Effective Models for the Specular Color Constituent Computing

Alexander N. Romanyuk, Yuri L. Lyashenko, Alexander V. Melnik and Alexander P. Goncharuk

Abstract — This paper describes the alternative BRDF (Bidirection reflectance distribution function) models. Main emphasis was done on the models development with simple hardware implementation, based on formulas with low computation complexity. The results, achieved by authors, testify the increase of BRDF approximation accuracy in more than 22 times in comparison with most widespread solutions.

Index Terms — Shading, BRDF, Bidirectional Reflectance Distribution Function, Rendering.

1 Introduction

THE requirements towards graphics systems are growing steadily together with necessity of resolving more and more complicated and complex tasks of real-time high-quality images formation.

The most complicated and resource-intensive computations at rendering stage occur when shading 3D graphic objects, that has volume effect created by the color gradation of points within the scene.

On shading, for all points of the surface their coordinates and color intensities are calculated, that leads to big time expenses. This especially takes place when use complicated illumination models, in particular those, operating with diffuse and specular color constituents.

In this connection, the problem of increasing the shading productivity in computer graphics systems becomes especially important. Solution of this problem provides a possibility to setup effective interactive channel and to ensure necessary realism and dynamics of 3D images.

2 Approaches Analysis and Problem Formulation

On the shading 3D scenes according to the Phong method [1-3] color intensities are computed with following formula:

$I = I_l k_s cos^n \gamma$,

where I_l - Incident light source intensity, k_s - specular light coefficient, n - surface brightness coefficient, $cos^n \gamma$ – BRDF – bidirectional Reflectance Distribution Function, that characterizes optical properties of the surface.

The $cos^n \gamma$ function, used in Blinn and Phong illumination models [1, 2] has big computational complexity. The coefficient n is known to vary within the range from 0 to 1000.

On BRDF models creation one has to consider the most exacting requirements for the highlight epicentre representation [4, 5, 8]. For the peripheral regions, which are responsible for blooming zones formation it's necessary to provide monotonous color intensity fading to exclude artifact [6, 7]. At the same time there're no especial accuracy requirements towards color intensity determination within these zones [4, 8].

According to the method [9], suggested by R.F.Lion $cos^n \gamma$ is expanded into Taylor series and instead of angle γ (angle between reflected light direction and direction to observer) the length of the chord between these vectors is used.

This substitute doesn't have considerable influence upon the accuracy only for small values of the angle γ. Using limited amount of Taylor series terms doesn't allow approximation of $cos^n \gamma$ with the necessary quality.

Sufficiently high approximation accuracy was reached at the $cos^n \lambda$ approximation by function $cos^k(\sqrt{n/k} \cdot \gamma)$ [10], where $k = 2,4,8,...$ and is chosen depending on the n range, $(\kappa << n)$. Unfortunately, this approach requires λ computation, which means resource-intensive arc cosine operation usage.

C. Schlick [8] has suggested approximating $cos^n \gamma$ by $cos\,\gamma / (n - n\,cos\,\gamma + cos\,\gamma)$ function. The analysis revealed that this approach provides sufficient accuracy only for the highlight epicentre and is hardly applicable for hardware implementation.

The paper [11] contains the idea of approximating $cos^n \gamma$ by polynomial $cos^n \gamma = a \cdot cos^2 \gamma + b \cdot cos\, x\gamma + c$.

To find a, b, c they use values of $cos^n \gamma$ at start and end points, and also at cusp point. It's proved that $b = 1 - a$, $c = 0$. The hardware solution has a values for every n are stored in memory unit, and 2 other coefficient are computed by simple formulas. This solution allows sufficiently accurate highlight epicenter approximating with relative error no more than 4,5 %). Unfortunately, the fast fading of this function at $\gamma \to 0$ leads to sharp borders of the blooming. Much better accuracy was achieved on approximating by the polynomial of 3rd power [12]. The values of function at maximum, minimum and $cos^n \gamma$ =0,5 points were used to find coefficients. This approach guarantees relative error of epicenter approximation no more than 0,67 % [12]. Unfortunately, using the

---

- A.N. Romanyuk is within the Vinnitsa National Technical University, Vinnitsa, Ukraine.
- Y.L.Lyashenko, is within the Vinnitsa National Technical University, Vinnitsa, Ukraine.

polynomial of 3rd power doesn't provide sufficient blooming accuracy. So that, for specular color formation in blooming zone they used another function $g(\gamma) = cos(\gamma)/(a - b\,cos(\gamma))$, where unknown coefficients are found from the eq. $cos^n(\gamma) = g(\gamma)$ at $\gamma = arccos(\sqrt[n]{0.5})$, and from the similar equations with derivatives of these functions. This approach gives good accuracy, but is not applicable for the hardware implementation.

The paper [13] covers the problem of the BRDF hardware implementation for its equivalent form

$$cos^n \gamma = 2 \wedge n \log_2(cos\,\gamma)$$

The disadvantage of this approach is the insufficient accuracy of blooming zone formation and the necessity to use several memory units of big relatively big size for hardware implementation. In recent years, the acceptable alternative BRDF model, providing realistic representation of either epicenter or blooming highlight zone, having simple hardware implementation was not found.

The purpose of paper is – development and investigation of the bidirectional reflectance distribution function model, having low computational complexity and simple hardware implementation

3 BRDF Approximation Models

Let's try to approximate $cos^n \gamma$ by the function $W(n, \gamma) = \left(\zeta \cdot \left(cos\,\gamma - 1\right) + 1\right)^m$ on conditions that $0 \le \gamma \le \pi / 2$.

This function is suggested because:

a) both functions are based on the cosine function;

b) at $\gamma = 0$ $cos^n \gamma = \left(\zeta \cdot \left(cos\,\gamma - 1\right) + 1\right)^m = 1$, that satisfies boundary conditions;

c) both functions are positive, when $0 \le \gamma \le \pi/2$ and $m = 2k$, where $k \in N$;

d) $\left(\zeta \cdot (cos\,\gamma - 1) + 1\right)^m$ function reaches zero value, that is pre-condition of blooming formation;

e) ζ coefficient changes the highlight size.

To determine ζ in $\left(\zeta \cdot (cos\,\gamma - 1) + 1\right)^m$ we expand $cos^n \gamma$ into Taylor series and take first 2 terms $cos^n \gamma \approx 1 - n \cdot \gamma^2 / 2$.

The same procedure on $W(n, \gamma)$ leads to equation:

$$\left(\zeta \cdot (cos\,\gamma - 1) + 1\right)^m \approx 1 - \zeta \cdot m \cdot x^2.$$

Let's find ζ. Right sides of these formulas are supposed to be equal, so that $\zeta = n/m$. Let's examine the situation, when $m = 2$, than:

$$cos^n \gamma \approx \left(n(cos\,\gamma - 1)/2 + 1\right)^2.$$

Let's analyze this BRDF model.

The comparison with Schlick formula was done. This formula is widespread in computer graphics devices, because of its sufficient accuracy and low computation complexity.

The graphics of

$$cos^n \gamma \approx \left(n(cos\,\gamma - 1)/2 + 1\right)^2.$$

and of Schlick functions on the condition that $n = 100$ are shown on the fig.1. The fig.1 shows that suggested BRDF model approximates highlight epicenter with sufficient accuracy and monotonously fades to zero after the cusp point. The part of BRDF function above the cusp point is considered to be responsible for the highlight epicenter formation. The behavior of this function fits the requirements to BRDF model. The fig.2 shows graphics of maximal relative errors of the approximation of $cos^n \gamma$ by $W(n, \gamma)$ for epicenter zone. These figures show the accuracy of epicenter approximation has increased in comparison with Schlick formula.

The $W(n, \gamma)$ function reaches zero value, when $cos(\gamma) = (n-2)/n$. $W(n, \gamma)$ is square of real number – and is never negative. Let's find when derivative of $W(\gamma, n)$ is equal to zero.

$$W'_x(n, \gamma) = -\left(\frac{n}{2}(cos\,\gamma - 1) + 1\right) \cdot n \cdot sin\,\gamma = 0.$$

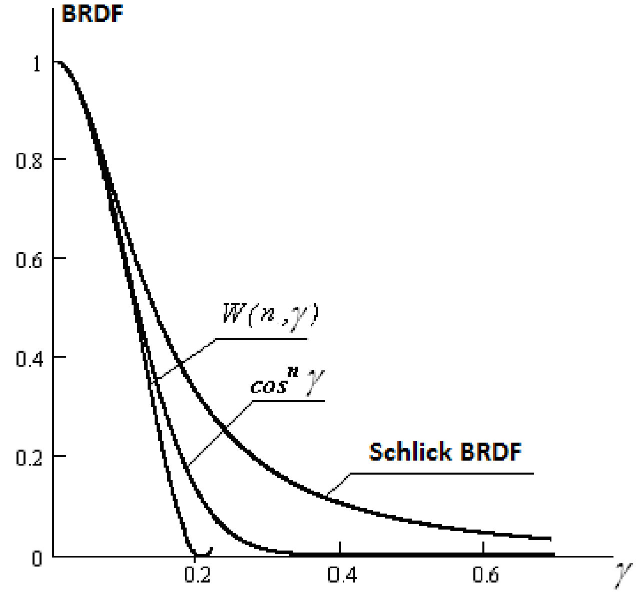

Fig. 1. BRDF graphics

Let's determine extremums of the function. From the last formula it is obtained that $cos(\gamma) = (n-2)/n$. The $W(n, \gamma)$ signs analysis from the left and right side shows that BRDF reaches zero value at

$$\gamma = ar\cos\frac{n-2}{n}$$

$W(n, \gamma)$ monotonously fades within the interval $0, ar\cos((n-2)/n)$, because its x derivative equals to

$$-\left(\frac{n}{2}(cos\,\gamma - 1) + 1\right) \cdot n \cdot sin\,\gamma$$

In its turn, $n \cdot sin\,\gamma$ is positive on interval from 0 to π. $-(n(cos\,\gamma - 1) + 1)/2 \le 0$ is true at the condition that $0 \le \gamma \le ar\cos((n-2)/n)$. Thus, within interval where function changes from 1 to 0, its derivative hasn't negative values, q. e. d.

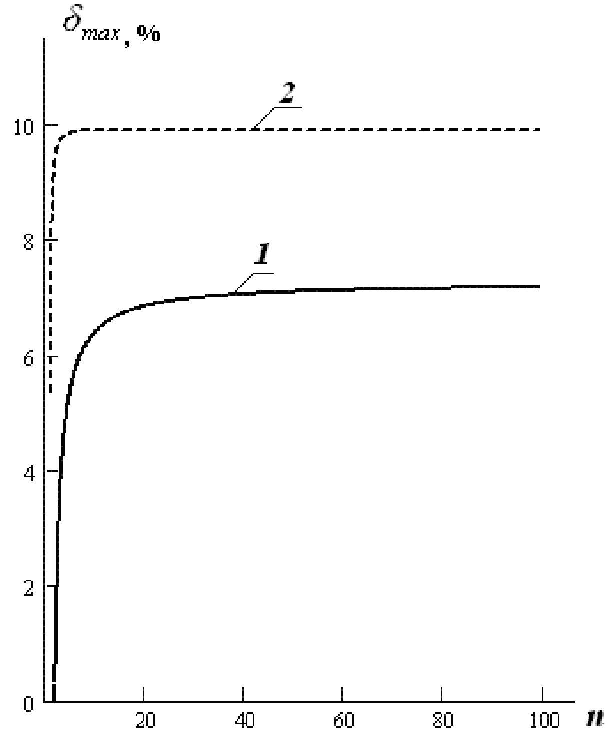

Fig. 2. The maximal relative error graphics for the highlight epicenter approximation by:
1– $W(n,\gamma)$ function; 2 –Schlick function

Let's examine how the raising of power m influences upon approximation accuracy. Three cases are chosen – $m=4$, $m=8$ ▯ $m=16$:

$$W_1(n,\gamma)=\left(\zeta_1\cdot(\cos\gamma-1)+1\right)^4,$$

$$W_2(n,\gamma)=\left(\zeta_2\cdot(\cos\gamma-1)+1\right)^8,$$

Considering the equation $\zeta=n/m$, one can find

$$\zeta_1=\frac{n}{4},\qquad \zeta_2=\frac{n}{8},\qquad \zeta_3=\frac{n}{16}.$$

It's significant, that coefficients ζ_1, ζ_2 and ζ_3 can be easily computed on the assembly level.

Considering values of ζ_1, ζ_2 and ζ_3 coefficients, it's possible to find $W_1(n,\gamma)$, $W_2(n,\gamma)$ and $W_3(n,\gamma)$:

$$W_1(n,\gamma)=\left(\frac{n}{4}\cdot(\cos\gamma-1)+1\right)^4,$$

$$W_2(n,\gamma)=\left(\frac{n}{8}\cdot(\cos\gamma-1)+1\right)^8,$$

$$W_3(n,\gamma)=\left(\frac{n}{16}\cdot(\cos\gamma-1)+1\right)^{16}.$$

Figures 3–5 show graphics of $\cos^n\gamma$ and $W_1(n,\gamma)$, $W_2(n,\gamma)$ ▯ $W_3(n,\gamma)$ functions when $n=35$. The analysis of graphics leads to conclusion that increase of m leads to approximation accuracy increase. The modeling helps to determine maximal relative error of the approximation within the range $n\in[1{,}1000]$. The results of such modeling are represented on fig.6.

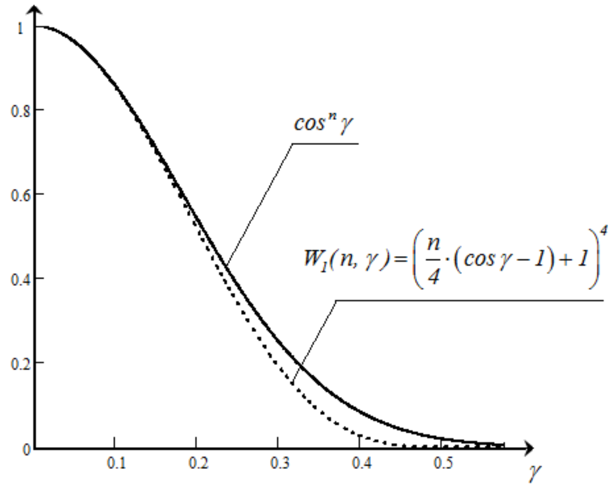

Fig.3 – Graphics of $\cos^n\gamma$ and $W_1(n,\gamma)$

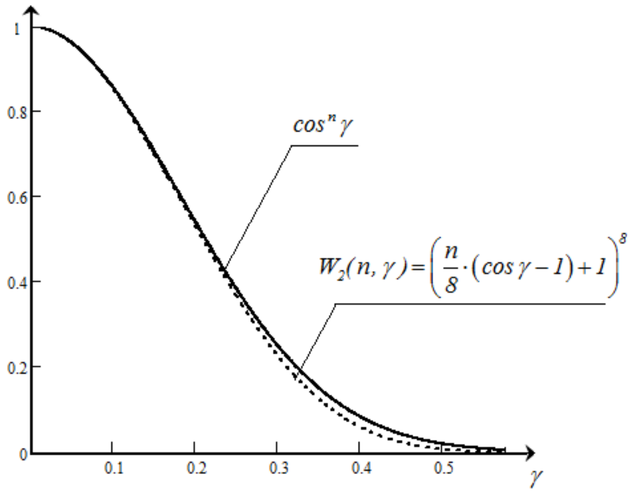

Fig.4 – Graphics of $\cos^n\gamma$ and $W_2(n,\gamma)$

It is significant, that special requirements to approximation accuracy are applied only for epicenter zone – part of BRDF from 0 to cusp point. The BRDF values after cusp point determine blooming zone formation.

The investigation has shown, that maximal relative error of the $\cos^n\gamma$ approximation before the cusp point, within interval $n\in[1{,}512]$ doesn't exceed: 8% – for $W_1(n,\gamma)$, 3,8% – for $W_2(n,\gamma)$ ▯ 1% – for $W_3(n,\gamma)$. The obtained results tell about considerable increase of the approximation accuracy in comparison with Schlick function. The maximal relative error for the Schlick function within this interval is 22,4%.

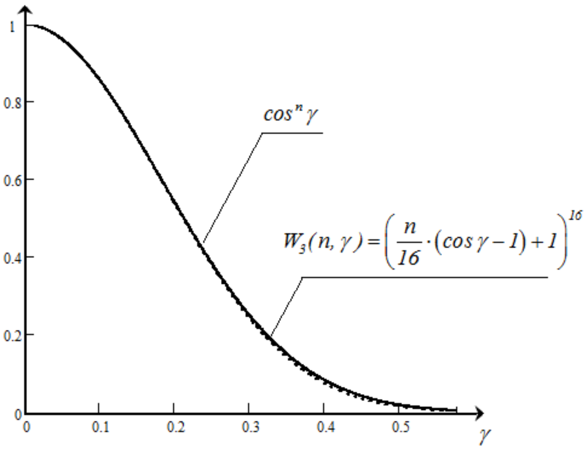

Fig.5 – Graphics of $\cos^n \gamma$ and $W_3(n, \gamma)$

Function $W_3(n, \gamma)$ has very good approximation quality even for the blooming zone. The maximal relative errors ε values for approximation functions $W_1(n, \gamma)$, $W_2(n, \gamma)$ are $W_3(n, \gamma)$ within various ranges a collected in table 1. The corresponding relative errors for the Schlick function are also provided in the table.

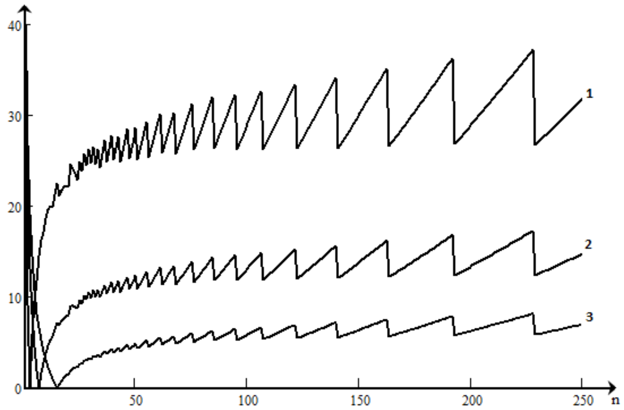

Fig.6 – maximal relative error graphics for $\cos^n \gamma$ approximation by: 1 - $W_1(n, \gamma)$, 2 - $W_2(n, \gamma)$ and 3 - $W_3(n, \gamma)$.

TABLE 1
MAXIMAL RELATIVE ERRORS OF FUNCTIONS $W_1(n, \gamma)$, $W_2(n, \gamma)$, $W_3(n, \gamma)$ AND SCHLICK FUNCTION

Function	ε,% before the cusp point	ε,% after the cusp point
$W_1(n, \gamma)$	8,0	35,2
$W_2(n, \gamma)$	3,8	16,2
$W_3(n, \gamma)$	1,0	7,6
Schlick	22,4	98,2

Fig. 7 represents graphics of maximal relative errors of $W_1(n, \gamma)$, $W_2(n, \gamma)$ and $W_3(n, \gamma)$ functions at blooming zone.
The analysis of these errors leads to conclusion, that the best result is obtained at approximation by $W_3(n, \gamma)$ function.

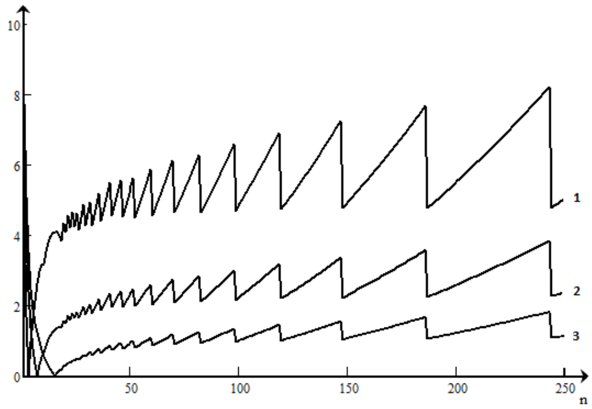

Fig.7 – Graphics of maximal relative error values for approximation of $\cos^n \gamma$ by functions: 1 - $W_1(n, \gamma)$, 2 - $W_2(n, \gamma)$ and 3 - $W_3(n, \gamma)$ for the blooming zone.

The relative error of $W_3(n, \gamma)$ in cusp point doesn't exceed 1%, at the same time this error for Schlick formula is more than 22 %. To calculate the Schlick's BRDF it's necessary to perform 1 multiplication operation, 1 dividing operation, 1 subtraction and 1 decrement operations. The dividing operation complicates hardware implementation of this formula. The suggested formula $W_3(n, \gamma)$ doesn't contain resource-intensive dividing operations. It's enough to perform only 4 multiplication operations, shift operation, increment and decrement. All these operations have easy hardware implementation. The software implementation $W_3(n, \gamma)$ takes in 2 times less time than Schlick formula. These facts lead to conclusion, that computation complexity of $W_3(n, \gamma)$ is considerably lower than that of Schlick function. It is important, that all 3 suggested functions provide better accuracy than Schlick formula.

There're peculiarities of blooming zone formation. The boundary value is chosen to be equal to 2^{-q}, where q is picked depending on the required specular constituent accuracy. Then $\cos^n \gamma$ has to be computed within this interval – $2^{-q} \le \cos^n \gamma \le 1$. From, which .

$$0 \le \gamma \le \arccos\left(2^{\left(-\frac{q}{n}\right)}\right)$$

For Schlick function we have

$$2^{-q} \le \frac{\cos\gamma}{n - n\cdot\cos\gamma + \cos\gamma} \le 1.$$

From this we can find that Schlick function is to be calculated only within this interval

$$0 \le \gamma \le \arccos\frac{n}{2^q + n - 1}.$$

Let's determine the γ vary interval for suggested BRDF. If solve the we find, that $2^{-q} \le \left(\frac{n}{m}(\cos\gamma - 1) + 1\right)^m \le 1$

$$0 \le \gamma \le \arccos\left(\frac{m \cdot 2^{-q/m} - m + n}{n}\right).$$

The significant disadvantage of Schlick formula is extremely slow fading in blooming zone leading to unnatural object illumination. Vary interval for γ in this case increases.

The ratio $\Re$, defines, in how many times the arguments in $\cos^n \gamma$ and in suggested functions differs on the condition that these functions reach value 2^{-q}. This ratio also determines the highlight sizes correlation.

For $\cos^n \gamma$ and $W_1(\gamma, n)$

$$\Re_1 = \frac{\arccos\left(2^{\left(-\frac{q}{n}\right)}\right)}{\arccos\left(\frac{4 \cdot 2^{-q/4} - 4 + n}{n}\right)}.$$

For $\cos^n \gamma$ and $W_2(\gamma, n)$

$$\Re_2 = \frac{\arccos\left(2^{\left(-\frac{q}{n}\right)}\right)}{\arccos\left(\frac{8 \cdot 2^{-q/8} - 8 + n}{n}\right)}$$

For $\cos^n \gamma$ and $W_3(\gamma, n)$

$$\Re_3 = \frac{\arccos\left(2^{\left(-\frac{q}{n}\right)}\right)}{\arccos\left(\frac{16 \cdot 2^{-q/16} - 16 + n}{n}\right)}$$

For $\cos^n \gamma$ and Schlick formula

$$\Re_4 = \frac{\arccos\left(2^{\left(-\frac{q}{n}\right)}\right)}{\arccos\frac{n}{2^q + n - 1}}$$

4 Conclusions

The suggested BRDF models have:

- Much better approximation accuracy in comparison with Schlick formula.
- More simple hardware implementation in comparison with Schlick formula.

The approximation accuracy is increased more than in 22 times for the epicenter zone and more than in 14 times for the blooming zone in comparison with Schlick formula. For the hardware implementation of the suggested functions only simple functions are required.

References

[1] D. Hern, P. Baker, Computer Graphics and OpenGL standard. — M.: Publishing house «Williams», 2005. — 1168 p. (Book)

[2] D.Rogers, "Mathematical Basics of Computer Graphics". — Moscow, Mir, 2001.—512. (Book)

[3] Frensis Hill. OpenGL. Computer Graphics Programming for Professionals. Piter, 2002. —1088 p. (Book)

[4] Hast A., Barrera T., Bengtsson E. Fast Specular Highlights by modifying the Phong-Blinn Model// International Conference on Computer Graphics and Interactive Techniques, —2003, —pp.1. (Conference proceedings)

[5] Kautz J., McCool M. Approximation of glossy reflection with prefiltered environment maps // Proceeding graphics interface. —2000. — pp. 119-126 (Technical report)

[6] Foley, Van Dam, Feiner, and Hughes Computer Graphics // Principles and Practice. Addison Wesley. —Ch. 16. — 1996. — pp. 800–87. (Book)

[7] Levoy M., Hanrahan P. Light Field Rendering // SIGGRAPH 96. - 1996. - pp. 31-42. (Conference proceedings)

[8] Chrictophe Schlick. A Fast Alternative to Phong's Specular Model // Graphics Gems IV. Academic Press. — 1994. — pp. 404–409. (Conference proceedings)

[9] Lyon R.F. Phong Shading Reformulation for Hardware Renderer Simplification // Apple Technical Report № 43. —1993. (Technocal report with number)

[10] Romanuyk O. Methods for Specular Color Component Accelerate Calculation / O. Romanuyk, A. Chernij // IEEE Workshop on Intelligent Data Acquistion and Advanced Computing systems: Technology and Applications. Sofia. — 2005. — P. 615— 619. (IEEE Transactions)

[11] Romanyuk A., Hast A., Lyashenko Y., "The Methods of Fast Shading"/ 9th Conference of Development and Application Systems, Suceava, Romania 2008.(Conference proceedings)

[12] Romanyuk A. Efficient Methods of 3D Objects Shading. Monography. / A. N. Romanyuk, A. V. Chorniy. - Vinnitsa : Universum-Vinnitsa, 2006. — 190 p. (Monography)

Alexander N. Romanyuk has receieved a PhD degree in 1986 and Grand PhD in 2009. At the moment he's author of 350 scientific publications, including 4 monographs and 60 patents. He was appointed on the position of Corresponding member of Telecommunications and Computer Science Academy in Russia.Within recent years he was scientific supervisor of 14 scientific projects. He applied into industry 2 devices – graphics accelerators. He has tight scientific relations with scientists in Russia, Canada, USA, Poland, Romania and Sweeden.

Yuri L. Lyashenko is a post-graduate student, Master of Computer Science. He owns about 10 Patents, 25 scientific publications. He is a winner of more than 30 All-Ukrainian Olympiads in math, physics, computer science, chemistry and English.

Alexander V. Melnik and Alexander P. Goncharuk are Masters of Computer Science.