May 2013

# What Next for Chaos Theory? From Metaphor to Phase Space

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## WHAT NEXT FOR CHAOS THEORY?
## FROM METAPHOR TO PHASE SPACE

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### ABSTRACT

In the management and social sciences literature, chaos theory has been used primarily as a metaphor to understand organizational phenomena. Using metaphors to understand organizations is a novel idea that has gained much acceptance, thanks to the pioneering work of Morgan (1986). However, chaos theory's value as a metaphor has been overused and offers little that cannot already be explained using existing theories and frameworks.

Because chaos theory is a mathematical theory, we believe its mathematical principles offer the greatest application to the management literature. In this paper, we offer the use of phase space, a tool of chaos theory, as a way to analyze firm performance.

### INTRODUCTION

The use of chaos theory in management research has been confined mainly to a metaphorical approach. In the management and organizational theory realms, the use of metaphor is well known, thanks to the pioneering work of Morgan (1986). However, none of Morgan's metaphors are based solely on a field of mathematics. In contrast, chaos theory is grounded in mathematical principles. Herein lies the problem; what are management scholars to do with this theory? Two options can be identified. First, scholars can continue to apply chaos theory primarily as a metaphor. Second, the mathematical concepts inherent to chaos theory, particularly its use of phase space, can be more actively applied. A review of the literature suggests that the first option has been overused while the second option is currently underutilized.

In this paper, we address the unwarranted enthusiasm and often imprecise application of chaos theory as a metaphor, as well as its underuse as a formidable mathematical application for management research. The paper begins with a brief history of chaos theory and its misapplication as a metaphor in management research. We then discuss how one relatively simple tool of chaos theory, phase space, can be used to examine firm performance history. We conclude with implications for management researchers.

### BACKGROUND

Although there are a number of important figures in the history of chaos theory, a key

starting point would begin with Edward Lorenz. As a meteorologist, Lorenz was working with equations in a weather forecasting model he was developing involving data on temperature, air pressure, and wind direction (Briggs & Peat, 1989).  At one point in his work he decided to take a shortcut. Instead of inputting the values to six decimal places, he used three. The results he obtained were considerably different from what he had anticipated.  This discovery later became the trigger event that led to a key component of chaos theory – sensitive dependence to initial conditions.  Lorenz had stumbled on a discovery that indicated a small change in the initial conditions of a system could lead to very different outcomes as the system evolved. This discovery evolved into an explanation known by many as the butterfly effect: When a butterfly flaps its wings in one part of the world, it can initiate a series of air currents that influence weather events that can eventually cause a hurricane in another part of the world, say Florida. Of course, should the butterfly flap its wings in a different direction, the hurricane could develop somewhere else, perhaps Cuba or Mexico. In the butterfly illustration, we see an explanation of how a slight change in initial conditions can lead to a vastly different outcome in the life of the system under study.

The bestselling book by James Gleick (1987) made chaos theory more understandable to those outside the mathematical and physics disciplines. Soon, social scientists, psychologists, and even a few management scholars found an interest in chaos theory. To these researchers, chaos theory offered an interesting, nonlinear framework that could be used as a lens to understand the complex social and psychological interactions that comprise individual and organizational behavior.

Chaos theory also inspired those who identified with the concept in nonmathematical ways. One viewpoint was that chaos carried with it a sense of mystery and excitement about life (Stoppard, 1995). The appeal of chaos theory has also been compared to a romantic appreciation of disorder that accompanies a corresponding reaction against the scientific appreciation for order and symmetry (Friedrich, 1988; Smith & Higgins, 2003). Polkinghorne (1993), a priest, articulated that chaos theory helps to describe the divine plan for the universe. While these viewpoints are interesting, they are amazingly off-base with the original intentions of chaos theory. In fact, in one sense, chaos theory is not actually a theory at all, but an extension of nonlinear mathematics (Bolland, & Atherton, 1999).  Put another way, chaos theory is somewhat of a mathematical anomaly.

One of the aspects of chaos theory that has also contributed to its increasing popularity is its visual dimension (Smith & Higgins, 2003). Attractors, a key component of chaos theory, can be graphed and often display an aesthetic appeal similar to computer art (Carey, 1995). A number of the papers reviewed for the present study depicted the famous Lorenz butterfly attractor. Add some color to this attractor and one can create an amazingly beautiful graphic. Certainly, this ability to take a seemingly difficult mathematical process and make it visually appealing adds to the mystique and popularity of chaos theory.

## THE TWO BASIC CONDITIONS OF A SYSTEM IN CHAOS

Chaos is not a state of randomness or disorder, but one whereby phenomena that appear to be unrelated actually follow an unknown or hidden pattern (Smith, 2002; Tetenbaum, 1998; van Staveren, 1999). This pattern is called an attractor and it can be visually observed through the plotting of data in phase space. Moreover, the relationships among the variables in a chaotic system are existent, but are "rather vague and at best, difficult to discern" (Kiel & Elliott, 1996:2). Chaotic systems possess two characteristics, sensitive dependence to initial conditions and unpredictability in the long run.

### Sensitive Dependence to Initial Conditions

Lorenz noted above that a slight change in the initial input of his data led to vastly different results in his weather model. In his book on chaos theory (Lorenz, 1993), he also discussed the paths of sleds descending down a snowy slope. In this example, he illustrated with diagrams how seven sleds can end up in different stopping areas at the bottom of a hill, even though they may have started their descent less than ten centimeters apart. Of course, the paths the sleds take change directions, depending on the location of small humps or moguls along the route of the descent. Nonetheless, the stopping point ultimately depends on the starting point, thus, the importance of sensitivity to initial conditions.

### Unpredictability in the Long Run

The behavior of a chaotic system cannot be predicted in the long run. At best, there may be some accuracy in making short-term predictions. The weather is an example of a chaotic system that cannot be determined on a long-term basis, but can be predicted successfully in the short run (Lorenz, 1993). For example, as the authors prepare this manuscript, the weather is cool and breezy in this location of the country. And yet, a year ago, nobody could have predicted with certainty the weather for this location of the country on this particular day. In fact, making such a prediction is all but impossible only a month in advance; hence, weather is unpredictable in the long-run, and so are systems in chaos.

## CHAOS THEORY AS A METAPHOR IN MANAGEMENT RESEARCH

During the 1990s, social scientists began to take an interest in chaos theory (Guastello, 2008). In the field of management research, its use has primarily been invoked as a metaphor. The practice of using metaphors to help explain the workings of an organization is not new. Gareth Morgan (1986) was a key player in generating enthusiasm for the use of metaphors to explain organizational behavior. However, Morgan did not use metaphors that were based on deep mathematics, an underpinning of chaos theory.

The dilemma of using chaos theory as a metaphor concerns the theory's intended use, to explain a unique mathematical state—chaos—where system behavior is neither completely orderly nor completely random. Herein lies the irony: Social scientists and organizational scholars do not readily use elaborate concepts from chemistry or physics to explain social behavior, so why the attraction to chaos theory? Indeed, the appeal of metaphors is a strong attracting force, one that can cause some misunderstandings, as we examine in the next section.

**The Metaphor Problem**

Misunderstandings concerning chaos theory stem primarily from its overuse as a metaphor, not in its mathematical use, because the latter is extremely limited in the management literature. While metaphors are useful to understanding complex systems, there is a tendency to extend them beyond their usefulness. Commenting on Gleick's book, Eigenauer (1993: 455) noted:

*While Gleick's work is solid, it has led some to be captivated by chaos theory's fecund metaphorical terminology and elegant computer aided graphical images. Although those images ... show striking instances of order hidden within chaotic systems, too often they are used to forward the thesis that there are other systems, ranging from modern literary theory to stock market fluctuations, that also house deep structure amid their apparent disorder. The result is, on occasion, analysis that is based only on metaphor.*

Smith (2002:523) also concurred with a similar thought, "… some disciplines have already displayed a tendency to rely too heavily on purely conceptual applications of chaos theory. This is in danger of reducing chaos theory to a collection of metaphors, or worse still reducing it to just semantic innovation if the application is trivial."

One problem with using chaos theory as a metaphor is that it offers little that cannot already be explained using existing theories or frameworks. For example, Bright and Pryor (2005) compare the four types of attractors to career decisions. While their work is interesting and describes the various career dilemmas employees face, their discussion of attractors does not augment our knowledge of careers. Put differently, the same paper without the discussion on attractors would have been sufficient. In another example, Sellnow, Seeger, & Ulmer (2002) offer an excellent assessment of the 1997 Red River flood in Minnesota and North Dakota. However, their attempts to tie in chaos theory to the discussion add little to our knowledge and even distracts from the central point; their analogy of the strange attractor to the support agencies involved in managing the flood is interesting, but not necessary.

While metaphors are useful in gaining new insights, their overuse can lead to problems. Metaphors begin to break down in their usefulness when exact parallels between the metaphor and the phenomena under study are either not possible or inappropriate (Barton, 1994, Chubb, 1990). While chaos theory can enable us to think about our research question in a different perspective, the metaphor must not be overextended. For example, the field of psychology was

one of the first social sciences to embrace chaos theory as a tool of analysis. However, as Barton (1994) noted, some misunderstandings of what chaos actually means have emerged. He cited Bütz's (1992) use of chaos as "overwhelming anxiety," a state that bears little resemblance to the mathematical state of chaos described in the original theory.

To summarize at this point, chaos theory is a mathematical concept and one that is difficult to use as a metaphor to social phenomena, particularly in management research. However, within chaos theory, there is a rather simple tool that does not require an advanced knowledge of mathematics to understand, the concept of phase space. It is in phase space where chaotic behavior can be deciphered and analyzed.

## EXAMINING FIRM PERFORMANCE IN PHASE SPACE

Chaos theory is a mathematical theory and therefore, its use should include mathematics (Elliott & Kiel, 1996; Faber & Koppelaar, 1994; Smith, 1995; Smith, 2002). In this section, we reflect on use of chaos theory in management research by offering an application of phase space analysis, a mathematical tool of chaos theory. To understand phase space, it is first necessary to review the concept of attractors.

### Attractors

In chaos theory, an attractor is a pattern that forms when the behavior of the system is plotted in phase space (Lorenz, 1993). When the points are joined by a line in a chronological order, a pattern develops that can resemble a point, orbit, or some kind of unusual pattern. The unusual pattern is also referred to as a strange attractor.

Attractors range from being fairly simple to vastly complex. Four types of attractors have been identified: Point, pendulum, torus (which is a type of orbit), and strange (Barton, 1994; Hudson, 2000; Stam, 2003). In phase space, a point attractor is depicted as a single plot on a graph. This occurs because the system behavior remains consistent over time. The pendulum attractor, also referred to as a period attractor, resembles a narrow back-and-forth pattern when graphed in phase space. The torus attractor is a more complex pattern that forms an orbit, but also contains sub-orbits within the orbit, thus resembling a donut when graphed in phase space. Finally, the strange attractor, sometimes referred to as a fractal, is a complicated pattern that exists when the system is in chaos. The attractor is called strange because its shape may or may not resemble any known pattern

### Graphing Variables in Phase Space

In phase space, the properties of the system under study are plotted at a point in time. With each iteration, another plot is made, which eventually results in a pattern (i.e., an attractor) when the plots are joined in chronological order by a line. "Diagramming the movement of a system's variables in phase space reveals the curious byways of an otherwise hidden reality" (Briggs & Peat, 1989: 32). Put another way, the pattern of a time series that looks haphazard

may actually have a hidden structure to it if we look at it in a different manner, through phase space.

Phase space can be graphed using variables that the researcher desires to study. With one variable, phase space is typically graphed by placing the current data point from a time series on the y-axis and the prior data point from the previous period on the x-axis. This one variable arrangement is also referred to as pseudo phase space (Williams, 1997). Within the medical field, Stam (2003) studied the single variable, EEG patterns (brainwaves), and plotted attractors in an attempt to identify conditions that can lead to an epileptic seizure. Yambe and associates (2003) looked at cardiac functions via the variable maximum ventricular elastance (E max) to determine phase space patterns that might indicate heart problems. In the production operations management literature, Giannelos and associates (2007) used the single variable, flow time, in assessing dispatching policies for manufacturing jobs.

Plotting with two variables is also possible in phase space. For example, mechanical systems have been examined in phase space using position and velocity while ecological systems have been studied in terms of the population size of the species being studied (Briggs & Peat, 1989). In medical research, Reidbord and Redington (1992) constructed a phase space with heart rate and the patient's behavior state as the study variables. In the area of public administration research, Kiel (1993) constructed an attractor in phase space using time series data involving labor costs associated with service requests. In the strategic management literature, Priesmeyer and Baik (1989) examined revenue and profit changes among retailers and identified attractors in phase space.

Priesmeyer and Baik's (1989) work set the stage for the following discussion. Their examination of two business variables in phase space is both conceptually easy to understand and useful in tracking patterns in firm performance. In the next section, we assess the movement of revenue and income variables in phase space. We then offer a glimpse into how one firm exhibited a strange attractor in phase space.

## Choosing Variables to Study

Management scholars typically seek to identify the influence of selected independent variables on a dependent variable. A standard approach is to conduct a correlation analysis to check for multicollinearity, and then multiple regression analysis to determine the strength of the independent variables on the dependent variable. However, in chaos research, scholars seek to identify the pattern of movement of the system under study through time by graphing the system variables in phase space.

In this paper, we graph the two variables of revenue and profits. These variables were selected based on the following criteria:

- Both variables can be captured in time series data.
- Revenue and profits are absolutely essential for the long-term sustainability of the firm.

- Both variables function as a proxy for how the firm is operating.
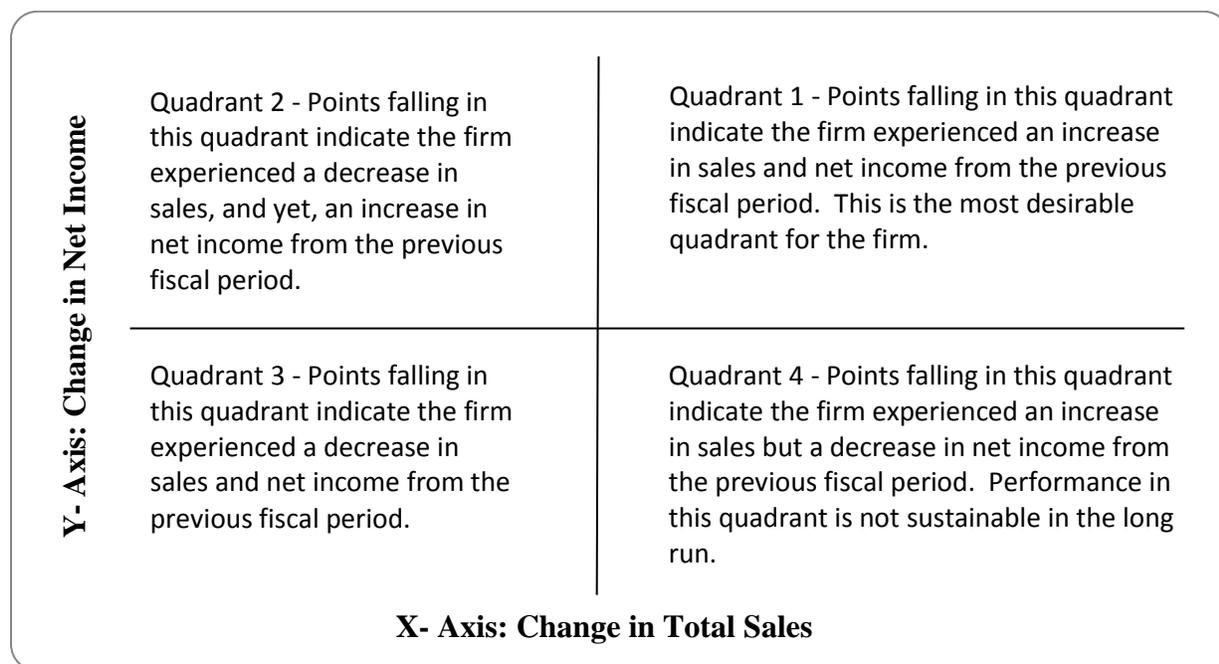
## Understanding Phase Space

The first requirement for analyzing a system in phase space is time series data, as it is the primary domain area for studying chaotic behavior (Haynes, Blaine, & Meyer, 1995; Hudson, 2000). In this paper, we examine total sales and net income as they appear in phase space. In phase space, we need to capture the movement of these variables through time. To accomplish this, we adjust our two study variables to reflect this requirement. Thus, we need to capture the variables as the "change in total sales", which will be shown on the x-axis, and the "change in net income", which will be depicted on the y-axis.

To obtain the change in total sales (x-axis coordinate), the difference between the present total sales for the fiscal quarter and the total sales for the previous quarter are calculated. The same procedure is used to calculate the change in net income (y-axis coordinate), using the net income (loss) figures.

$$\text{Change in Total Sales} = \text{Total Sales}_{current} - \text{Total Sales}_{previous}$$
$$\text{Change in Net Income (loss)} = \text{Net Income}_{current} - \text{Net Income}_{previous}$$

Figure 1 depicts the two study variables, change in total sales (X-axis) and change in net income (Y-axis). The figure also depicts the resulting four quadrants. Note that the upper right quadrant would be the most desirable for the firm, as it depicts a situation where the firm increased both sales and net income from the *previous* fiscal period. Points in the lower right quadrant indicate the firm increased sales, but experienced a decrease in net income from the previous fiscal period, an undesirable situation with any company. The upper left quadrant depicts a situation where sales decreased, but income increased from the previous fiscal period. Such a scenario might occur when a firm is downsizing and/or divesting one or more of its business units. Finally, the lower left quadrant depicts a decrease in sales and net income from the previous fiscal period. Points in this quadrant do not necessarily indicate the firm is performing poorly. A more likely scenario is that the company is experiencing a seasonal factor in its business cycle. Many companies rebound the next fiscal period and actually vacillate between increasing and decreasing sales and net income. However, there would be cause for concern if the company continued to experience decreased sales and income over successive fiscal periods. Such performance would not be sustainable in the long-run.

**Figure 1 – The Four Quadrants in Phase Space**



| | Quadrant 2 - Points falling in this quadrant indicate the firm experienced a decrease in sales, and yet, an increase in net income from the previous fiscal period. | Quadrant 1 - Points falling in this quadrant indicate the firm experienced an increase in sales and net income from the previous fiscal period. This is the most desirable quadrant for the firm. |
|---|---|---|
| **Y - Axis: Change in Net Income** | Quadrant 3 - Points falling in this quadrant indicate the firm experienced a decrease in sales and net income from the previous fiscal period. | Quadrant 4 - Points falling in this quadrant indicate the firm experienced an increase in sales but a decrease in net income from the previous fiscal period. Performance in this quadrant is not sustainable in the long run. |

**X- Axis: Change in Total Sales**

Adapted from Priesmeyer, H.R. (1992). *Organizations and Chaos: Defining the Methods of Nonlinear Management*. Westport, CT: Quorum Books, page 29.

**An Example of Firm Performance in Phase Space**

Suppose that a firm increased sales for three fiscal periods by exactly $400, while net income by exactly $200. If this oversimplified situation were graphed in phase space, it would be plotted as a single point. The resulting point attractor is shown in Figure 2.

**Figure 2 - A Point Attractor**
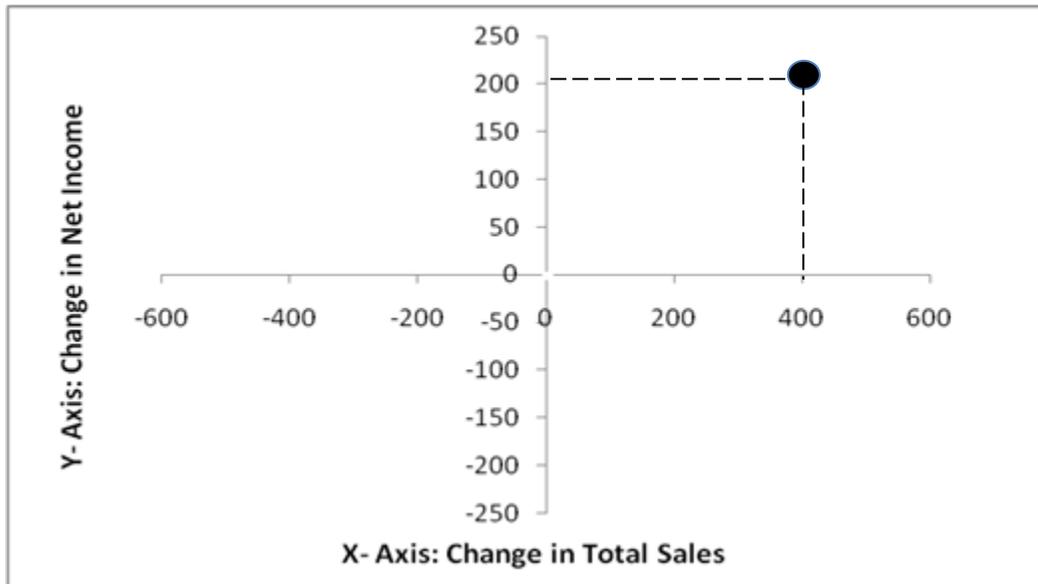


Now, suppose that in the next fiscal quarter, the firm experienced a decrease in sales by $400 and a decrease in net income of $200. The resulting plots would look like the ones in Figure 3. When the two points are joined, the line slopes from the top right to the bottom left. Plots in the lower left or right quadrants do not necessarily mean the firm experienced a net loss, only that it experienced a *decrease* in net income for that fiscal period.
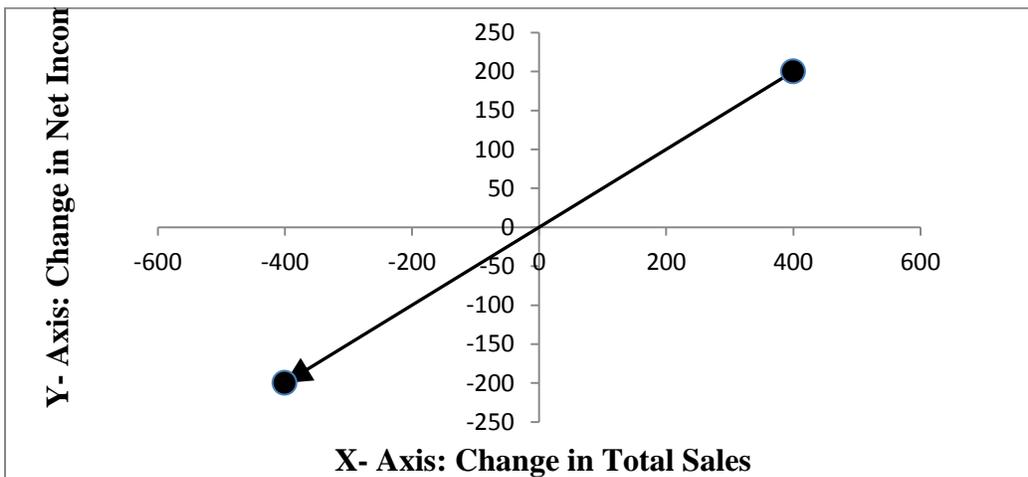


**Figure 3 - Movement through Phase Space after One Fiscal Period**

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Assume that in the next fiscal quarter, the firm experienced an increase in total sales of $200 and an increase in net income of $200. Remember, these are changes from the *previous* fiscal period. This is not the same as saying that the firm's sales and net income were $200, an all but impossible situation. Figure 4 illustrates this change, which is a return to the original quadrant.

**Figure 4 - Two Fiscal Periods Graphed in Phase Space**
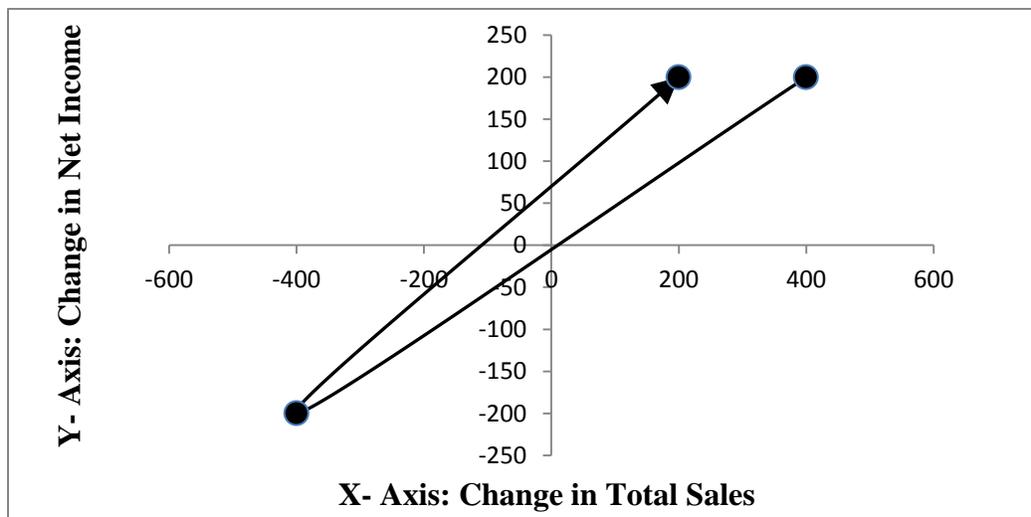


Assume also that in the subsequent quarter the firm experienced a decrease in total sales of $175 and a decrease in net income of $200. When the plots are joined, one can see the beginnings of a pendulum or more accurately, a period two attractor as shown in Figure 5.

**Figure 5 - Three Fiscal Periods Graphed and the Beginning of a Period Two Attractor**
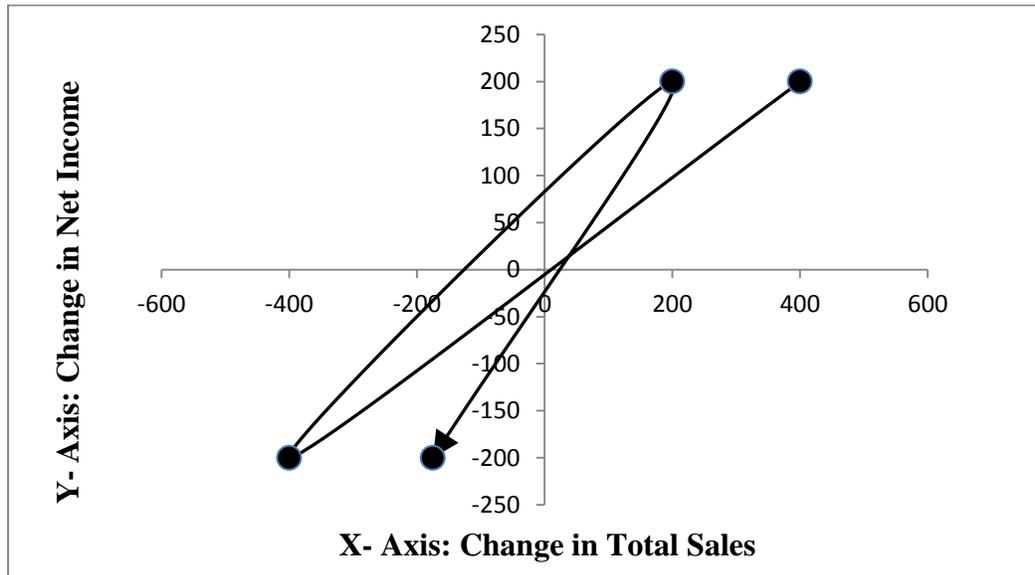
When the remaining points in this example are graphed, what results is an obvious pattern, an attractor that cycles between the two quadrants. In fact, this pattern could be quite normal for any business that experiences seasonal business cycles where sales and net income fluctuate. One of the characteristics of the period two attractor is that is the subsequent quadrant is easy to determine. In addition, we propose this particular attractor is typical for a firm that is operating well. The completed example of the period two attractor appears in Figure 6. The dataset used to generate this hypothetical example is in Table 1.
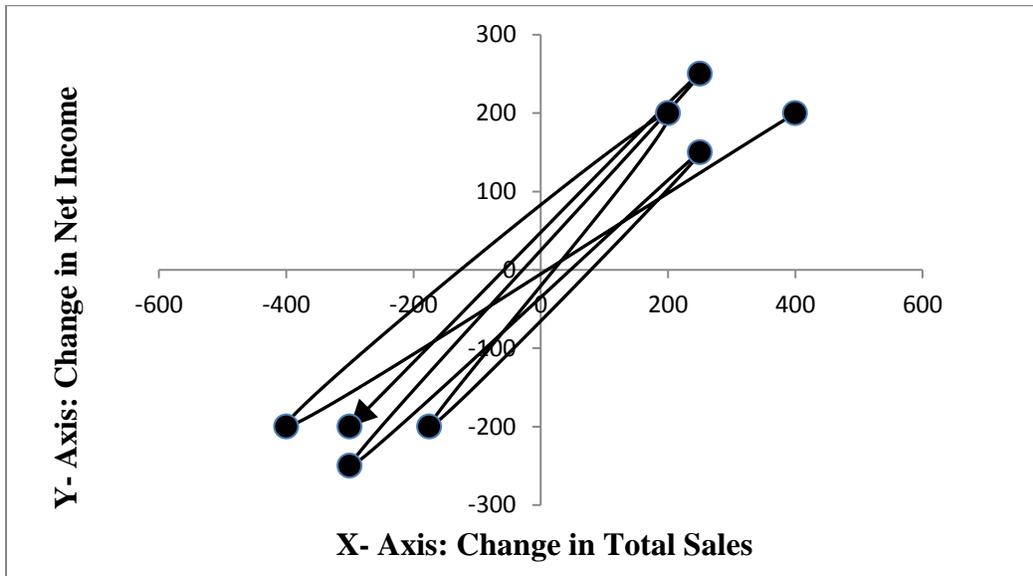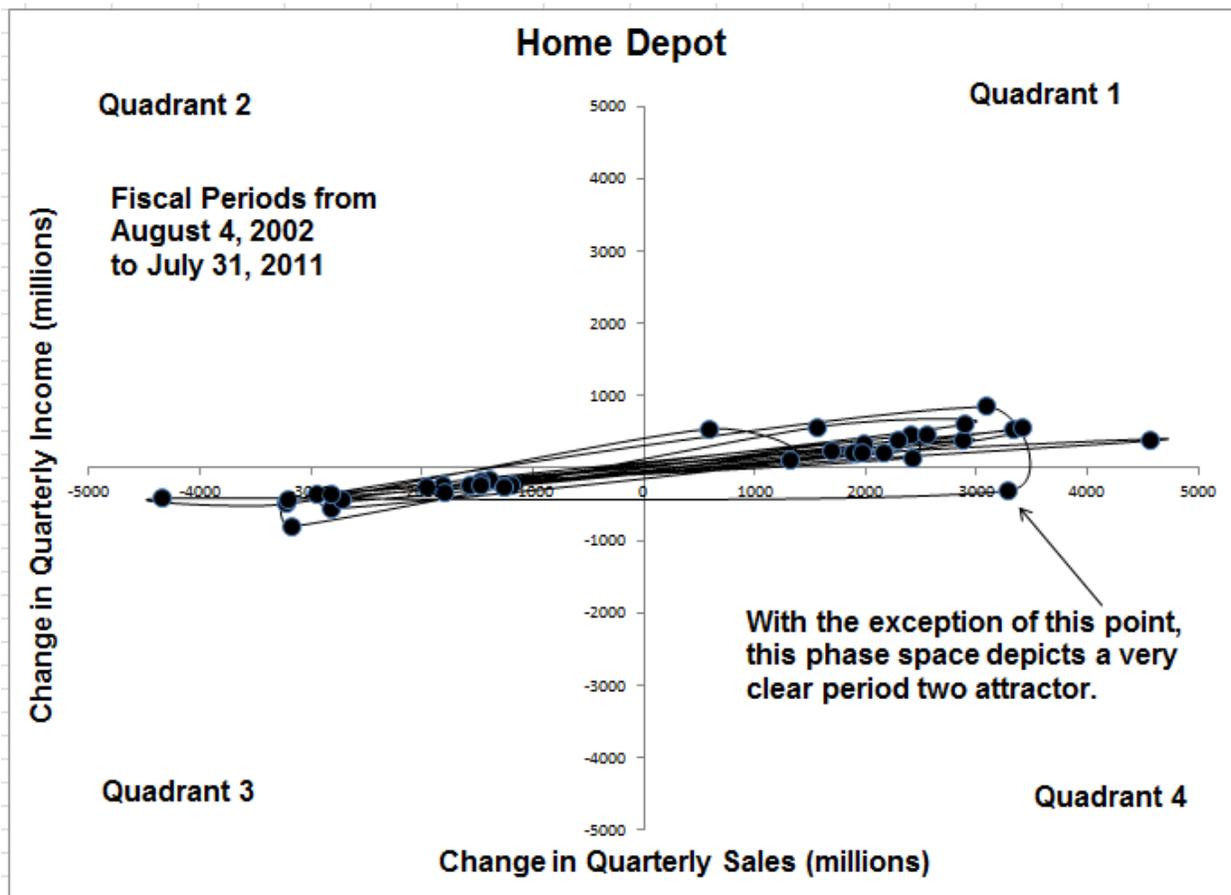
**Figure 6 – A Period Two Attractor**



**Table 1 - Data Set for the Above Example**

| Fiscal Period | Change in Total Sales | Change in Net Income |
|---|---|---|
| 1 | 400 | 200 |
| 2 | -400 | -200 |
| 3 | 200 | 200 |
| 4 | -175 | -200 |
| 5 | 250 | 150 |
| 6 | -300 | -250 |
| 7 | 250 | 250 |
| 8 | -300 | -200 |

Consider another example of a period two attractor using data from an actual company. Home Depot exhibits a relatively consistent two phase oscillation from the upper right to the lower left quadrants; an indication that sales and net profits are moving in a cyclical pattern. Figure 7 illustrates Home Depot's performance in phase space.

**Figure 7 – Home Depot's Performance in Phase Space**



**Data Source – Mergent Online**

The performance for Home Depot shows visits to quadrants 1 and 3 oscillating in a clear period two attractor. There is one visit to quadrant 4, indicating an increase in sales and a slight decrease in income from the previous fiscal period. Other than this one exception, the phase space is depicts a regular pattern of performance and we can conclude that the system is not in chaos.

Figure 8 depicts the performance for Apple.

**Figure 8 – Apple's Performance in Phase Space**
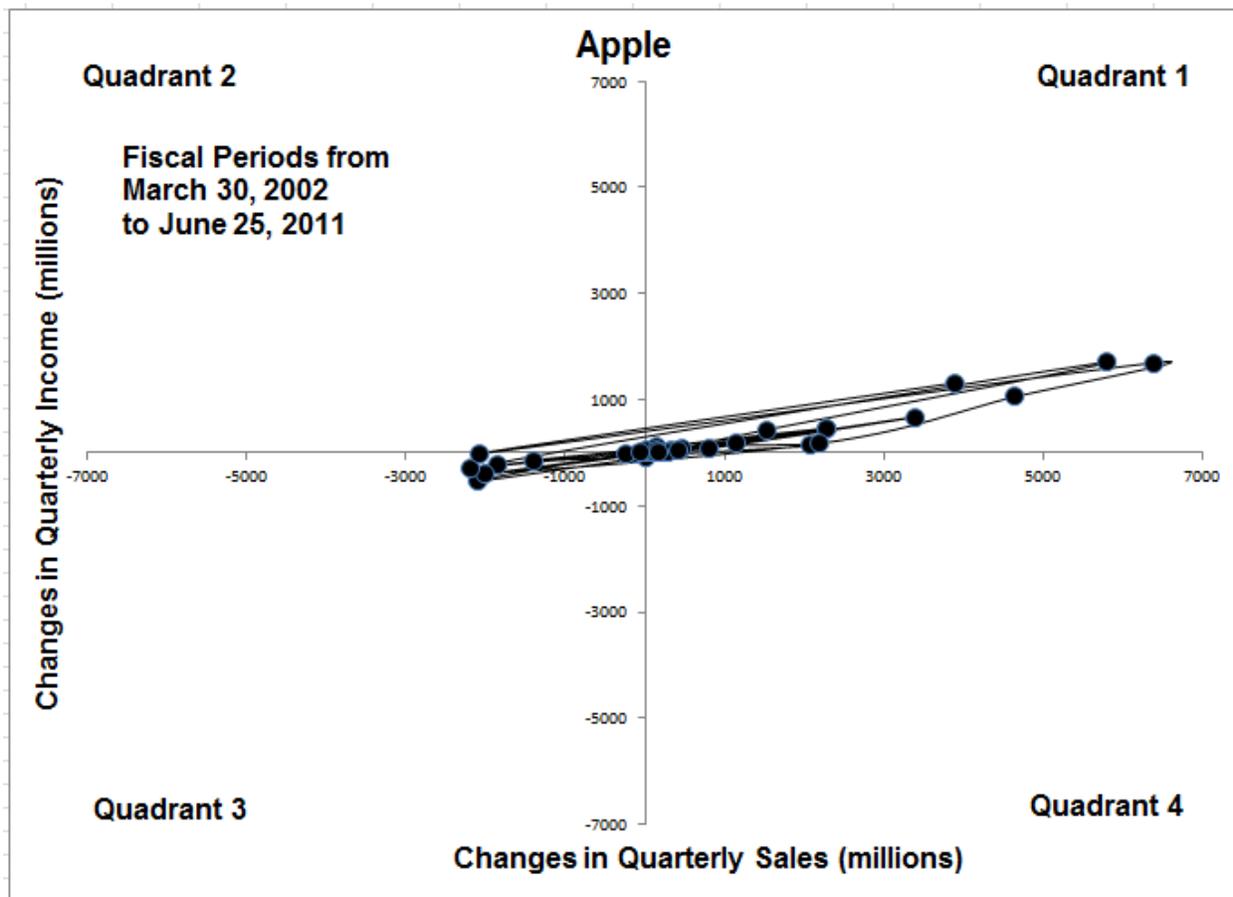


The performance for Apple also depicts a clear period two attractor. However, there are some distinct differences from its performance compared to Home Depot's period two attractor. First, the attractor is positioned more to the right of the phase space than Home Depot. In other words, the points go deeper into quadrant 1 relative to quadrant 3. Recall that Home Depot's attractor oscillated evenly from quadrants 1 and 3. Put differently, the visits into quadrant 1 went about as deep as into quadrant 3. With Apple, the visits are deep into quadrant 1 and shallow into quadrant 3, an indication of strong performance.

Another difference from the Home Depot attractor is that Apple experiences "consecutive visits" within quadrant 1. In other words, there are several trajectories that contain consecutive points within quadrant 1. This indicates that both sales and net income are increasing marginally

over two consecutive fiscal periods. Again, this observation is an indication of strong firm performance. We can conclude that Apple is not in chaos.

## Chaos and Phase Space

Those who study phase space will ultimately seek to determine if time series data exhibits the characteristics of chaos. It should be noted that systems that exhibit a point or period two attractor (discussed above) are *not* in chaos because the system remains predictable over a period of time; recall that a condition of chaos is unpredictability in the long run. The question now becomes, what does a system in chaos look like in phase space? It will resemble a complex shape or pattern, often referred to as a strange attractor. We should remember though that the strange shape is more than just an object to observe; it is an indication that the system is experiencing the two conditions of chaos, 1) sensitive dependence to initial conditions, and 2) unpredictability in the long run. In the next section, we analyze a firm that exhibits chaotic behavior as indicated by its strange attractor.

## CHAOTIC PERFORMANCE IN PHASE SPACE

In this section, we analyze the performance of the company, Pizza Inn, in phase space. Specifically, we suspected that the company would depict a chaotic attractor when the variables of sales and income (loss) were graphed in phase space. This observation is based on the fact that Pizza Inn's performance has been suboptimal over the past decade.

We suspected that a poor performing firm would be more difficult to predict in terms of performance from one fiscal quarter to the next. Hence, its phase space history would be chaotic. The result would be a strange attractor that did not appear to follow a point or period two attractor. This methodology is consistent with Priesmeyer and Baik's (1989) assessment of retail firms.

To test our proposition, we collected nine years of time series data from Mergent Online for the company, Pizza Inn. Next, we plotted two variables, change in sales (x-axis) and change in income (y-axis), in phase space.

## RESULTS

Figure 9 depicts performance for Pizza Inn for fiscal years 2002 to 2010. Immediately, it can be seen that this graph does not depict a period two attractor like was seen with Home Depot and Apple. Indeed, there are plots in all four quadrants and the trajectory paths do not seem to

follow any organized pattern. The phase space attractor takes on a strange shape, and based on this, we can make a preliminary conclusion that the system is chaotic.

**Figure 9 – Pizza Inn Firm Performance in Phase Space**
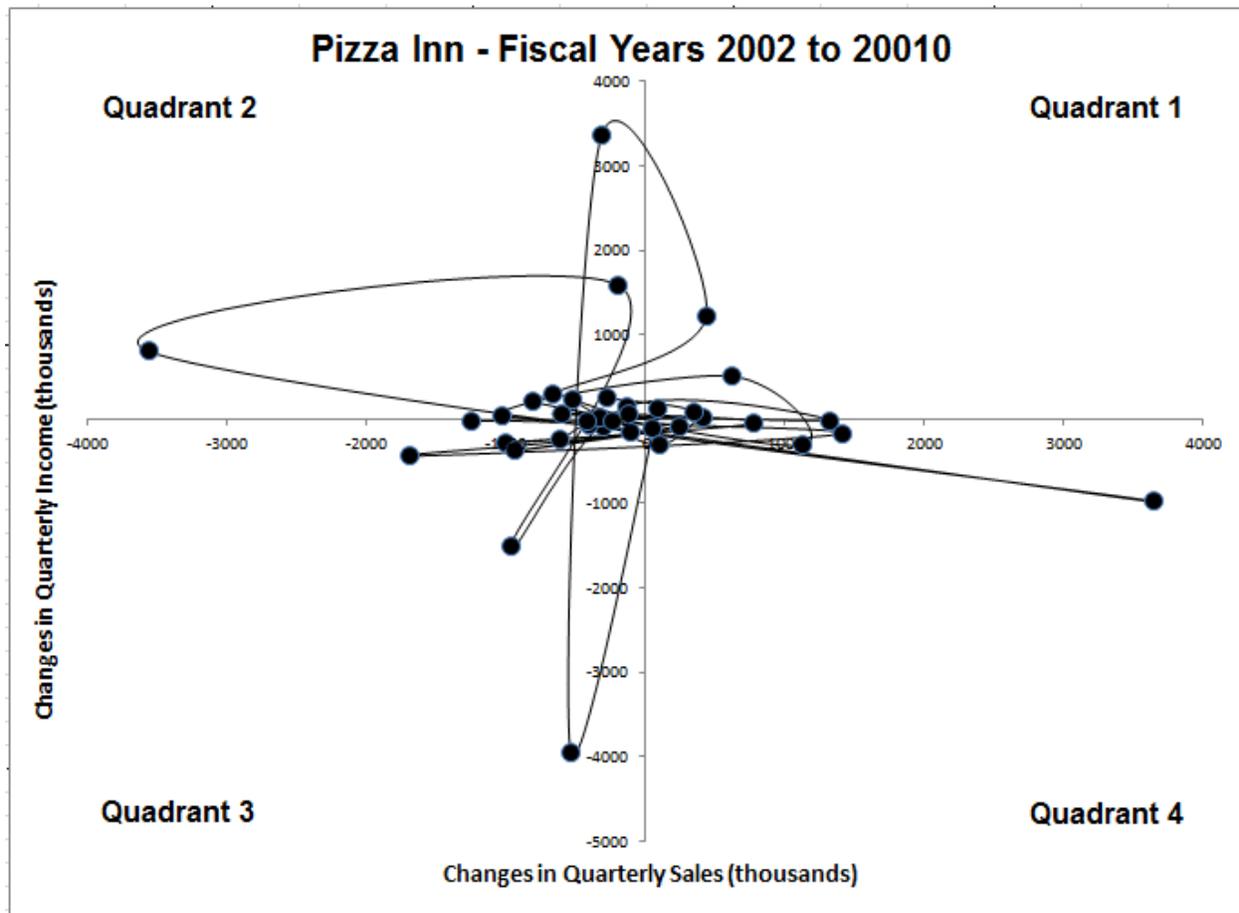


Chaos theorists seek to examine the characteristics of the attractor to determine the nature of the system under study. While this system appears to be chaotic, there is additional information available if we look at the phase space on a smaller scale. Figure 10 shows that there is a sub-attractor that exhibits a fairly constant pattern. This area is shown within the dashed oval pattern on the graph. Note also that those points outside the attractor boundary have been identified by the large arrows.

**Figure 10 – The Sub-Attractor and the Outlier Points**
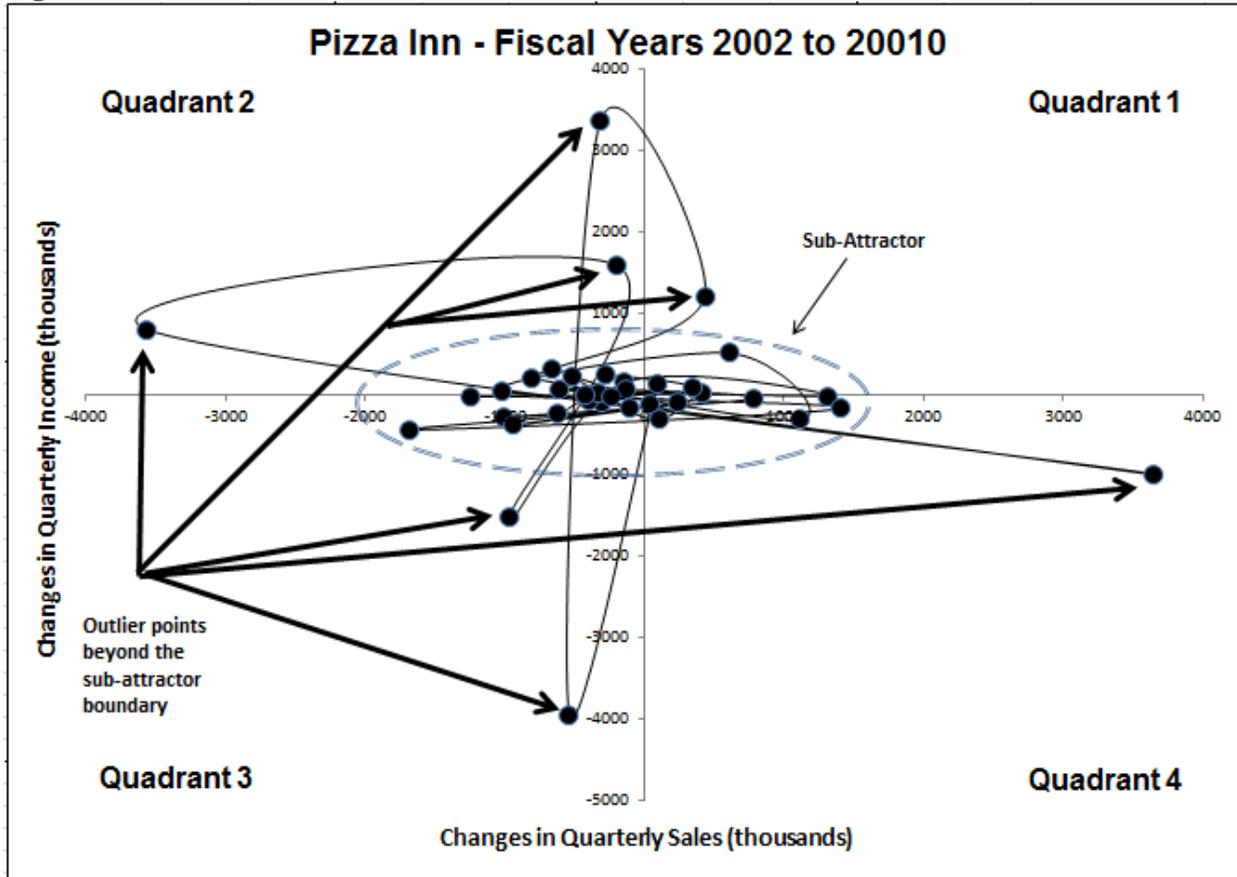


The sub-attractor displays a consistent pattern of change for sales and income. The long and narrow shape of the attractor indicates that variation in sales exceeded the variation in income. For sales, the variation within the attractor went from - 1689 to +1317 (in thousands). For income, the various was much lower, from - 436 to +617 (in thousands). Intuitively, this should not surprise us since sales are always going to be a higher figure than income. However, a phase space graph shows the change in sales and income from the previous fiscal period. Hence, it is possible that some points will still lie outside the attractor. Indeed, that is the case in this phase space graph. For example, the outlier point in quadrant 4 displays a large increase in sales, and yet, a decrease in income. A somewhat similar situation exists in quadrant 2, where three points lie outside the attractor. The two points along the y-axis indicate an increase in income despite a decrease in sales. The one point along the x-axis depicts a large decrease in sales, and a slight increase in income. Recall that points in quadrant 3 should be considered undesirable if they continue to exist over consecutive fiscal periods.

The structure of the sub-attractor in figure 10 is difficult to determine. In other words, the trajectories from point to point cannot be determined. In order to see this structure more clearly, we must examine the attractor in greater detail so we can determine the patterns of movement that lie within (see figure 11). Note that the outlier points are not depicted, but the paths to those points are indicated.
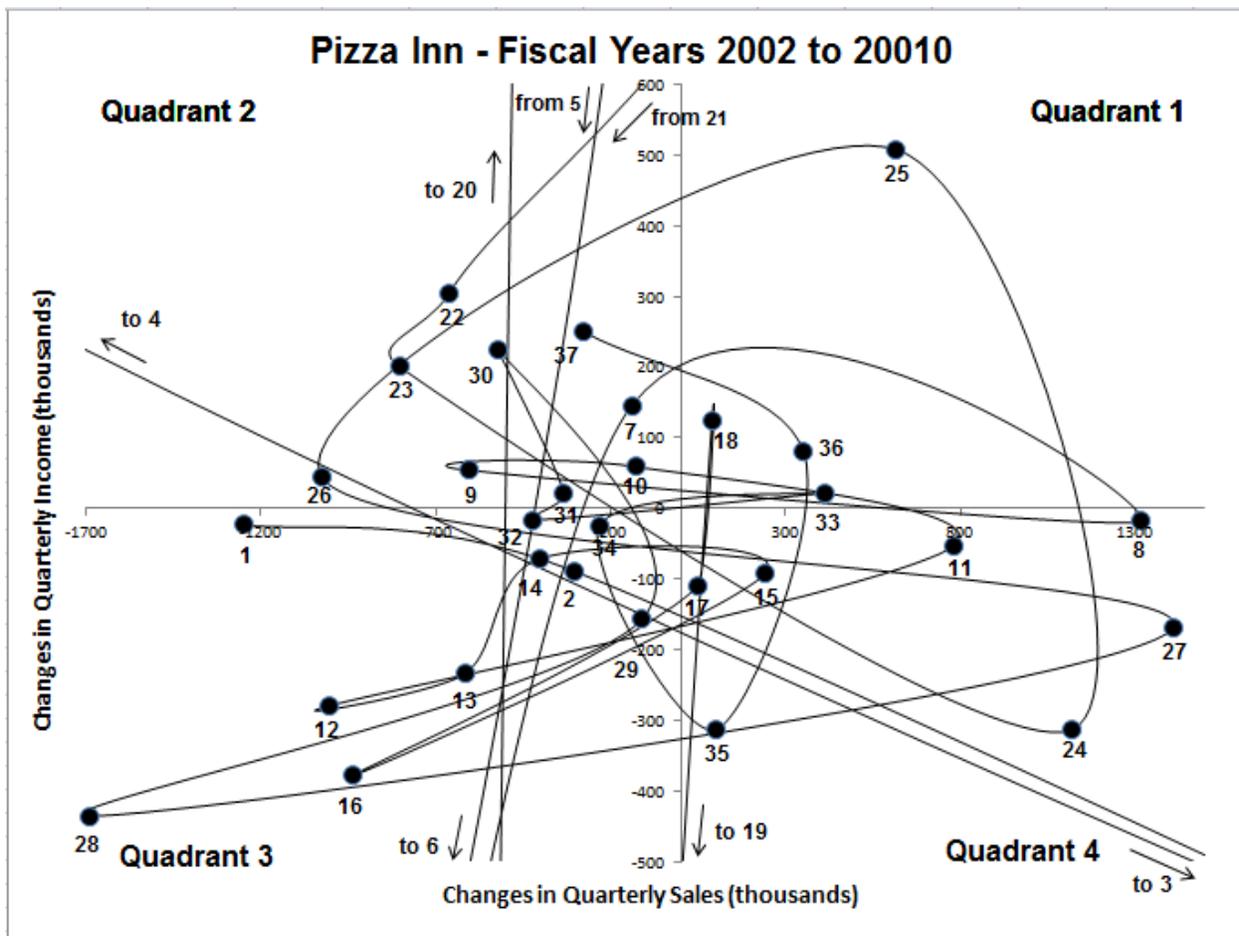
**Figure 11 – The Sub-Attractor in Detail**



Figure 11 begins at point 1 in quadrant 3, which is the starting point for fiscal year 2002. The trajectory moves to point 2, also in quadrant 3, an indication that sales and income have both decreased during the first quarter of 2002. During the second quarter, sales increase to an outlier, point 3, which lies beyond the attractor. Unfortunately for Pizza Inn, net income declined despite this sales increase. During the third quarter, there is a dramatic swing back to another outlier, point 4, which indicates a decrease in sales accompanied by an increase in

income. Finally, the 2002 fiscal year ends at point 5, another outlier, in which income increases but sales decrease.

The first quarter of 2003 ends at point 6, another outlier, which depicts a decrease in both sales and income. At point 7, the second quarter ends with an increase in income, but a continued decline in sales. Points 8 to 18 lie within the attractor and the paths can be traced fairly easily. The trajectories are not consistent however; instead, they follow a haphazard pattern that is impossible to predict. In other words, the system is chaotic. Recall that for a consistent period two attractor, we could discern a clear back and forth pattern of trajectories between quadrants 1 and 3. That is not the case here.

Points 19 to 21 are outliers and not shown on the graph. The trajectory returns to point 22 in quadrant 3, which corresponds with the end of the first quarter in 2007. The remaining points lie in the attractor. The final fiscal quarter in 2010 ends with point 37.

## DISCUSSION

In the previous section, we illustrated how a tool of chaos theory, phase space, can be used to track the firm performance variables of sales and income. We identified a strange attractor in the ten year operating period of Pizza Inn and concluded that it was a system that was chaotic. In the following discussion, we examine more closely what it means to be in chaos.

**A Chaotic System**

We have concluded that Pizza Inn's performance in terms of sales and income is chaotic. This conclusion meets the two criteria of chaos, (1) sensitive dependence to initial conditions, and (2) unpredictability in the long run. In regards to the first condition, chaos researchers have learned that when systems move from a period two to a period four cycle, that system is also moving towards chaos (Briggs & Peat, 1989; Gleick, 1987). We illustrated a period two cycle with Home Depot and Apple, which is considered a stable cycle and non-chaotic. It should be acknowledged that period four cycles can also exist, but these are considered less stable. A period four cycle would indicate a system is visiting four distinct areas in phase space in an alternating manner. Certainly, there are business cycles that can fall into this category, but they are less likely to occur and more fragile than a period two cycle. A period eight cycle is less stable than a period four cycle. When systems are pushed beyond eight period cycles, then chaos is fast approaching. In fact, Priesmeyer (1992) maintains that for all practical purposes, businesses operating in a period eight cycle are already in chaos.

As for the second condition, unpredictability in the long-run, our data does support this conclusion with the Pizza Inn chain. The haphazard or strange shape of the attractor displays a pattern of firm movement in phase space that is difficult to determine. It is impossible to predict

what Pizza Inn will look like next year. Indeed, we cannot even predict which quadrant the phase space plot will fall on in the next fiscal quarter.

## Phase Space Quadrants

The plots in the phase space quadrants can reveal important information about the performance of Pizza Inn. Each quadrant represents the degree to which sales and income increase or decrease. Assume that the period two cycle from quadrant 1 to quadrant 3 and back to 1, and so on, is the normal healthy cycle. If the system departs from that cycle by going to quadrants 2 or 4, then it will need to correct itself in order to return to the quadrants 1 − 3 sequence.

As mentioned above, quadrant 1 is the most desirable because points in this area indicate an increase in both sales and income. However, of the 37 possible points, only five reside in quadrant 1 (points 18, 21, 25, 33, and 36). The low presence in quadrant 1 indicates Pizza Inn's inability to sustain growth.

Although consecutive points plotted in quadrant 1 would be ideal, healthy firms in a normal business cycle would most likely move to quadrant 3 during their next fiscal quarter. This back and forth movement is the path that marks the now familiar period two cycle discussed above. For Pizza Inn however, only two points (18 and 33) move to quadrant 3. The others three points (21, 25, and 36) depart from the healthy period two cycle and move to quadrant 2 indicating a decrease in sales, but an increase in income. While the increase in income is positive, it also indicates the firm performance is chaotic, that is, it is not approaching a regular period two.

Quadrant 2 contains 12 of the 37 points in the system. The strong presence in this quadrant is not sustainable because of the problem of decreasing sales. If we assume that a period two is the normal cycle, then what would follow is a countermovement from quadrant 2 over to quadrant 1. Once the system is reset at quadrant 1, it would move to quadrant 3, and then back to 1. However, this is not the case with Pizza Inn as the trajectories continue to move all over the graph in various locations in an inconsistent manner.

Quadrant 3 contains 12 of the 37 points in the system. If Pizza Inn operated in a normal period two cycle, we would expect an equal number of points in quadrants 3 and 1. As mentioned, this is not the case. What exists is a disproportionately high number of points in quadrant 3 relative to quadrant 1, a sign of chaotic financial performance. This does not necessarily imply poor financial performance, as phase space does not indicate if Pizza Inn is in the black or the red in term of net income. However, chaotic performance is of concern since there is an inability to establish a consistent pattern in terms of sales and income.

Quadrant 4 contains 8 of the 37 points. This is of concern since this quadrant indicates an increase in sales and a decrease in net income. Again, this does not mean Pizza Inn went into the

red; only a traditional time series graph can identify such a situation. Instead, it displays a pattern of movement that may be signaling a problem if it continues.

## ADVANTAGES, LIMITATIONS AND FUTURE DIRECTIONS

### Advantages

The main advantage of using phase space is its ability to illustrate changes in system behavior in an interesting visual format. Because phase space graphs changes in the system variables, it is well suited for showing shocks to the system that are not as noticeable using traditional time series data. Phase space is also suitable for identifying patterns in system behavior that are periodic, random, or chaotic.

Phase space is not a technique to replace any of the more traditional approaches to time series analysis; instead, it can supplement our understanding of firm performance. Hence, we believe the use of chaos theory is not a superior approach to analysis, but rather represents another valuable technique for scholars.

### Limitations

The primary limitation of this study was the limited number of data points, 37, within the time series under study (see Williams, 1997). Moreover, some theorists maintain that chaos is difficult to identify with real world data (Williams, 1997). In this study we used the term *chaotic* instead of chaos to describe the Pizza Inn time series data.

### Future Directions

Two key research opportunities have been identified.

- *Are there common phase space attractors for specific industries?* We know that certain industries can be volatile due to market influences and supply chain uncertainty (e.g. the airline industry). Is it possible that a number of companies in the same industry exhibit a common attractor?
- *What other variables should be displayed in phase space?* For example, if research and development (R&D) expenditures were graphed in phase space with net income or net sales as accompanying variables, would the resulting attractors yield meaningful information? R&D is mentioned because of its use as a means of differentiation. R&D is particularly characteristic of the prospector business strategy and the differentiation strategy (Parnell, 2008).

## CONCLUSION

The use of chaos theory as a metaphor is limited and, at times, inappropriate. Rather, chaos theory should be interpreted as a mathematical phenomenon, and more accurately, a subset of nonlinear dynamics. We propose that using phase space analysis is a suitable way to

incorporate chaos theory into management research. However, chaos theory is not a superior tool of analysis, but an additional tool with specific strengths that can aid in the understanding of firm performance over time.

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