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Published in:
Book of Abstract

Publication date:
2022

Document Version
Peer reviewed version

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1 Spectral identification of networks

Spectral network identification aims at estimating the eigenvalues of the Laplacian matrix L of the network from data. This allows to infer global information on the network structure from local measurements at a few number of nodes **Mauroy2017**.

2 Scalar diffusive coupling framework

We consider a dynamical system defined over a network. The local dynamics of the unit attached to node k are directly influenced by its neighbors through input and output signals. In the particular case of n identical units with diffusive coupling, the dynamics of node k are described by

$$\begin{cases} \dot{x}_k = F(x_k) + G(x_k)u_k & \in \mathbb{R}^m \\ u_k = \sum_{j=1}^n W_{kj}(y_k - y_j) & \in \mathbb{R}^r \\ y_k = H(x_k), & \in \mathbb{R}^r \end{cases} \tag{1}$$

with the (continuously differentiable) functions $F : \mathbb{R}^m \to \mathbb{R}^m$, $G : \mathbb{R}^m \to \mathbb{R}^{m\times r}$ and $H : \mathbb{R}^m \to \mathbb{R}^r$. Note that the coupling coefficients W_{kj} are the entries of the adjacency matrix $W \in \mathbb{R}^{n\times n}$ of the network. The network Laplacian matrix is given by $L = D - W$, where D denotes the degree matrix defined by $d_i = \sum_{k=1}^n W_{ik}$. We further assume that the units asymptotically reach a synchronized state $x_1 = \cdots = x_n = x^*$, *i.e.* $\lim_{t\to\infty} x_k(t) = x^*$.

The Jacobian matrix associated with the whole system (1) and evaluated at x^* writes

$$J = I_n \otimes A - L \otimes BC^T, \tag{2}$$

with $A = \partial F/\partial x(x^*)$, $B = G(x^*)$ and $C = \nabla H(x^*)$. In the previous work **Mauroy2017**, it was proved that the relationship between the spectrum of the Jacobian matrix $\sigma(J)$ and the spectrum of the Laplacian matrix $\sigma(L)$ is one-to-one when $r = 1$ and satisfies

$$(A - \mu I_m)w = \lambda BC^T w, \tag{3}$$

where $\mu \in \sigma(J)$ and $\lambda \in \sigma(L)$. It follows that one can retrieve the spectrum of L from the spectrum of J. Moreover, states measurements can be used to compute the spectrum of the Jacobian matrix through the DMD algorithm **Schmid2010**.

It is noticeable that the results of **Mauroy2017** are limited to the case $r = 1$ in (1). This corresponds to scalar-valued input and output signals, a condition that is not satisfied in many situations, such as reaction-diffusion networks. If $r > 1$, the rank of the matrix BC^T can be larger than 1 and the one-to-one relation between μ and λ obtained from (3) does not hold. In this context, the main contribution of the present work is to provide a generalized framework for spectral identification, which is valid for the case of vector-valued inputs and outputs ($r > 1$).

3 Generalization of the framework

We will show that the spectral identification problem is feasible under a mild assumption based on the spectral moments of BC^T. More precisely, one can obtain a linear system of equations which allows to compute the spectral moments of L from those of J. Finally the spectrum of L can be recovered from its spectral moments. However, this process might lead to significant numerical errors on the individual eigenvalues and should therefore be avoided.

We will propose an alternative method for solving the spectral identification problem, which is based on the properties of the characteristic polynomial of the generalized eigenvalues problem (3). This method will be illustrated with numerical simulations.