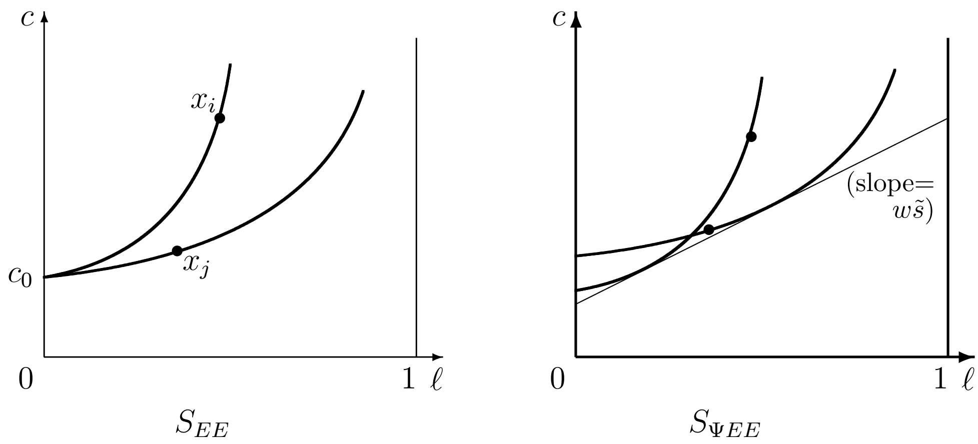
Budget Egalitarian Equivalence ($S_{\Psi EE}$):
$\forall\, e = (s_N, R_N, f) \in \mathcal{E}$,

$$S_{\Psi EE}(e) = \left\{ \begin{array}{l} x_N \in PE(e) \mid \exists w \in W(x_N), \exists x_0 \in X, \\ \forall\, i \in N,\ u_i(x_i) = \max u_i\left(B(\tilde{s}, w, x_0)\right) \end{array} \right\},$$

where $\tilde{s}$ is such that $w\tilde{s} = \Psi(ws_1, ..., ws_n)$.[47]

[46]Kolm (1968) attributes this idea to Lange (1936), who proposed to compensate workers for the disutility of their particular jobs, in his scheme of market socialism. But it is not clear in Lange's writings that such compensation would render all workers indifferent to one and the same bundle $(0, c_0)$.

[47]The function Ψ bears on the ws_i, because these values are independent of any joint rescaling of the skills and the production function that leaves unchanged the set of feasible allocations.



- Figure 1: S_{EE} and $S_{\Psi EE}$ -

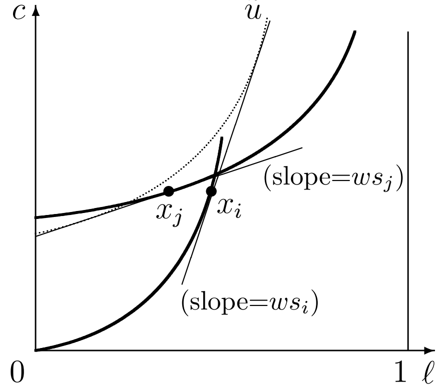
One has $S_{\Psi EE} = S_{EE}$ when $\Psi \equiv 0$. Among the $S_{\Psi EE}$ family, S_{EE} is the most favorable to agents who have a strong aversion to labor, by making all indifference curves cross at a point $(0, c_0)$. This automatically entails envy on behalf of "hard-working" agents toward "lazy" agents (whose indifference curve will lie everywhere above the "hard-working" agents' curve, except at $(0, c_0)$). The other $S_{\Psi EE}$ rules favor other preferences, depending on Ψ. Since $w\tilde{s}$ is the slope of the reference budget in the (ℓ, c)-space, the greater $w\tilde{s}$ the better it is for agents who are willing to work. These allocation rules are therefore not very neutral with respect to preferences.

Second, the Conditional Equality rule selects Pareto-efficient allocations having the property that an agent with reference preferences would be indifferent between having to choose among any of the budget sets of all the agents. This is a rather immediate adaptation of the similar solution from the previous section.

Conditional Equality ($S_{\Phi CE}$, Fleurbaey and Maniquet 1996a):
$\forall e = (s_N, R_N, f) \in \mathcal{E}$,

$$S_{\Phi CE}(e) = \left\{ \begin{array}{c} x_N \in PE(e) \mid \exists w \in W(x_N), \forall\, i, j \in N, \\ \max \tilde{u}\,(B(s_i, w, x_i)) = \max \tilde{u}\,(B(s_j, w, x_j)) \end{array} \right\},$$

where $\tilde{u} = \Phi(u_1, ..., u_n)$.



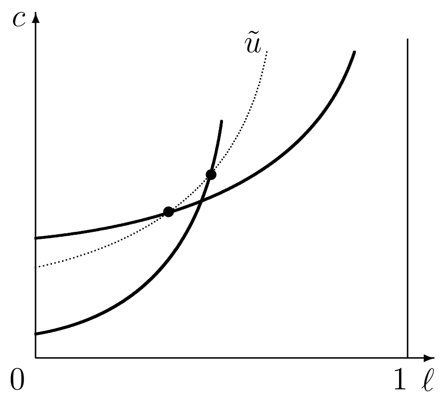
- Figure 2: $S_{\Phi CE}$ -

Third, Objective Egalitarianism selects Pareto-efficient allocations having the property that an agent with reference preferences would be indifferent between the bundles assigned to all agents. This is another possible adaptation of the Conditional Equality rule of the previous section.

Objective Egalitarianism ($S_{\Phi OE}$, Gaspart 1996, 1998):
$\forall e = (s_N, R_N, f) \in \mathcal{E}$,

$$S_{\Phi OE}(e) = \{x_N \in PE(e) \mid \forall i, j \in N,\ \tilde{u}(x_i) = \tilde{u}(x_j)\},$$

where $\tilde{u} = \Phi(u_1, ..., u_n)$.



- Figure 3: $S_{\Phi OE}$ -

It is worth examining the redistributive consequences of the choice of $\tilde{u}$ in $S_{\Phi CE}$ and $S_{\Phi OE}$. With a function $\tilde{u}$ displaying a strong aversion to labor, $S_{\Phi CE}$ selects allocations which perform little redistribution from high-skilled to low-skilled. Indeed, all individual budget sets being equivalent for $\tilde{u}$, they must then be similar at low levels of labor, which corresponds to a situation where profits of the firm are approximately equally divided, and no compensation is made for differential skills. Conversely, with a $\tilde{u}$ displaying a very low aversion to labor, $S_{\Phi CE}$ selects allocations which are very favorable to the low-skilled, and quite unfavorable to the high-skilled.

For $S_{\Phi OE}$ the consequences are not exactly the same. With a $\tilde{u}$ displaying a very low aversion to labor, for instance, consumption levels in selected allocations will be substantially equalized, which is favorable to low-skilled agents, but also, among equally skilled agents of any skill level, favorable to agents with a strong aversion to labor (they will work less, without being penalized in terms of consumption). Conversely, a $\tilde{u}$ displaying a strong aversion to labor is favorable to the high-skilled but also to the hard-working agents. This shows that Objective Egalitarianism is not really neutral.

All the rules satisfy ***P**areto-**E**fficiency*, ***R**eplication **I**nvariance* and ***C**ontraction **I**ndependence*, and all but $S_{\Phi OE}$ satisfy ***E**qual **T**reatment of **E**quals* and **N**o **D**iscrimination.

The following proposition examines how these rules fare in terms of compensation and natural reward, and also depicts the conflict between these two principles.

Proposition 9 *The following table describes the incompatibilities (marked by* $\bigotimes$*) between the main axioms of compensation and natural reward, under the assumption that the allocation rule satisfies* **P**areto-**E**fficiency*. The table also shows what axioms are satisfied by the various solutions. Rule* $S_{\Phi OE}$ *does not satisfy* NEES*,* NEUS *or* NEZS.[48]

	PPI	M	NEES	NEUS	NEZS
SS	$\bigotimes$	$\bigotimes$	$\bigotimes$	$\bigotimes$	S_{EE}
EUEF	$\bigotimes$	$\bigotimes$	$\bigotimes$	$S_{\Psi EE}$ (Ψ *idempotent*)	
EUUF	$\bigotimes$	$\bigotimes$	$S_{\Phi CE}$ (Φ *idempotent*)		
EURF	$S_{\Phi OE}$ (Φ *constant*)	$S_{\Phi CE}$ (Φ *constant*)			

Proof. We focus on the impossibilities and omit the rest.

SS and NEUS are incompatible. Let $e = ((1,1,1),(u_1,u_2,u_3),f) \in \mathcal{E}$ be defined as follows: $u_1(\ell,c) = c - \ell/10$, $u_2(\ell,c) = c - 9\ell/10$, $u_3(\ell,c) = c - \ell$, $f(q) = \min\{q,2\}$, and $\overline{\ell} = 1$. Let S satisfy the axioms. Let $x_N \in S(e)$. By PE, $\ell_1 = \ell_2 = 1$, $\ell_3 = 0$ and $c_1 + c_2 + c_3 = 2$. By NEUS, $c_1 = c_2$ and $c_3 \leq c_2 - 9/10$. Therefore $c_1 \geq c_3 + 9/10$, and replacing c_3 by $2 - 2c_1$, one obtains $c_1 \geq 29/30$. Now, consider $e' = ((20,20,20),(u_1,u_2,u_3),f) \in \mathcal{E}$ and $x'_N \in S(e')$. By PE, $\ell'_1 = 1/10$, $\ell'_2 = \ell'_3 = 0$ and $c'_1 + c'_2 + c'_3 = 2$. By NEUS, $c'_2 = c'_3$ and $c'_2 \geq c'_1 - 9/100$. Replacing c'_2 by $1 - c'_1/2$, one obtains $c'_1 \leq 218/300$. The problem is that $u_1(1/10, 218/300) < u_1(1, 29/30)$, which implies that $u_1(x'_1) < u_1(x_1)$, in contradiction with SS (by PE, at least one of the agents is better off in x'_N).

EUEF and NEES are incompatible. Let $e = ((s_1,s_2,s_3,s_4),(u_1,u_2,u_3,u_4),f) \in \mathcal{E}$ be defined as follows: $s_1 = s_2 = 0$, $s_3 = s_4 = 1$, $u_1(\ell,c) = u_3(\ell,c) = c - \ell/4$ and $u_2(\ell,c) = u_4(\ell,c) = c - \ell/2$, $f(q) = q$ and $\overline{\ell} = 1$. Let S satisfy the axioms. Let $x_N \in S(e)$. By PE, $\ell_1 = \ell_2 = 0$ and $\ell_3 = \ell_4 = 1$; by EUEF, $c_1 = c_3 - 1/4$ and $c_2 = c_4 - 1/2$. By NEES, $c_1 = c_2$ and $c_3 = c_4$, which is incompatible with the previous equalities.

M and EUUF are incompatible. Let $e = ((s_1,s_2),(u_1,u_2),f) \in \mathcal{E}$ be defined as follows: $s_1 = 0$, $s_2 = 1$, $u_1(\ell,c) = c - \ell/4$ and $u_2(\ell,c) = c - \ell/2$, $f(q) = q$ and $\overline{\ell} = 1$. Let S satisfy the axioms. Let $x_N \in S(e)$. By PE, $\ell_1 = 0$ and $\ell_3 = 1$. Therefore, by M, $x_N \in S(s_N,(u_1,u_1),f)$ and $x_N \in S(s_N,(u_2,u_2),f)$. By EUUF, in $(s_N,(u_1,u_1),f)$ one must have $c_1 = c_2 - 1/4$, whereas in $(s_N,(u_2,u_2),f)$ one must have $c_1 = c_2 - 1/2$. These two equalities cannot be satisfied simultaneously by x_N. ■

In the above proof of the incompatibility between ***E****qual* ***U****tility for* ***E****qual* ***F****unction* and ***N****o-****E****nvy among* ***E****qually* ***S****killed*, a very weak part of ***N****o-****E****nvy among* ***E****qually* ***S****killed* has been used, namely, the condition that among two equally skilled agents, no one must have more consumption than the other, when they work the same. This shows how strong

[48] Notice that, since it does not satisfy ***E****qual* ***T****reatment of* ***E****quals*, the fact that it satisfies ***M****onotonicity* does not guarantee that it satisfies these other axioms.

the incompatibility with ***E**qual **U**tility for **E**qual **F**unction* is. Since the ***E**qual **U**tility for **E**qual **F**unction* and ***N**o-**E**nvy among **E**qually **S**killed* axioms involved in this theorem are logical consequences of the No-Envy axiom, the negative result obtained by Pazner and Schmeidler (1974) about the existence of Pareto-efficient and envy-free allocations is just a corollary of this proposition, and the difficulties with No-Envy can be explained by the fact that this requirement combines compensation and natural reward in a too demanding way.[49]

The lack of neutrality of S_{EE}, $S_{\Psi EE}$ and $S_{\Phi OE}$ with respect to individual preferences, which has been noticed above, can be explained by the fact that these rules fail to satisfy important natural reward axioms, as stated in the proposition.

Characterization results have been provided for some of these rules. Some of them involve a Consistency axiom which is the proper adaptation of the Consistency defined above to the current framework. Its precise definition is somewhat technical and is omitted here.[50]

Proposition 10 *(Fleurbaey and Maniquet 1999) The* Egalitarian Equivalent rule S_{EE} *is the smallest rule, with respect to inclusion, satisfying* **P**areto **E**fficiency*,* **E**qual **U**tility for **E**qual **F**unction*,* **N**o-**E**nvy among **Z**ero **S**killed, **N**o **D**iscrimination*, and* **C**onsistency*.*[51]

Proposition 11 *(Fleurbaey and Maniquet 1996a) If S satisfies* **P**areto **E**fficiency*,* **E**qual **U**tility for **R**eference **F**unction, **M**onotonicity*,* **N**o **D**iscrimination *and* **C**ontraction **I**ndependence*, then there is a constant* Φ *such that for all* $e \in E$, $S_{\Phi CE}(e) \subseteq S(e)$*. The same result is obtained if* **M**onotonicity *is replaced by* ***N**o-**E**nvy among **E**qually **S**killed and* **C**onsistency*.*

Proof. The structure of the proofs of the results involving Consistency is very similar to that of Propositions 3 and 4 above. We only provide the proof of the first part of the latter proposition.

Let S satisfy PE, EURF, M, ND and CI. Let $e \in \mathcal{E}$. Let $\tilde{u} \in \mathcal{U}$ be such that EURF is satisfied for it, and consider the constant function $\Phi \equiv \tilde{u}$. Let $x_N \in S_{\Phi CE}(e)$ with a

[49] As an alternative direction of research, one can also try and identify subdomains of economies where the incompatibilities presented above do not hold. For instance, Piketty (1994) identified a domain restriction under which ***No-Envy*** and ***Pareto-Efficiency*** are compatible. Any $e = (s_N, u_N, f) \in \mathcal{E}^L$ satisfies this domain restriction if and only if for all $i, j \in N$ such that $s_i \leq s_j$, for all $x, x' \in X$ such that $u_i(x) = u_i(x')$, $x' < x \Rightarrow u_j(x) \geq u_j(x')$.

This condition, which says that the less productive agents have a lower willingness to work as well, is quite strong. It implies the Spence-Mirrlees single crossing condition (usually defined on earnings-consumption bundles), but is much stronger.

[50] When one considers a subeconomy, one has to delete the consumption and labor of the rest of the population. This deletion may render the resulting production function non-concave. The consistency condition can then only be applied when the reduced economy is still in the domain $\mathcal{E}$. See Fleurbaey and Maniquet (1996a, 1999) for details, and also Thomson (1988), Moulin and Shenker (1994).

[51] See Fleurbaey and Maniquet (1999), Prop. 7. This proposition relies, instead of ***No-Envy among Zero Skilled***, on ***Work Alone Lower Bound***, an axiom defined in Subsection 3.4. But ***No-Envy among Zero Skilled*** would play the same role in the proof as ***Work Alone Lower Bound***.

supporting price $w \in W(x_N)$. Let $e' = (s_N, u'_N, g) \in \mathcal{E}$ be defined by: for all $i \in N$, $u'_i(\ell, c) = c - s_i w\ell$, and $g(q) = \sum_{i\in N}(c_i - s_i w\ell_i) + wq$.

We claim that $S(e') = S_{\Phi CE}(e')$. Suppose not. Let $x'_N \in S(e') \setminus S_{\Phi CE}(e')$. Let x''_N be defined by: for all $i \in N$, $x''_i \in \arg\max_{x\in B(s_i,w,x'_i)} \tilde{u}(x)$. One has $x''_N \in F(e')$. By ND, $x''_N \in S(e')$. By M, $x''_N \in S(s_N, (\tilde{u}, \ldots, \tilde{u}), g)$, violating EURF because by construction $x''_N \notin S_{\Phi CE}(s_N, (\tilde{u}, \ldots, \tilde{u}), g)$, so that there are i, j such that $\tilde{u}(x''_i) \neq \tilde{u}(x''_j)$. Therefore $S(e') \subseteq S_{\Phi CE}(e')$. By ND again, one actually has $S(e') = S_{\Phi CE}(e')$.

By ND, $x_N \in S(e')$ as well. By M, $x_N \in S(s_N, u_N, g)$. By CI, one finally concludes that $x_N \in S(e)$. ■

As in the monetary compensation problem, the allocation rules can be interpreted as derived from social ordering functions which rationalize them. In particular, the following social ordering function rationalizes the Egalitarian Equivalent rule S_{EE}. It applies the leximin criterion to numerical representations of preferences which are constructed by measuring the distance between the $(0, 0)$ bundle and the intersection of an indifference curve with the axes.

Egalitarian Equivalent ordering function (R_{EE}, Fleurbaey and Maniquet 2005): $\forall\, e \in \mathcal{E}$, $\forall x_N, x'_N \in X^n$,

$$x_N\, R_{EE}(e)\, x'_N \Leftrightarrow (v_i(x_i))_{i\in N} \geq_{\text{lex}} (v_i(x'_i))_{i\in N}$$

where for all $i \in N$, $v_i(x)$ is defined by $u_i(x) = u_i(0, v_i(x))$ or $u_i(x) = u_i(-v_i(x), 0)$.

The definition of the appropriate social ordering functions rationalizing $S_{\Phi CE}$ and $S_{\Phi OE}$ over $\mathcal{E}$ is yet an open question, but things are easier for $S_{\Phi CE}$ on the domain $\mathcal{E}^L$. For this domain, one can rely on the notion of implicit budget $IB(s, u, x)$, which is the smallest budget set which enables an agent with skill s and utility function u to obtain the utility level $u(x)$. For $s \in \mathbb{R}_+$, $u \in \mathcal{U}$, $x \in X$, the implicit budget $IB(s, u, x)$ is defined by

$$IB(s, u, x) = \{(\ell', c') \in X \mid c' - s\ell' \leq e_u(s, u(x))\},$$

where $e_u(.,.)$ is the expenditure function

$$e_u(s, v) = \min\{c - s\ell \mid (\ell, c) \in X,\ u(\ell, c) \geq v\}.$$

Notice that the bundle x need not belong to the implicit budget $IB(s, u, x)$. Then, we have the following social ordering function, which rationalizes $S_{\Phi CE}$ by applying the leximin criterion to the implicit budgets of the agents evaluated with the reference utility function.

Conditional Equality ordering function ($R_{\Phi CE}$, Fleurbaey and Maniquet 2005): $\forall\, e \in \mathcal{E}^L$, $\forall x_N, x'_N \in X^n$,

$$[x_N\, R_{\Phi CE}(e)\, x'_N] \Leftrightarrow (\max \tilde{u}\,(IB(s_i, u_i, x_i)))_{i\in N} \geq_{\text{lex}} (\max \tilde{u}\,(IB(s_i, u_i, x'_i)))_{i\in N}\ ,$$

where $\tilde{u} = \Phi(u_1, ..., u_n)$.

Fleurbaey and Maniquet (2005) provide characterizations of these and other social ordering functions in the domain $\mathcal{E}^L$, relying on axioms which are inspired by the above axioms of compensation and natural reward. We do not go into details here, and only explain how the idea of compensation can suggest various axioms, which bear interesting relations with other parts of the literature.

The ***E**qual **U**tility for **E**qual **F**unction* axiom, for allocation rules, requires equalizing utilities of agents having the same utility function. A social ordering function, similarly, may be required to be averse to welfare inequality among agents having the same utility functions. This may be captured by the following property, which is essentially an application of Hammond's equity axiom (Hammond 1976), expressing an infinite degree of inequality aversion. The difference with Hammond's equity axiom is the restriction to agents with identical utility functions.

Hammond Compensation (Fleurbaey and Maniquet 2005):
$\forall e = (s_N, u_N, f) \in \mathcal{E}^L$, $\forall x_N, x'_N \in X^n$, if there exist $i, j \in N$ such that:
i) $\forall k \in N$, $k \neq i, j \Rightarrow x_k = x'_k$,
ii) $u_i = u_j$,
iii) $u_j(x_j) > u_j(x'_j) > u_i(x'_i) > u_i(x_i)$,
then $x'_N \, P(e) \, x_N$.

As shown in Fleurbaey and Maniquet (2002), one can also rely on the Pigou-Dalton principle of transfer in order to express the same idea, albeit with a milder (arbitrarily small) degree of inequality aversion. The Pigou-Dalton principle is usually applied in studies of one-dimensional inequalities in income.[52] Here income (c) inequality is also reduced, but the principle is applied only to agents with identical utility functions and with identical and unchanged labor (ℓ).[53]

Pigou-Dalton Compensation (Fleurbaey and Maniquet 2002):
$\forall e = (s_N, u_N, f) \in \mathcal{E}^L$, $\forall x_N, x'_N \in X^n$, if there exist $i, j \in N$ and $\delta > 0$ such that:
i) $\forall k \in N$, $k \neq i, j \Rightarrow x_k = x'_k$,
ii) $u_i = u_j$,
iii) $\ell_i = \ell'_i = \ell_j = \ell'_j$,
iv) $c'_j = c_j - \delta > c_i + \delta = c'_i$,
then $x'_N \, P(e) \, x_N$.

The study of social ordering functions is particularly relevant for applications of the ethical principles of compensation and natural reward to contexts where the Pareto-efficient allocation rules cannot be implemented. The most important case for applications is income taxation, when neither s_i nor ℓ_i is observed by the redistributive agencies and redistribution must be made only on the basis of earned income $s_i \ell_i$. Such applications are discussed below.

[52] See e.g. Sen (1973).

[53] With less restrictions, this axiom would clash with the Pareto principle. See e.g. Fleurbaey and Trannoy (2003).

3.3 Bundle equality and welfare lower bounds

The incompatibility between compensation and natural reward has been dealt with above by weakening the requirements. One may find, however, that the weakenings of the main axioms ***E**qual **U**tility for **E**qual **F**unction* and ***N**o-**E**nvy among **E**qually **S**killed*, presented above, are unsatisfactory. For instance, ***E**qual **U**tility for **E**qual **F**unction* requires welfare equality, that is, an infinite inequality aversion, whenever two agents have identical utility functions. By weakening it into either ***E**qual **U**tility for **U**niform **F**unction* or ***E**qual **U**tility for **R**eference **F**unction*, we have kept the infinite inequality requirement but restricted its application in a sharp way. In two approaches presented now, the infinite inequality aversion is dropped. In the first approach, developed by Gaspart (1996), the emphasis is put on bundle equality. In the second approach, developed by Maniquet (1998), the emphasis is put on opportunity sets of bundles. The two approaches also have in common that each axiom is now intended to simultaneously capture (imperfect) compensation and natural reward requisites.

We begin with two examples of properties in the first approach. These properties are consistent with the idea that bundles should be equalized (though not at the expense of efficiency). ***N**o **D**omination by the **A**verage **B**undle* requires that no bundle be composed of both a larger labor time than the average labor time and a lower consumption than the average consumption. The idea is that no agent should so badly treated (e.g. because of a low skill —compensation— or a particular utility function —natural reward) that he must work more than average for a lower than average consumption.

No Domination by the Average Bundle (Gaspart 1996):
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$, $\forall i \in N$,

$$\ell_i \leq \frac{\sum_{j \in N} \ell_j}{\#N} \text{ or } c_i \geq \frac{\sum_{j \in N} c_j}{\#N}.$$

The axiom of ***S**election of **E**fficient **E**galitarian **A**llocations* requires that whenever a Pareto-efficient allocation composed of equal bundles exists, it is selected. When all agents have the same bundle, it seems clear that no advantage is given to anyone on the basis of his skill (compensation) or utility function (natural reward). Equality of resources and No-Envy cannot be more simply and clearly satisfied.

Selection of Efficient Egalitarian Allocations (Gaspart 1996):
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in F(e)$, $\forall x \in X$,

$$[\forall i \in N,\ x_i = x \text{ and } x_N \in PE(e)] \Rightarrow [x_N \in S(e)].$$

As explained above, bundle equality is consistent with responsibility for one's utility function and the principle of natural reward. In addition, by considering labor time-consumption bundles (ℓ, c) rather than efficient labor-consumption bundles $(\hat{\ell}, c)$, the two axioms above also convey an idea of skill compensation.

It is interesting to examine the compatibility scheme between these axioms and the compensation and responsibility properties previously presented. The results are presented in the following theorem. Two main lessons may be drawn from those results.

First, axioms based on the desirability of material equality are more easily combined with responsibility and reward properties than with compensation properties. Secondly, they highlight a new solution, the proportional allocation rule.

Proposition 12 *(Gaspart 1996, 1998) There exist* Pareto-efficient *allocation rules satisfying* **R**eplication **I**nvariance*,* **C**ontraction **I**ndependence*, and each of the following lists of axioms:*
1. **N**o **D**omination by the **A**verage **B**undle*,* **S**election of **E**fficient **E**galitarian **A**llocations *and* **P**areto **P**reserving **I**ndependence*;*
2. **S**election of **E**fficient **E**galitarian **A**llocations *and **E**qual **U**tility for **E**qual **F**unction.*

On the other hand, no Pareto-efficient *allocation rule can satisfy any of the following pairs of axioms:*
1. **N**o **D**omination by the **A**verage **B**undle *and* **E**qual **U**tility for **R**eference **F**unction*;*
2. **S**election of **E**fficient **E**galitarian **A**llocations *and* **S**kill **S**olidarity*.*

The first statement of the theorem is proved by an example. The Proportional rule selects Pareto-efficient allocations having the property that the consumption-labor ratio c_i/ℓ_i is identical among agents.

The Proportional Rule (S_P, Roemer and Silvestre 1989, Gaspart 1998):
$\forall e = (s_N, u_N, f) \in \mathcal{E}$,

$$S_P(e) = \{x_N \in PE(e) \mid \exists r \geq 0, \forall i \in N,\ c_i = r\ell_i\}.$$

The Proportional rule satisfies ***N**o **D**omination by the **A**verage **B**undle*, ***S**election of **E**fficient **E**galitarian **A**llocations* and ***P**areto **P**reserving **I**ndependence*. This rule fails to satisfy ***N**o **D**iscrimination* and ***E**qual **T**reatment of **E**quals*. It is given axiomatic characterizations, on the basis of ***P**areto **P**reserving **I**ndependence* but also by reference to the family of solutions belonging to Objective Egalitarianism (defined above), in Gaspart (1998).

The compatibility between ***S**election of **E**fficient **E**galitarian **A**llocations* and ***E**qual **U**tility for **E**qual **F**unction* is an immediate consequence of the fact that a Pareto-efficient egalitarian allocation often fails to exist in economies where two agents have equal preferences but unequal skills. Therefore, these two requirements bear on two essentially disjoint sets of economies.[54]

Now, we turn to the second approach, due to Maniquet (1998), in which allocation rules are required to guarantee "*equal rights*". The idea is to grant all agents an identical opportunity set in X, and to require the allocation rule to give every agent a bundle which she weakly prefers to her best choice in the opportunity set. The common opportunity set in X is called an equal *right*, since every agent has a definite right to be as well-off as if her choice was really to be made in this opportunity set.

[54] As an example, consider the rule which selects egalitarian allocations whenever they are efficient, and coincides with S_{EE} otherwise. This rule satisfies ***S**election of **E**fficient **E**galitarian **A**llocations* and ***E**qual **U**tility for **E**qual **F**unction*, in addition to ***P**areto **E**fficiency*, ***R**eplication **I**nvariance* and ***C**ontraction **I**ndependence*.

This equal right is, formally, a compact subset of X. Moreover, Maniquet (1998) suggests that, as an unconditional right, this opportunity set should not depend on the profile u_N. Feasibility constraints impose, however, to take account of s_N and f. In an economy, $e = (s_N, u_N, f)$, an equal right must then be a subset $E(s_N, f) \subset X$ such that $E(s_N, f)^n \in F(e)$.

The guarantee of equal right then takes the form of the following axiom. Following Maniquet (1998), we restrict the domain to the set $\mathcal{E}_+ \subset \mathcal{E}$ of economies where agents have a strictly positive skill.[55]

Guarantee of equal right E (Maniquet 1998):
$\forall e = (s_N, u_N, f) \in \mathcal{E}_+$, $\forall x_N \in S(e)$,

$$\forall i \in N,\ u_i(x_i) \geq \max u_i(E(s_N, f)).$$

Let us note that if an equal right E does not satisfy the feasibility constraint $E(s_N, f)^n \in F(e)$, then it cannot be guaranteed by any allocation rule.

An equal right is compensating low skilled agents, since the lower bound levels $\max u_i(E(s_N, f))$ guaranteed to two agents having the same utility function are the same. Moreover, in line with natural reward, it leaves the agents responsible for their choice over the opportunity set, so that the difference between the minimal welfare levels guaranteed to two agents having the same skill only reflects their different utility function. The compensation it carries out, however, may be rather low, if the equal right itself is small. This suggests trying to find large equal rights.

Let us summarize the main results obtained by this approach. First, an equal right cannot be very large. In other words, we come back here to the fundamental trade-off between compensation and natural reward. Second, an equal right can easily be combined with strong requirements of either skill compensation or natural reward. Third, when combining equal rights with skill compensation or natural reward properties, we come back, essentially, to standard allocation rules.

Looking for an equal right which is as large as possible, Maniquet (1998) shows that a prominent family of equal right correspondences $E(.,.)$ is the following. Let $\ell^\star \in [0, \overline{\ell}]$ be given.

$\ell^\star$-equal right (Maniquet 1998):

$$E_{\ell^\star}(s_N, f) = \left\{ (\ell, c) \in X \,|\, \ell \geq \ell^\star,\ c \leq \frac{1}{n} f\left(\sum_i s_i \ell^\star \right) \right\}.$$

Imposing some $\ell^\star$-equal right is compatible with almost any compensation or natural reward requirement. Let $\mathcal{E}_+^{\overline{\ell}} \subset \mathcal{E}_+$ be the subdomain such that for every $e = (s_N, u_N, f) \in \mathcal{E}_+^{\overline{\ell}}$, every $i \in N$, every $x \in X$, there exists $c' \in \mathbb{R}_+$ such that $u_i(\overline{\ell}, c') \geq u(x)$, that is, working maximal time $\overline{\ell}$ can always be compensated by enough consumption.

[55] This additional assumption simplifies the analysis, since it implies that the sets of weak and strong Pareto efficient allocations coincide. Moreover, the two last results mentioned in this subsection hold true only if this assumption is made.

Proposition 13 *(Maniquet 1998) Let $\ell^\star \in [0, \overline{\ell}]$ be given. There exist allocation rules guaranteeing the* $\ell^\star$-equal right *which satisfy* **P**areto-**E**fficiency*,* **R**eplication **I**nvariance*,* **E**qual **T**reatment of **E**quals*,* **C**ontraction **I**ndependence*, and each one of the following axioms or lists of axioms:*
1. **S**kill **S**olidarity *over $\mathcal{E}_+^{\overline{\ell}}$;*
2. ***E**qual **U**tility for **E**qual **F**unction* *and* **N**o-**E**nvy among **U**niformly **S**killed*;*
3. **E**qual **U**tility for **U**niform **F**unction *and* **N**o-**E**nvy among **E**qually **S**killed *over $\mathcal{E}_+^{\overline{\ell}}$;*
4. **M**onotonicity*.*

We restrict our attention to examples of compatibility 1 and 4. The $\ell^\star$-Egalitarian Equivalent rule $S_{\ell^\star EE}$ selects Pareto-efficient allocations having the property that there is some consumption level c_0 such that all agents are indifferent between their assigned bundle and the $(\ell^\star, c_0)$ bundle.

$\ell^\star$-Egalitarian Equivalent rule ($S_{\ell^\star EE}$, Maniquet 1998):
$\forall e = (s_N, R_N, f) \in \mathcal{E}_+^{\overline{\ell}}$,

$$S_{\ell^\star EE} = \{x_N \in PE(e) \mid \exists\, c_0 \in \mathbb{R}_+, \forall i \in N,\ u_i(x_i) = u_i(\ell^\star, c_0)\}.$$

The Egalitarian Equivalent rule S_{EE} is obviously an element of the $S_{\ell^\star EE}$ family of rules. For a given $\ell^\star$, the $S_{\ell^\star EE}$ rule satisfies ***P**areto-**E**fficiency*, ***S**kill **S**olidarity*, ***C**onsistency*, ***R**eplication **I**nvariance*, ***N**o **D**iscrimination* and guarantees the *$\ell^\star$-Equal Right.* Moreover, Maniquet (1998) shows that it is the only one to satisfy such properties on a relevant domain (extended to non-concave production functions). The argument is similar to the proof of Proposition 10.

The $\ell^\star$-equal budget rules select Pareto-efficient allocations having the property that all budget lines cross at a point of abscissa $\ell^\star$.

$\ell^\star$-Equal Budget rule ($S_{\ell^\star EB}$, Kolm 1996a, Maniquet 1998):
$\forall e = (s_N, R_N, f) \in \mathcal{E}_+$,

$$S_{\ell^\star EB}(e) = \left\{ \begin{array}{c} x_N \in PE(e) \mid \exists w \in W(x_N), \forall i, j \in N, \\ c_i - s_i w(\ell_i - \ell^\star) = c_j - s_j w(\ell_j - \ell^\star) \end{array} \right\}.$$

Notice that for any e, there is Φ such that $S_{\ell^\star EB}(e) = S_{\Phi CE}(e)$. Therefore $S_{\ell^\star EB}$ can almost be seen as a member of the Conditional Equality family of allocation rules. This rule satisfies ***P**areto-**E**fficiency*, ***M**onotonicity*, ***C**onsistency*, ***R**eplication **I**nvariance*, ***N**o **D**iscrimination* and guarantees the *$\ell^\star$-Equal Right.* Moreover, Maniquet (1998) shows that it is the smallest one, with respect to inclusion, to satisfy this list of axioms, on the subdomain of $\mathcal{E}_+$ with differentiable production functions. The argument is similar to the proof of Proposition 11.

Kolm (1996a, 2004a,b) defends the $\ell^\star$-Equal Budget rule as intuitively appealing, since it corresponds to an equal sharing of the agents' earnings $s_i\ell_i$ on the first $\ell^\star$ units of labor, the rest being left to every agent. Kolm suggests that $\ell^\star/\overline{\ell}$ can be interpreted as the degree of egalitarianism of the allocation rule, with respect to earnings. With $\ell^\star/\overline{\ell} = 0$, the $\ell^\star$-Equal Budget rule coincides with Varian's (1974) Equal Wealth rule, consisting in an equal sharing of profits and no sharing of earnings. With $\ell^\star/\overline{\ell} = 1$, the allocation coincides with Pazner and Schmeidler's (1978b) Full-Income-Fair rule, with all budget lines crossing at $\ell = \overline{\ell}$.

The choice of $\ell^\star$ for the $\ell^\star$-Egalitarian Equivalent rule has radically different implications. Whatever $\ell^\star$, this rule performs a substantial compensation at the benefit of low-skilled agents. But with a low $\ell^\star$, $S_{\ell^\star EE}(e)$ is more favorable to the agents who have a strong aversion to labor, and conversely with a high $\ell^\star$. For instance, it is clearly better for an agent with strong aversion to labor to have all indifference curves crossing at $\ell^\star = 0$ than at a greater $\ell^\star$. Unsurprisingly, this rule therefore fails to be neutral with respect to various kinds of individual preferences. This is just a consequence of the fact that, by being good at compensation, this rule is not so good in terms of natural reward.

3.4 Limited self-ownership

The compensation principle may be criticized for contradicting the idea that agents, in particular high-skilled agents, should be free to take advantage of their skill, at least to the extent that society should not force them to work to the benefit of low skilled agents. We will show that some of the proposed solutions pass some reasonable tests of self-ownership.

Pazner and Schmeidler's (1978b) Full-Income-Fair rule, which is at the same time an $\ell^\star$-Equal Budget rule with $\ell^\star = \overline{\ell}$ and a Conditional Equality rule with $\tilde{u}$ defined by: $\tilde{u}(\ell, c) = c$, has been criticized by Varian (1974) and Dworkin (1981b), on the basis that this rule is very hard for the skilled agents, since they are heavily taxed, and may actually be forced to work in order to be just able to pay taxes and have a zero consumption (the "slavery of the talented"). On the other hand, Varian's (1974) Equal Wealth rule, which is the $\ell^\star$-Equal Budget rule with $\ell^\star = 0$, does not display the same shortcoming, but, as noticed above, does not perform any compensation of skill inequalities.

We can formalize the criticism by noting that the Full-Income-Fair rule does not satisfy the following participation property.

Participation:
$\forall e = (s_N, u_N, f) \in \mathcal{E}, \forall x_N \in S(e), \forall i \in N,\ u_i(x_i) \geq u_i(0, 0).$

Actually, this criticism can be addressed to almost all members of the Conditional Equality and $\ell^\star$-Equal Budget families. Moreover, a similar criticism can be addressed to almost all $\ell^\star$-Egalitarian Equivalent rules, even if in this case agents who are likely to be worse off than at $(0, 0)$ are the "lazy" agents (that is, agents with a low willingness to work), rather than the talented.

Proposition 14 *(Fleurbaey and Maniquet 1996a, Maniquet 1998) The* Equal Wealth *rule is the only Conditional Equality rule and the only ℓ^*-Equal Budget rule satisfying* **P**articipation. *The* Egalitarian Equivalent *rule is the only* $\ell^\star$-Egalitarian Equivalent *rule satisfying* **P**articipation.

Since the Equal Wealth rule simply selects the equal-dividend laisser-faire allocations and does not perform any compensation for skill inequalities, this result is quite negative for the families of Conditional Equality rules and $\ell^\star$-Equal Budget rules. In contrast, the Egalitarian Equivalent rule does satisfy substantial compensation axioms.

The participation welfare level is quite low and ***Participation*** is consistent with a very limited idea of self-ownership. A higher guarantee is provided by the following property.

Work alone lower bound (**WALB**, Fleurbaey and Maniquet 1999):
$\forall e = (s_N, u_N, f) \in \mathcal{E}$, $\forall x_N \in S(e)$,

$$u_i(x_i) \geq EUUF_i(e),$$

where

$$EUUF_i(e) = \max\left\{\bar{u} \mid \exists x'_N \in F(e), \forall j \neq i, \ell_j = 0,\ \bar{u} = u_i(x'_i) = u_i(x'_j)\right\}.$$

This axiom guarantees that no agent is worse off than in the hypothetical situation in which he alone works and feeds the other agents just enough to be indifferent between his bundle and theirs. The two rules identified in the previous theorem satisfy this individual rationality constraint. Moreover, we have the following results.

Proposition 15 *(Fleurbaey and Maniquet (1999)) The* Egalitarian Equivalent *rule is the only rule satisfying* **P**areto-**E**fficiency*,* **N**o **D**iscrimination*,* **S**kill **S**olidarity *and* **W**ork **A**lone **L**ower **B**ound*.*

Proof. Let S satisfy the axioms. First, we prove that for all $e \in \mathcal{E}$, $S(e) \subseteq S_{EE}(e)$. Suppose not. Then there is $e \in \mathcal{E}$, $x_N \in S(e) \setminus S_{EE}(e)$. Recall v_i from the definition of the Egalitarian Equivalent ordering function R_{EE}. By PE, $\max_{i \in N} v_i(x_i) \geq 0$. Since $x_N \notin S_{EE}(e)$, for some $j \in N$, $u_j(0, \max_{i \in N} v_i(x_i)) > u_j(x_j)$. Let $s'_j \in \mathbb{R}_+$ and $e' = ((0, \ldots, s'_j, \ldots, 0), u_N, f)$ be chosen so that there is $x'_N \in PE(e')$ such that for all $i \in N$, $u_i(x'_i) = u_i(x_i)$ (such a s'_j can be found, by continuity of the u_i and of f). By PE, SS, and ND, $x'_N \in S(e')$, violating WALB for j.

Second, we prove that $S(e) = S_{EE}(e)$. By SS, S is single-valued in utility. Therefore, by ND, $S(e) = S_{EE}(e)$. ■

Proposition 16 *The* Equal Wealth *rule is the smallest allocation rule satisfying* **C**ontraction **I**ndependence*,* **N**o **D**iscrimination*,* **M**onotonicity *and* **W**ork **A**lone **L**ower **B**ound*.*

The proof is similar to that of Proposition 11, noting that in a linear economy like $e' = (s_N, u'_N, g)$ defined there, ***Work Alone Lower Bound*** imposes that for all $j \in N$, $u'_j(x'_j) \geq u'_j(0, \frac{1}{\#N}\sum_{i \in N}(c_i - s_i w \ell_i))$.

4 The utilitarian approach to responsibility

We now turn to the literature which has adopted a radically different approach, based on the utilitarian reward principle.[56] This literature combines egalitarian and utilitarian

[56] As mentioned in the introduction, an early application of an unmodified utilitarian criterion to the issue of compensation has been made by Arrow (1971), and Sen (1973) emphasized the fact that this can lead to transfers in the wrong direction, because handicapped agents are likely to have a lower marginal utility. A more egalitarian approach, but still with comparable welfare, has been studied by Otsuki (1996).

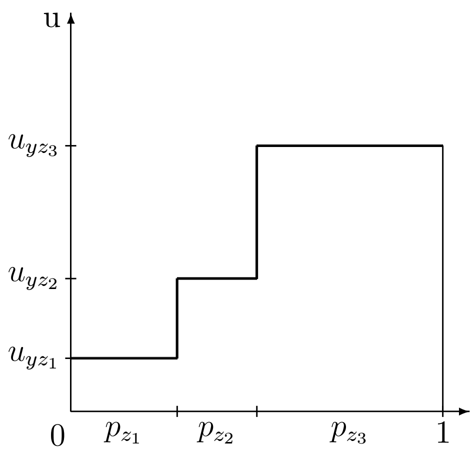
social welfare functions in the construction of a new kind of complex social welfare functions. Such social welfare functions display inequality aversion only in the dimensions of differential talents and circumstances that call for compensation.[57]

4.1 Framework

A distinctive feature of this approach is that it disregards the economic structure of the allocation problem and even the functional form of the function $u(x, y, z)$, in order to focus on the outcomes eventually obtained by agents with different characteristics (y, z). The objects of evaluation are, typically, functions or matrices $(u_{yz})_{y \in Y, z \in Z}$ where u_{yz} is the outcome (an index of utility, or functionings, etc.) achieved by agents with talent y and effort z. For simplicity we will assume that y and z are independently distributed.[58] Let p_y (resp. p_z) be the proportion of the population with talent y (resp. effort z). Let $u_y = (u_{yz})_{z \in Z}$ (a row-vector) and $u_z = (u_{yz})_{y \in Y}$ (a column-vector). One can interpret u_y as describing the "opportunities" open to agents with talent y, and one can even think of a related opportunity set as defined by (assuming non-negative values for outcomes)

$$O_y = \{u \in \mathbb{R}_+^Z \mid u \leq u_y\}.$$

A graphical representation of such a set, that takes account of the distribution of z in the population, is presented in Figure 4.



- Figure 4 -

In this setting, the compensation principle remains applicable and implies that inequalities within vectors u_z are bad. On the other hand, in this abstract framework it is impossible to formulate the idea of equal resources or of neutrality with respect to z, so that natural reward cannot even be conceived. It is then understandable that most of the

[57]The literature reviewed in this section includes Bossert, Fleurbaey and Van de gaer (1999), Goux and Maurin (2002), Hild and Voorhoeve (2004), Mariotti (2003), Ooghe and Lauwers (2003), Ooghe, Schokkaert and Van de gaer (2003), Peragine (2002, 2004), Roemer (1993, 1998, 2002), Schokkaert and Van de gaer (1994), Van de gaer (1993).

[58]Roemer constructs z so that this always holds. See below.

literature has adopted a utilitarian kind of reward, translating the idea that inequalities within vectors u_y are acceptable into the idea that the social evaluation may have no aversion to inequality over such vectors.

Recall that one consequence of natural reward was ordinalism, discarding information about utilities as distinct from purely ordinal preferences. Another distinctive feature of the approach reviewed here is that individual preferences are no longer sufficient information, and a cardinally measurable and interpersonally comparable index of achievement u is needed and plays a key role in the evaluation of social situations.

One may wonder whether this framework should be described as welfarist (in the formal sense[59]). It occupies a sort of middle position. A pure welfarist approach would simply rank vectors $(u_i)_{i\in N}$ and would take account neither of individual characteristics y_i, z_i nor of resource allocation. This framework ranks matrices $(u_{yz})_{y\in Y, z\in Z}$ but still ignores how the individuals are treated in terms of resources. These differences can be illustrated with Example 1.

Example 1 (pursued). Recall that $u = (x + y)\, z$, where x, y and z are real numbers. Suppose that each category of individuals contains only one person and that the four individuals have the following characteristics in two different situations:

$y \setminus z$	1	3
1	individual 1	individual 2
3	individual 3	individual 4
Situation S1		

$y \setminus z$	1	3
1	individual 1	individual 3
3	individual 2	individual 4
Situation S2		

Suppose that one declares policy B to be better than policy A in situation S1. Then, under pure welfarism, one should also declare policy B̄ to be better than policy Ā in situation S2 (if they were feasible), because they yield the same distribution of utilities as policies B and A, respectively:

Policy Ā	
i	x and u
1	$x = 5$ $u = 6$
2	$x = 15$ $u = 18$
3	$x = 1$ $u = 6$
4	$x = 3$ $u = 18$

Policy B̄	
i	x and u
1	$x = 2$ $u = 3$
2	$x = 24$ $u = 27$
3	$x = 0$ $u = 3$
4	$x = 5$ $u = 27$

In contrast, by looking at the distribution of u_{yz} one is able to say that policy B̄ is worse than Ā in situation S2, because it offers individuals with a low y the grim opportunities $(3, 3)$ instead of $(6, 6)$ with Ā.

[59] Welfarism of the formal sort is compatible with any subjective or objective interpretation of the utility index u. A more substantive (or philosophical) brand of welfarism focuses on subjective utility. On this distinction, see Mongin and d'Aspremont (1998), d'Aspremont and Gevers (2002).

Now, consider the alternative technology $u = (x + y)z^2/2$. Under this technology, the same distributions of u_{yz} as with policies A and B can be obtained with the following policies:

Policy Â

$y \setminus z$	1	3
1	$x = 11$ $u = 6$	$x = 3$ $u = 18$
3	$x = 9$ $u = 6$	$x = 1$ $u = 18$

Policy B̂

$y \setminus z$	1	3
1	$x = 5$ $u = 3$	$x = 5$ $u = 27$
3	$x = 3$ $u = 3$	$x = 3$ $u = 27$

If one looks only at the distributions of u_{yz}, one is forced to rank Â and B̂ in the same way as A and B. In contrast, a non-welfarist criterion can prefer A to B and B̂ to Â, because A and B̂ are more neutral with respect to individuals' exercise of responsibility. One has to look at the allocation of x in order to see that B strongly rewards a high z, while Â strongly punishes it.

4.2 Social welfare functions

Two prominent social welfare functions have been proposed in order to rank distributions of u_{yz}. Van de gaer (1993) introduced the following:

$$\sum_{y \in Y} p_y \; \varphi \left(\sum_{z \in Z} p_z u_{yz} \right),$$

where φ is a concave function. The idea is to compute the average outcome for each class of y, which can also be interpreted as the population-weighted area of the opportunity set O_y :

$$\sum_{z \in Z} p_z u_{yz}$$

(this is the area delineated in Figure 4) and the concavity of φ is meant to embody a social aversion to inequality of such average outcomes. Schokkaert and Van de gaer (1994) and Peragine (2002, 2004)[60] rely on this criterion in order to study various types of Lorenz dominance of distributions of characteristics.

Here we will focus on the maximin version of Van de gaer's social welfare function, which exhibits an infinite aversion to inequalities across y, which is better in line with the

[60] Van de gaer (1993) actually considered a continuum, so that his social welfare function writes

$$\int_Y \varphi \left(\int_Z u_{yz} g(z) dz \right) f(y) dy,$$

where $f(y)$ and $g(z)$ the density functions of y and z. Peragine considers a population with a finite number of y, a continuum of z, and a linearized version of the criterion:

$$\sum_y \frac{n_y}{n} \alpha_y \int_Z u_{yz} g(z) dz,$$

where α_y is an ethical weight expressing priority for subpopulations with a "low" y.

compensation principle. We will call it the "min of means" social welfare function:

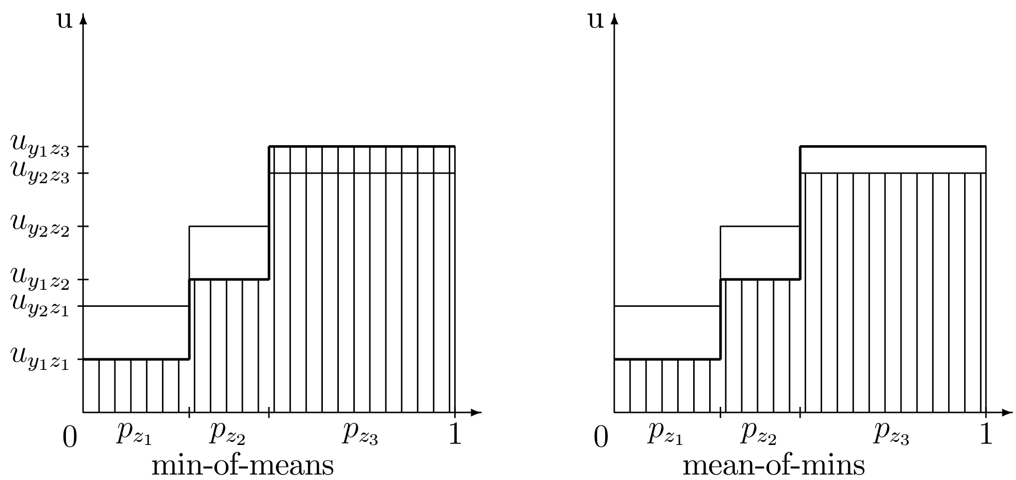
$$W_{\min M} = \min_{y \in Y} \left(\sum_{z \in Z} p_z u_{yz} \right),$$

The second prominent social welfare function, due to Roemer (1993, 1998), is computed as

$$W_{M \min} = \sum_{z \in Z} p_z \min_{y \in Y} u_{yz}.$$

One sees that it inverts the position of min and mean, so that we may call it the "mean of mins" social welfare function. It still applies an infinite inequality aversion across y and no inequality aversion across z.[61] Interestingly, it can be interpreted as the population-weighted area of the intersection of opportunity sets

$$\bigcap_{y \in Y} O_y.$$



- Figure 5 -

Both criteria boil down to ordinary utilitarianism

$$W_U = \sum_{y,z} p_y p_z u_{yz}$$

if all agents in the population have the same y. They may therefore advocate transferring resources to agents with parameter z inducing higher marginal utility.[62] This illustrates

[61] Roemer actually focused on the case of a discrete y and continuous z, so that

$$W_{M \min} = \int_Z \left(\min_{y \in Y} u_{yz} \right) g(z) dz.$$

This has to do with his particular statistical measurement of z, which is explained below.

[62] In the production model of Section 3, when agents have identical utility functions $u_i(\ell, c)$, a utilitarian planner would choose a Pareto-efficient allocation such that agents with higher skill obtain a lower utility, as shown by Mirrlees (1974). In Roemer (1998, p. 68), in an example dealing with unemployment insurance, the optimal policy for $W_{M \min}$ produces more inequalities than the laisser-faire.

the contrast with the solutions relying on the natural reward principle, which would advocate equal resources in this case.

It is also worth noting that the summation over z featured in these social welfare functions tends to enter in conflict with the principle of compensation, even though the maximin criterion is applied over y. This conflict can be shown in Example 1.

Example 1 (pursued). The utilitarian-like policy B presented in Example 1 is actually not the best according to the min-of-means and mean-of-mins social welfare functions. One computes that, under policy B, $W_{\min M} = W_{M\min} = 15$ (recall that y and z are independently distributed, with equal population size for each value). An alternative policy can do better for both criteria:

Policy BB

$y \setminus z$	1	3
1	$x = 0$ $u = 1$	$x = 9$ $u = 30$
3	$x = 0$ $u = 3$	$x = 7$ $u = 30$

Under policy BB, one obtains $W_{\min M} = W_{M\min} = 15.5$. What happens is that it is better to sacrifice the undeserving poor and redistribute the two units of resources they receive in policy B to deserving agents, who are able to transform one unit of resource into three units of utility. This is done at the cost of forgoing compensation among the undeserving subpopulation.

In the quasi-linear case of Subsection 2.4, marginal utility is always equal to one, independently of the agent's characteristics y and z. Then the opposition between natural reward and utilitarian reward is diminished and the conflict between utilitarian reward and compensation is also reduced, as the utilitarian criteria tend to entail a large indifference between allocations (ordinary utilitarianism is then indifferent between all feasible allocations[63]). Bossert, Fleurbaey and Van de gaer (1999) notice that, in this particular framework, the S_{OACE} rule, which satisfies ***E****qual* ***R****esource for* ***E****qual* ***T****alent*, always selects a best allocation for $W_{\min M}$, which shows a relative compatibility between natural reward and utilitarian reward in this case. On the other hand, all egalitarian-equivalent rules $S_{\Psi EE}$, $S_{\Psi PAE}$, S_{AEE}, S_{APAE} and S_{OAEE} select best allocations for $W_{M\min}$. Actually, the only thing that $W_{M\min}$ then requires is that agents with equal z obtain equal utilities, proving a compatibility between compensation and utilitarian reward in this particular case.

4.3 Min-of-means or mean-of-mins?

The last paragraph in the previous subsection, and related analysis in an optimal taxation problem by Schokkaert et al. (2004), suggest that $W_{M\min}$ is somehow stronger in compensation, and therefore more egalitarian, than $W_{\min M}$. This is not always true, however, as the following example shows.

[63] Recall that we assumed no free disposal in Section 2.

Example 4. Consider the following two policies.

Policy B1

$y \setminus z$	1	3
1	$u = 2$	$u = 30$
3	$u = 20$	$u = 26$

Policy B2

$y \setminus z$	1	3
1	$u = 1$	$u = 29$
3	$u = 25$	$u = 29$

One computes $W_{\min M}(B1) = 16 > W_{\min M}(B2) = 15$, whereas $W_{M\min}(B1) = 14 < W_{M\min}(B2) = 15$. One sees that $W_{M\min}$ prefers B2, even though it worsens the opportunities of the worst-off group, $y = 1$, and hurts the worst-off among the worst-off. This paradox occurs because the mean-of-mins criterion may seek to improve the situation of high effort agents even when they belong to well-off groups of talents, just because they happen to have the (relatively) smallest outcome in their subgroup of effort level.

Ooghe, Schokkaert and Van de gaer (2003) have provided axiomatic characterizations of (leximin versions of) $W_{\min M}$ and $W_{M\min}$ which clarify the ethical underpinnings of these criteria.[64] Their framework deals with matrices $(u_{yz})_{y\in Y, z\in Z}$, as defined above. The domain $\mathcal{D}$ of such matrices is the unrestricted domain $\mathbb{R}^{YZ}$. Let $\mathcal{D}^Y$ the subdomain of $\mathcal{D}$ such that $u_{yz} = u_{yz'}$ for all y, z, z' (flat reward for all talents, effort has no effect); and $\mathcal{D}^Z$ the subdomain of $\mathcal{D}$ such that $u_{yz} = u_{y'z}$ for all y, y', z (equal prospects for all talents, talent has no effect). The problem is to find a complete ordering R over the domain $\mathcal{D}$. They focus on the particular case when $p_y p_z = p_{y'} p_{z'}$ for all y, y', z, z' (i.e. each component of the matrix $(u_{yz})_{y\in Y, z\in Z}$ represents the same number of individuals) —this simplifies the formulation and justification of permutation axioms presented below.[65]

The leximin extensions of the min-of-means and mean-of-mins criteria are defined as follows:

Leximin of Means ($R^L_{\min M}$):
$\forall u, v \in \mathcal{D}$, $u \; R \; v$ if and only if

$$\left(\sum_{z\in Z} p_z u_{yz}\right)_{y\in Y} \geq_{lex} \left(\sum_{z\in Z} p_z v_{yz}\right)_{y\in Y}.$$

Mean of Leximins ($R^L_{M\min}$):
$\forall u, v \in \mathcal{D}$, $u \; R \; v$ if and only if

$$\left(\sum_{z\in Z} p_z u_{(y)z}\right)_{y\in Y} \geq_{lex} \left(\sum_{z\in Z} p_z v_{(y)z}\right)_{y\in Y},$$

where $\left(u_{(y)z}\right)_{y\in Y}$ is a reranked u_z such that $u_{(1)z} \leq u_{(2)z} \leq ...$

[64] Ooghe and Lauwers (2003) also provide a slightly different axiomatization of a leximin version of $W_{M\min}$.

[65] With a finite number of individuals and an independent distribution of y and z, a matrix with cells of unequal population sizes can always be expanded into a matrix with one individual in each cell (some columns may have the same value of z, and some lines the same value of y). Therefore it is not really restrictive to consider matrices with the same number of individuals for each cell.

Let us briefly introduce their main axioms. There is a standard Strong Pareto axiom, satisfied by both orderings.[66]

Strong Pareto:
$\forall u, v \in \mathcal{D}$,

$$u \geq v \Rightarrow u\ R\ v;\ u > v \Rightarrow u\ P\ v.$$

They obtain the utilitarian feature of these orderings by an axiom of translation-scale invariance saying that changing u_{yz} into $u_{yz} + w_z$ for an arbitrary vector $(w_z)_{z \in Z}$ does not change the ranking of matrices. This kind of axiom is common in characterizations of utilitarianism (see e.g. Bossert and Weymark 2004).

Translation-Scale Invariance in z (Ooghe et al. 2003)
$\forall u, v \in \mathcal{D}$, $\forall w \in \mathcal{D}^Z$,

$$u\ R\ v \Leftrightarrow u + w\ R\ v + w.$$

Both orderings also satisfy the following egalitarian axiom, which embodies the compensation principle in a mild way. It says that when a type of talent has prospects which dominate another, it is not bad to reduce the gap between them.

Dominance Compensation (Ooghe et al. 2003):
$\forall u, v \in \mathcal{D}$, $\forall y, y' \in Y$,

$$\left.\begin{array}{l} v_y > u_y \geq u_{y'} > v_{y'} \\ \forall y'' \neq y, y',\ u_{y''} = v_{y''} \end{array}\right\} \Rightarrow u\ R\ v.$$

We now introduce permutation axioms, referred to as "Suppes indifference". The first two are rather weak and permute the prospects of talents and the achievements of effort. They are satisfied by both orderings. The first axiom simply permutes the whole opportunities of the various talents.

Suppes Indifference for Talent (Ooghe et al. 2003):
$\forall u, v \in \mathcal{D}$, $\forall \sigma$ permutation on Y,

$$\forall y,\ v_y = u_{\sigma(y)} \Rightarrow u\ I\ v.$$

The second axiom is more complex and applies different permutations of outcomes for different y subgroups, but it does so only for matrices in which prospects for different talents are dominated, i.e. for any y, y', $u_y \leq u_{y'}$ or $u_y \geq u_{y'}$ (before and after permutations). For such matrices $R^L_{\min M}$ and $R^L_{M \min}$ coincide.

Suppes Indifference for Effort (Ooghe et al. 2003):
$\forall u, v \in \mathcal{D}$ such that $\forall y, z$, $u_{yz} = u_{(y)z}$ and $v_{yz} = v_{(y)z}$, $\forall \left(\sigma_y\right)_{y \in Y}$ permutations on Z,

$$\forall y, z,\ v_{yz} = u_{y\sigma_y(z)} \Rightarrow u\ I\ v.$$

The most interesting part of this axiomatic analysis lies in the distinction between the two orderings, which is deciphered through stronger versions of these axioms performing separate permutations for every effort level or every talent.

[66] Hild and Voorhoeve (2004) explain, however, that the mean-of-mins social welfare function may violate the (weak) Pareto principle when agents change their z in response to policy.

Strong Suppes Indifference for Talent (Ooghe et al. 2003):
$\forall u, v \in \mathcal{D}$, $\forall (\sigma_z)_{z \in Z}$ permutations on Y,

$$\forall y, z,\ v_{yz} = u_{\sigma_z(y)z} \Rightarrow u\ I\ v.$$

Strong Suppes Indifference for Effort (Ooghe et al. 2003):
$\forall u, v \in \mathcal{D}$, $\forall (\sigma_y)_{y \in Y}$ permutations on Z,

$$\forall y, z,\ v_{yz} = u_{y\sigma_y(z)} \Rightarrow u\ I\ v.$$

These axioms are better explained and critically assessed with examples.

Example 5. The first of these axioms, satisfied by the mean-of-leximins ordering, says that permuting the outcomes obtained by a subgroup of given effort level does not change the evaluation. For instance, the following social situations are deemed equivalent, even though the prospects for the two groups of y are comparable in one case and in clear domination in the other.

Policy ML1

$y \setminus z$	1	3
1	$u = 2$	$u = 30$
3	$u = 6$	$u = 26$

Policy ML2

$y \setminus z$	1	3
1	$u = 2$	$u = 26$
3	$u = 6$	$u = 30$

In contrast, the leximin-of-means ordering prefers policy ML1.

The second axiom, satisfied by $R^L_{\min M}$, says that permuting the outcomes obtained by a subgroup of given talent does not change the evaluation. For instance, the following social situations are deemed equivalent, even though the prospects for the two groups of y are equal in one case and very different in the other.

Policy LM1

$y \setminus z$	1	3
1	$u = 2$	$u = 30$
3	$u = 2$	$u = 30$

Policy LM2

$y \setminus z$	1	3
1	$u = 2$	$u = 30$
3	$u = 30$	$u = 2$

In contrast, $R^L_{M \min}$ strongly prefers policy LM1. In other words, it seems that each of these axioms deletes a relevant part of information, and as a result each of the orderings may be criticized for ignoring the respective part.

We may now state the result.

Proposition 17 *(Ooghe, Schokkaert, Van de gaer 2003) Let R be a complete ordering over $\mathcal{D}$, satisfying* **S**trong **P**areto*,* **T**ranslation-**S**cale **I**nvariance in z*, and* **D**ominance **C**ompensation*. If, in addition, it satisfies* **S**uppes **I**ndifference for **T**alent *and* **S**trong **S**uppes **I**ndifference for **E**ffort*, then $R = R^L_{\min M}$. On the other hand, if in addition, it satisfies* **S**uppes **I**ndifference for **E**ffort *and* **S**trong **S**uppes **I**ndifference for **T**alent*, then $R = R^L_{M \min}$.*

The same authors also study variants of these orderings. For instance, they consider CES versions, which can be defined as follows.[67]

$$W^{CES}_{\min M} = \left(\sum_{y \in Y} p_y \left(\sum_{z \in Z} p_z \left(u_{yz} \right)^{\mu} \right)^{\frac{\rho}{\mu}} \right)^{\frac{1}{\rho}},$$

$$W^{CES}_{M \min} = \left(\sum_{z \in Z} p_z \left(\sum_{y \in Y} p_y \left(u_{yz} \right)^{\rho} \right)^{\frac{\mu}{\rho}} \right)^{\frac{1}{\mu}}.$$

Hild and Voorhoeve (2004) try to avoid the utilitarian reward and adopt a dominance criterion which covers various possible reward approaches (although the abstract framework itself makes it impossible to apply the natural reward approach). Their criterion may be formulated as an axiom, saying that if for every effort level, the observed outcome distribution is better (according the leximin criterion), then the overall matrix is better.

Opportunity Dominance (Hild and Voorhoeve 2004**):**
$\forall u, v \in \mathcal{D}$,

$$\left.\begin{array}{l} \forall z,\ u_z \geq_{lex} v_z \\ \exists z,\ u_z >_{lex} v_z \end{array}\right\} \Rightarrow u\ P\ v.$$

This axiom is satisfied by $R^L_{M \min}$, but not by $R^L_{\min M}$, as shown in the following example.

Example 5 (pursued). This axiom imposes to prefer MLL2 to ML1 in the following example, even though, as in ML1 vs ML2, the prospects for the two groups are comparable in ML1 and in clear domination in MLL2, as recognized by $R^L_{\min M}$.

Policy ML1

$y \setminus z$	1	3
1	$u = 2$	$u = 30$
3	$u = 6$	$u = 26$

Policy MLL2

$y \setminus z$	1	3
1	$u = 2$	$u = 27$
3	$u = 6$	$u = 30$

In other words, the Opportunity Dominance criterion is agnostic about reward but clearly sides with the mean-of-mins approach and against the min-of-means approach on whether the evaluation should be on ex ante opportunities for various y or on ex post inequalities within z subgroups. It can therefore also be criticized for ignoring some relevant information.

The discussions in Example 5 strongly suggest that it would be better to take account of the two kinds of considerations (ex ante and ex post), instead of ignoring one of them totally. Goux and Maurin (2002) address this issue, and propose an interesting partial criterion, according to which a matrix u is at least as good as another matrix u' if $W_{M \min}(u) \geq W_{\min M}(u')$. This criterion itself does not yield the desired strict preference

[67] Kolm (2001) and Martinez (2004) made similar proposals.

in the above examples. But their results suggest a refined criterion, according to which a matrix u is strictly better than u' if[68]

$$\begin{aligned} W_{M\min}(u) &> W_{\min M}(u') \\ \text{or } W_{\min M}(u) &> W_{M\min}(u) \geq W_{\min M}(u') \\ \text{or } W_{M\min}(u) &\geq W_{\min M}(u') > W_{M\min}(u'). \end{aligned}$$

In Example 5, this partial criterion enables us to rank LM1 above LM2, since

$$W_{M\min}(LM1) = W_{\min M}(LM2) = 16 > W_{M\min}(LM2) = 2;$$

and ML1 above ML2, since

$$W_{\min M}(ML1) = 16 > W_{M\min}(ML1) = 14 = W_{\min M}(ML2).$$

Let us briefly present their axiomatic results underlying this proposal. The domain they deal with is that of non-negative matrices $\mathcal{D} = \mathbb{R}_+^{YZ}$. They rely on a Weak Pareto axiom.

Weak Pareto:
$\forall u, v \in \mathcal{D}$,

$$u \gg v \Rightarrow u\ P\ v.$$

They use a scale invariance axiom, which is satisfied by both $R^L_{\min M}$ and $R^L_{M\min}$.

Scale Invariance:
$\forall u, v \in \mathcal{D}$, $\forall \alpha \in \mathbb{R}_{++}$,

$$u\ R\ v \Leftrightarrow \alpha u\ R\ \alpha v.$$

They also invoke a separability axiom which has to do with decomposing the effect of talent and effort. It says that if the effect of talent remains the same, a change in the multiplicative effect of effort does not alter the ranking of two matrices. This axiom is also satisfied by both $R^L_{\min M}$ and $R^L_{M\min}$.

Fixed Effort Effect Separability (Goux and Maurin 2002):
$\forall u, v, u', v' \in \mathcal{D}$, if $\forall y, z,\ u_{yz} = \alpha_y \beta_z,\ v_{yz} = \gamma_y \beta_z,\ u'_{yz} = \alpha_y \beta'_z,\ v'_{yz} = \gamma_y \beta'_z$, then

$$u\ R\ v \Leftrightarrow u'\ R\ v'.$$

The most relevant axiom they introduce is meant to force R to take account of ex ante prospects and ex post inequalities at the same time, because it says that if prospects are better for every talent, and outcomes are better for every effort, then the matrix is better overall.

Talent-Effort Dominance (Goux and Maurin 2002):
$\forall u, v \in \mathcal{D}$,

$$\left.\begin{array}{l} \forall y \in Y, \begin{pmatrix} u_y \\ \vdots \\ u_y \end{pmatrix} R \begin{pmatrix} v_y \\ \vdots \\ v_y \end{pmatrix} \\ \forall z \in Z, (u_z, ..., u_z)\ R\ (v_z, ..., v_z) \end{array}\right\} \Rightarrow u\ R\ v$$

[68]Notice that one always has $W_{M\min}(u) \leq W_{\min M}(u)$.

(with strict preference in case of strict preference for some i or some j).

This axiom is not satisfied by $R^L_{\min M}$, and this can be illustrated by the following example.

Example 6. Consider the two policies:

Policy LMM1

$y \setminus z$	1	3
1	$u = 4$	$u = 18$
3	$u = 8$	$u = 14$

Policy LMM2

$y \setminus z$	1	3
1	$u = 2$	$u = 20$
3	$u = 10$	$u = 12$

.

The ordering $R^L_{\min M}$ is indifferent between LMM1 and LMM2. But one sees that

$$\begin{array}{|c|c|}\hline 4 & 18 \\ \hline 4 & 18 \\ \hline\end{array} \; I^L_{\min M} \; \begin{array}{|c|c|}\hline 2 & 20 \\ \hline 2 & 20 \\ \hline\end{array}$$

$$\begin{array}{|c|c|}\hline 8 & 14 \\ \hline 8 & 14 \\ \hline\end{array} \; I^L_{\min M} \; \begin{array}{|c|c|}\hline 10 & 12 \\ \hline 10 & 12 \\ \hline\end{array}$$

$$\begin{array}{|c|c|}\hline 4 & 4 \\ \hline 8 & 8 \\ \hline\end{array} \; P^L_{\min M} \; \begin{array}{|c|c|}\hline 2 & 2 \\ \hline 10 & 10 \\ \hline\end{array}$$

$$\begin{array}{|c|c|}\hline 18 & 18 \\ \hline 14 & 14 \\ \hline\end{array} \; P^L_{\min M} \; \begin{array}{|c|c|}\hline 20 & 20 \\ \hline 12 & 12 \\ \hline\end{array}$$

which should imply, by ***T**alent-**E**ffort **D**ominance*, that LMM1 is strictly better, as it indeed seems obvious, since inequalities ex post are reduced for every level of effort.

Somewhat surprisingly, however, the ***T**alent-**E**ffort **D**ominance* axiom is satisfied by $R^L_{M\min}$, and this suggests that this axiom fails to fully capture the concern for the relevance of ex ante evaluations. In other words, this axiom is compatible with indifference between ML1 and ML2, and with ***S**trong **S**uppes **I**ndifference for **T**alent*, which appears questionable. The $R^L_{M\min}$ ordering is excluded from the theorem below only by an additional continuity condition.

Proposition 18 *(Goux and Maurin 2002, Gajdos and Maurin 2004) If the ordering R, defined on domain $\mathcal{D} = \mathbb{R}^{YZ}_+$, is continuous and satisfies* **W**eak **P**areto*,* **T**alent-**E**ffort **D**ominance *and* **F**ixed **E**ffort **E**ffect **S**eparability*, then there exist three continuous, increasing, homogeneous functions $I : \mathcal{D} \to \mathbb{R}_+$, $I_y : \mathbb{R}^Y_+ \to \mathbb{R}_+$ and $I_z : \mathbb{R}^Z_+ \to \mathbb{R}_+$ such that:*
1. I represents R;
2. If $I_y\left(I_z(u_y)_{y\in Y}\right) = I_z\left(I_y(u_z)_{z\in Z}\right)$ then $I(u) = I_y\left(I_z(u_y)_{y\in Y}\right)$;
3. If $I_y\left(I_z(u_y)_{y\in Y}\right) \neq I_z\left(I_y(u_z)_{z\in Z}\right)$ then $I(u)$ is strictly in between.

Relevant examples of such functions are $I_z(u_y) = \sum_{z\in Z} p_z u_{yz}$ and $I_y(u_z) = \min_y u_{yz}$, yielding

$$\begin{aligned} I_y\left(I_z(u_y)_{y\in Y}\right) &= \min_{y\in Y}\left(\sum_{z\in Z} p_z u_{yz}\right) = W_{\min M}(u) \\ I_z\left(I_y(u_z)_{z\in Z}\right) &= \sum_{z\in Z} p_z \left(\min_{y\in Y} u_{yz}\right) = W_{M\min}(u). \end{aligned}$$

Other examples are $I_z(u_y) = \left(\sum_{z\in Z} p_z\,(u_{yz})^{\mu}\right)^{\frac{1}{\mu}}$ and $I_y(u_z) = \left(\sum_{y\in Y} p_y\,(u_{yz})^{\rho}\right)^{\frac{1}{\rho}}$ which then lead to combining the CES versions of the min-of-means and mean-of-mins social welfare functions. One limitation of this theorem is that it implies very little about the functions, and for instance utilitarianism W_U does satisfy all the axioms (with related I_y and I_z which both compute the mean).

Goux and Maurin (2002) emphasize the formal similarity between this issue and the well-known social choice problem of combining ex ante and ex post social evaluations of risky prospects.[69] Assume that z is unknown ex ante. Then the min-of-means criterion may be viewed as focusing on ex ante prospects for y subgroups, while the mean-of-mins criterion focuses on ex post inequalities for various possible realizations of z. From the literature on risky social choice, a simple combination of the two approaches would simply add the two criteria:

$$W_C = W_{\min M} + W_{M\min}.$$

Whether this can be given a convincing justification in the context of opportunities remains to be seen.[70]

Mariotti (2003) also exploits the similarity with the risk context in a different way, and examines the particular case in which an opportunity for agent i is defined as a probability π_i of success. In other words, there are only two possible outcomes, success and failure.[71] If success is valued a unit (1) and failure is valued zero (0), then π_i also measures the expected value of the outcome for i. In this context, and interpreting i as a talent subgroup with a proportion π_i of success, both $R^L_{\min M}$ and $R^L_{M\min}$ would apply the leximin to the π vector. Instead of the leximin criterion, Mariotti advocates the Nash-product criterion

$$W_N = \pi_1.\cdots.\pi_n.$$

He excludes the maximin by requiring the ranking to obey an independence condition.[72]

5 Related literature

In this section we present some related literature which studies similar issues in different ways, or which applies or extends some of the ideas reviewed above.

[69] On this problem, see Gajdos and Maurin (2004), Broome (1991), Deschamps and Gevers (1979), Ben Porath, Gilboa and Schmeidler (1997). The "min-of-means" and "mean-of-mins" terminology used here is borrowed from this literature.

[70] There seem to be significant differences in the two problems. In the risk problem, only one value of z will eventually be realized, and its determination is outside the agents' control, whereas in the opportunity context, all values of z will eventually be observed in the population, and every agent is responsible for his own value of z. These differences may limit the transposition of ethical principles from one problem to the other.

[71] Vallentyne (2002) adopts a similar probabilistic interpretation of opportunity sets, but with more than two possible outcomes.

[72] For two agents, this condition says that if $(\pi_1,\pi_2)\,R\,(\pi_1',\pi_2')$ and $(\pi_1'',\pi_2)\,R\,(\pi_1''',\pi_2')$, then $(\alpha\pi_1 + (1-\alpha)\,\pi_1'',\pi_2)\,R\,(\alpha\pi_1' + (1-\alpha)\,\pi_1''',\pi_2')$ for all $0<\alpha<1$.

5.1 Ranking distributions of opportunity sets

In many contexts it is considered that the characteristics that the agents are responsible for are chosen by them, and that compensating for the non-responsible characteristics is tantamount to providing agents with "equal opportunities". As recalled in the introduction, this is actually the way some philosophers have introduced these general egalitarian principles.

Thomson (1994) has studied the idea of equal opportunities in the simple fair division model, and has made an important distinction between actual opportunities (the consumed bundle belongs to the opportunity set) and "equivalent" opportunities (the consumed bundle yields the same satisfaction as the best bundle in the opportunity set).

Kranich (1996) has initiated an important branch of the literature which studies how to rank distributions of opportunity sets $(O_1, ..., O_n)$, where O_i denotes individual i's opportunity set, in terms of inequality. Kranich (1996) characterizes inequality measures that are linear, such as, in the case of two agents, the absolute value of the cardinality difference, $|\#O_1 - \#O_2|$. Kranich (1997a) extends this analysis to opportunity sets which are not finite, such as compact subsets of the Euclidean space, for instance budget sets in a market economy. Ok (1997) and Ok and Kranich (1998) analyze the possibility to adapt the basic results of the theory of income inequality, which connect Lorenz dominance to Pigou-Dalton transfers, to this new framework where income is replaced by opportunity sets. They show that the difficulty lies in adapting the notion of transfer and their results suggest that it is hard to avoid relying on cardinality-based evaluations of opportunity sets if one wants to replicate the classical theorems of income inequality. See also Arlegi and Nieto (1999) and Weymark (2003), the latter revisiting the results of Kranich (1996) with a version of the Pigou-Dalton transfer principle that only applies when the opportunity sets are nested. Other parts of this literature simply assume that there is a given measure of the value of sets (Herrero 1997 proposes to measure inequality by the difference between the greatest and the smallest value of the individual sets in the considered distribution, Herrero 1996 defines social welfare in terms of capability indices) or a given preordering over opportunity sets (Herrero, Iturbe-Ormaetxe and Nieto 1998, Bossert, Fleurbaey and Van de gaer 1999). In particular the last two references provide characterizations of social preorderings which rank distributions of opportunity sets on the basis of the intersection of individual sets, $\bigcap_{i=1}^n O_i$, a notion that has been already encountered in the above discussion of the mean-of-mins criterion. This intersection criterion is also axiomatically defended, in an economic model and in terms of "common capabilities", by Gotoh and Yoshihara (1999, 2003).[73]

The earlier parts of the literature presented in the previous sections seldom explicitly referred to opportunity sets but, as emphasized by Bossert et al. (1999), it is sometimes easy to interpret the solutions in terms of equalizing some appropriately defined opportunity sets. One can even argue that this literature provides ethical justifications for specific and concrete definitions of opportunity sets in economic contexts. Consider for instance

[73] More detailed surveys of this literature can be found in Peragine (1999), Barbera, Bossert and Pattanaik (2004).

the model of Section 2. The Egalitarian Equivalent allocation rule can be interpreted as equalizing the virtual opportunity sets:

$$\{(x,y) \mid x \leq \tilde{x}, y = \tilde{y}\}.$$

In general, most Egalitarian Equivalent allocation rules can be interpreted as applying the maximin criterion to "equivalent" opportunity sets that individuals would accept to choose from. This is clear for instance with the Budget Egalitarian Equivalent rules in Section 3.

The conditional egalitarian allocation rule, again in the model of Section 2, can be read in terms of opportunity sets

$$\{(x,y) \mid x \leq x_i, y = y_i\}.$$

and equality is sought between these heterogeneous sets by referring to a particular utility function $\tilde{u}$: The value of an opportunity set is the value taken by the indirect utility function on this set. A similar interpretation can be made of the Conditional Equality allocation rule in the model of Section 3.

If one refers to the philosophical theories, however, it seems more relevant to define opportunity sets in terms of pairs (responsibility variable, achievement). This is actually how we proceeded in Example 1 in the introduction. In the setting of Section 2, this suggests defining the opportunity set of an agent as:

$$\{(u, \bar{u}) \mid u \in \mathcal{U},\ \bar{u} = u(x_i, y_i)\}$$

or in the quasi-linear case:

$$\{(z, \bar{u}) \mid z \in Z,\ \bar{u} = x_i + v(y_i, z)\}.$$

It might be desirable to take account of the potential dependence of x_i on u_i (or z_i) via the allocation rule, in these definitions.[74] Notice that the conditional egalitarian allocation rule amounts to equalizing the sections of these opportunity sets taken at $u = \tilde{u}$, or $z = \tilde{z}$.

5.2 Extended insurance

As explained in Subsections 2.2 and 3.2, the No-Envy condition is too demanding in the context of compensation for non-transferable internal resources, because it is often impossible to find allocations satisfying it. Whereas most of the economic literature tried to escape this difficulty by imagining weaker requirements, Dworkin (1981b) suggested to retain the No-Envy condition but to apply it to a modified framework, namely, imagining a hypothetical insurance market in which agents could insure themselves, ex ante, against the prospect of ending up with a handicap. No-Envy could easily be satisfied ex ante by

[74] But then one runs into the following problem. In a small economy, the opportunities for a given agent depend on the values of the responsibility variables chosen by the other agents, so that opportunities become interdependent and there is no way to define them ex ante. See Kolm (1993), Fleurbaey (1995b), Barbera et al. (2004) for brief explorations of this matter.

granting all agents the same income in this hypothetical market. Ex post, the resource transfers performed between agents would just reflect their free choices of insurance. Obviously, this extended insurance market would have to operate under a veil of ignorance, the agents ignoring their personal talents.[75] A similar device was considered by Kolm (1985), and it is rather natural to consider that the extended insurance is just an extension of ordinary insurance markets and social insurance schemes, in the direction of increasing the scope of redistribution from the "lucky" toward the "unlucky".

Roemer (1985a) showed that this intuition is likely to be incorrect, and provided several examples showing that an extended insurance may behave like ordinary utilitarianism and entail the same paradoxical anti-redistribution from the unlucky (handicapped) to the lucky (talented).[76] The reason for this paradox is simple. First, an agent who maximizes his expected utility ex ante is doing a similar formal computation as a utilitarian planner maximizing a weighted sum of utilities. Insurance markets therefore generally produce results which bear a close relationship to a utilitarian allocation of resources. When agents have similar utility functions, with decreasing marginal utility of income, utilitarianism produces egalitarian consequences in the allocation of resources. Similarly, insurance against ordinary damages which reduce income or wealth enables agents to transfer resources from the states where they are rich to the states where they are poor.

Now, contrary to ordinary damages, personal talents typically alter utility functions. When describing agents in an extended insurance market, one then has to deal with state-dependent utility functions. In such a market, if a damage (low talent, handicap) reduces the marginal utility of income, then agents will typically want to transfer resources from states with damage toward states without damage, and will therefore want to insure against the more favorable state, in order to obtain more resources in this state, thereby sacrificing their welfare in the unlucky state.

As Roemer shows, in the case of extended insurance for production skills, one also gets unpalatable consequences. We have seen above that when all agents have the same utility functions, the principle of compensation calls for an equalization of utility levels. This will generally not happen with the extended insurance market, precisely for the same reason which leads a utilitarian planner to choose allocations with unequal utility levels in such a case.[77]

[75]Unlike Rawls's veil of ignorance, the agents would know their preferences and ambitions. Someone with musical ambitions, for instance, could then take a special insurance against short fingers or unperfect pitch. The fact that preferences are often influenced by actual talents makes Dworkin consider the possibility of having agents know their talents but still ignore the market value of their talents.

[76]In addition, it is not obvious that the hypothetical choices made in a virtual ex ante state are relevant to justify transfers of resources between individuals who have never lived in this ex ante state. Consider someone who belongs to the unlucky two percent of the population with a genetic disease. Is it an acceptable consolation for him to be told that, if given the possibility, he would have taken the two percent risk of getting those bad genes and ending up with his current share of resources? It is far from clear that trade-offs made by an individual over his possible future selves are relevant to trading-off the conflicting interests of separate individuals who have to share resources. See Kolm (1996b, 1998).

[77]More recent discussions of the extended insurance scheme can be found in Kolm (1996b, 1998), Roemer (1996, 2002a), Fleurbaey (2002), Dworkin (2002).

5.3 Extended sympathy and interpersonal comparisons of utility

The notion of envy-freeness that is central to the approach presented in this chapter can be considered as intermediate between ordinary envy-freeness, which bears only on external resources, and a more extensive egalitarian criterion which encompasses all personal characteristics and essentially seeks equality of utilities. This range of possibilities was identified by Kolm (1972), and more recently the same author (Kolm 1996a,b) argued that the location of the cut between characteristics submitted to compensation and other characteristics is a central feature of various theories of justice. More precisely, assume that individual utility is a function of m personal characteristics: $u_i(x_i, \theta_{i1}, ..., \theta_{im})$. Deciding to compensate for the first k characteristics can be enforced for instance by applying the No-Envy test in the following way:

$$u_i(x_i, \theta_{i1}, ..., \theta_{ik}, \theta_{ik+1}, ..., \theta_{im}) \geq u_i(x_j, \theta_{j1}, ..., \theta_{jk}, \theta_{ik+1}, ..., \theta_{im}).$$

Notice that, as shown in the previous section, this general scheme is also relevant for such characteristics as productive skills, which are not a direct argument of the ordinary utility function, but are an argument of a suitably defined utility function.

This generalized envy test seems to introduce a kind of interpersonal comparison of utility, although, as stressed in Section 2, it is perfectly immune to any increasing transformation of the individual utility functions. Only ordinal preferences over vectors of "extended resources" $(x, \theta_1, ..., \theta_k)$ do matter. When all characteristics are subject to compensation, the No-Envy test reads like equalizing utilities if the individual utility functions are identical and interpersonally comparable, that is, if there is a unique function $u(x_i, \theta_{i1}, ..., \theta_{im})$ that describes all individuals' utilities in an interpersonally comparable way.

A related notion is that of extended sympathy, and a joint use of the concepts of extended sympathy and No-Envy has been made in particular by Suzumura (1981a,b, 1983). In the extended sympathy framework, every individual is endowed with preferences over extended pairs (x, i), where x describes the social alternative (say, an allocation) and i is the name of an individual. In order to compare this with our formalism, assume that every agent has a utility function $v_i(x_j, j)$, where x_j is simply agent j's consumption (preferences are self-centered). With this convention, the No-Envy test as applied by Suzumura corresponds to

$$v_i(x_i, i) \geq v_i(x_j, j).$$

This suggests that it is equivalent to the test

$$u_i(x_i, \theta_{i1}, ..., \theta_{ik}, \theta_{ik+1}, ..., \theta_{im}) \geq u_i(x_j, \theta_{j1}, ..., \theta_{jk}, \theta_{ik+1}, ..., \theta_{im})$$

if in $v_i(x_j, j)$, the second argument depends only on $(\theta_{j1}, ..., \theta_{jk})$, and if the shape of the function v_i is molded by $(\theta_{ik+1}, ..., \theta_{im})$. Therefore the extended sympathy framework is equivalent to ours, albeit with a less explicit description of individual characteristics.

But there remains an important difference. A standard axiom, which is often relied upon in the extended sympathy framework, is the ***Axiom of Identity***, meaning that every agent's extended preferences respects others' tastes over ordinary resources:

Axiom of Identity (Sen 1970):
$\forall i,j \in N, \forall x,x',\ v_i(x,j) \geq v_i(x',j) \Leftrightarrow v_j(x,j) \geq v_j(x',j).$

Translated into our notations, this axiom would mean

$$\begin{aligned} u_i(x,\theta_{j1},...,\theta_{jk},\theta_{ik+1},...,\theta_{im}) &\geq u_i(x',\theta_{j1},...,\theta_{jk},\theta_{ik+1},...,\theta_{im}) \Leftrightarrow \\ u_j(x,\theta_{j1},...,\theta_{jk},\theta_{jk+1},...,\theta_{jm}) &\geq u_j(x',\theta_{j1},...,\theta_{jk},\theta_{jk+1},...,\theta_{jm}), \end{aligned}$$

and it is clear that this condition has little appeal in our framework,[78] even if we assume that $u_i = u_j$, since the difference between characteristics $(\theta_{ik+1},...,\theta_{im})$ and $(\theta_{jk+1},...,\theta_{jm})$ may plausibly entail different preferences over x. For instance, in the production model of Section 3, the ***Axiom of Identity*** would require that all agents have identical preferences over (ℓ, c). Indeed, for this model the equivalence in the ***Axiom of Identity*** translates into

$$u_i(\hat{\ell}/s_j, c) \geq u_i(\hat{\ell}'/s_j, c') \Leftrightarrow u_j(\hat{\ell}/s_j, c) \geq u_j(\hat{\ell}'/s_j, c'),$$

or equivalently,

$$u_i(\ell, c) \geq u_i(\ell', c') \Leftrightarrow u_j(\ell, c) \geq u_j(\ell', c').$$

The ***Axiom of Identity*** seems reasonable only in the case when all characteristics are subject to compensation. Therefore it is probably a safer reading of the extended sympathy approach that it usually deals with the extreme case of full compensation for all individual characteristics.

5.4 Bargaining

Yoshihara (2003) studies a variant of the production model of Section 3, with several goods being produced with unequal skills, and focuses on bargaining solutions (which, unlike allocation rules, select a single utility vector for any given utility possibility set). He provides characterizations of three bargaining solutions (egalitarian, Nash, Kalai-Smorodinsky) and highlights the interpretation of some of the axioms in terms of responsibility and compensation. Unlike the solutions presented above, the bargaining solutions violate Ordinalism and rely on utility figures. However, the Nash and Kalai-Smorodinsky solutions do exhibit some independence with respect to rescaling of utilities, which can be interpreted as reflecting some ascription of responsibility to individuals for their utility functions. Conversely, the egalitarian solution satisfies stronger solidarity properties (such as Skill Solidarity). Yoshihara shows that no bargaining solution satisfies Skill Solidarity and an independence axiom with respect to utility rescaling, in addition to Pareto efficiency and Equal Treatment of Equals. Since the Egalitarian-Equivalent allocation rule S_{EE} satisfies all of these axioms, this suggests that bargaining solutions suffer from a stronger dilemma between compensation and responsibility than allocation rules.

[78] Except, trivially, when x is one-dimensional, as in Section 2.

5.5 Surplus-sharing and cost-sharing approach

The literature presented in Section 3 studies allocation rules which define a subset of first best allocations for every economy, or social ordering functions which rank all allocations. Another interesting object is a surplus-sharing rule, which defines a sharing of the product for any vector of input contributions. More precisely, consider an economy $e = (s_N, u_N, f) \in \mathcal{E}$. A surplus-sharing rule is a function $\psi : [0, \overline{\ell}]^n \to \mathbb{R}_+^n$ such that for all $\ell_N \in [0, \overline{\ell}]^n$,

$$\sum_{i=1}^{n} \psi_i(\ell_N) = f\left(\sum_{i \in N} s_i \ell_i\right).$$

A surplus-sharing rule defines a game form, in which input contributions are the strategies, and consumptions (paired with input contributions) are the payoffs.

Kranich (1994) studies how to make the rule ψ perform some kind of compensation by considering the following axiom:

Equal Share for Equal Work (Kranich 1994):
$\forall \ell_N \in [0, \overline{\ell}]^n$, $\forall i, j \in N$,

$$\ell_i = \ell_j \Rightarrow \psi_i(\ell_N) = \psi_j(\ell_N).$$

He studies the case of a two-person economy, and gives sufficient conditions for a Pareto-efficient allocation to be obtainable as a Nash equilibrium of the game defined by a surplus-sharing rule satisfying ***Equal Share for Equal Work***. Notice that in this result the choice of the surplus-sharing rule must be tailored to the particular profile of preferences in the economy.

Gotoh and Yoshihara (1999, 2003) study a variant of the same model in which leisure and consumption are used by the agents in order to construct capability sets. In their model the agents have different productive skills but also different skills in the making of capability sets. These authors rely on compensation axioms similar to ***Equal Share for Equal Work***, and on axioms of solidarity with respect to changes in input contributions and changes in capability skills, in order to characterize a family of surplus-sharing rules that maximizes the (appropriately defined) value of the common capability set of the agents.[79]

Moulin and Sprumont (2002) study the cost-sharing problem, which is dual to the surplus-sharing problem and consists in choosing a function $\phi : \mathbb{R}_+^n \to \mathbb{R}_+^n$ such that for all $c_N \in \mathbb{R}_+^n$,

$$\sum_{i=1}^{n} \phi_i(c_N) = C(c_N),$$

where $C : \mathbb{R}_+^n \to \mathbb{R}_+$ is a cost function. The problem of compensation may arise in this setting when the cost function is asymmetric in the individuals demands c_i (some individuals are more costly to serve than others), and one considers that agents are responsible for the quantities demanded but not for the fact that serving them is more or less costly.

[79] Extending this analysis, Gotoh, Suzumura and Yoshihara (2005) study the construction of extended social ordering functions which rank pairs of game forms and allocations on the basis of principles of procedural fairness, capability egalitarianism, and efficiency.

Moulin and Sprumont study the following axiom, whose connection with the principle of compensation and ***E**qual **U**tility for **E**qual **F**unction* is quite transparent:

Strong Ranking (Moulin and Sprumont 2002):
$\forall c_N \in \mathbb{R}^n_+$, $\forall i, j \in N$,

$$c_i \geq c_j \Rightarrow \phi_i(c_N) \geq \phi_j(c_N).$$

In particular, this axiom entails that when two agents formulate the same demand, they pay the same share, irrespectively of the differential cost induced by their demands. This equity principle is actually a cornerstone of the pricing policy of some public monopolies (post services for instance).

Focusing on the case of demands expressed in discrete units ($c_i \in \mathbb{N}$), Moulin and Sprumont show that many reasonable sharing rules satisfy this axiom, but their axiomatic analysis singles out the *cross-subsidizing serial method* defined as follows, when individuals are re-ranked so that $c_1 \leq ... \leq c_n$:

$$\begin{cases} \phi_1(c_N) = \frac{1}{n} C(c_1, ..., c_1) \\ \phi_2(c_N) = \phi_1(c_N) + \frac{1}{n-1} \left[C(c_1, c_2, ..., c_2) - C(c_1, ..., c_1) \right] \\ \vdots \\ \phi_i(c_N) = \phi_{i-1}(c_N) + \frac{1}{n-i+1} \left[C(c_1, c_2, ..., c_i, ..., c_i) - C(c_1, c_2, ..., c_{i-1}, ..., c_{i-1}) \right]. \end{cases}$$

To the best of our knowledge, these three works are the only exceptions to a rather intriguing general neglect of the compensation problem in the surplus-sharing and cost-sharing literature. The tradition in this literature is to assign full responsibility to the agents for their demands and for the induced cost. This is expressed in particular in a dummy axiom, according to which any agent i who generates no cost (the cost function is constant with respect to y_i) should pay nothing. This axiom is incompatible with the ***S**trong **R**anking* (SR) axiom. For a general survey of the cost sharing literature, see Moulin (2002).

5.6 Opinions

Although the layman should not necessarily dictate the exact list of equity principles that ought to be studied in political philosophy and in normative economics, it is nonetheless interesting to submit our axioms to the test of questionnaires in which simple situations are presented to uninformed people in order to see what basic principles are spontaneously accepted or rejected.

A number of empirical works have to do with the ethical principles mentioned here. First, the classical paper by Yaari and Bar-Hillel (1984) is well known as showing that the traditional welfarist solutions perform badly because different contexts seem to call for different choices of allocation rules. It turns out the principles of compensation and of natural reward provide a simple explanation of the pattern of answers in that work. Let us briefly recall the setting. A bundle of two goods $(\overline{x_1}, \overline{x_2}) = (12, 12)$ has to be shared between Jones, whose utility function is $u_J(x_1, x_2) = 100x_1$, and Smith, whose utility function is $u_S(x_1, x_2) = 20x_1 + 20x_2$. The main result obtained by Yaari and Bar-Hillel is that when the utility function describes the nutritional metabolism of the individuals, the answers are mostly in favor of the allocation $(4, 0)$ for Jones, $(8, 12)$ for Smith, which yields equal utilities, whereas when it is a matter of tastes, the answers shift to the allocation $(12, 0)$ for Jones, $(0, 12)$ for Smith, which corresponds to either utilitarianism, the Nash bargaining solution or the equal-income Walrasian equilibrium.

Now, suppose that individuals are not deemed responsible for their metabolism but are deemed responsible for their tastes. When they are not responsible for the difference in the utility functions, then application of the compensation principle requires equalizing the utilities. When they are responsible for the difference, then the principle of natural reward would be satisfied by applying the equal-income Walrasian equilibrium. The answers are perfectly compatible with this pattern. They are also compatible with adopting a utilitarian reward in the latter case.

The simple examples in this case involve only two agents and therefore do not raise the issue of the incompatibility between compensation and natural reward, or between compensation and utilitarian reward. The first incompatibility has been studied by Schokkaert and Devooght (1998). They study examples in the quasi-linear case. In the first subcase, the function is separable in the two parameters y and z (resp., non-responsible and responsible parameters): $v(y, z) = 50y + 150z$, while in the second subcase it is not separable: $v(y, z) = 200y + 150z + 100yz$. Two different contexts are proposed to the respondents, each with four individuals. The first one deals with health expenditures (y describes medical needs, z describes choosing an expensive doctor), and the second one with income redistribution (y is an innate talent, z measures effort). In the context of health expenditures, and in the separable subcase, the answers are very much in favor of application of the "canonical" allocation rule (which, as described in Section 2, coincides in the separable case with the main allocation rules)

$$x_i = -50y_i + \frac{1}{4}\sum_{j=1}^{4} 50y_j,$$

while in the non-separable subcase, the respondents tend to favor allocations respecting the natural reward principle (more precisely, the ***E**qual **R**esource for **E**qual **T**alent* axiom), at the expense of compensation.[80]

In the context of income redistribution, the answers are much less attuned to the compensation and natural reward principles, and show an exaggerated willingness to reward less talented agents who make a high effort and to punish more talented agents who exert little effort, but may also be interpreted as expressing a desire to maintain a remuneration of talent itself, especially under effort. One may suspect that respondents bring in incentive considerations which are absent in the model of Subsection 2.4.

[80] An experimental analysis about dental expenditures has also been made by Clement and Serra (2001), and similarly shows some acceptance of the idea of responsibility. See also Schokkaert and Devooght (2003) for an international comparison of opinions on compensation and responsibility.

5.7 Applications

Applications of social ordering functions and social welfare functions to the design of optimal income tax have been numerous. The setting is the classical optimal taxation problem, as formulated by Mirrlees (1971). Assuming the planner knows the distribution of types (skills and preferences) in the population, but does not observe individuals' types, they study income tax schemes leading to allocations that maximize a social ordering function, under the usual incentive compatibility constraints (these constraints amount here to letting agents freely choose their labor time on the budget set defined by the tax scheme). Bossert, Fleurbaey and Van de gaer (1999) and Fleurbaey and Maniquet (1998, 2001, 2002) use the social ordering functions introduced in Section 3, and a few others of a similar kind. Roemer (1998), Roemer et alii (2000), Vandenbroucke (2001), Schokkaert, Van de gaer, Vandenbroucke and Luttens (2004) rely on various versions of the min-of-means and mean-of-mins social welfare functions. van der Veen (2004) directly compares budget sets, viewed as opportunity sets. Most of these works focus on the linear tax, with the exception of the papers by Fleurbaey and Maniquet.[81] Vandenbroucke (2001) and van der Veen (2004) consider the combination of labor subsidies with income tax. In a different vein, the implementation problem (in which the planner does not know the distribution of types) for economies with unequal skills is studied in Yamada and Yoshihara (2004).

Under richness assumptions on the distribution of types (in particular, the smallest skill is 0), and considering non-linear taxes, Fleurbaey and Maniquet (1998) show that maximizing any social ordering functions satisfying Weak Pareto[82] and ***H**ammond **C**ompensation* entails maximizing the minimum income.[83] On the other hand, the tax scheme obtained by maximization of a Conditional Equality ordering function $R_{\Phi CE}$ depends on the choice of the reference function $\tilde{u}$. For some $\tilde{u}$ (corresponding to hard-working preferences), the result is, again, to maximize the minimum income. This shows that the compensation-natural reward conflict may disappear in second-best problems, in the sense that different social objectives based on these two principles may lead to the same allocations.[84] However, Fleurbaey and Maniquet (2001, 2002) show that when the minimum wage is positive, a slight dose of natural reward in social ordering functions

[81] In a related work, Boadway et al. (2002) study an ordinary weighted utilitarian objective on the same model, considering various possible weights for agents with different utility functions, with the idea of spanning diverse reward principles.

[82] Weak Pareto requires an allocation to be socially preferred if it is preferred by all agents to another allocation.

[83] An analysis of the basic income proposal in terms of similar considerations is made in Gotoh and Yoshihara (2004).

[84] Another connection between the taxation setting and the ethical principles deserves to be noted. Recall that in the study of allocation rules, a link between incentive conditions (***I**ndependence of **U**tilities*, ***M**onotonicity*) and natural reward axioms already suggested a congruence, noted above, between natural reward and incentives. Similarly, in a context of taxation, it appears that the allocations which are achievable under incentive constraints do satisfy significant features of natural reward. For instance, in the linear version of the model of Section 3 (domain $\mathcal{E}^L$), all allocations which are achievable by a redistributive tax on earnings $s_i\ell_i$ do satisfy ***N**o-**E**nvy among **E**qually **S**killed*, because agents with equal skills automatically have the same budget set in this setting.

based on compensation leads to the conclusion that the marginal tax rate for incomes below the minimum wage must be zero or even negative on average. The reason is that the social ordering function then advocates maximization of the net income of agents who work full time at the minimum wage (the working poor).

Roemer et al. (2000) and Vandenbroucke (2001) study the application of the mean-of-mins and/or min-of-means criteria, with a measure of individual "advantage" $u(x, y, z)$ which is objective. For instance, in a study of various Western countries with the purpose of comparing actual tax rates with the optimal tax rate for Roemer's criterion, Roemer et al. (2000) take individual advantage to be the logarithm[85] of income, disregarding the disutility of labor. The idea is to measure inequalities of opportunity for income. But with such an objective approach, they obtain the possibility of choosing linear taxes which are Pareto-inefficient (tax rates beyond 80% are advocated in this paper for some European countries). Schokkaert et al. (2004) make a thorough comparison of the results obtained with various objective and subjective measures of individual utility, and with the two social welfare functions. They show in particular that mean-of-mins generally leads to higher tax rates than min-of-means. Vandenbroucke (2001) shows that the results are bound to change substantially when the possibility of subsidizing labor (assuming labor hours are observable) at a fixed rate is introduced in supplement to the linear income tax.

Under the specific assumptions on the distribution of skills and on agents' utility functions used by Roemer (1998), Bossert, Fleurbaey and Van de gaer (1999) compute the optimal linear tax associated to different social ordering functions, including those presented in Section 3,[86] as well as the min-of-means and mean-of-mins social welfare functions (with a subjective measure of individual utility, avoiding Pareto-inefficient results). Their result is that the highest income tax rate, and therefore the largest redistribution, follows from maximizing the Egalitarian Equivalent ordering function R_{EE}.

Kranich (1997b) studies a slightly different model where tax revenues are used by the government to finance education policies. An education policy consists of allocating different amounts of money to the schooling of agents, thereby enhancing their ability to earn income. Under several assumptions (in particular the assumption that each agent is capable of achieving any earning ability level ex post), this model has the interesting feature that, under complete information, an education policy exists such that the resulting Pareto-efficient allocation satisfies both ***Equal Utility for Equal Function*** and ***Equal Resource for Equal Talent***. Kranich proves, however, that even in this context but under incomplete information, information/incentive constraints also prevent the planner from achieving full compensation, so that ***Equal Utility for Equal Function*** is no longer achievable. This negative outcome also holds, obviously, in ordinary taxation models.

In Roemer (1998, 2002), an empirical computation of the mean-of-mins criterion is made for the allocation of educational resources in the USA, considering only race as a non-responsible parameter, and measuring the achievement of individuals by the logarithm of earnings, econometrically estimated as a function of educational resources and "effort"

[85] Recall that these criteria do not satisfy ***Ordinalism*** and are sensitive to the particular cardinal utility function adopted to measure individual outcomes.

[86] Actually, they use a different variant of Conditional Equality.

(measured by the percentile of the number of years of attendance in a subgroup of agents with the same race and similar amount of educational resources —see below). Policy is described simply by two instruments, the average amount of educational resources given to each of the two types of pupils. The optimal allocation of resources obtained by this computation implies a high transfer since resources devoted to black pupils should be about ten times as much as resources allotted to white pupils. Llavador and Roemer (2001) also apply the same criterion to the issue of the allocation of international aid across countries.

Roemer (1993, 1998) also makes a specific proposal about how to construct a measure of a responsibility variable for a given population. First the variables representing talent have to be chosen, and by sample surveys or any statistical means the joint distribution of the talent variables and the other characteristics that influence people's outcomes must be estimated. Then two agents in the population are considered as identical in terms of "responsibility" (or "effort") if their other characteristics are at the same percentile of the conditional distribution of these characteristics in the respective class of talent of these two agents. Of course, this method essentially requires that the other characteristics can be ranked on the real line and are a monotonic function of the underlying "effort" variable that one tries to estimate.[87] An interesting consequence of this method is that the distribution of the responsibility variable is then automatically independent of talent.

This method has been applied by Van de gaer, Martinez and Schokkaert (2001) in order to define new measures of social mobility. The idea is that social mobility can be viewed as congruent to equality of opportunities, when opportunities are measured on the basis of the conditional distribution of a given generation's outcomes (e.g. income), for a given level of parental outcome. Similarly, O'Neill, Sweetman and Van de gaer (2000) make a non-parametric estimation of the distribution of the earnings in the USA, conditional on parental earnings[88]. They draw "opportunity sets" by representing the graph of the inverse of the cumulative distribution function, which tells the income obtained at any percentile in this distribution. In their analysis the only non-responsible variable that serves to compute the conditional distribution is parental income. Their results show that different parents' incomes entail unequal opportunities to achieve a given income. For instance, the same level of income is obtained by children of poor families who are at the 65th percentile of their conditional distribution, but is obtained by children of rich families who are only at the 40th percentile of their conditional distribution.[89]

A different empirical application of the axioms of compensation has been made by Schokkaert, Dhaene and Van de Voorde (1998) and Schokkaert and Van de Voorde (2004). It deals with the joint management of health insurance agencies (such as the allocation of budget allowances to regional mutual insurances in Belgium). A basic distinction is

[87] For a discussion of assumptions underlying this method, see Fleurbaey (1998), Roemer (1998, 2002), Kolm (2001), Hild and Voorhoeve (2004).

[88] See Lefranc, Pistolesi and Trannoy (2003) for similar applications to French data and Goux and Maurin (2002) for an application combining the Van de gaer and Roemer criteria.

[89] For a general comparison of mobility measures with measures based on the idea of equal opportunities, see Van de gaer, Martinez and Schokkaert (2001), Schluter and Van de gaer (2002). See also Roemer (2004) and Fields et al. (2005).

made, in the set of variables explaining the level of medical expenditures, between those for which the insurer should be held responsible, and those which do elicit compensation. Compensation in this setting has implications in terms of incentives for risk selection: Full compensation should prevent insurers from selecting among their potential customers those who display favorable risks. Natural reward, on the other hand, has a direct effect on incentives for efficient management of resources. Therefore the axioms of compensation and natural reward seem relevant in this application, albeit with a reinterpretation. The authors show how to rely on an econometric estimation of the equations of medical expenditures in order to choose the allocation rule for the global budget. They discuss the problem of possible non-separability of the exogenous variables in the equation, and show the unavoidable ethical choices that must be made by the public decision-maker in this respect.

A similar kind of application has been made, for the problem of interregional budget transfers, by Cappelen and Tungodden (2003a).[90] The idea is that regions are responsible for their policy, in particular their tax rate, but not for characteristics of the region influencing the tax base. They show that two prominent transfer schemes in the fiscal federalism literature, namely the foundation grant and the power equalization grant, can be related to conditional equality and egalitarian-equivalence, respectively. Calsamiglia (2004) studies how policies which attempt to achieve equality of opportunity separately in various sectors of individual life (e.g. health, labor market, education) may ultimately achieve equality of opportunity globally. She shows that, for some contexts, this obtains if and only if rewards to effort are equalized across individuals in each sector.

6 Conclusion

Let us put the various approaches described in this chapter in perspective. One of the lessons from the surveyed literature is that anyone who embarks in social evaluations in the compensation-responsibility context must make two key decisions: 1) How should responsibility be rewarded (the two prominent options being liberal neutrality —natural reward— or zero inequality aversion —utilitarian reward)? 2) Is compensation or reward the priority? This second question is inseparable from, and in some contexts equivalent to, a third one: Is the focus on ex post inequalities or on ex ante prospects? The following table places the main evaluation criteria which have been presented above in their respective positions with respect to these questions.

	Natural reward	Utilitarian reward
Priority on compensation (ex post)	egalitarian-equivalent	mean-of-mins
Priority on reward (ex ante)	conditional equality	min-of-means

In the rest of this conclusion, we highlight a few open questions. The distinction between various reward principles is not clearly made in many parts of the literature, some authors spontaneously adopting a natural reward approach while other authors

[90] This is also the main motivating topic in Iturbe and Nieto (1996).

choose a utilitarian reward without much reflection. Initially, the different approaches were linked to the fact that some authors studied allocation rules and Pareto-efficient redistribution, while other authors, interested in inequality measurement and dominance analysis or in taxation and redistribution under incentive constraints, looked for social welfare functions and therefore took welfarist social welfare functions as their point of departure. In the theory of fair allocation rules, the Walrasian equilibrium and the general idea of equality of resources (as expressed e.g. in the No-Envy condition) are absolutely pre-eminent, and this led authors toward natural reward. In contrast, the theory of social welfare functions identifies neutrality about welfare inequalities with a zero inequality aversion as displayed by a utilitarian social welfare function. Now that social welfare functions embodying natural reward and geared toward Walrasian allocations have been produced,[91] such as the Conditional Equality ordering function $R_{\Phi CE}$ (see Subsection 3.2), it is possible to compare natural reward and utilitarian reward not only in the field of first-best redistribution, but also in the field of second-best redistribution. A preliminary comparative study is made in Bossert, Fleurbaey and Van de gaer (1999), but a general confrontation between natural reward and utilitarian reward remains to be done.

Propositions 3 and 4 in Subsection 2.3 are typical of many results in this literature, in which a family of allocation rules (or social ordering functions) is characterized, but this family contains an infinity of very different solutions, which differ by some crucial parameter (such as $\tilde{y}$ or $\tilde{u}$). The choice of a precise member in such a family presumably requires invoking additional ethical principles, such as more precise reward principles[92] or totally different notions. For instance, self-ownership as studied in Subsection 3.4 was shown to restrict the range of admissible parameters. But a theory about the choice of the reference parameters remains to be elaborated. In addition, it is rather disturbing when the Consistency axiom forces solutions to stick to one and the same reference parameter whatever the profile of the population. It would seem more sensible to have the reference parameter depend on the profile, and this has been shown to be important in order to satisfy some compensation or natural reward axioms (e.g. it is necessary, as stated in Proposition 2, that Φ be idempotent so that $S_{\Phi CE}$ may satisfy ***Equal Utility for Uniform Function***). There exist alternative solutions, such that the average rules S_{AEE}, S_{ACE} or the balanced rules S_{BE}, S_{BCE} presented in Subsections 2.2 or 2.4.1. But such solutions have their limitations, since they can be computed only in special contexts (and their characterizations rely either on contrived axioms or on special domain assumptions).

More generally, we conjecture that in the future a more refined theory will make a distinction between different contexts. For instance, in $u(x, y, z)$, z may be an effort or merit variable, or simply a taste parameter, and the discussions of Examples 2 and 3

[91] A general study of the construction of social ordering functions which are oriented toward equalizing resources and only rely on individual ordinal non-comparable preferences is made in Fleurbaey and Maniquet (1996b), in relation to the theory of allocation rules and also in relation to the presumed impossibility of social choice without interpersonal comparisons of utility.

[92] For instance, Tungodden (2005) argues in favor of the flattest (most egalitarian) reward, advocating a particular $S_{\Psi EE}$ rule. In the production model, Fleurbaey and Maniquet (2005) give related arguments in favor of R_{EE} (and an ordering function rationalizing $S_{\Psi EE}$ in $\mathcal{E}^L$, with $\Psi(ws_N) = w \min_i s_i$), which can also be interpreted as the most egalitarian in a similar sense.

have shown that moral intuition may be sensitive to this. Moreover, Examples 1 through 3 rely on different utility functions. In Example 1, x and y are perfect substitutes, which makes the principle of compensation and the principle of natural reward not only compatible, but also equally attractive. Example 2, in which agents are totally responsible for their sensitivity to transfers, has been chosen to criticize the principle of compensation. Example 3, in which talent, but not transfers, affects the productivity of effort, was cooked to criticize the principle of natural reward. The shape of the utility function may then be relevant to the choice of ethical principles.

Another drawback of all the solutions which are not direct extensions of the No-Envy allocation rule S_{NE} is that they may fail to select envy-free allocations when some exist. In other words, axioms like ***E**qual **U**tility for **E**qual **F**unction* and ***E**qual **R**esource for **E**qual **T**alent* (or ***N**o-**E**nvy among **E**qually **S**killed*) are too weak to guarantee that whenever envy-free (and Pareto-efficient) allocations exist, some of them must be selected. On the other hand, the direct extensions of S_{NE} presented in Subsection 2.2 do not have clear ethical properties in terms of compensation and reward outside the subdomain of economies in which ***N**o-**E**nvy* can be satisfied (although Prop. 2 above does provide some answers), and do not have any axiomatic justification. In brief, some work remains to be done around ***N**o-**E**nvy* in the compensation problem.

The literature has focused on the two models presented here, and therefore has neglected issues which are not described in these models. In particular, it remains to study the general problem of talents and handicaps which simultaneously alter an individual's productive abilities, her quality of life and her preferences over consumption goods. The model of Section 3 deals only with the first dimension, the model in Section 2 with the second dimension and may be considered to cover the third dimension only when x is interpreted as income, u as indirect utility and when consumption prices are fixed. In-kind transfers cannot be studied in this setting. In a different direction, one may also want to study more specific problems such as spatial inequalities (living somewhere may be seen as a handicap in terms of living cost, access to public goods, etc.), family size, and so on. The production model could also be refined so as to take account of the fact that agents may be deemed responsible for some of their skill parameters, and/or the fact that they may not be responsible for some of their preference parameters.

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