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Published in:
39th Benelux Meeting on Systems and Control

Publication date:
2020

Document Version
Peer reviewed version

Local exponential stability of nonlinear distributed parameter systems: Application to a nonisothermal tubular reactor

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Deducing exponential stability of an equilibrium of a nonlinear distributed parameter, i.e. infinite-dimensional, system on the basis of the stability of a linear approximation of it is in general quite challenging. Some of the existing theories, see e.g. [4, 1], rely on the Fréchet differentiability of the nonlinear semigroup generated by the nonlinear operator dynamics. However, checking Fréchet differentiability for nonlinear operators defined on an infinite-dimensional space is difficult or even impossible if these are unbounded. In many cases, the general theory cannot be applied and a case-by-case study has to be performed by working directly on the semigroup instead of its generator.

The approach that is proposed here is based on an adapted concept of Fréchet differentiability which takes different spaces and norms into account. This is called the $(Y,X)-$Fréchet differentiability, where X is the state (Hilbert) space and Y is an auxiliary space chosen to handle more easily norm-inequalities when working in infinite-dimensions (typically L^∞ or Sobolev spaces ($H^p, p \in \mathbb{N}_0$), which are all multiplicative algebras). The systems that are considered here are governed by the following abstract ODE:

$$\dot{x}(t) = Ax(t) + N(x(t)),\ x(0) = x_0, \tag{1}$$

where $A : D(A) \subset X \to X$ and $N : D(N) \subset X \to X$ are linear and nonlinear operators, respectively. Let $x^e \in D(A) \cap D(N)$ be an equilibrium of (1).

Definition : Let $(Y, \|\cdot\|_Y)$ be an infinite-dimensional (possibly Banach) space such that $D(A) \cap D(N) \subset Y \subseteq X$. The operator N is called $(Y,X)-$Fréchet differentiable at x^e if there exists a bounded linear operator $dN(x^e) : X \to X$ such that for all $h \in D(A) \cap D(N), N(x^e+h) - N(x^e) = dN(x^e)h + R(x^e,h)$ where $\lim_{\|h\|_Y \to 0} \|R(x^e,h)\|_X / \|h\|_X = 0$.

This allows more easily checkable adapted Fréchet differentiability conditions, provided that local exponential stability of the equilibrium of (1) holds in a weaker sense, see [3]. Based on this new concept, our approach to deduce exponential stability of the equilibrium of (1) can be summurazied as in Figure 1. Let $(S(t))_{t\geq 0}$ be the nonlinear semigroup generated by the operator $A+N$ on X. The standard concept of Fréchet differentiability is needed for $(S(t))_{t\geq 0}$ on Y, with $(T_{x^e}(t))_{t\geq 0}$ as Fréchet derivative, the linear semigroup generated by the Gâteaux derivative $A + dN(x^e)$ of $A+N$. After showing that $(T_{x^e}(t))_{t\geq 0}$ satisfies some Lyapunov-type stability condition and that it is exponentially stable on X, the new concept of $(Y,X)-$Fréchet differentiability pops up to make the connection between Y and X, in order to deduce

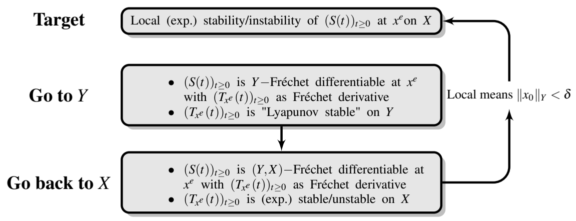

Figure 1: Illustration of the new theoretical framework.

local exponential stability of the equilibrium of (1) on X, see [3]. Local means here that the $Y-$norm of the initial condition has to be small instead of its $X-$norm.

Our theoretical approach is illustrated on a nonisothermal axial dispersion tubular reactor which exhibits different numbers of equilibria depending on the diffusion coefficients: see [2] for the existence and multiplicity of the equilibria. Our main result states that, in the case of only one equilibrium profile, the latter is locally exponentially stable for the nonlinear system governing the dynamics. Moreover, in the case where the reactor exhibits three equilibria, local exponential bistability is established, that is, the pattern "stable – unstable – stable" is highlighted, see [3].

Acknowledgment

This research was conducted with the financial support of F.R.S-FNRS. Anthony Hastir is a FNRS Research Fellow under the grant FC 29535. The scientific responsibility rests with its authors.

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