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NUMERICAL SIMULATION OF THE QUANTUM STATES OF SQUEEZED LIGHT

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Анотація. Розглянуто квантову концепцію кодування інформації. З метою зменшення складності приготування квантових станів розроблено обчислювальний метод симуляції квантових станів світла отриманих раніше томографічними методами. Наведено результати комп'ютерної симуляції когерентних та розмитих станів.

Аннотация. Рассмотрена квантовая концепция кодирования информации. С целью уменьшения сложности приготовления квантовых состояний разработан численный метод симуляции квантовых состояний света ранее полученных и проанализированных томографическими методами. Наводятся результаты компьютерной симуляции когерентных и размытых состояний.

Abstract. The quantum concept of the information encoding is examined. States of a light with explicit quantum character were generated and analyzed by tomographical methods are hold on. A numerical method for simulation of the light quantum states by compare of a sine function with it's conversion to a digital one (the sine is discretized, quantized and delayed) is developed. Some simulate results are presented.

Key words: information encoding, light, quantum state, homodyne experiment, computing simulation.

INTRODUCTION

Obtaining information about the states of quantum fields of matter and radiation is a central theme in many fields of application of quantum physics, e.g. as a quantum computing, high resolution measurements in nanotechnologies etc. Light fields are those have many advantages in of such applications. Recently has been achieved experimental success in generating and determining states of various quantum mechanical systems by quantum state reconstruction (QSR) method. Historically, maybe the light field was the first on which QSR was performed [1].

A single spatial monochromatic mode of light represents a harmonic oscillator system for which non-classical states can be generated very efficiently using the interaction of laser light with nonlinear optical media. Squeezed states have a reduced uncertainty in a specific quadrature compared to that of the vacuum state. They have been typically characterized by measuring the variance of the electric field with a homodyne detector state. A study of all types of squeezed states of light (squeezed vacuum, amplitude squeezed states and states squeezed in an arbitrary quadrature) with construct portraits contained the all about the quantum mechanical properties of the squeezed optical states are given in [1].

Nevertheless, experimental simulations because of their complexity are less convenient in use for study, develop and research of quantum hardware. More appropriate for that would be numerical tools.

In this paper a numerical method for the quantum states simulation by compare of a sine function with it's conversion to a digital one (the sine is discretized, quantized and delayed) is presented.

REQUIREMENTS FOR THE SIMULATION OF QUANTUM STATE

Like in [2] is able to make deduce three (plus two at least) requirements for the implementation of simulation of quantum states. So, there are needs:

a) a scalable numerical process with a well characterized states;

b) the ability initialized the state;

c) long relevant decoherence times much longer than for the simulation of the quantum state,

and

d) a universal set of quantum gates (logical gates);

e) a qubit-space determining capability – for simulation of the computation, measurement etc applications.

In this paper are presented (a, b, c) aspects rather then (c, d) ones.

NUMERICAL PROCESS WITH A WELL CHARACTERIZED STATES

Recently was researched the harmonic test signal noise in the channels with digital signal processing [3].

We hold the

$$\xi_k = s_k^1 - s_k^2, \tag{1}$$

where $s_k^1 = \Xi_1\left[A \cdot sin(2\pi k\mu)\right]$, $s_k^2 = \Xi_2[A \cdot sin 2\pi k\mu']$, $\Xi_{1,2}[\cdot]$ – quantized operators with different weight c of the least significant bit; if T_d – period of discrete, then $\mu = \frac{T_d}{T_s}$, $\mu' = \frac{T_d + \theta}{T_s}$, $0 < \theta < T_d$, $k = \overline{0,\infty}$.

Quantized operators Ξ are given by expression

$$\Xi_\bullet : s(kT_d) \in R \to \hat{s}(kT_d) \in \left\{e_m, m = \overline{0,n-1}\right\}_k,\ e_m \in GF(2).$$

The operator $\Xi'_\bullet : \hat{s} \in GF(2^n) \to s \in R$ is not exists — $\Xi'_\bullet \neq \Xi_\bullet^{-1}$, so, an error appears:

$$\xi_k = s\left(kT_d\right) - \tilde{s}\left(kT_d\right).$$

We hold the $c = \frac{A}{2^n}$, , where c — the weight of the least significant bit of n-binary digital code, and $e_0, e_{1\cdot i} \in GF\left(2^n\right)$, the zero and bases elements of the finite extended field of Galois with properties $e_{1i} \wedge e_{1j} = \begin{cases} e_0, & i \neq j; \\ e_{1i}, & i = j; \end{cases}$ and $e_0 \wedge e_{1i} = e_0,\ i = \overline{1,n}$ are true. We put on for getting $\alpha_i \in \left\{e_0, e_{1i}, i = \overline{1,n}\right\}$ such recursive algorithms

$$y_i = s(mT_d), y_{i+1} = Ent\left(\frac{y_i}{2}\right);$$

$$\alpha_{i+1} = \begin{cases} e_0, & (y_{i+1} < 1) \vee (y_i - 2y_{i+1} = 0) \\ e_{1i}, & (y_{i+1} = 1) \vee (y_i - 2y_{i+1} = 1) \end{cases},$$

where $Ent(\cdot)$ — entire part of number, $i = \overline{0,n-1}$ — a number. Then

$$\Xi_1\left[s(\cdot)\right] = \begin{cases} \bigcup_i^n e_{1i}, & |s(\cdot)| \geq A; \\ \bigcup_i^n (\alpha_i \wedge e_{1i}), & \frac{c}{2} \leq |s(\cdot)| < A; \\ e_{0,} & |s(\cdot)| < \frac{c}{2}. \end{cases} \tag{2}$$

Put on $\beta_{im} = \begin{cases} 0, \ \alpha_{im} = e_0; \\ 1, \ \alpha_{im} = e_{1i.}. \end{cases}$. Then inverse transform

$$\Xi_1' : \tilde{s}_m = c\sum_{i=1}^{n} \beta_{im} 2^{i-1} .$$

The error depends from the speed of change of *s* value in the time:

$$\left|\xi_{\bullet}\right| \in \begin{cases} (0, c), & \left.\dfrac{ds(t)}{dt}\right|_{\max} \le c / T_d \\ (0, q \cdot c), & \left.\dfrac{ds(t)}{dt}\right|_{\max} > c / T_d \end{cases},$$

where $q \in Z$.

In dependence of μ type (natural, rational or irrational) will be different means of cycle $\nu = [\phi(Q(\mu))]^{-1}$, where Q — complement of fraction to entire number, $\phi(\cdot)$ — Riemann function of values ξ_k repeating []. That is why values ξ_k are taken across period $N = Ent(\mu^{-1})$, where $Ent(\cdot)$ — entire part of number can be gathered to group $\xi_{i\eta}$, $\eta = \overline{1, N}$, $i = \overline{1, \nu}$. Because non-commensurable of sine's periods T_s with period of discreet T_d appears a "run" of a phase of the sine sample. For a finite interval of observation *T* the set of values $\xi(iT_d)$ in dependence from a class of a number μ (natural, rational, irrational), is split by the closing cycle condition $Q(\frac{m}{T_d}) = \frac{\mu}{\nu}$ (where $m = T_d Fr(\mu), Q(\bullet) = 1 - Fr(\bullet), Fr(\bullet)$ — fraction part of a number) on subsets of power M_Ω that depend from value *T* (number of cycles). Values M_Ω for one cycle are equal:

$$M_\Omega = \begin{Bmatrix} \mu \\ \nu Ent(\mu) \\ \infty \end{Bmatrix} \quad \text{if} \quad \frac{T_s}{T_d} \in \begin{Bmatrix} \mathrm{N} \\ \mathrm{R} \\ \mathrm{Q} \end{Bmatrix}.$$

These quantities of error values are representative and generate sequence of vectors $\xi(m, \omega), m = \overline{1, M}, M = Ent(\mu), \omega \in \Omega$. It was fundament for definition of indicators mapping of indexes $\mathrm{I} : \{i\} \xrightarrow{\mathrm{I}^i_{m,\omega}} \{m, \omega\}$. After this indicator influence the set of realisations is received (see also [103])

$$\xi(iT_d) \xrightarrow{\mathrm{I}^i_{m,\omega}} \xi^{(n)}_{p_1,\ldots,p_n}(m, \omega), \tag{3}$$

where $p \in K$. We can say nothing about any influence of permutations of components of every *m*-vector because the invariant property for these.

The fig. 1 showed mapping result (3) obtained by an experiment in which continuous sine function is minuses the digital to analogues conversion (showed on the top) of its analogue to digital conversion. Then the quantities $\xi_{i\eta}$ are scattered on continuous slash lines (to see below). These illustrations show that the probability

distribution function of $\xi_{i\eta}$ values depends on the time, i.e., the probabilistic model of the noise in this case is not stationary and should have periodic properties. For example, the correlation function is periodic, as is obvious visually.

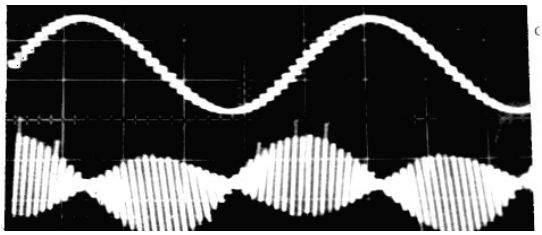


Fig. 1. Photography of $\xi_{i\eta}$ (below) and s_k^2 (high) have been obtained under the experiment (scales on ordinate axe are differ)

A periodically correlated random sequence (PCRS) therefore is a model of the (1). By definition, it is a stochastic sequence whose mean m_n and covariance r_n are such that $m_{n+N} = m_n$ and $r_{n+N,j+N} = r_n$. The number N is then called the correlation period. By its properties a PCRS is the process with a complex values and it is essential to be capable to enlargement it into more simple processes. In a stationary model of stochastic sequences this enlargement is possible because of its invariance relative to the time shifts interpreted as a group of transformations, which automatically leads to spectral expansion into harmonic components as group characters. In conformity with the general trend of research in the field of non-stationary processes, methods of analysis of stationary processes are used to study PCRS as sequences of a certain class of non-stationary, introducing the appropriate modifications into these methods. To this end, on a set of generally complex-valued random sequences are defined a scalar product $(\cdot,\cdot)_{\mathrm{H}}$, which appears, for example, as applied to the values of the sequence in the following form

$$(\xi_{j+n},\xi_j)_{\mathrm{H}} = m_j(\overset{\circ}{\xi}_{j+n}\,\overline{\overset{\circ}{\xi}}_j) = \lim_{K\to\infty}\frac{1}{K-j+1}\sum_{k=1}^{K}(\overset{\circ}{\xi}_{k+n}\,\overline{\overset{\circ}{\xi}}_k) = b_{j,n},$$

where $\overline{\overset{\circ}{\xi}}$ is the conjugate of $\overset{\circ}{\xi}_{\cdot} = \xi_{\cdot} - m_{\cdot}^{\xi}$, m_j^{ξ} is the value of the expectation, defined generally from the ensemble of realizations.

For the space H of the sequences in which such scalar product is defined, it is natural to define the norm as $\|\xi\|_{\mathrm{H}}^2 = (\overset{\circ}{\xi}_j,\overset{\circ}{\xi}_j)_{\mathrm{H}} = b_{j,0}$. Since $b_{j,0}$ also defines the mean power which, because of the physical nature, is finite, the space H is a Hilbert space. Proceeding from the general properties of representations of groups and functions on groups, we see that the definition of PCRS leads to cyclic properties of the group of shifts of their values, where the shift operator on an additive cyclic group of addition of integers (indexes) modulo N will be unitary, i.e., the values b_{jn} are independent of j. This means that PCRS in the Hilbert space H is weakly stationary, indicating the possibility of defining ergodicity on such a class of sequences, denoted by it. The ergodicity of PCRS allows computing the estimates of this sequence (the mean and the covariance) from its realizations.

According to the theory of integral representations, a given PCRS can be obtained from a process with uncorrelated values if we subject this process to a covariance operator with the kernel defined as $B_n = (\overset{\circ}{\xi}_{j+n},\overset{\circ}{\xi}_j)_{\mathrm{H}}$.

The stationarity of PCRS in the Hilbert space H allows its spectral representations — expansions into a series of orthonormalized eigenfunctions φ_{kn} of the covariance operator kernel which corresponds to the eigenvalues λ_k. The convergence of such series is considered with respect to the norm $\|\xi\|_{\mathrm{H}}^2$, the processes differ solely in the power, i.e., for the Hilbert space H, statistical estimates of mean characteristics are defined in the case of processes belonging to the class π^N. The relation $m_j\{\xi_j\} = \sum_{r=0}^{N-1} m_j\{\xi_{r+jN}\}/N$, which holds for

periodic sequences, makes it possible, however, to find not only the statistical estimates of mean characteristics, but also to determine the phase structure of these characteristics. Then PCRS is regarded as a random finite-dimensional vector (for a finite definition set), or as a random vector in H (when the definition set is countable), where the vector components are stationary Hilbert sequences. One must then examine the respective covariance matrices, and for spectral representations, two-dimensional spectral functions.

Thus distinguish of a digital sine signal from usual sine is so that it has decomposition of it's probably parts looked like quantum particle state. So, here is obviously to find correspondence with them.

DISCUSSION OF RESULTS

In the theory of the quantum mechanics the states of quantum particle are determines as the resolutions of Srödinger equation. Practically they are computed by the results of a homodyne experiment with harmonic waves as a reconstruction.

In this paper we use the numerical simulation when sine and digital sine are compare is look like in the homodyne experiment when bright squeezed light and squeezed vacuum is generate with an parametric optical oscillator and while scanning the phase.

Under consideration (1-3) we obtain portraits well coincide with [1] but is digitized:

a). Coherent states (fig. 2, a,b,c), when weight the least significant bit $c \equiv const$ and very small, $\mu = \frac{T_s}{T_d} \equiv const$ and very big, but time delay θ varied and its increasing lead to increasing ξ_n with constant noise value.

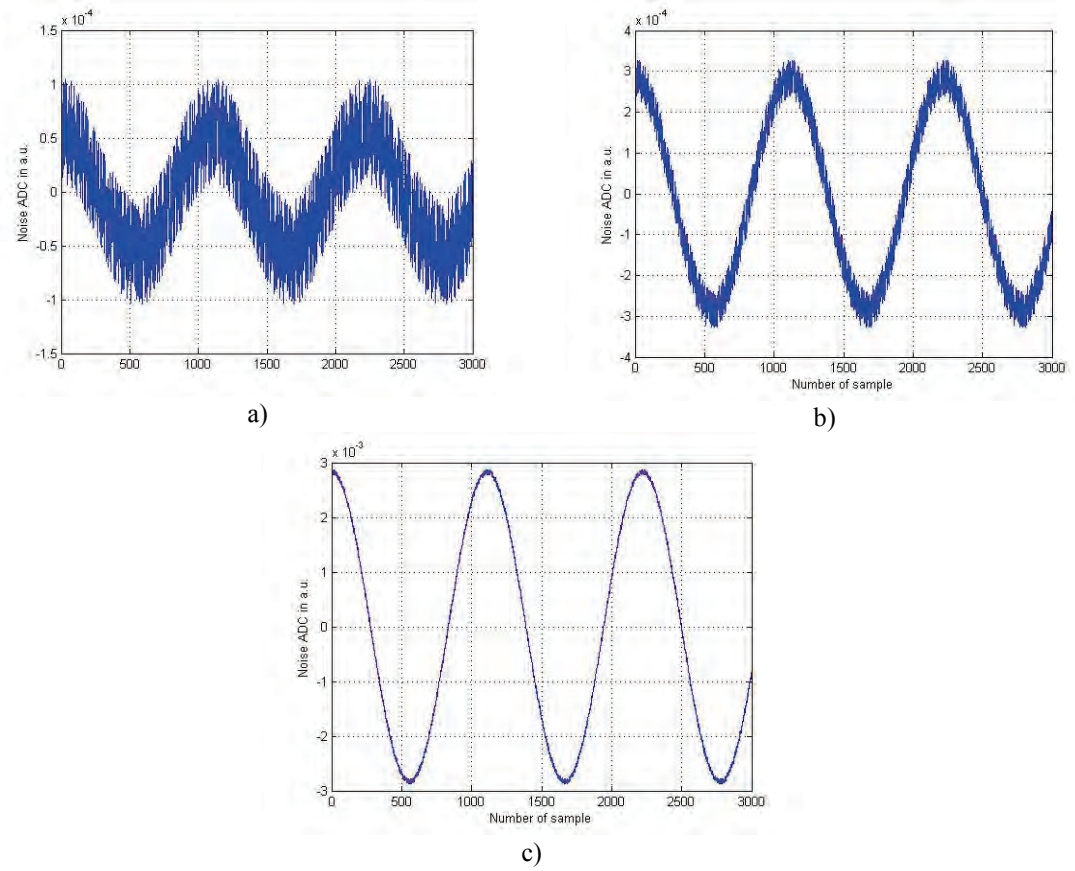


Fig. 2. Coherent states:
a) $A = 1$, $T_s = 1.11$, $T_d = 0{,}0001$, $c = 0{,}0001$, $\theta = 0.00001$;
b) $A = 1$, $T_s = 1.11$, $T_d = 0{,}0001$, $c = 0{,}0001$, $\theta = 0.00005$;
c) $A = 1$, $T_s = 1.11$, $T_d = 0{,}0001$, $c = 0{,}0001$, $\theta = 0.0005$.

b). Thermal state (fig. 3), when zero time delay.

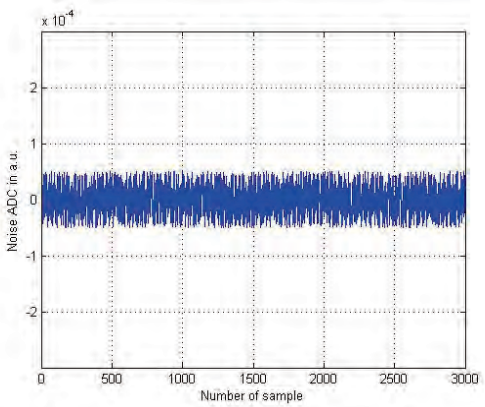
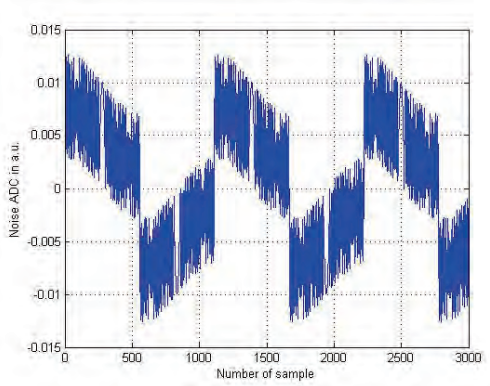
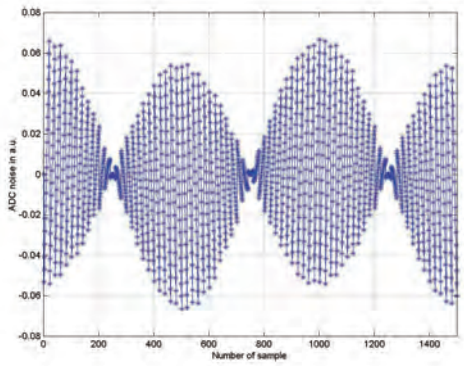


Fig. 3. Thermal state:
$A = 1$, $T_s = 1.11$, $T_d = 0{,}001$, $c = 0{,}0001$, $\theta = 0$

c). Squeezed states (fig. 4).



a)



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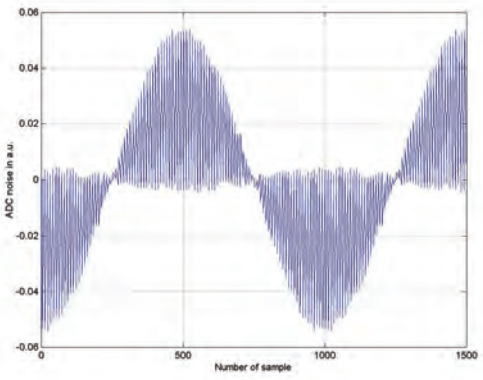
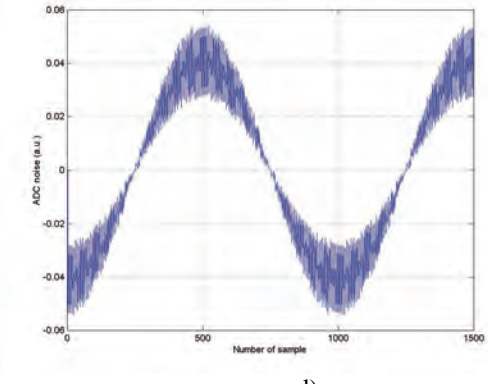


c)



d)

Fig. 4. Squeezed states:
a) amplitude squeezed state ($A = 1$, $T_s = 1.11$, $T_d = 0.001$, $c = 0.01$, $\theta = 0.0005$);
b) squeezed vacuum state ($A=1$, $T_s = 1$, $T_d = 0.02$, $c = 0.0035$, $\theta = 0.0095$);
c) phase squeezed state ($A =1$, $T_s = 1$, $T_d = 0.01$, $c = 0.0035$, $\theta = 0.0095$);
d) a state squeezed at a some angle of the phase of a quadrature ($A =1$, $T_s = 1$, $T_d = 0.005$, $c = 0.0035$, $\theta = 0.0095$).

CONCLUSION

So, three requirements for the implementation of simulation of quantum states are satisfied:

a) A scalable numerical process with well characterized states is obtained;
b) The ability for states initialize is determined;
c) Decoherence times are much long for the simulation of the quantum state are proved.

These results would be useful for inference of numerical algorithms — from quantum logical gates to simulation of the quantum computation, measurement etc. The complexity of quantum states preparations is decreased significantly [4] in comparison with resolution methods of Srödinger equation.

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