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Research Article

Inventory Policy for Retail Stores: A Multi-Item EOQ Model Considering Permissible Delay in Payment and Limited Warehouse Capacity

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A B S T R A C T

The retail industry such as minimarkets has many products consisting of several types of products that have expiration dates. Their warehouses have limited capacity, making it difficult to make decision about optimum inventory. Most of the suppliers will give permissible delay in payment, that can be used to increase income potential through earned by considering the risk of fines imposed if payments are exceeded and help companies raise capital before generating sales. These three factors must be considered when developing the inventory model. The purpose of this study is to develop a multi-item inventory model by considering perishable or damaged products, permissible delay in payment in limited warehouse. Model development is carried out in 2 stages. The first stage was the development of a multi-item EOQ model by considering product defects and permissible delay in payment. The second stage model is by adding a capacity constraint factor to the model. The results obtained are getting the optimal order quantity by considering the number of product types, product damage factors, late payments in limited warehouses, the best ordering policy can be found, and it is known that the total inventory costs to changes in parameters are good and sensitive to changes in percentage, interest percentage, payment allowances, and warehouse capacity through sensitivity tests.

INTRODUCTION

The effective management of materials is crucial to the performance of many organizations [1]. The existence of inventory is not only considered as a liability that must be eliminated but is also necessary to ensure the fulfillment of demand. When inventory is eliminated, it can cause losses including loss of potential income, idle machinery and equipment, increasing potential loss of customers who move to other companies. Therefore, inventory needs to be managed with good inventory management to obtain optimal performance [2].

For retail companies such as minimarkets that sell food or the chemical industry which have the expiration date of goods is one of the factors that affect the total cost of inventory [3]. Referring to the Law no. 8 of 1999 in Indonesia concerning on consumers' protection, companies are prohibited to sell damaged products, so damaged products have no resale value [4]. Inventories that are stored but not used for too long will cause costs due to expired goods and losses due to damaged goods that cannot be sold [5].

Minimarkets have suppliers who provide permissible delay in payments until the Allowable time limit to increase demand for supplier's products [6]. Inventory policy is also decided by factor from the characteristics of the supplier. In general, the permissible delay regarding the payment period can potentially give extra income that will compensate the cost of inventory. The retail industry also utilizes this when there is a lack of fund for inventory cost. But if the company can't complete the transaction to the set time there is a penalty cost for the company [7].

Retail Companies such as minimarkets will also always have product inventory especially food products where products that is sold have more than one item number for one supplier and have an expiration date, a multi-item single source system [8]. This will make the order schedule to be more complex because the optimum midpoint in the order period must be determined for many items.

It is important for a good inventory system to consider the limitations in the company. Retail industry such as minimarkets has a significant lack of inventory space so that it effects the optimal order that must be in accordance with the storage capacity.

Some studies have been done to develop Economic Order Quantity (EOQ) models that consider the permissible delay in payment time. Goyal [7] did the research considering the permissible delay of the payment time. Yang and Wee [8]

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developed a collaborative inventory system of single vendor and single buyer to maximize the total profit using permissible delay in payment. Duary et al. [10] develop a model where the suppliers offer some price-discounts for advance payments made by their retailers. As advance payments put a constraint on the capital position of the retailers, the retailers meanwhile, enjoy some delay in the final payment. Jaggi et al. [11] developed an EOQ model for deteriorating items with initial inspection, allowable shortage under the condition of permissible delay in payment. Silitonga and Iskandar [12] developed single item inventory model considering the damage factor in product and payments delay.

Many studies also have been done to develop models that consider perishable items and warehouse capacity constraint. Sargut and Isik [13] developed a dynamic lot sizing model for a single perishable item under production, while William et al. [14] developed an inventory model for determining optimum replenishment time and order quantities and space requirements for multi-item medicines in a hospital. Silitonga, Kristiana and Parley [15] developed probabilistic demand inventory model considering perishable products and warehouse constraint. There is also a fuzzy EOQ model developed by Chou, Julian, and Hung [16], with demand-dependent unit cost under limited storage capacity. Meanwhile Rahman et al. [17] demonstrated the optimal strategy of an inventory system for perishable goods with hybrid demand dependent on selling price and stock under the partial backlogging with a certain fixed ratio. Lesmono, Limansyah and Loedy [18] developed a multi-item perishable inventory model with deterministic demands, return and all-units discount. Duan et al. [19] developed an inventory model for perishable items with inventory level dependent demand rate, where with and without backlogging are studied. Silitonga and Moses [20] developed a multi-item EOQ model that consider discounts and warehouse capacity.

The purpose of the study is to develop an inventory model to be applied to retail industry such as minimarket. A minimarket has product inventory with the characteristics: composed of many types of product item that have an expiration date, suppliers that give permissible delay in payment until the allowable time limit, and limited storage capacity. Here, deterministic model can be applied to the level C products in ABC classification, where the demand variation impact to the cost is negligible.

MODEL

Model Description

In general, the model can be implemented in retail industry such as Minimarkets with characteristics of the product inventory that is derived from many types of product item that have an expiration date. Supplier with the characteristic of giving a permissible delay of payment until the allowable time limit and limited warehouse capacity. There is some problem boundary to implement the model with the right analysis which is the characteristic of the demand is known and constant, low variation in products, the goods come from one supplier so that the product that is ordered will come at the same time while the order is done, fine is given according to the number of leftover products in the warehouse after the permissible time limit has passed, damaged product have no value, and the expiration date is known. This model can be implemented directly by the company either from the management level or from the field worker using computer application such as spreadsheet to recapitulate the sales data, expiration date data and data of each order.

The model is developed to accommodate the condition of a retail store with and without limited warehouse capacity, hence the model development is carried out in 2 stages. The first stage was the development of a multi-item EOQ model by considering product defects and permissible delay in payment. The second stage or final model is by adding a capacity constraint factor to the model.

Model Assumption

1. Quantity of order lots and ordering costs are fixed for each order.
2. The cost of storage is proportional to the number of products stored, the length of storage time, and the price of the product per unit.
3. The cost of the penalty is proportional to the remaining product in the warehouse after the time limit has been passed.
4. Shortage of goods occurs after there are no more goods in the warehouse.
5. Defective products will be discarded after the undamaged products in the warehouse are used up so that there is no storage fee in the shortage period.
6. The length of time allowed by the supplier is known at the beginning of the planning period.
7. The percentage of good products is known and constant to the optimal order quantity.
8. The expiration time for each type of product is the same.
9. Each product sold will be directly credited to a bank account and taken after the time limit has been reached.
10. Products that are subject to fines will be paid to the supplier at the end of the good product expiration period.
11. Interest income received is simple interest (not compound interest).
12. The fund used for payment of the purchasing cost to the supplier is only as big as the level of inventory that is saved.
13. There is no minimal limit concerning the amount of balance saved.

Model Scenario

There are 3 possible scenarios that occur in the development of the model scenarios 1 and 2 are derived from the model of Silitonga dan Iskandar [9] that has developed the factors of product damage and permissible delay in payment and the third scenario is derived from the development of Goyal’s [7] first model that developed the model that considered the permissible delay in payment. Scenario 1 shows that a good product is sold out (t_1) before the time limit is passed (t_3) and before the order period (T), the scenario 2 shows that a good product sold out (t_1) after the time limit is passed (t_3) and before the order period (T), so that a penalty fee is charged from the supplier, and scenario 3 show that a good product sold out (t_1) after the time limit is passed (t_3) but the supplier provides a limit of time allowance (t_3) which is longer than the period of ordering (T). The diagrams of three possible scenarios are as follows:

Scenario 1

Occurs when $t_1 \leq t_3 \leq T$

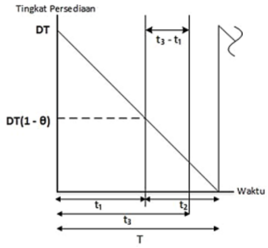

Figure 1. Scenario 1

Scenario 2

Occurs when $t_3 < t_1 < T$

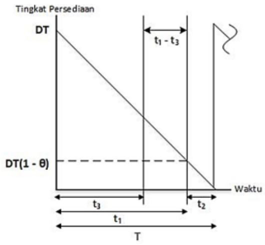

Figure 2. Scenario 2

Scenario 3

Occurs when $t_1 < T < t_3$

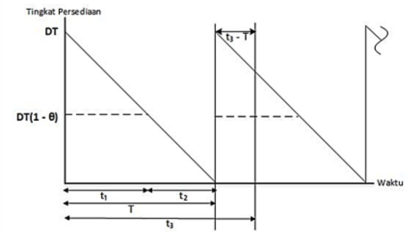

Figure 3. Scenario 3

Model Formulation

The model development uses the multi-item EOQ model with the variable of decision being the total cost for inventory. The determination of the optimal value of the variable is done to minimize the target objective function, which is the total cost of inventory.

The total inventory cost is as follows:
Z = Ordering Cost + Holding Cost + Shortage Cost + Damage Cost + Fine Cost – Interest Revenue

$$Z = O_p + O_s + O_{so} + O_d - P_i \qquad (1)$$

Each cost components and its value are described as follows:

Ordering Cost (O_p)

Ordering cost is the cost for one order in one year. The amount of the ordering cost with a joint order can be calculated from the cost per one order (S^*) divided by period between orders (T^*). Mathematically modeled as follows:

$$Ordering\ Cost\ (O_p) = S^* \times \frac{1}{T^*} \qquad (2)$$

Holding Cost (O_s)

This cost is the cost of maintenance needs to maintain the product during the period t_1 in a year. Holding cost is generated by multiplying the demand for item i (D_i), the period between orders (T^*), and the percentage of the price of holding costs for item-i per unit in a planning time horizon. With the amount of storage costs per unit of each item expressed by a fraction of the purchase price of each item per unit, which is equal to $P_i h_i$, so that mathematically the holding cost per unit of goods is as follows:

$$When\ N = \frac{D_l}{Q_l}\ and\ T^* = \frac{1}{N} \rightarrow T^* = \frac{Q_l}{D_l}, Q_l = T^*\ x\ D_l \qquad (3)$$

$$Holding\ cost\ (O_s) = \sum_{l=1}^{n} \frac{D_l T^* \theta_l P_{il} h_l (2 - \theta_l)}{2} \qquad (4)$$

During the period t_2 no products are stored because 2 all damaged products will be immediately destroyed.

Shortage Cost (O_{so})

This is the cost due to expired products so produce a run out condition and demand cannot be met by the company. The amount of the shortage cost is the result of multiplying the cost of the shortage of item i (U_i) with the average shortage of item i and the length of time of product shortages (t_2). Mathematically modeled as follows:

$$Shortage\ Cost\ (O_{so}) = \sum_{l=1}^{n} \frac{D_l T^* U_l (1 - \theta_l)^2}{2} \qquad (5)$$

Damage Cost (O_{kd})

Damage cost is a cost when the product is damaged so that it has no resale value in a year. The amount of the damage cost is the result of multiplying the number of damaged item-i with the cost of purchasing for item-i (P_l).

Mathematically modeled as follows:

$$Damage\ cost (O_{kd}) = \sum_{l=1}^{n} D_l P_l (1 - \theta_l) \qquad (6)$$

Fine Cost (O_d)

This cost is a cost when the company is late in paying the purchase value of unsold products from suppliers in a year. This cost only appears if scenario 2 occurs because the good product is sold out after passing the payment allowance in a year. The amount of the penalty fee is the result of multiplying the cost of the fine per unit item i (purchasing cost multiplied by the percentage of the product being fined) with the average product i that has not been sold in the warehouse and the length of time until the product is sold out ($t_1 - t_3$).

Mathematically modeled as follows:

$$O_d = \sum_{l=1}^{n} P_l I_c \frac{(2 D_l T^* - D_l T^* \theta_l - D_l t_3))}{2}\ x \left(\theta_l - \frac{t_3}{T^*}\right) \qquad (7)$$

Interest Revenue (P_I)

Because there is a permissible delay in payment, the cash to pay for the products is deposited in a bank account. The interest will reduce the inventory cost. Based on scenario 1, the interest income is the result of multiplying the interest income per unit with the average number of good products sold in t_1 and t_2, based

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on scenario 2 interest revenue is calculated based on the amount of interest income per unit times the average number products sold during the time limit that has not been reached in t_3, while based on scenario 3 interest revenue is calculated based on the amount of interest revenue per unit multiplied by the average number of products sold during the time slack limit has not been reached in t_3.

Scenario 1, $t_1 \leq t_3 \leq T$

$$P_I = \sum_{l=1}^{n} \left(D_l \theta_l P_l I_d t_3 - \frac{D_l T^* \theta_l^2 P_l I_d}{2} \right) \tag{8}$$

Scenario 2, $t_3 < t_1 < T$

$$P_I = \sum_{l=1}^{n} \frac{D_l P_l I_d \, t_3^2}{2T^*} \tag{9}$$

Scenario 3, $t_1 < T < t_3$

$$P_I = \sum_{l=1}^{n} D_l P_l I_d \left(t_3 - \frac{T^*}{2} \right) \tag{10}$$

By substituting equations (2), (4), (5), (6), and (8) into equation (1), the total inventory cost for scenario 1 is

$$Z = \frac{S^*}{T^*} + \sum_{l=1}^{n} \left[\begin{array}{c} \frac{D_l T^* \theta_l P_l h_l (2-\theta_l)}{2} + \\ \frac{D_l T^* U_l (1-\theta_l)^2}{2} + \\ D_l P_l (1-\theta_l) \\ - \left(D_l \theta_l P_l I_d t_3 - \frac{D_l T^* \theta_l^2 P_l I_d}{2} \right) \end{array} \right] \tag{11}$$

Next, to find the cost of minimum total inventory and optimal T in scenario 1 is achieved if $\frac{\delta Z}{\delta T} = 0$, then obtained as follows:

$$T^* = \sqrt{\frac{2S}{\sum_{l=1}^{n} \left[\begin{array}{c} D_l \theta_l P_l h_l (2-\theta_l) + \\ D_l U_l (1-\theta_l)^2 + D_l \theta_l^2 P_l I_d \end{array} \right]}} \tag{12}$$

By substituting equations (2), (4), (5), (6), (7), and (9) into equation (1), the total inventory cost for scenario 2 is

$$Z = \frac{S^*}{T^*} + \sum_{l=1}^{n} \left[\begin{array}{c} \frac{D_l T^* \theta_l P_l h_l (2-\theta_l)}{2} + \frac{D_l T^* U_l (1-\theta_l)^2}{2} \\ + D_l P_l (1-\theta_l) + \\ \left(\begin{array}{c} P_l I_c \frac{(2 D_l T^* - D_l T^* \theta_l - D_l t_3))}{2} \\ x \left(\theta_l - \frac{t_3}{T^*} \right) \end{array} \right) \\ - \left(\frac{D_l P_l I_d \, t_3^2}{2T^*} \right) \end{array} \right] \tag{13}$$

Next, to find the cost of minimum total inventory and optimal T in scenario 2 is achieved if $\frac{\delta Z}{\delta T} = 0$, then obtained as follows:

$$T^* = \sqrt{\frac{2S + \sum_{l=1}^{n} \left[P_l D_l t_3^2 (I_c - I_{d)} \right]}{\sum_{l=1}^{n} \left[\begin{array}{c} D_l \theta_l P_l h_l (2-\theta_l) + \\ D_l U_l (1-\theta_l)^2 + P_l I_c \left(2 D_l \theta_l - D_l \theta_l^2 \right) \end{array} \right]}} \tag{14}$$

By substituting equations (2), (4), (5), (6), and (10) into equation (1), the total inventory cost for scenario 3 is as follows:

$$Z = \frac{S^*}{T^*} + \sum_{l=1}^{n} \left[\begin{array}{c} \frac{D_l T^* \theta_l P_l h_l (2-\theta_l)}{2} + \frac{D_l T^* U_l (1-\theta_l)^2}{2} \\ + D_l P_l (1-\theta_l) - \left(D_l P_l I_d \left(t_3 - \frac{T^*}{2} \right) \right) \end{array} \right] \tag{15}$$

Next, to find the cost of minimum total inventory and optimal T in scenario 2 is achieved if $\frac{\delta Z}{\delta T} = 0$, then obtained as follows:

$$T^* = \sqrt{\frac{2S}{\sum_{l=1}^{n} \left[\begin{array}{c} D_l \theta_l P_l h_l (2-\theta_l) + \\ D_l U_l (1-\theta_l)^2 + D_l P_l I_d \end{array} \right]}} \tag{16}$$

If there is warehouse limitation, then the total volume of each item i should be smaller or equal to the warehouse capacity, so the following limitation is made:

$$\sum_{i=1}^{n} Qw_i^* \leq W \tag{17}$$

The need for the area of each item of goods (Qw_i^*) is calculated by the following equation:

$$\sum_{i=1}^{n} Qw_i^* = Q_i^* \text{ x } w_i \tag{18}$$

If $\sum_{i=1}^{n} Qw_i^* > W$, then Q_i^* must be reduced according to the proportion of area requirement of each item so that orders will be made more frequently. The percentage of each item QwP_i^* can calculated by the following equation:

$$QwP_i^* = \frac{Qw_i^*}{\sum_{i=1}^{n} Qw_i^*} \tag{19}$$

Furthermore, to find the economic order quantity for item i in units after considering capacity, it is carried out with the following equation:

$$Qg_i^* = \frac{QwP_i^* \; x \; W}{w} \tag{20}$$

After calculating the economic order quantity for item i (units) by considering the capacity, then the inter-order period is calculated using the following equation [20]:

$$G = \frac{\sum_{i=1}^{n} Qg_i^*}{\sum_{i=1}^{n} D_i} \tag{21}$$

The quantity of economic order while in the period from each order must fulfill the number of demand (D) while in the period T^* is notated as such:

$$Q_i^{**} = D_i \, T^* \tag{22}$$

Procedure/Algorithm

The algorithm used to find the solution to achieve the optimal quantity of product order while considering the number of item types, damage factor, and permissible delay in payment time in the first stage and adding the problems of capacity constraint for the final model can be described as follows:

Algorithm of First Model

1. Compute T^* using equation (12) according to Scenario 1.

2. If $t_3 \leq T^*$, compute t_1, then check for result validity with the stated condition in Scenario 1 ($t_1 \leq t_3$). If it is valid, then T* is valid.
3. If t1 does not valid with condition in scenario 1 compute T^* using equation (14) according to Scenario 2.
4. If $t_3 \leq T^*$, compute t_1, then check for result validity with the stated condition in Scenario 2 ($t_3 < t_1$). If valid, then T* is valid.
5. If T^* does not valid with scenario 1 and 2, compute T^* using equation (16) according to Scenario 3.
6. If $t_3 > T^*$, compute T^* using equation (16) according to Scenario 3, so that T^* valid.
7. Compute Z for each validated scenario.
8. Compare the Z for each validated T^*.
9. Choose T^* with the lowest amount of Z.
10. Compute Q_i^* with equation $Q_i^* = D_i\, T^*$.

Algorithm of Final Model

1. Compute T_{joint} using equation (12) according to Scenario 1.
2. If $t_3 \leq T_{joint}$, compute t_1, then check for result validity with the stated condition in Scenario 1 ($t_1 \leq t_3$). If Valid, then T_{joint} valid.
3. If the result does not valid, compute T_{joint} using equation (14) according to Scenario 2.
4. If $t_3 \leq T_{joint}$, compute t_1, then check for result validity with the stated condition in Scenario 2 ($t_3 < t_1$). If Valid, Then T^* valid.
5. If $t_3 > T^*$, compute T^* using equation (16) according to Scenario 3.
6. If, $t_3 > T_{joint}$, Compute T^* using equation (16) according to scenario 3, then T^* valid.
7. Compute Z for each validated T_{joint}.
8. Compare the Z for each validated T_{joint}.
9. Choose (T_{joint}) with the lowest amount of Z.
10. Compute Q_i^* with equation $Q_i^* = D_i\ T_{joint}$
11. Compute requirement of area each item using equation $Qw_i^* = Q^* x w_i$. If $\sum_{i=1}^{n} Qw_i^* \leq W$, Q_i^* is valid and stops in stage 10.
12. If $\sum_{i=1}^{n} Qw_i^* > W$, compute the percentage of the requirement area each item using equation (18).
13. Compute the economic order quantity for item i using equation (19).
14. Compute ordering period using equation (20).
15. Compute Q_i^* using equation $Q_i^* = D_i T^*$.

RESULT AND DISCUSSION

First Model

Data used in the first model of this study is secondary data taken from research by Limansyah and Lesmono [21] that developed the model that consider the number of product types and product expiration date and variable data that are assumed by Silitonga and Iskandar [12] that developed the model that consider the factors of expiration date and permissible delay in payment time in the same time such as the time of the permissible delay in payment given by the supplier, the fraction of products in good condition, percentage of fines and percentage in interest. The amount of interest and penalty can be calibrated according to the company, but the writer assumed that the percentage is according to what is implemented right now in Bank Central Asia [22] with 0.01 and fine with 0.03 [23]. There are also variables that were changed from Limansyah namely the absence of the selling price of the product used in this study and the all-unit discount factor not being considered, so that the product price data was taken based on the results of calculations Q_i^* which were adjusted to the price range on the data of Limansyah and Lesmono [21]. The following is a table of the number of goods and prices per unit can be seen in Table 1. The product prices are adjusted with the optimal quantity that resulted from the calculation.

Table 1. Quantity of Product and Price per Unit

Product A		**Product B**		**Product C**	
Qty	**Price**	**Qty**	**Price**	**Qty**	**Price**
≤ 115	Rp 11,500	≤ 175	Rp 9,500	≤ 250	Rp 15,000
> 115	Rp 10,000	> 175	Rp 8,000	> 250	Rp 14,000

Parameter Data

Parameter data are according to Limansyah and Lesmono [21] for demand data, inventory cost, and fraction of good product. Permissible delay in payment time data, interest percentage, and fine percentage are according to data gathered from Silitonga and Iskandar [12]. Data of warehouse capacity and volume of every item are derived from the writer assumption. The parameter data can be seen in Table 2.

Table 2. Data Parameter Model

No	**Parameter**	**Product A**	**Product B**	**Product C**
1	$D_i\ (unit)$	500	800	1250
2	$h_i\ (Rp)$	0.80	0.90	0.95
3	θ_i	0.80	0.80	0.80
4	$t_3\ (year)$	0.08		
5	I_c	0.03		
6	I_d	0.01		
7	W (m^3)	1000		
8	$w_i\ (m)$	3.5	3	4

Data Components of Inventory Cost

Data of inventory cost component is composed of purchase cost, order cost, and stock out cost. The purchasing cost are according to Limansyah and Lesmono [21] and according to the quantity calculation, the order cost data are according to Silitonga and Moses [20], and the stock out cost are from Silitonga and Iskandar data [12].

Table 3. Data Components of Inventory Cost

No	**Components of Inventory Cost**	**Product A**	**Product B**	**Product C**
1	$P_i\ (Rp)$	Rp 11,500	Rp 9,500	Rp 15,000
2	$S\ (Rp)$			Rp275,000
3	$U_i\ (Rp)$	Rp 50	Rp 100	Rp 150

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Data Processing of First Model

Firstly, perform calculations using algorithms based on three scenarios and compare the total inventory costs between each scenario, then select the scenario with the lowest total inventory cost. Results using the developed model are shown in Table 4.

Table 4. Result of The First Model Processing

No	Result of The Model Processing	Product A	Product B	Product C
1	Total inventory cost			Rp10,352,879
2	Optimal order quantity (unit)	69	111	173
3	Damage product quantity (unit)	14	22	35
4	Fine product quantity (unit)	15	24	38
5	The optimal joint order (year)	0.138		

According to the result, It showed that t_1 is not valid with Scenario 1 ($t_1 \leq t_3 \leq T$) because $t_1 > t_3$, so that it is continued with calculation according to scenario 2. Each component cost will be given in Table 5. The processing of the first model resulted in total inventory cost of Rp 10,352,879.

Table 5. Component Cost of The First Model

No	Component Cost	Scenario 1	Scenario 2	Scenario 3
1	Validation Q according to scenario (Y/N)	N	Y	N
No	**Component Cost**	**Type of Product**	**Cost**	**Total Cost**
2	Ordering Cost	All product with joint order policy	Rp 275,000	Rp 1,990,092
3	Holding Cost	Product A	Rp 305,111	Rp 1,940,277
		Product B	Rp 453,687	
		Product C	Rp 1,181,478	
4	Shortage Cost	Product A	Rp 69	Rp 808
		Product B	Rp 221	
		Product C	Rp 518	
5	Damage Cost	Product A	Rp 1,150,000	Rp 6,420,000
		Product B	Rp 1,520,000	
		Product C	Rp 3,750,000	
6	Fine Cost	Product A	Rp 1,636	Rp 9,135
		Product B	Rp 2,163	
		Product C	Rp 5,336	
7	Interest Revenue	Product A	Rp 1,332	Rp 7,434
		Product B	Rp 1,760	
		Product C	Rp 4,342	
Total Inventory Cost (Z)				**Rp 10,352,879**

Final Model

The data used in this final model research such as model parameter data and inventory cost component data are secondary data taken from Limansyah and Lesmono [21] and the variable data assumed by Silitonga dan Iskandar [12] is the same as the data taken for the data collection of the first stage of the model. However, in this final model there are added data that is assumed by the author, namely the large warehouse capacity is 1000 m^2 and the unit area of each item is 3.5 m^2, 3 m^2, dan 4 m^2.

Data Processing of Final Model

Based on the results of the first model data processing that has taken into account the number of types of goods, expiration, and permissible delay in payment, the optimal order quantities for the three types of goods are 69 units, 111 units, and 173 units, respectively. Thus, a calculation of the number of items that must be ordered is carried out by considering the capacity constraints following the algorithm starting from step 11 and using the equation in the algorithm so that the following results are obtained. The results using the final developed model are shown in Table 6. According to that, it is known that t_1 is not valid with scenario 1. Each component cost is given in Table 7.

Table 6. Result of The Final Model Processing

No	Result of The Model Processing	Product A	Product B	Product C
1	Total inventory cost	Rp	10,463,722	
2	Optimal order quantity (unit)	55	87	137
3	Damage product quantity (unit)	11	17	27
4	Fine product quantity (unit)	4	6	9
5	The optimal joint order (year)	0.109		

Table 7. Component Cost of The Final Model

No	Component Cost	Scenario 1	Scenario 2	Scenario 3
1	Validation Q according to scenario (Y/N)	N	Y	N
No	**Component Cost**	**Type of Product**	**Cost**	**Total Cost**
2	Ordering Cost	All product with joint order policy	Rp 275,000	Rp 2,516,250
3	Holding Cost	Product A	Rp 241,311	Rp 1,534,557
		Product B	Rp 358,820	
		Product C	Rp 934,426	
4	Shortage Cost	Product A	Rp 55	Rp 639
		Product B	Rp 175	
		Product C	Rp 410	
5	Damage Cost	Product A	Rp 1,150,000	Rp 6,420,000
		Product B	Rp 1,520,000	
		Product C	Rp 3,750,000	
6	Fine Cost	Product A	Rp 300	Rp 1,675
		Product B	Rp 396	
		Product C	Rp 978	
7	Interest Revenue	Product A	Rp 1,684	Rp 9,399
		Product B	Rp 2,225	
		Product C	Rp 5,490	
Total Inventory Cost (Z)				**Rp 10,463,722**

The processing of the first model resulted in total inventory cost of Rp. 10,463,722. Furthermore, a comparison of the costs of inventories from the model developed by the author with previous researchers is carried out. The cost components between inventory models 1 and 2 have differences, namely the number of types of products considered in the inventory model 2. The cost components between inventory models 2 and 3 also have differences, namely the capacity constraint factor that is not considered in the inventory model 2.

The following are the results of calculating the optimal order quantity and reorder point (Table 8), as well as costs in the previous inventory model and the inventory model that has been developed (Table 9):

Table 8. Comparison of Data Processing Results

No	Result	Type of Product	Iskandar's Model (Individual Order)	Writer's First Model (Joint Order)	Writer's Final Model (Joint Order)
1	Minimize total inventory cost	All Types of Product	Rp 11,028,315	Rp 10,352,879	Rp 10,463,722
2	Optimal order quantity (unit)	Product A	137	69	55
		Product B	168	111	87
		Product C	164	173	137
3	Damage product quantity (unit)	Product A	27	14	11
		Product B	34	22	17
		Product C	33	35	27
4	Fine product quantity (unit)	Product A	70	15	4
		Product B	71	24	6
		Product C	31	38	9
5	The optimal joint order (year)	Product A	0.275	0.138	0.109
		Product B	0.211		
		Product C	0.131		

The model from the first stage of development focus on the factors of expiration date and permissible delay in payment time by considering the number of item types have an advantage to accommodate the ideal condition of a company with factors of high number of item type, expiration date, and have an unlimited capacity of warehouse for inventory.

The model from the first stage development is also used for analyzing the ratio of loss if capacity becomes a problem so that the best alternative with the minimum inventory cost can be decided. Does expanding the warehouse or calibrating the quantity according to capacity and deciding the best policy in ordering that give the minimum total cost of inventory for an unlimited warehouse. The final model or stage 2 model is developed to accommodate the ideal condition of the retail industry such as minimarkets if it have an inventory with the factor of expiration date and permissible delay in payment time by considering the number of item type and the constraint of warehouse capacity.

Table 9. Comparison of Inventory Cost

No	Result	Type of Product	Iskandar's Model (Individual Order)	Writer's First Model (Joint Order)	Writer's Final Model (Joint Order)
1	Ordering Cost	Product A	Rp 546,112	Rp 1,990,092	Rp 2,516,250
		Product B	Rp 712,300		
		Product C	Rp 1,145,725		
		Total	**Rp 2,404,137**	**Rp 1,990,092**	**Rp 2,516,250**
2	Holding Cost	Product A	Rp 527,364	Rp 305,111	Rp 241,311
		Product B	Rp 691,394	Rp 453,687	Rp 358,820
		Product C	Rp 1,119,378	Rp 1,181,478	Rp 934,426
		Total	**Rp 2,338,137**	**Rp 1,940,277**	**Rp 1,534,557**
3	Shortage Cost	Product A	Rp 137	Rp 69	Rp 55
		Product B	Rp 168	Rp 221	Rp 175
		Product C	Rp 164	Rp 518	Rp 410
		Total	**Rp 469**	**Rp 808**	**Rp 639**
4	Damage Cost	Product A	Rp 1,000,000	Rp 1,150,000	Rp 1,150,000
		Product B	Rp 1,520,000	Rp 1,520,000	Rp 1,520,000
		Product C	Rp 3,750,000	Rp 3,750,000	Rp 3,750,000
		Total	**Rp 6,270,000**	**Rp 6,420,000**	**Rp 6,420,000**
5	Fine Cost	Product A	Rp 9,524	Rp 1,636	Rp 300
		Product B	Rp 8,271	Rp 2,163	Rp 396
		Product C	Rp 4,097	Rp 5,336	Rp 978
		Total	**Rp 21,892**	**Rp 9,135**	**Rp 1,675**
6	Interest Revenue	Product A	Rp 583	Rp 1,332	Rp 1,684
		Product B	Rp 1,155	Rp 1,760	Rp 2,225
		Product C	Rp 4,583	Rp 4,342	Rp 5,490
		Total	**Rp 6,320**	**Rp 7,434**	**Rp 9,399**
Total Inventory Cost (*Z*)			**Rp 11,028,315**	**Rp 10,352,879**	**Rp 10,463,722**

The comparison of cost has been done and the result is the total cost of the developed model is the smallest compared to the whole model because it implemented the joint order policy and consider product damage and permissible delay in payment time. next, the Silitonga and Iskandar [12] model have a higher total cost compared to the model that is being developed because the model doesn't consider the number of item types that have the same supplier so that the order policy must be individual order while the model that is developed does the order together for many product types of one supplier or joint order policy. According to that, the final model developed is more suitable with the actual condition of retail company such as minimarkets that consider the factor of multi-item, product damage, permissible delay in payment and warehouse capacity.

The comparison of the mathematical model structure between Iskandar's model and the model being developed can be seen in Figure 4.

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Component Cost	Iskandar's Model [12]	First Model	Second Model
Ordering Cost (O_p)	$O_p = s \times \frac{1}{T}$	$O_p = S^* \times \frac{1}{T^*}$	$O_p = S^* \times \frac{1}{T^*}$
Holding Cost (O_s)	$O_s = \frac{DT\theta Ph(2-\theta)}{2}$	$O_s = \sum_{i=1}^{n} \frac{D_i T^* \theta_i P_{i1} h_1 (2-\theta_i)}{2}$	$O_s = \sum_{i=1}^{n} \frac{D_i T^* \theta_i P_{i1} h_1 (2-\theta_i)}{2}$
Shortage Cost (O_{so})	$O_{so} = \frac{DTU(1-\theta)^2}{2}$	$O_{so} = \sum_{i=1}^{n} \frac{D_i T^* U_i (1-\theta_i)^2}{2}$	$O_{so} = \sum_{i=1}^{n} \frac{D_i T^* U_i (1-\theta_i)^2}{2}$
Damage Cost (O_{kd})	$O_{kd} = DP(1-\theta)$	$O_{kd} = \sum_{i=1}^{n} D_i P_i (1-\theta_i)$	$O_{kd} = \sum_{i=1}^{n} D_i P_i (1-\theta_i)$
Fine Cost (O_d)	$O_d = PI_c \frac{(2DT - DT\theta - DT_3))}{2} \; x \; \theta - \frac{t_3}{T}$	$O_d = \sum_{i=1}^{n} P_i I_c \frac{(2D_iT^* - D_iT^*\theta_i - D_it_3))}{2} \; x \left(\theta_i - \frac{t_3}{T^*}\right)$	$O_d = \sum_{i=1}^{n} P_i I_c \frac{(2D_iT^* - D_iT^*\theta_i - D_it_3))}{2} \; x \left(\theta_i - \frac{t_3}{T^*}\right)$
Interest Revenue (P_I)	**Scenario 1, $t_1 \le t_3 \le T$** $P_I = D\theta PI_d\, t_3 - \frac{DT\theta^2 PI_d}{2}$	**Scenario 1, $t_1 \le t_3 \le T$** $P_I = \sum_{i=1}^{n} \left(D_i\theta_iP_iI_dt_3 - \frac{D_iT^*\theta_i^2P_iI_d}{2}\right)$	**Scenario 1, $t_1 \le t_3 \le T$** $P_I = \sum_{i=1}^{n} \left(D_i\theta_iP_iI_dt_3 - \frac{D_iT^*\theta_i^2P_iI_d}{2}\right)$
	Scenario 2, $t_3 < t_1 < T$ $P_I = \frac{D\theta PI_d\, t_3^2}{2T}$	**Scenario 2, $t_3 < t_1 < T$** $P_I = \sum_{i=1}^{n} \frac{D_iP_iI_d\, t_3^2}{2T^*}$	**Scenario 2, $t_3 < t_1 < T$** $P_I = \sum_{i=1}^{n} \frac{D_iP_iI_d\, t_3^2}{2T^*}$
	-	**Scenario 3, $t_1 < T \le t_3$** $P_I = \sum_{i=1}^{n} D_iP_iI_d\left(t_3 - \frac{T^*}{2}\right)$	**Scenario 3, $t_1 < T \le t_3$** $P_I = \sum_{i=1}^{n} D_iP_iI_d\left(t_3 - \frac{T^*}{2}\right)$
Component Cost	**Iskandar's Model [12]**	**First Model**	**Second Model**
The Optimal Joint Order (T^*)	**Scenario 1, $t_1 \le t_3 \le T$** $T^* = \sqrt{\frac{2S}{D\theta Ph(2-\theta) + DU(1-\theta)^2 + D\theta^2PI_d}}$	**Scenario 1, $t_1 \le t_3 \le T$** $T^* = \sqrt{\frac{2S}{\sum_{i=1}^{n}\left[D_i\theta_iP_1h_1(2-\theta_i) + D_iU_i(1-\theta_i)^2 + D_i\theta_i^2P_iI_d\right]}}$	**Scenario 1, $t_1 \le t_3 \le T$** $T^* = \sqrt{\frac{2S}{\sum_{i=1}^{n}\left[D_i\theta_iP_1h_1(2-\theta_i) + D_iU_i(1-\theta_i)^2 + D_i\theta_i^2P_iI_d\right]}}$
	Scenario 2, $t_3 < t_1 < T$ $T^* = \sqrt{\frac{2S + DP\, t_3^2(I_c - I_d)}{D\theta Ph(2-\theta) + DU(1-\theta)^2 + 2D\theta PI_c + D\theta^2PI_c}}$	**Scenario 2, $t_3 < t_1 < T$** $T^* = \sqrt{\frac{2S + \sum_{i=1}^{n}\left[P_iD_it_3^2(I_c - I_d)\right]}{\sum_{i=1}^{n}\left[D_i\theta_iP_1h_1(2-\theta_i) + D_iU_i(1-\theta_i)^2 + P_iI_c(2D_i\theta_i - D_i\theta_i^2)\right]}}$	**Scenario 2, $t_3 < t_1 < T$** $T^* = \sqrt{\frac{2S + \sum_{i=1}^{n}\left[P_iD_it_3^2(I_c - I_d)\right]}{\sum_{i=1}^{n}\left[D_i\theta_iP_1h_1(2-\theta_i) + D_iU_i(1-\theta_i)^2 + P_iI_c(2D_i\theta_i - D_i\theta_i^2)\right]}}$
	-	**Scenario 3, $t_1 < T \le t_3$** $T^* = \sqrt{\frac{2S}{\sum_{i=1}^{n}\left[D_i\theta_iP_1h_1(2-\theta_i) + D_iU_i(1-\theta_i)^2 + D_iP_iI_d\right]}}$	**Scenario 3, $t_1 < T \le t_3$** $T^* = \sqrt{\frac{2S}{\sum_{i=1}^{n}\left[D_i\theta_iP_1h_1(2-\theta_i) + D_iU_i(1-\theta_i)^2 + D_iP_iI_d\right]}}$
Order Size Optimal	$Q^* = DT^*$	$Q_i^* = D_iT^*$	$Q_i^* = D_iT^*$
Order Size Optimal Considering Warehouse	-	-	$Qg_i^* = \frac{QwP_i^* \; x\, W}{W}$
The Optimal Joint Order Considering Warehouse	-	-	$G = \frac{\sum_{i=1}^{n} Qg_i^*}{\sum_{i=1}^{n} D_i}$

Figure 4. Comparation of mathematical equation in each the model

There are differences between Iskandar's inventory model and the two model that is developed. The difference of mathematical equation of the whole cost component of the first inventory model compared to second model is caused by the lack of product type number that is considered so that each product in model 1 is ordered with individual order policy so that it have a variation of optimal order time, on the other hand, the mathematical equation of the second inventory model considered item type, so that with joint order policy where order time and arrival time of goods are the same for every type of product. In the third model, there is an addition of mathematical equation that model 1 and 2 doesn't have which is the equation that determine quantity of an optimal order in relation to the available capacity with an approach in the demand proportion according to the boundary equation from Silitonga and Moses [21] model, so that the equation for an optimal order time while considering warehouse capacity exist.

Sensitivity analysis is proceeded to check the influence of uncontrollable parameter to the output of the model [24]. The result of sensitivity analysis to find the effects of parameters used to the result of the model that is being developed can be seen in Figure 5. These parameters are fine percentage, interest percentage, and permissible delay in payment time, the proportion of good products and warehouse capacity tested against the total inventory cost.

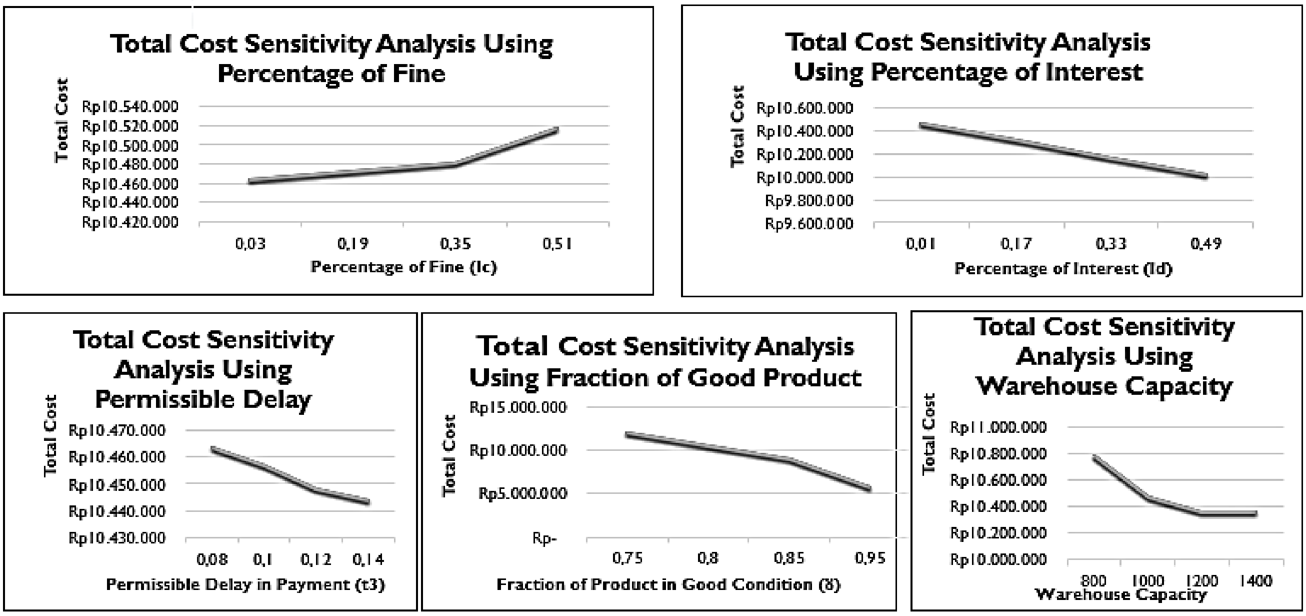

Figure 5. Sensitivity Analysis

According to the sensitivity analysis, it was found that the higher the fine percentage the higher the total inventory cost. The higher the interest percentage, longer permissible delay, promising proportion of good product, and capacity limitation that can contain and Q optimum size, then the total inventory cost is smaller. The parameter that affected the cost the most is the proportion of good products. The increase in good product proportion of up to 95% decrease the total cost up to 52.54%.

CONCLUSIONS

This study developed a multi-item economic order quantity model that considers expiration factor, permissible delay in payment, and warehouse capacity constraints. The development of the model is carried out in 2 stages, starting with the development of a model taking damage factor and permissible delay in payment by considering the number of types of items. The second stage is developing a model taking capacity constraints. This aims to accommodate the actual conditions of minimarkets when faced with limited warehouse capacity, they can conduct a comparison analysis to the best alternative with the minimum inventory cost, whether to expand the warehouse or adjust the quantity according to capacity.

Based on the results of the comparative analysis of total inventory costs, it can be concluded that the more factors from the inventory system are considered, the higher the total inventory costs. Because of this, the total cost of the first model that is developed is cheaper by margin of Rp110,843 compared to the last model developed because it doesn't account the limited warehouse capacity. Through this inventory model, a company can obtain the minimum total inventory cost that can save the total cost of inventory by Rp.675,436. by selecting an alternative policy for ordering goods, whether with an individual order policy or a joint order policy.

The result of the sensitivity analysis shows that a change in good product proportion have a major effect to the total cost of inventory. The increase in good product percentage by 20% decrease the total cost of inventory by 52.54%. The direction of further research can be directed by doing research of expired goods effected by expiration time and conducting research that consider multi supplier factor.

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NOMENCLATURE

D_i	: Total demand for item i in one planning horizon (unit/year)
Q_i	: Order size optimal for item i (unit)
P_i	: Purchase price item i per unit (Rp/unit)
S^*	: Cost of placing one order with joint order policy (Rp/unit)
h_i	: Holding cost item i for one planning horizon (Rp/unit/year)
U_i	: Stock out cost item i for one planning horizon (Rp/unit/year)
T^*	: The optimal joint order (year)
t_1	: Time period until product in good condition is sold out (year)
t_2	: Time period in shortage (year)
t_3	: Permissible delay in payment (year)
I_d	: Percentage of interest can be earned
I_c	: Percentage of fine given by vendor in Rupiahs per unit
θ_i	: Fraction of product in good condition $(0 < \theta_i < 1)$
$1 - \theta_i$	: Fraction of damage product
w_i	: Volume size for item i (unit of volume)
QwP_i^*	: Percentage of order size optimal for item i considering warehouse capacities
Qw_i^*	: Required area for item i (volume)
QW_i^*	: Volume for item i considering warehouse capacities (volume)
Qg_i^*	: Order size optimal for item i considering warehouse Capacities (unit)
W	: Total warehouse capacities (unit of volume)
G	: The optimal joint order considering warehouse capacities (year)
Z	: Total inventory cost for during one planning horizon (Rp)