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ABSTRACT

MPS is a moving particle semi-implicit method for modeling incompressible fluid in the computational fluid dynamics. The numerical technique can be used to simulate the flow pattern, water surface profile, critical flow location, velocity profile, and pressure profile in open channels. This study is to use MPS method to model the flow system in rectangular weir and compare numerical results with experimental observations to show reasonable agreement.

KEY WORDS: MPS, mesh-free particle method, free-surface, narrow-crested, water surface profile, velocity profile, pressure profile.

INTRODUCTION

The moving particle semi-implicit method (MPS) is a computational fluid dynamics technique for solving flow problem. Due to mesh adaptability and connectivity problems in the mesh-based numerical methods (finite difference, finite element, or finite volume method), the MPS has some advantages in the modeling complicated flow problems where water surface varied violently such as in breaking waves and dam breaks. The mesh-free methods use a set of arbitrarily distributed particles instead of mesh to solve governing equations in all kinds of boundaries and interfaces. In the mesh-free particle methods, governing equations are discretized over a set of arbitrarily distributed particles moving in the Lagrangian coordinate systems. A finite number of moving particles are employed to fill the system domain. Each particle possesses a set of field variables such as mass, concentration and momentum. To improve the computational efficiency, the weakly compressible MPS method was developed for modeling of incompressible fluids and validated through engineering applications

Koshizuka and Oka (1996) introduced MPS in nuclear engineering to model the collapse of a water column. The MPS is able to provide the approximations to Navier-stokes equations by particles representation and spatial discretization. The simulation of the open-boundary and free-surface flow gets easier if the mesh free method is used (Liu & Liu, 2003). Nowadays, MPS has been successfully applied to solve many fluid flow problems, such as dam break (Fu and Jin 2014), hydraulic jump (Nazari et al. 2012), 2D fluid structure interaction (Sun et al. 2015) and free flow over broad-crested weir (Xu and Jin, 2015). In the above engineering applications, weir flow is an interesting subject since weir is a major hydraulic structure used to either measure the fluid flow rate or adjust the discharge in an open channel. Simply, the weir can be installed across the channel to allow the fluid flow over the weir crest.

There is variety of weirs available in engineering practices such as rectangular weir, trapezoidal weir, crump weir, and ogee weir. Basically, there are two major types of flow regimes for weir flow including free flow and submerged flow. Free flow means the fluid can move freely as long as the downstream tailwater level is below the weir crest. Under the free flow condition, the critical depth can exist on weir of finite crest weir, and a region of parallel flow can be observed on such broad-crest weir crest. Critical flow corresponds to the Froude number equal to one. For Froude number greater than one, the flow is supercritical. While with the Froude number less than one, the subcritical flow exists. When the tailwater starts to back up and makes the water depth above crest finally exceed the critical depth, this flow is regarded as submerged (Hyatt, Skogerboe & Austin, 1966). Under submerged flow condition, the flow discharge is affected by the backwater and the upstream water level increases as tailwater increases. Between above two conditions, a modular limit is introduced to express the condition at which flow changes from free flow to submerged flow. When flow reaches modular limit, the upstream water level starts to increase, and this is defined by Hager and Schwalt as an increment of 1mm (1994). For the variation of submergence, a ratio y/h is used to represent different submergence level, where y is the downstream tailwater head above weir and h is upstream water head over weir. All of the three flow conditions are investigated in this study.

In this paper, a study of a numerical method for simulating flow over a narrow-crested weir is conducted. This study is aiming to use the MPS method with LES-SPS turbulence model for the simulation and compare the numerical results with the available experimental observations on the water surface profiles. The velocity and pressure profiles are plotted to show the distribution from upstream of weir, on the weir, and downstream of weir. Then, the capability of MPS for simulating the open-boundary surface-flow over weir is verified and validated.

MPS METHOD

Governing equation

The behavior of incompressible and viscous flow over weir of finite crest length is described by the Navier-Stokes equation, which expresses as mass continuity equation and conservation of momentum.
Conservation of mass (mass continuity):

$$\frac{D\rho}{Dt} + \rho\nabla \cdot u = 0 \tag{1}$$

Conservation of momentum:

$$\frac{Du}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2 u + f \tag{2}$$

Where ρ represents the fluid density, u represents the velocity vector, P stands for the pressure, ν denotes the kinetic viscosity, t denotes the time, ∇P is the pressure gradient term, $\nu\nabla^2 u$, is the viscosity term, and f represents the external body force term. This momentum equation describes that the incompressible viscous fluid flow under the force of viscosity, pressure, and the body force such as the gravitational force.

MPS Particle Interaction

In MPS method, the system domain is represented by a finite number of moving particles in continuum. Total number of particles in the simulation for this study includes 10000 storage particles and particles initially building the weir model by solid particles, ghost particles, and fluid particles with predefined size DL. In the particle domain, one particle can be affected by the particles in its vicinity. MPS defines that this particle interaction happens if and only if its neighboring particles are within the interaction area with a radius of r_e, which in this simulation r_e is equal to 3.5DL, where DL represents the average particle size.

The MPS particle interaction is covered by a chosen Kernel function also known as weighting function. Here, the 3rd order polynomial spiky function is selected for the MPS simulation for improved accuracy as follows (Shakibaeinia & Jin, 2010):

$$W(r_{ij}, r_e) = \begin{cases} (1-\frac{r_{ij}}{r_e})^3, & 0 < r_{ij} \le r_e \\ 0 \quad , & r_{ij} > r_e \end{cases} \tag{3}$$

where r_e represents the support area radius, i represents any particle i, and j represents one of its neighboring particles as particle j, so r_{ij} equals to the distance between particle i and particle j which is defined as $r_{ij} = |r_j - r_i|$.

MPS Discretization

MPS spatial discretization of the equations on the simulation domain is achieved by three models including gradient model, divergence model, and Laplacian model. The gradient formula used in this simulation is first described by Toyota et al. (2005). This formula is developed on the original gradient model from Kozushika to increase the stability of method (Lee et al., 2010). The gradient formula is written as:

$$\langle\nabla\varphi\rangle_i = \frac{d}{n^0}\sum_{j\neq i}\left(\frac{\varphi_i + \varphi_j}{r_{ij}^2} r_{ij} W(r_{ij}, r_e)\right) \tag{4}$$

Then it is integrated to calculate the pressure gradient term as follows:

$$\langle\nabla P\rangle_i = \frac{d}{n^0}\sum_{j\neq i}\left(\frac{p_i + p_j}{r_{ij}^2} r_{ij} W(r_{ij}, r_e)\right) \tag{5}$$

where φ is a scalar, d is the number of spatial dimension, n^0 is the average initial particle number density, which is constant for incompressible fluid, r is the position vector that r_i is the position for particle i, and r_j is the position for particle j.

The divergence model is described as following equation：

$$\langle\nabla\cdot u\rangle_i = \frac{d}{n^0}\sum_{j\neq i}\left(\frac{u_j - u_i}{r_{ij}^2} r_{ij} W(r_{ij}, r_e)\right) \tag{6}$$

The Laplacian formula is used to compute the viscosity term as follows,

$$\langle\nabla^2\varphi\rangle_i = \frac{2d}{\lambda n^0}\sum_{j\neq i}\left((\varphi_j - \varphi_i) W(r_{ij}, r_e)\right) \tag{7}$$

$$\langle\nabla^2 u\rangle_i = \frac{2d}{\lambda n^0}\sum_{j\neq i}\left((u_j - u_i) W(r_{ij}, r_e)\right) \tag{8}$$

where λ is a correction parameter for approximation to balance calculation for variance increase equals to the analytical solution. Here, λ is described by an equation:

$$\lambda = \frac{\int_V W(r_{ij}, r_e) r^2 dv}{\int_V W(r_{ij}, r_e) dv} \tag{9}$$

Boundary Treatment

In particle representation, fluid particle simply represents the fluid. Solid boundary is made up of the solid particles, which are represented by the wall particles in this simulation. Ghost particle treatment is then launched to satisfy particles searching in the vicinity of solid boundary. In order to make sure constant number of particles is available to be searched within the interaction radius, ghost particles are created outward from wall particles for three layers as r_e=3.5DL. Also, ghost particles are given the same properties as wall particles in terms of the velocity and pressure.

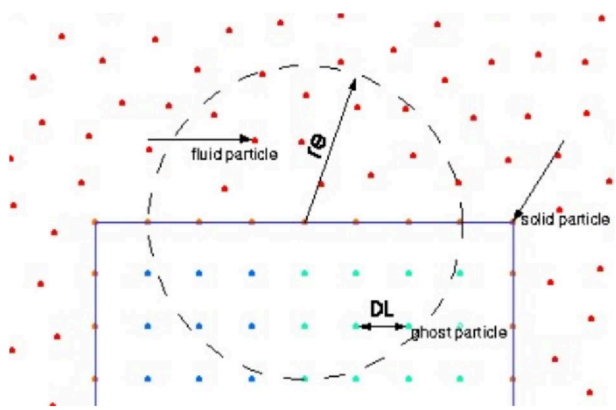

Fig. 1 Particle representation and solid boundary treatment

Weir flow is an open-boundary and free-surface flow. Free surface defines the fluid is subject to atmospheric pressure which is equal to zero. The determination of the free surface in MPS is implemented by an equation:

$$\langle n^*\rangle_i \le n^0\beta \tag{10}$$

This equation serves as criteria for identifying if the particle belongs to the free surface particle. It gives that when the temporal particle number density is not greater than a value $n^0\beta$, then the free surface condition can be assigned to that particle. Here, β is the coefficient for determining free surface, which equals to 0.97 in this model.

NUMERICAL RESULTS

The numerical simulation is completed by using WC-MPS model to simulate flow of a narrow-crested weir. Water surface profiles, velocity profiles, and pressure profiles are plotted to show the hydraulic characteristics of flow over weir with square-edge respectively at modular limit, free flow and submerged flow conditions. The numerical results of water surface are compared with the experimental observations (Azimi et al. 2014). Different time steps are randomly chosen. All of results from the MPS method shown in comparison have reached the steady-state. Five sections along the weir including middle of the upstream channel, upstream edge of weir, middle above the weir crest, downstream edge of weir, and middle of the downstream channel are selected for examining the velocity and pressure distribution. To minimize the upstream and downstream effect, both of upstream and downstream lengths are adjusted correspondingly by several trials

The narrow-crested weir of 0.40m in breadth, 0.076m in height, and 0.076m in crest length is simulated to discharge the flow of 22.45L/s. The depth of upstream flow is initiated at 0.171m for free flow, which enables water to leave the outlet at a depth of 0.0479m. Later, the tailwater depth will gradually increase to 0.1445m with an upstream water depth reaching at 0.1757m, which addresses the flow at modular limit condition. The downstream water level will keep increasing to 0.164m to generate submerge flow.

Five numerical simulation results at different time step are presented to show the flow development of a submerged condition with y = 0.205m as an example. The static water condition is first illustrated in Fig. 2(a) at t=0s with no flowing velocity and having 0.2037m water height. Over the time, water starts to flow down, passes over the weir crest, then ejects into the downstream with high velocity, and finally back up behind the weir (Fig. 2a-d). For a period of time, the water remains as free flow condition even though the tailwater depth increases. It can be observed that the upstream water level stays the same at t=2.5s and t=10.0s with both sloping downward to weir as demonstrated in Fig. 2(b) and Fig. 2(c) respectively. As long as the flow passes the modular limit condition, the upstream depth starts to rise accordingly (Fig. 2d). The flow pattern for submerged flow at t=25.0s in Figure 2(d) and t=45.1s in Fig. 2(e) are the same, thus the flow has achieved the steady state.

The water surface profiles from MPS results are plotted to compare with the experimental data. From the Fig. (3a-3c), the simulated water surface profiles for free flow, modular limit, and submerged flows are shown in good agreement with the experimental results, especially in the upstream of weir. By increasing the tailwater depth several times until the weir crest, the downstream of simulated free flow remains similar pattern showing a minor underestimation. For the modular limit flow in Fig. 3(b), it is shown that the MPS method is able to replicate a series of surface waves at the downstream as the experimental measurements. As water gets submerged, Fig. 3(c) shows some inconsistence of water level at downstream between the numerical data and experimental data, yet, the flow trend remains similar.

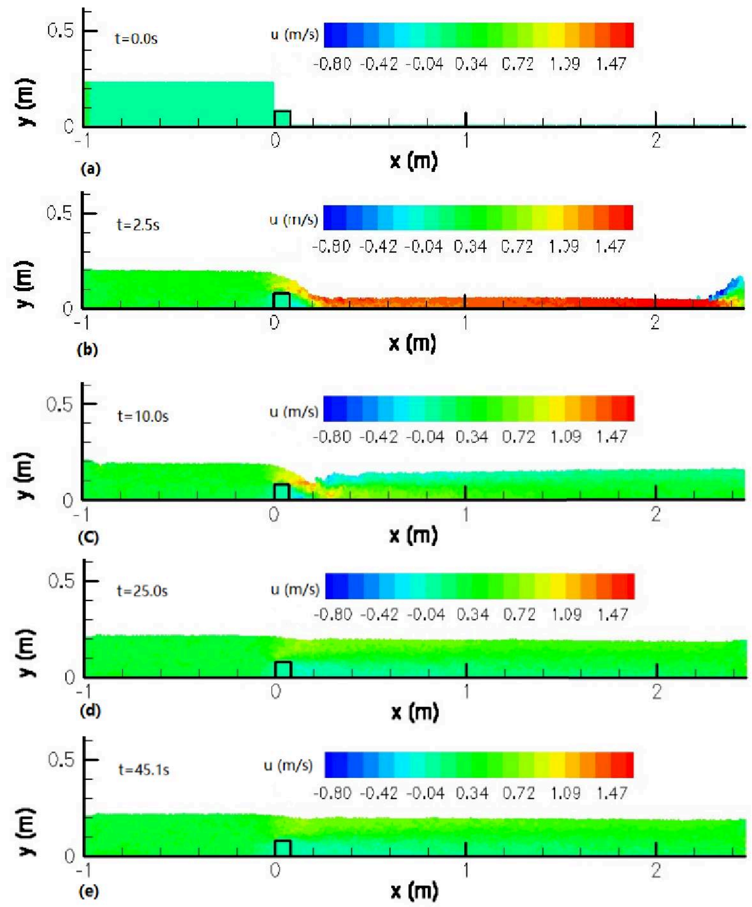

Fig. 2 MPS simulation of flow over narrow-crested weir at different time: (a)-(e)

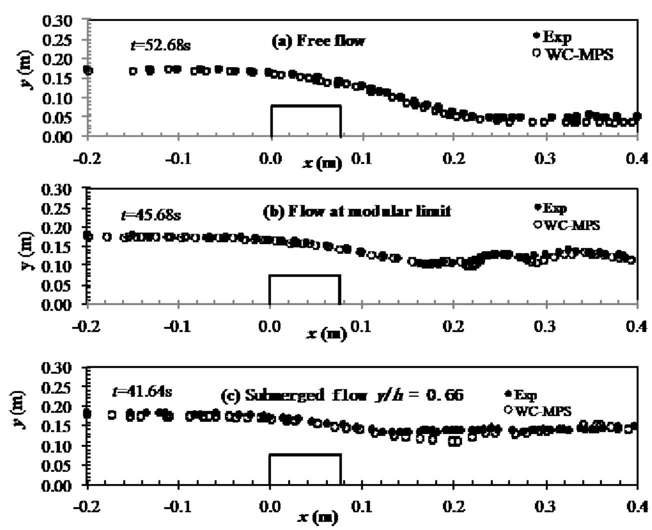

Fig. 3 Water surface profiles of flow over narrow-crested weir at free flow condition (a), modular limit (b), and submerged conditions (c).

Fig. 4 shows the horizontal velocity profiles for a submerged narrow-crested flow at five sections: *x*=-0.1m, 0m, 0.04m, 0.076m, and 0.24m. The submerge ratio of the downstream tailwater head above weir and upstream water head over weir is 0.89. The maximum velocity for each section varies from 0.4m/s to 0.7m/s. Relatively low velocity is observed at *x*=-0.1m. The velocity profile at the section x=0m shows the non-uniform pattern, and somewhat disturbed. This indicates the flow recirculation zone may develop at the entrance edge of weir. The developing recirculation zone is likely to affect the velocity distribution over the crest. At *x*= 0.04m, *x*=0.0.76m, and *x*=0.24m, the flow velocity profiles have the similar trend with velocity increasing toward the water surface. Velocities at the downstream are shown higher value than at the upstream. Due to the no-slip condition, the fluid velocity is zero at the bottom of channel and the weir crest. The flows shown in five sections are considered non-uniform, an increase in the horizontal velocity is observed along the flow direction.

The pressure distribution of simulated submerged flow over narrow-crested weir at the five section locations are shown in Fig.5. At the sections *x*=-0.1m, *x*=0.04m, *x*=0.076m, and *x*=0.24m, the pressure distribution all behaves similar to hydrostatic pressure under submerged flow condition. At *x*=0m, small difference in pressure distribution from hydrostatic pattern over the crest is believed to be caused by the recirculation zone at the entrance edge of weir crest.

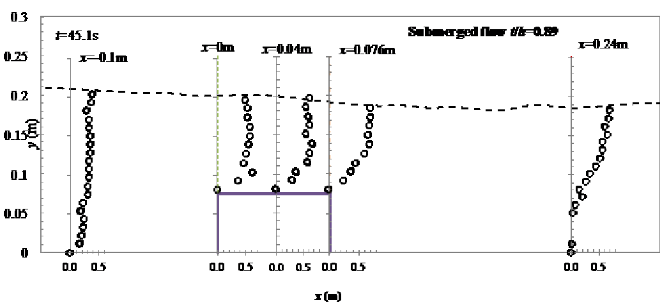

Fig. 4 Horizontal velocity profiles at different sections.

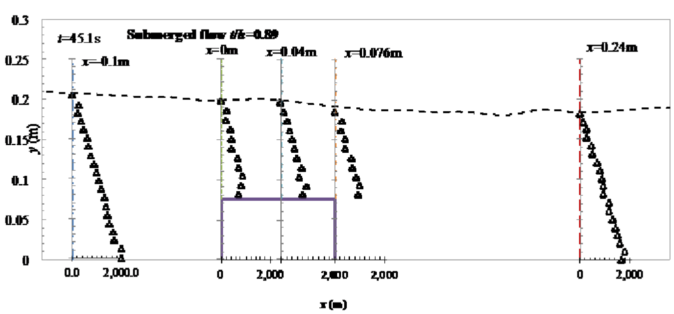

Fig. 5 Pressure profiles at different sections.

SUMMARY

The numerical study of open-boundary free-surface flow over weir of finite crest length with squared-edge is explored by using the MPS method. In the study, MPS method has addressed its capability of successfully modeling the narrow-crested weir flow. For water surface, the simulation results from the MPS method are shown in a good agreement. Horizontal velocity and pressure profiles are plotted to show the distribution pattern at different locations along the flow direction. Despite of some patterns of horizontal velocity and pressure distribution varied from theoretical distribution, the simulation results from the MPS method are considered reasonable.

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