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ABSTRACT

Arrival processes of sediment particles at the reservoir are temporally and spatially random in the occurrences of natural events such as landslides and mudflows. Changes of sediment concentration in reservoirs can easily affect the water quality, resulting in a shortage of water supply in the human society. In this paper, a stochastic framework is proposed to quantify the uncertainty of particle arrival processes, random-sized batch arrivals (RSBA). Random-sized batch arrivals including random occurrences and random magnitude of particles simulated by the batch Poisson process are believed to account for the randomness of particle arrivals more comprehensively. The number of sediment particles in each arrival defined as random magnitude is a binomially-distributed random variable, while random occurrences of particle arrivals are simulated by Poisson process. Random particle trajectories are simulated by the stochastic diffusion particle tracking model (SD-PTM), considering the deposition and resuspension process. A probabilistic description of particle concentrations and transport rates can be displayed through ensemble statistics.

KEY WORDS: sediment transport; random arrival processes; particle tracking model; batch Poisson process; random-sized batch arrival.

INTRODUCTION

Random occurrences of nature disasters such as typhoons, extreme rainfalls and floods often lead to random sediment supply to the reservoirs due to landslides, mudflows and dam breaches. Changes of sediment concentration in reservoirs can easily affect the water quality, resulting in a shortage of water supply in the human society. Therefore, an accurate estimation of sediment concentration should include both the uncertainties of sediment supply and sediment transport. This study aims to quantify the randomness of sediment arrival process both in occurrences and magnitude. In the present study, random–sized batch arrivals (RSBA) including random occurrences and random magnitude of incoming sediment particles simulated by batch Poisson process are believed to account for the randomness of particle arrivals more comprehensively. The number of sediment particles in each arrival, defined as random magnitude, is a binomially-distributed random variable, while random occurrences of particle arrivals are simulated by Poisson process. Subsequently, random trajectories of suspended sediment particles are simulated by stochastic diffusion particle tracking model (SD-PTM) proposed by Man and Tsai (2007). Probabilistic description results of ensemble statistics of sediment concentrations and particle transport rates can provide more information and assist in risk assessment for decision making in water quality control.

METHODOLOGIES

Random-Sized Batch Arrivals (RSBA)

Referred to Ross (2007), the arrivals occurring in accordance with a Poisson process, consisting a random number of customers, are defined as random-sized batch arrivals. In the perspective of batch Markovian arrival process (BMAP), random-sized batch arrivals can be simulated by batch Poisson process (Cordeiro and Kharoufeh, 2011). In batch Poisson process, the phase process *J(t)* is a collection of random variables; and the counting process *N(t)* is governed by the Poisson process. The theoretical mean of a batch Markovian arrival process $(N,J)\equiv\{N(t),J(t),t\geq 0\}$ is equal to **λ**B. In this study, the random-sized batch arrival process (RSBA) of sediment particles is applied to more comprehensively quantify the randomness of particle arrival process, including the random occurrences and random magnitude. The random occurrences are simulated by a Poisson process with a mean Poisson rate **λ**. The random magnitudes for arrivals are random variables of a binomial distribution *B(n, p)* where n represents the number of trails and p is the probability of event occurrence.

Fig. 2 is provided to better illustrate the perspective of RSBA. Fig. 2(a) illustrates the random number of arrivals according to the Poisson process with λ = 5 events per second. Fig. 2(b) shows the numbers of particles in each arrival along with simulation time. The random quantity of suspended sediment particles is generated by the binomial distribution *B(100, 0.02)*. In other words, one arrival would include two particles on the average. The joint event of Fig. 2(a) and Fig. 2(b) will lead to the numbers of particles that arrive at the control volume along with simulation time shown in Fig. 2(c). Theoretically, the ensemble means of incoming rate of particles would be 10 beads per second. It must be noted that Fig. 2 only displays one realization of the incoming particle arrival patterns.

Stochastic Particle Tracking Model

The stochastic diffusion particle tracking model (SD-PTM) addresses the random trajectories of suspended sediment particles in turbulent flows (Man, 2007; Man and Tsai 2007; Oh and Tsai 2010; Fan et al. 2014). The governing equation of sediment particle position $\mathbf{X}_t=\{X(t),Y(t),Z(t)\}^T$ is shown as the Eq.1:

$$d\mathbf{X}_t=\bar{\mathbf{u}}(t,\mathbf{X}_t)dt+\boldsymbol{\sigma}(t,\mathbf{X}_t)d\mathbf{B}_t \qquad (1)$$

where $\bar{\mathbf{u}}(t,\mathbf{X}_t)$ denotes the drift velocity vector; $\boldsymbol{\sigma}(t,\mathbf{X}_t)$ symbolizes a diffusion coefficient tensor; and $d\mathbf{B}_t$ stands for the Wiener process at time t in three-dimensional vector form. The first term of the right-hand side of Eq.1, mean drift term $\bar{\mathbf{u}}(t,\mathbf{X}_t)$, comprises two components, mean drift flow velocity $\bar{\mathbf{U}}$ and turbulence diffusivity $\mathbf{D}$. The mean drift term can be expressed as follow (Man and Tsai, 2007):

$$\bar{\mathbf{u}}(t,\mathbf{X}_t)=\bar{\mathbf{U}}+\nabla\mathbf{D}=\begin{cases}\bar{U}(t,x,y,z)+\partial D_x/\partial x\\ \bar{V}(t,x,y,z)+\partial D_y/\partial y\\ \bar{W}(t,x,y,z)-w_s+\partial D_z/\partial z\end{cases} \tag{2}$$

The relationships between the diffusion coefficients $\boldsymbol{\sigma}$ and the turbulent diffusivities $\mathbf{D}$ can be described as $\mathbf{D}_i=1/2\boldsymbol{\sigma}_{ii}^2$, $i=x,y,z$ (Man and Tsai, 2007).

Pickup Probability

Einstein (1950) first proposed the pickup probability as the probability of the dynamic lift greater than the submerged weight of a sediment particle. Based on this concept, Wu and Lin (2002) proposed pickup probability for sediment entrainment by assuming the instantaneous velocities are log-normally distributed. Resuspension criteria of the instantaneous velocities can be obtained by comparing the lift force and the submerged weight of a particle:

$$u_b^2 \geq \frac{4d}{3C_L}\frac{(\rho_s-\rho_f)}{\rho}g \tag{3}$$

where u_b denotes the instantaneous flow velocity approaching the particle on the bed; $\boldsymbol{d}$ represents the particle diameter; C_L is the lift coefficient; and $\boldsymbol{\rho_f}$ and $\boldsymbol{\rho_s}$ are densities of fluid and sediment particles, respectively. Thus, after taking logarithm of u_b, a normally distributed random variable $v_b=\ln(u_b)$ can be obtained. The mean $\bar{v_b}=\ln(5.52u_*)$ and variance $\sigma_v^2=0.123$ of u_b approximated by the analytical method with optimal lift coefficient C_L = 0.21 are suggested by Wu and Lin (2002).

MODEL SIMULATIONS

In the simulation, the batch-sized arrivals of incoming suspended particles are simulated by batch Poisson process. Random magnitude in each arrival is a random variable and is modeled by the binomial distribution *B(100, 0.02)*. The random arrivals are simulated by a Poisson process with rate λ = 5 (arrival/second). The mean incoming rate of suspended sediment particles is 10 beads per second. In the simulation, a control volume with 1 meter in longitudinal direction is set to examine particle movement. The size of the control volume is not constrained; it could be any size based on the demand of the survey region. Random trajectories of suspended sediment particle in the control volume are simulated by SD-PTM (Mean and Tsai, 2007). After a sediment particle deposits on the reference height which is equal to twice the particle diameter (2***d***), the log-normally distributed instantaneous flow velocities approaching the particles are used to determine whether the particle will resuspend or not (Wu and Lin, 2002). In the simulations, flow velocities are assumed to follow the logarithmic velocity profile $\bar{U}=(u_*/\kappa)\ln(z/z_0)$. Flow conditions and particle properties used in stochastic transport models are laboratory observations proposed by Kaftori et al. (1995) (Table 1). With random trajectories, the concentrations and transport rates can be calculated based on (Table 2). For instances, a particle with position lager than zero and smaller than the size of control volume in longitudinal direction is calculated and summed as concentration $\boldsymbol{N}$ which denotes the total number of particles in the control volume. $\boldsymbol{N_d}$ and $\boldsymbol{N_m}$ are the number of deposition and moving sediment particles respectively. In the simulation, transport rates are calculated based on the comparison with previous particle position. Deposition is defined when the position of a sediment particle in the vertical direction (Z_{n-1}) is above the reference height (Z_0) at time *n-1,* while at time *n*, it is at reference height ($Z_n= Z_0$). Conversely, resuspend of a particle is when the opposed condition is satisfied ($Z_n> Z_0| Z_{n-1}= Z_0$). Based on 10,000 simulations, ensemble means and ensemble variances of the concentrations and transport rates can be calculated. In the simulations, a time step of 0.1 seconds is used for a total simulation time up to 30 seconds. Moreover, the equilibrium of the system is defined as when the number of moving particles in the control volume becomes a constant.

Table 1 Flow conditions and particle parameters.

Variables	value	units
Particle diameter, $\boldsymbol{d}$	**275**	μm
Particle density, $\boldsymbol{\rho_s}$	**1050**	kgm^{-3}
Flow density, $\boldsymbol{\rho_f}$	**1000**	kgm^{-3}
Settling velocity, $\boldsymbol{W_s}$	**0.0025**	m/s
Roughness height	**0.000550**	m
Reference height	**0.000275**	m
Von Karman constant, $\boldsymbol{\kappa}$	**0.4**	
Shear velocity, $\boldsymbol{u_*}$	**0.0086**	m/s
Flow height, $\boldsymbol{H}$	**0.0284**	m

Table 2 Conditions for transport rates.

Concentration (bead/meter)	Condition
Total number, $\boldsymbol{N}$	$X_n\leq L\| X_n>0$
Deposited particles, $\boldsymbol{N_d}$	$Z_n=Z_0\| X_n\leq L$ & $X_n>0$
Moving particles, $\boldsymbol{N_m}$	$N-N_d$
Transport rates (bead/sec)	**Condition**
Depart Rate, $\boldsymbol{\mu}$	$X_n>L\| X_{n-1}\leq L$
Deposition Rate, $\boldsymbol{\mu_d}$	$Z_n=Z_0\| Z_{n-1}>Z_0$
Resuspension Rate, $\boldsymbol{\lambda_s}$	$Z_n>Z_0\| Z_{n-1}=Z_0$

SIMULATION RESULTS

As listed in Table 3, simulation results of concentrations and transport rates are calculated when the system reaches equilibrium after 16 seconds. Transport rates in the control volume are depicted in Fig. 3 to better present the dynamic equilibrium. The sum of arrival rate and resuspension rate is almost equal to deposition rate plus the depart rate. Fig. 4 presents the simulation results of concentrations and transport rates along with the simulation time.

Table 3 Simulation Results.

Results (at steady state)	**value**	**units**
Concentration N	**84.71**	bead/meter
Concentration N_m	**78.05**	bead/meter
Concentration N_d	**6.66**	bead/meter
Arrival rate, λ_a	**9.99**	bead/sec
Depart rate, μ	**9.99**	bead/sec
Deposition rate, μ_d	**34.42**	bead/sec
Resuspension rate, λ_s	**33.89**	bead/sec

The ensemble means and ensemble variances of concentrations N, N_d, N_m are presented in Fig. 4 (a) and Fig. 4 (b). In the beginning of the simulation, the control volume is empty. Therefore, as the random amount of particles being released into the control volume at random time, concentrations increase. As soon as the system reaches dynamic equilibrium, ensemble means of concentrations N, N_d, N_m in Fig. 4 (a) would be approximately constant. Fig. 4 (c) shows the ensemble means of depart rates along with simulation time. At first, it takes a particle some time to transport through the control volume, which is reflected on the zero values at the beginning of the simulation. The time when zero values of the depart rate are present can be used as the representative mean time for particles to transport through the control volume. After particles start to leave the control volume, the depart rates start to climb up and asymptotically reach the incoming rates when the system becomes steady. Fig. 4 (d) presents the ensemble variances of depart rates with the similar tendency as ensemble means of depart rates in Fig. 4 (c). Fig. 4 (e) and Fig. 4 (f) schematize the curves of ensemble means and ensemble variances of deposition and resuspension rates. Probability for a particle to deposit and resuspend will increase when the number of particles increase at the beginning of simulation. Results of resuspension rate are almost a little bit smaller than the deposition rate in Fig. 4 (e) and Fig. 4 (f). It indicates that when deposition happens, resuspension is likely to occur. However, not all the deposited particles would resuspend, and therefore the resuspension rates are slightly smaller than the deposition rates.

Table 4

segment	**1**	**2**	**3**	**4**	**5**	**6**
region ≤	0.000275	0.000275	0.0031	0.0059	0.0087	0.0115
region >		0.0031	0.0059	0.0087	0.0115	0.0143
segment	**7**	**8**	**9**	**10**	**11**	
region ≤	0.0143	0.0172	0.0200	0.0228	0.0256	m
region>	0.0172	0.0200	0.0228	0.0256	0.0284	m

As for spatial results, concentrations in z-direction along with simulation time are presented in Fig. 5. We divided ten segments above reference height and below the flow depth (Table 5). The last bottom segment represents the number of deposited particles. Concentrations (number of particles per longitudinal length[1], bead/ meter) represent by different colors in Fig. 5. Fig. 6 illustrates the concentration in vertical distance. The lowest points of all the curves in Fig. 6 (a) and Fig. (b) represent the number of particles which have deposited on the reference height at corresponding time. Fig. 6 (a) presents the concentrations in vertical direction at the beginning of the simulation (0.1, 0.2, 0.3, 0.4, and 0.5 second). Notice that random numbers of particles are released at random time, therefore the areas under the curves in Fig. 6 are not the same. In Fig. 6 (a), particles concentrate at the middle at the beginning of the simulation due to the released position which is the middle of the water depth. As the simulation time goes by, particles tend to settle to the lower region as shown in Fig. 6 (b) and Fig. 5. The highest concentration focuses on the upper segment 2. It is possibly due to the deposition and resuspension of particles at this region.

[1] The unit of concentrations is the number of particle per longitudinal length. Therefore, concentrations in longitudinal direction are spatially averaged.

CONCLUSION

The random-sized batch arrivals simulated by batch Poisson process are applied to quantify the randomness of the incoming sediment particles. In this study, the number of particles in each arrival is modeled by the binomial distribution B(100, 0.02) while the occurrences of events are generated by a Poisson process with rate λ = 5 (event/second). Flow conditions and particles characteristics are from experimental data by Kaftori et al. (1995). The stochastic diffusion particle tracking model (Man and Tsai, 2007) is used for particle transport in the control volume. Particle deposition and resuspension process is considered (Wu and Lin, 2002). Ensemble means and variances of concentrations and transport rates from 10,000 model simulations are presented. There are four random variables taken into consideration in the simulations, the random arrival time, the random number of incoming particles in each arrival, and the particle Brownian motion caused by turbulence in SD-PTM and the log-normally distributed instantaneous velocity approaching the deposited particles. Thus the ensemble variances will fluctuate along with the simulation time. The spatial ensemble mean of concentrations in the vertical direction are also displayed to examine particle movement. Compared to the deterministic models, the probabilistic descriptions of the simulation results (spatial and temporal concentrations and transport rates) can better describe the movement of suspended sediment particle when the input of sediment particles are uncertain both in occurrences and magnitude. Such scientific information can provide a more scientific and comprehensive information to assist decision makers in risk assessment for hydraulic structure design, sedimentation and water quality control.

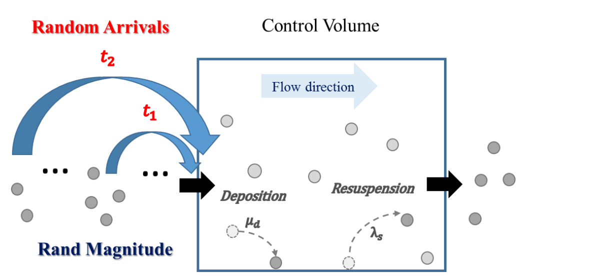

Fig. 1. Illustration of random-sized batch arrivals (RSBA) of incoming sediment particles.

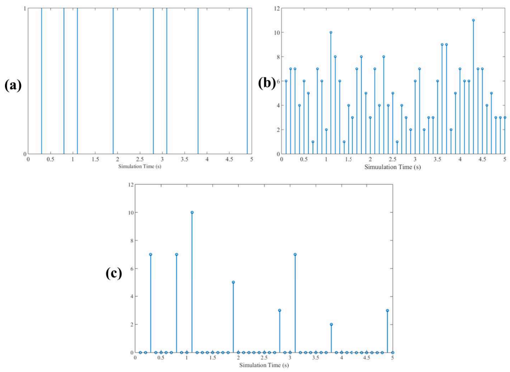

Fig. 2 (a) The number of arrivals along with simulation time in one realization. (b) The number of particles in each arrival of one realization. (c) The number of particles arriving at control volume versus simulation time of one realization.

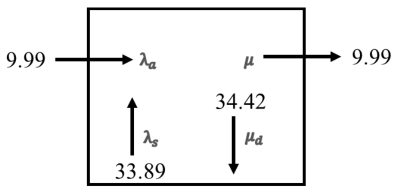

Fig. 3 Transport rates in the control volume.

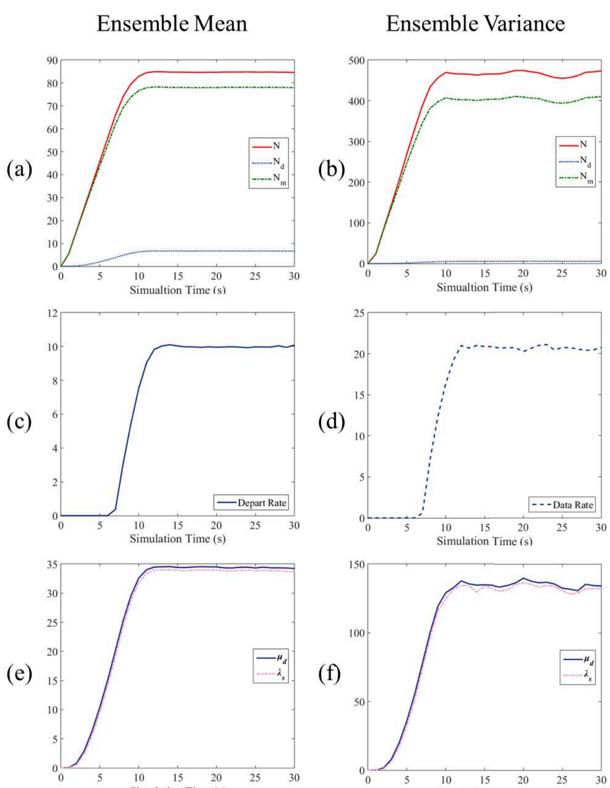

Fig. 4 (a) Ensemble means of concentrations (bead/meter). (b) Ensemble variances of concentrations. (c) Ensemble means of depart rate (bead/sec). (d) Ensemble variances of depart rate. (e) Ensemble means of deposition and resuspension rates (bead/sec). (f) Ensemble variances of deposition and resuspension rates.

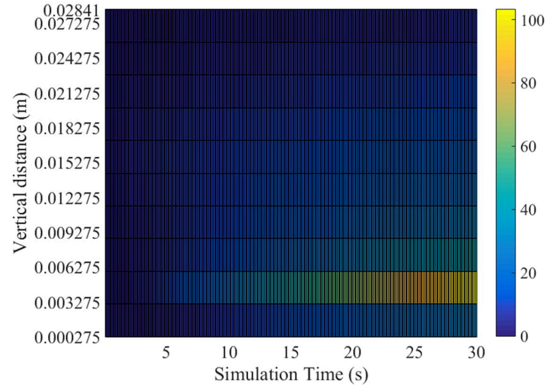

Fig. 5 Concentrations in vertical distance versus simulation time.

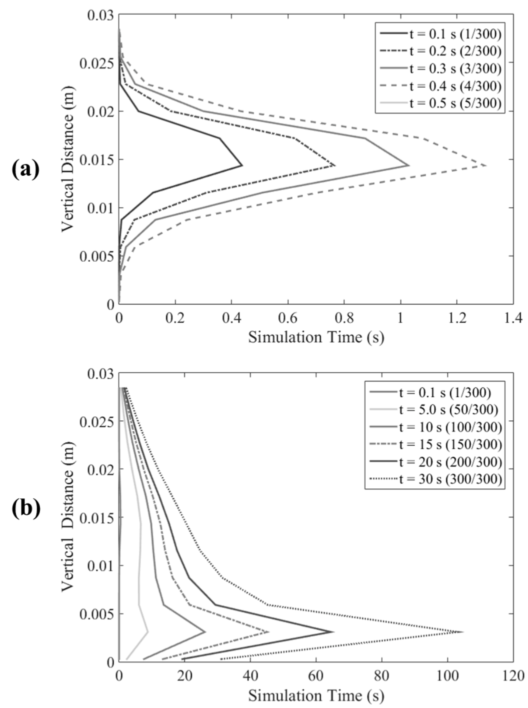

Fig. 6 Concentrations in vertical distance at different times.

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