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# Network-Constrained Optimal Scheduling of Multi-Carrier Residential Energy Systems: A Chance-Constrained Approach

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**ABSTRACT** This paper presents a day-ahead scheduling approach for a multi-carrier residential energy system (MRES) including distributed energy resources (DERs). The main objective of the proposed scheduling approach is the minimization of the total costs of an MRES consisting of both electricity and gas energy carriers. The proposed model considers both electrical and natural gas distribution networks, DER technologies including renewable energy resources, energy storage systems (ESSs), and combined heat and power. The uncertainties pertinent to the demand and generated power of renewable resources are modeled using the chance-constrained approach. The proposed model is applied on the IEEE 33-bus distribution system and 14-node gas network, and the results demonstrate the efficacy of the proposed approach in the matters of diminishing the total operation costs and enhancing the reliability of the system.

**INDEX TERMS** Chance-constrained method, combined heat and power, multi-carrier residential energy systems, thermal load, uncertainty.

## NOMENCLATURE

### A. SETS AND INDICES

$\Psi_T$	Set of time intervals, indexed by $t$
$\Psi_N$	Set of electrical buses, indexed by $n$ , $m$
$\Psi_I$	Set of natural gas nodes, indexed by $i$ , $j$
$\Psi_H$	Set of residential houses, indexed by $h$
$\Psi_H^n$	Set of houses accessible to the heat pumps at node $n$ .
$\Psi_{CHP}^n$	Set of houses accessible to the heat production of combined heat and power (CHP) units at node $n$ .
$k$	Index of sample scenarios in sample average approximation (SAA) method
$i_S, n_S$	Indices of gas and electricity upstream network node, respectively

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### B. PARAMETERS AND INPUT DATA

$\lambda_t^E, \lambda_t^G$	Electricity and natural gas prices at time $t$ , respectively
$\Delta t$	Scheduling time step
$P_{CHP}^{min}, P_{CHP}^{max}$	Minimum and maximum power production of CHP units, respectively
$H_{CHP}^{max}$	Maximum heat production of CHP units
$\eta_{CHP}$	Efficiency of CHP units
$x_{A:D}, y_{A:D}$	Corner points of feasible working region of heat and power output of CHP units
$M$	Sufficiently large positive number
$\gamma_h^{HP}, \gamma_h^{GF}$	Identifier of heat pump and gas furnace
$\eta_h^{HP}, \eta_h^{GF}$	Efficiency of heat pump and gas furnace
$C_h^{in}, C_h^{sf}$	Heat capacities of house interior and surface
$H_{h,t}^{in,rad}, H_{h,t}^{sf,rad}$	House interior and surface heat radiation

$\zeta_h^{in}, \zeta_h^{sf}, \zeta_h^{ex}$	Heat transfer capacities among interior, surface, and exterior, respectively	$P_{h,t}^{HP}, q_{h,t}^{GF}$	Consumption of heat pump and gas furnace
$T_h^{in,min}, T_h^{in,max}$	Minimum and maximum of indoor temperatures of house $h$ , respectively	$H_{h,t}^D$	Household heating power
$\eta^{ES,ch}, \eta^{ES,dis}$	Charge/discharge efficiency of electric storage systems	$T_{h,t}^{in}, T_{h,t}^{sf}$	Household interior and surface temperature
$\eta^{GS,in}, \eta^{GS,out}$	Input/output efficiency of gas storage devices	$H_{h,t}^{ext}$	Household Heating power comes from CHP units
$\epsilon_n^{ES,min}, \epsilon_n^{ES,max}$	Minimum and maximum state of charge of electric storage unit at electric node $n$	$\epsilon_{n,t}^{ES}, \epsilon_{i,t}^{GS}$	The stored energy level in electricity and gas storage units, respectively
$\epsilon_i^{GS,min}, \epsilon_i^{GS,max}$	Minimum and maximum stored energy in gas storage unit at gas node $i$	$P_{n,t}^{ES,ch}, P_{n,t}^{ES,dis}$	Electric energy storage charge and discharge powers, respectively
$P_n^{ES,max}$	Nominal power of electric storage unit at electric node $n$	$q_{i,t}^{GS,in}, q_{i,t}^{GS,out}$	Gas storage input and output flow
$q_i^{GS,max}$	Nominal power of gas storage unit at gas node $i$	$u_{n,t}^{ES}, u_{i,t}^{GS}$	Binary variables for electricity and gas energy storage systems
$P_{n,t}^{RG}$	Active power generation of renewable energy sources	$P_{n-m,t}, Q_{n-m,t}$	Active and reactive power flow through line between nodes $n$ and $m$
$P_{n,t}^D, Q_{n,t}^D$	The amount of active and reactive demand at bus $n$ and time $t$ , respectively	$V_{n,t}, \theta_{n,t}$	Voltage magnitude and phase angle at node $n$ and time $t$
$G_{n-m}, B_{n-m}$	Conductance and susceptance of branch between buses $n$ and $m$ , respectively	$q_{i-j,t}$	Gas flow through pipeline between nodes $i$ and $j$ at time $t$
$P_t^{min}, P_t^{max}$	Minimum and maximum of exchanged active powers with upstream electricity network at time $t$ , respectively	$\pi_{i,t}$	gas flow pressure at node $i$ and time $t$
$Q_t^{min}, Q_t^{max}$	The minimum and maximum of exchanged active powers with upstream electricity network at time $t$ , respectively	$\mu_{i-j,t}$	Auxiliary binary variable
$V_n^{min}, V_n^{max}$	The minimum and maximum of voltage magnitude at bus $n$ and time $t$ , respectively	$L_{i-j,t}$	Natural gas linepack of pipeline between node $i$ and $j$
$S_{n-m}^{max}$	The maximum capacity of line between electric nodes $n$ and $m$		
$\phi_{i-j}$	The pipeline constant for the pipeline between nodes $i$ and $j$		
$\kappa_{i-j}$	The linepack constant for pipelines		
$L_{i-j}^{min}, L_{i-j}^{max}$	Minimum and maximum of linepack capacity of pipeline between nodes $i$ and $j$ , respectively		
$\lambda_h^{pen}$	Penalty price for the heating appliance		
$\alpha_P, \alpha_Q, \alpha_V, \alpha_S$	Confidence coefficient of chance constraints		

**C. PARAMETERS AND INPUT DATA**

$P_t^{up}, q_t^{in}$	The amount of purchased active power and gas from upstream networks at time $t$ , respectively
$Q_t^{up}$	The amount of exchange reactive power with upstream grid at time $t$
$P_{n,t}^{CHP}, H_{n,t}^{CHP}$	Generated electric power and heat by CHP unit at node $n$ and time $t$
$q_{i,t}^{CHP}$	Consumed gas by CHP unit at electric bus $n$ fed from gas node $i$ and time $t$

**I. INTRODUCTION**

Multi-carrier energy systems (MCESS) equipped with high-efficiency technologies have been noted increasingly around the world due to their operational flexibility to supply consumers' demands. MCESS are generally implemented through energy hubs in which numerous energy carriers as input ports (e.g., electricity and natural gas) can be converted, stored, and transformed to provide necessary various types of energy demands such as electricity, heating, cooling, etc. [1]–[3]. Therefore, MCESS enable the interaction between electricity and gas networks, as the most important energy carriers, such that their operation of each network affects the other one [4]. Hence, the simultaneous scheduling of these networks (i.e. gas and electricity) is necessary for optimal operation MCESS.

Several research works have been conducted to investigate the interaction between natural gas and electricity networks. The authors in [5] investigate the impacts of natural gas network constraints on the contribution of natural gas-fired power plants in a day-ahead market without considering renewable energy resources and system uncertainties. In another research, [6], the authors investigate the synchronous exploitation of electricity and natural gas systems, comprising electric vehicles (EVs) and variable renewable energy sources (VRESs) formulated on a continuous-time model method. It proposes a stochastic framework to expand the integration of VRESs into the power system scheduling problem. In [7], a new model is proposed for optimal operation of integrated energy systems considering the dynamic characteristics and uncertainties of wind resources.

This paper utilizes the CVaR method to model the uncertain parameters in the scheduling model of natural gas and electricity networks. The study in [8] presents optimal day-ahead scheduling of combined gas and electricity networks with regard to reciprocal energy flow between these two networks. In this work, the hourly topology of electricity and gas networks and the operation of power to gas (P2G) units are optimized to minimize the total operation costs. In [9], the security constraint unit commitment (SCUC) problem is analyzed considering the impacts of the natural gas network's constraints ignoring the presence of renewable energies. The research in [10] investigates the effect of the uncertainty of natural gas supply for gas generation units along with its price fluctuations on the contribution of natural gas power plants and daily operation costs under a two-stage stochastic scheduling problem, disregarding the existence of renewable energy resources. However, this study presents a simple and incomplete model for a natural gas network such that it neglects the existence of gas pressure limitations and the non-linear model of natural gas flow transmitted by pipelines. In another study, [11], a scenario-based SCUC problem is solved to manage the wind power generation fluctuations in integrated natural gas and electricity networks. The role of P2G technology is researched in [12] for optimal operation of the integrated electricity and natural gas networks. The optimal scheduling of MCESs in the presence of P2G storage is investigated in [13] to supply the electrical, thermal, and gas loads while considering the uncertainties of renewable energies and electricity price as well as exchange energy with both electrical and natural gas networks. In [14], the scheduling problem of electricity and gas network coordinators is studied which considers the natural gas storage systems and in the absence of renewable energy resources. The authors in [15] propose a simultaneous optimization problem of natural gas and electricity systems with regard to renewable energies and P2G technology. In [16], an interval-based robust chance-constrained (IBRCC) optimization model is proposed for assigning demand response programs to efficacious buses of the power system considering wind uncertainty and equipment malfunctions. Reference [17] presents an optimal scheduling formulation for commitment of natural gas generation units in a flexible ramp market.

Moreover, the literature on integrated electricity and gas networks includes research and studies in low and medium voltage distribution systems. In [18], the optimal scheduling of electrical and natural gas networks via energy hub platform is investigated which takes advantage of distribution network reconfiguration ability as a control variable to solve the optimization problem with the aid of a heuristic algorithm. Reference [19] presents an optimal scheduling model for interconnected energy networks with electrical and thermal loads under a game-theoretical approach. The study in [20] proposes an energy dispatch method for multi-carrier energy microgrids in the presence of electricity and gas carriers in both islanded and the grid-connected modes. The presented model is formulated as a mixed integer linear

programming (MILP) problem which minimizes the operation costs of the microgrid while improving the dispatch flexibility in supplying all types of demands. In [21], a multi-objective scheduling model is introduced for merged thermal-natural gas-electrical energy distribution systems to minimize the total operation costs and power imbalance in the electrical network. The research in [22] proposes a mixed integer dynamic approximation scheme to ease the real-time optimization of interconnected natural gas and electricity networks. The study in [23] investigates the optimal scheduling of MCESs considering wind resources and the uncertainties associated with load, electricity price, and renewable energy production to minimize the total operation costs using a fuzzy-logic approach. In [24], a multi-objective framework is established for operation of coordinated gas and electrical networks considering dynamic security constraints for both networks. The study in [25] presents a MILP scheduling formulation to mitigate the negative effects of renewable energy fluctuations on the operation of Multi-Energy Residential Systems. The uncertainties associated with renewable energy generation and demand variations are handled by chance constrained programming method (CCPM) based on Normal probability distribution functions (PDF). In [26], a deterministic MILP model is proposed for optimal scheduling of multi-energy microgrids considering technical and economic relations of both electricity and natural gas systems. The research in [27] proposes a rolling dispatch strategy and operational flexibility metric to quantify the ability of multi-energy microgrids equipped with CHP units to deal with uncertainties. The authors in [28] present a scenario-based MILP formulation for the optimal scheduling of multi-energy microgrid systems with integrated electrical and natural gas networks. In [29], a coordinated scheduling model is proposed for optimal operation of multi-energy microgrids in which the associated heterogeneous uncertainties are characterized using a new hybrid stochastic-interval method. The study in [30] proposes a mixed-integer second-order cone programming optimization programming (MISOCP) formulation for optimal operation of multi-carrier energy microgrids considering coupled electricity, heating, and natural gas networks. In [31], a deterministic scheduling model is proposed for an electric-thermal-gas coupling microgrid considering demand response programs and consumer satisfaction which is solved by the particle swarm optimization algorithm.

Table 1 presents a summary of the related research works and the proposed model. In this table, HSD and RD are the abbreviations for heat source devices and responsive loads, respectively. Looking upon the prior research works regarding this area reveals that power-to-heat and gas-to-heat concepts in the scheduling problem have not been fully explored. Moreover, the scheduling problem of MCESs based on the natural gas and electricity networks contemplates simple models for both electricity and gas networks. Besides, according to Table 1, the optimal scheduling of the integrated electricity and gas networks, considering thermal load, both

TABLE 1. Taxonomy of related research works.

Ref.	MRES devices					Power grid	Natural gas grid	Uncertainty modeling	Optimization problem
	CHP	HSD	ESS	RES	RD				
[18]	✓	-	-	✓	✓	✓	✓	-	MINLP
[19]	-	-	✓	✓	-	-	-	Scenario-based	MINLP
[20]	✓	-	✓	✓	-	-	-	-	MILP
[21]	✓	✓	✓	-	✓	✓	✓	-	NLP
[22]	-	-	✓	-	✓	✓	✓	Stochastic	MILP
[23]	✓	-	✓	✓	✓	-	-	Fuzzy Method	MILP
[25]	✓	✓	✓	✓	✓	✓	✓	CCPM-Normal PDFs	MISOCP
[26]	✓	✓	✓	✓	-	✓	-	-	MILP
[27]	✓	-	✓	✓	✓	-	-	-	MILP
[28]	✓	✓	✓	✓	✓	✓	✓	Scenario-based	MILP
[29]	✓	✓	✓	✓	✓	-	-	Stochastic-Interval	MILP
[30]	✓	✓	✓	-	-	✓	✓	Scenario-based	MISOCP
[31]	-	✓	✓	✓	✓	-	-	-	MILP
Proposed model	✓	✓	✓	✓	✓	✓	✓	CCPM-SAA	MINLP

electrical and gas storage devices, capturing the steady-state power and gas flow have been rarely investigated in the literature. Hence, this paper seeks to investigate the optimal day-ahead scheduling problem for a multi-carrier residential energy system (MRES). In this model, the residential system operator attempts to minimize the cost of purchased energy from both gas and electricity markets by optimally utilization of its equipment such as heating boilers, combined heat and power resources, and the power to gas storage units. The proposed model introduces a new penalty cost for the MRES scheduling problem based on air and environmental temperatures to consider the response of flexible thermal loads. The proposed model in this study is formulated as a Mixed Integer Non-Linear Programming (MINLP) optimization problem due to the non-linear AC power flow and gas flow equations in the electricity and natural gas distribution networks. This consideration presents a more realistic scheduling model compare to research works that have not considered these networks (e.g., [19], [20], [23]) or have used a simplified models (e.g., [22], [28]). Furthermore, the uncertainties associated with power production of wind and solar resources as well as load are modeled using the chance-constrained approach. Unlike, work in [25] which considers uncertainties follows the Normal probability distribution functions (PDFs), the proposed CCPM-based scheduling model in this paper utilizes the sample average approximation (SAA) method that can be used for all types of PDFs.

Hence, the main contributions of this paper can be summarized as follows:

- Proposing a more realistic scheduling model based on the non-linear AC power flow and gas flow in the electricity and natural gas distribution networks.
- Introducing a new penalty cost for the MRES scheduling problem based on air and environmental temperatures.
- Investigating the power-to-heat and gas-to-heat concepts in the scheduling problem by incorporating heat pumps, CHP units, and gas furnaces.
- Utilization of the chance-constrained approach for optimal operation of MRES to guarantee the confidence level of the system.

The rest of the paper is structured as follows. Section II describes the proposed MRES scheduling model including general model description and the mathematical formulation of objective function and constraints associated with existing resources and both electricity and gas networks. The uncertainty characterization and its modeling by chance-constrained approach along with relevant constraints are then discussed in section III. The simulation results are given in section IV. Finally, findings and concluding remarks are shown in section V.

## II. PROBLEM DESCRIPTION

### A. MULTI-CARRIER RESIDENTIAL ENERGY SYSTEM (MRES)

According to world statistics, electricity and gas consumption comprises nearly 85% of the whole energy utilization across the globe. Moreover, almost a moiety of the used gas and electrical energy is utilized for heating purposes in which the residential buildings make up the most important share of energy users. Regarding this point, the scheduling of gas networks must follow the importance of demands, thereby giving preference to residential customers.

The overall schematic of a MRES is shown in Fig. 1. As can be viewed, electricity and gas energy carriers supply the MRES by the means of power and gas distribution grids. In the MRES, the wind and solar renewable energy resources besides battery energy storage and purchased power from up-stream networks are the main sources of electric power supply, although some part of the electric power is provided by CHP unit. On the other hand, natural gas not only can supply residential gas load but also it can be used as a primary fuel for CHP and Gas Furnace (GF). The generated heat by the CHP, GF, and Heat Pump can be harnessed to secure the required thermal demands. The day-ahead operation of the MRES is managed by the residential energy system operator. Indeed, the operator has the responsibility to purchase the required amount of energy and deliver it to various residential customers by participating in the day-ahead energy market. However, since the capacity of residential energy systems is much smaller than other systems like industrial energy systems, we suppose that the MRES operator is a price taker market participant.

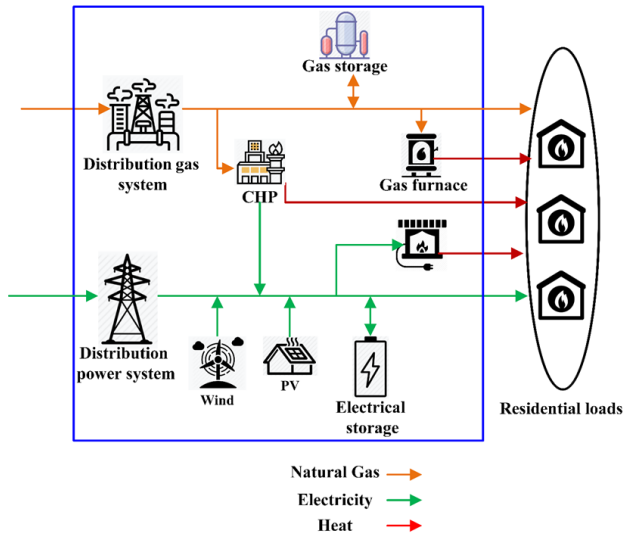


FIGURE 1. Schematic of an MRES.

**B. PROBLEM FORMULATION**

The proposed model aims to find the optimal scheduling of an MRES comprises from electrical and thermal loads, CHP units, gas furnace, heat pumps, electric and gas energy storage systems as well as considering both electricity and natural gas networks. The general structure of the proposed model is depicted in Fig. 2 which consists of input data, objective function, constraints and result output. The objective function of the model is to minimize the total scheduling costs including purchased electricity and gas as well as a penalty cost that will be imposed on the MRES operator in case of incapability of supplying demand. The penalty cost in the objective function is determined based on the rolling penalty function of real-time pricing. According to this approach, a penalty cost assigns for each residential building when the flexible loads of each building (e.g., heat sources) work outside the specified ranges. The constraints of this model mainly include electricity and gas network equations, CHP operational constraints, electricity and gas energy storage equations, operational constraints of heat pumps and gas furnaces devices. The constraints electricity network are based on AC-power flow equations and the non-linear equations are utilized for representing the flow of natural gas in the pipe lines of gas network. The mathematical representation of the objective function and constraints are provided in the following subsections.

**1) OBJECTIVE FUNCTION**

The optimal scheduling of MRES aims to reduce the costs of purchased energy from gas and electricity networks in addition to penalty costs that will be imposed on operators in case of incapability of supplying the demand. Thus, the objective function of the suggested model is formulated as follows:

$$\min \sum_{t \in \Psi_T} (\lambda_t^E P_t^{up} + \lambda_t^G q_t^{in}) \Delta t + \vartheta \quad (1)$$

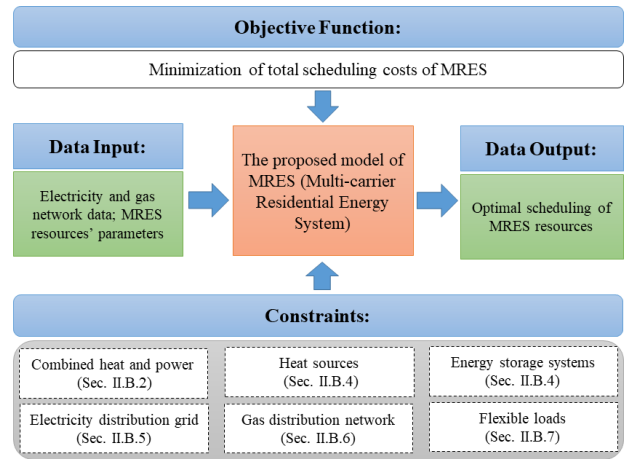


FIGURE 2. Structure of the proposed MRES scheduling model.

where the first and second terms represent the cost of purchasing electricity and gas from the upstream electricity and natural gas networks, respectively. The last term in (1) is referred to penalty cost if the MRES operator could not provide the required demand. The proposed MRES scheduling problem under different types of constraints is related to the performance of all its constituents, which will be discussed in the following subsections.

**2) CHP UNIT CONSTRAINTS**

One of the main components of an MRES is Combined Heat and Power (CHP) units. The main point in the scheduling of CHP units is the correlation between electrical and thermal power, which brings difficulty to find optimal operating point of these units. The operation region of a CHP unit is generally a function of a series of constraints such as minimum and maximum of incoming gas flow, the maximum generated heat, and so on. Figure 3 demonstrates the operation zone of a typical CHP unit [32].

According to Fig. 3, the operation region of a CHP unit located at electric node  $n \in \Psi_N$  and supplied from gas node  $i \in \Psi_I$  can be modeled by the following constraints  $\forall t \in \Psi_T$  as:

$$P_{n,t}^{CHP} - y_A - \frac{y_A - y_B}{x_A - x_B} \times (H_{n,t}^{CHP} - x_A) \leq 0 \quad (2)$$

$$P_{n,t}^{CHP} - y_B - \frac{y_B - y_C}{x_B - x_C} \times (H_{n,t}^{CHP} - x_B) \geq M (I_{n,t}^{CHP} - 1) \quad (3)$$

$$P_{n,t}^{CHP} - y_C - \frac{y_C - y_D}{x_C - x_D} \times (H_{n,t}^{CHP} - x_C) \geq M (I_{n,t}^{CHP} - 1) \quad (4)$$

$$P_{CHP,n,t}^{min} I_{n,t}^{CHP} \leq P_{n,t}^{CHP} \leq P_{CHP,n,t}^{max} \quad (5)$$

$$0 \leq H_{n,t}^{CHP} \leq H_{CHP}^{max} \quad (6)$$

$$q_{i,t}^{CHP} = P_{n,t}^{CHP} \times \eta_{CHP} \quad (7)$$

$$I_{n,t}^{CHP} \in \{0, 1\} \quad (8)$$

where (2)–(4) determines the power and heat production values of the CHP unit at node  $n$  based on the CHP operational region in Fig. 3. The power and heat value of CHP are limited by the minimum and maximum values as represented in (5) and (6). The consumed natural gas of the CHP unit at  $n^{th}$  electric node and the  $i^{th}$  gas node is calculated in (7) using the CHP efficiency.

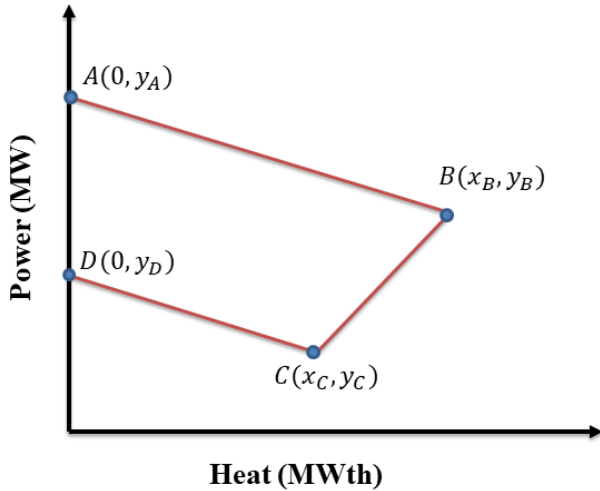


FIGURE 3. Performance area of a CHP unit or convex characteristic.

### 3) HEAT SOURCES OF RESIDENTIAL UNITS

Heating demand comprises a major share of energy consumption at home. Heat pumps and gas furnaces which uses electricity and natural gas respectively are considered the two heat sources of home energy systems. The generated heat by gas furnaces and heat pumps are calculated  $\forall h \in \Psi_H$  and  $t \in \Psi_T$  as follows [25]:

$$H_{h,t}^D = \gamma_h^{HP} \cdot \eta_h^{HP} \cdot P_{h,t}^{HP} + \gamma_h^{GF} \cdot \eta_h^{GF} \cdot Q_{h,t}^{GF} \quad (9)$$

where first term is related to heat pumps and the second one is related to gas furnaces. The presence of heating equipment in a house brings the internal temperature of the house to be kept in an appropriate range so as to provide the needs of customers. As stated in [33], to simulate the indoor temperature profile we use the linear thermal model  $\forall t \in \Psi_T$  as follows:

$$\frac{T_{h,t}^{in} - T_{h,t-\Delta t}^{in}}{\Delta t} = (c_h^{in})^{-1} \left[ H_{h,t}^D + H_{h,t}^{ext} + H_{h,t}^{in,rad} + \zeta_h^{is} (T_{h,t}^{sf} - T_{h,t}^{in}) + \zeta_h^{ie} (T_t^{ext} - T_{h,t}^{in}) \right]; \quad \forall h \in \Psi_H \quad (10)$$

$$\frac{T_{h,t}^{sf} - T_{h,t-\Delta t}^{sf}}{\Delta t} = (c_h^{sf})^{-1} \left[ H_{h,t}^{sf,rad} + \zeta_h^{is} (T_{h,t}^{in} - T_{h,t}^{sf}) + \zeta_h^{se} (T_t^{ext} - T_{h,t}^{sf}) \right]; \quad \forall h \in \Psi_H \quad (11)$$

$$\sum_{h \in \Psi_{CHP}^n} H_{h,t}^{ext} = H_{n,t}^{CHP} \quad (12)$$

$$T_h^{in,min} \leq T_{h,t}^{in} \leq T_h^{in,max}; \quad \forall h \in \Psi_H \quad (13)$$

where (10) and (11) calculate temperature inside and on the floor of the house, respectively. Equation (12) indicates that the total external heat of the house  $h$  connected to bus  $n$  is supplied by the CHP at that bus. Constraint (13) ensures that the average house temperature should be between the minimum and maximum possible temperatures.

### 4) ENERGY STORAGE SYSTEMS

In the presented model, apart from the electric energy storage systems, natural gas storage units are considered, too. According to the storage capacity, charge and discharge limitations, and also charge and discharge efficiency, the equations of electrical and natural gas storage units  $\forall t \in \Psi_T$  can be shown as follows [34].

$$\epsilon_{n,t}^{ES} - \epsilon_{n,t-1}^{ES} = \left( P_{n,t}^{ES,ch} \cdot \eta^{ES,ch} - \frac{P_{n,t}^{ES,dis}}{\eta^{ES,dis}} \right) \Delta t; \quad \forall n \in \Psi_N \quad (14)$$

$$\epsilon_{i,t}^{GS} - \epsilon_{i,t-1}^{GS} = \left( q_{i,t}^{GS,in} \cdot \eta^{GS,in} - \frac{q_{i,t}^{GS,out}}{\eta^{GS,out}} \right) \Delta t; \quad \forall i \in \Psi_I \quad (15)$$

$$\epsilon_n^{ES,min} \leq \epsilon_{n,t}^{ES} \leq \epsilon_n^{ES,max}; \quad \forall n \in \Psi_N \quad (16)$$

$$\epsilon_i^{GS,min} \leq \epsilon_{i,t}^{GS} \leq \epsilon_i^{GS,max}; \quad \forall i \in \Psi_I \quad (17)$$

$$0 \leq P_{n,t}^{ES,ch} \leq u_{n,t}^{ES} P_n^{ES,max}; \quad \forall n \in \Psi_N \quad (18)$$

$$0 \leq P_{n,t}^{ES,dis} \leq (1 - u_{n,t}^{ES}) P_n^{ES,max}; \quad \forall n \in \Psi_N \quad (19)$$

$$0 \leq q_{i,t}^{GS,in} \leq u_{i,t}^{GS} q_i^{GS,max}; \quad \forall i \in \Psi_I \quad (20)$$

$$0 \leq q_{i,t}^{GS,out} \leq (1 - u_{i,t}^{GS}) q_i^{GS,max}; \quad \forall i \in \Psi_I \quad (21)$$

$$u_{n,t}^{ES}, u_{i,t}^{GS} \in \{0, 1\}; \quad \forall n \in \Psi_N, \quad \forall i \in \Psi_I \quad (22)$$

Equations (14) and (15) show the amount of energy stored in the electric and gas storage units, which are a function of the amount of charge and discharge power as well as the heat transfer rates. Equation (16) and (17) limit the amount of stored energy in electric and gas storage between the minimum and maximum capacity, respectively. Constraints (18) and (19) indicate the minimum and maximum charge/discharge rates of electric storage, while equations (20) and (21) demonstrate the minimum and maximum input/output rates of gas storage units. Finally, constraint (22) emphasizes that both electric and gas storage can only be in one of charge or discharge operation status.

### 5) ELECTRICITY GRID CONSTRAINTS

The MRES operators supply the required electricity by participating in the day-ahead electricity market using the electricity distribution networks as well as renewable wind and solar sources, CHP units, and electrical energy storage systems. The mathematical representation of electricity grid constraints  $\forall n \in \Psi_N$  and  $t \in \Psi_T$  is as follows.

$$\begin{aligned} P_{n,t}^{RG} + P_{n,t}^{CHP} + P_{n,t}^{ES,dis} - P_{n,t}^{ES,ch} - P_{n,t}^D \\ = \sum_{h \in \Psi_H^n} P_{h,t}^{HP} + \sum_{m \in \Psi_N} P_{n-m,t}; \quad \forall n \neq n_S \end{aligned} \quad (23)$$

$$\sum_{m \in \Psi_N} Q_{n-m,t} + Q_{n,t}^D = 0; \quad \forall n \neq n_S \quad (24)$$

$$\begin{aligned} P_t^{up} + P_{n,t}^{RG} + P_{n,t}^{CHP} + P_{n,t}^{ES,dis} - P_{n,t}^{ES,ch} - P_{n,t}^D \\ = \sum_{h \in \Psi_H^n} P_{h,t}^{HP} + \sum_{m \in \Psi_N} P_{n-m,t}; \quad \forall n = n_S \end{aligned} \quad (25)$$

$$Q_t^{up} - Q_{n,t}^D = \sum_{m \in \Psi_N} Q_{n-m,t}; \quad \forall n = n_S \quad (26)$$

$$\begin{aligned} P_{n-m,t} = V_{n,t} V_{m,t} \left( G_{n-m} \cos(\theta_{n,t} - \theta_{m,t}) \right. \\ \left. + B_{n-m} \sin(\theta_{n,t} - \theta_{m,t}) \right) \end{aligned} \quad (27)$$

$$\begin{aligned} Q_{n-m,t} = V_{n,t} V_{m,t} \left( G_{n-m} \sin(\theta_{n,t} - \theta_{m,t}) \right. \\ \left. - B_{n-m} \cos(\theta_{n,t} - \theta_{m,t}) \right) \end{aligned} \quad (28)$$

$$P_t^{min} \leq P_t^{up} \leq P_t^{max} \quad (29)$$

$$Q_t^{min} \leq Q_t^{up} \leq Q_t^{max} \quad (30)$$

$$V_n^{min} \leq V_{n,t} \leq V_n^{max} \quad (31)$$

$$P_{n-m,t}^2 + Q_{n-m,t}^2 \leq (S_{n-m}^{max})^2 \quad (32)$$

Active and reactive power balances in all electric nodes of the MRESS excluding node  $n_S$  (point of common coupling (PCC)) are formulated using (23) and (24), respectively. Moreover, the active and reactive power balances for PCC are formulated as equations (25) and (26), respectively. In addition, equations (27) and (28) represent the active and reactive power flows through branches between nodes ( $m$ ) and ( $n$ ) according to the AC load flow model, consecutively. Constraints (29) and (30) confine the active and reactive exchanged power with the upstream grid at each time interval  $t$ . The range of voltage deviation at each node and the limitation of electric power passed by the line between nodes ( $m$ ) and ( $n$ ) are constrained in (31) and (32), respectively.

## 6) GAS NETWORK CONSTRAINTS

Natural gas distribution networks are considerably distinct from electricity networks. The flow of natural gas in gas pipes is a complex phenomenon that is a function of environmental conditions, pipe length, diameter, and material. Usually, the equations under stable conditions are used for modeling gas network. In [35], a comprehensive review of existing models is presented for gas networks. The amount of flowing gas through the gas pipe lines between gas nodes ( $i$ ) and ( $j$ ) is modeled using (33)  $\forall i, j \in \Psi_I$  and  $t \in \Psi_T$  as [36]:

$$q_{i-j,t} = \text{sgn}(\pi_{i,t}, \pi_{j,t}) \phi_{i-j} \sqrt{|\pi_{i,t}^2 - \pi_{j,t}^2|} \quad (33)$$

where in (33),  $\text{sgn}(\cdot)$  is a sign function that is defined using as:

$$\text{sgn}(\pi_{i,t}, \pi_{j,t}) = \begin{cases} 1, & \pi_{i,t} \geq \pi_{j,t} \\ -1, & \pi_{i,t} < \pi_{j,t} \end{cases} \quad (34)$$

where  $\phi_{i-j}$  is the pipeline constant that depends on temperature, length, diameter, friction, and gas composition [36].

Constraint (33) is a non-linear equation that includes sign function and root square, and it can be relaxed by defining binary variables as [25]:

$$q_{i-j,t}^2 \leq \phi_{i-j}^2 (\pi_{i,t}^2 - \pi_{j,t}^2) + M(1 - \mu_{i-j,t}) \quad (35)$$

$$q_{i-j,t}^2 \leq M \mu_{i-j,t} \quad (36)$$

$$\pi_{i,t} - \pi_{j,t} \leq M \mu_{i-j,t} \quad (37)$$

$$\mu_{i-j,t} + \mu_{j-i,t} = 1 \quad (38)$$

$$\mu_{i-j,t} \in \{0, 1\} \quad (39)$$

where (35) and (36) are the relaxed equations for (33), and (37)-(39) indicate gas flow direction using binary variables  $\mu_{i-j,t}$ . Accordingly, (40) and (41) illustrate the gas network nodal balance equations at the reference point ( $i = I^s$ ) and other nodes, respectively, and  $\forall t \in \Psi_T$ . These equations represent that entire gas flow injection is equivalent to total gas dissipated at each gas node. However, it should be noted that the compressor and pressure drop compensation are not considered in this study due to short pipelines of MRESS.

$$\begin{aligned} q_t^{in} + q_{i,t}^{GS,out} - q_{i,t}^{GS,in} - q_{i,t}^{CHP} - q_{i,t}^{GF} \\ = \sum_{j \in \Psi_I} q_{i-j,t}; \quad \forall i = i_S \end{aligned} \quad (40)$$

$$\begin{aligned} q_{i,t}^{GS,out} - q_{i,t}^{GS,in} - q_{i,t}^{CHP} - q_{i,t}^{GF} \\ = \sum_{j \in \Psi_I} q_{i-j,t}; \quad \forall i \in \Psi_I \& i \neq i_S \end{aligned} \quad (41)$$

In addition, unlike electric distribution systems, the natural gas production and consumption may not be equal at the same time. More specifically, natural gas pipelines will spontaneously perform as gas storage systems, which is generally denoted as linepack [12]. Constraint (42) calculates the linepack of pipeline  $i - j$ , i.e.,  $L_{i,j,t}$ ,  $\forall i, j \in \Psi_I, t \in \Psi_T$  as:

$$L_{i-j,t} = \frac{2}{3} \kappa_{i-j} \left( \pi_{i,t} + \pi_{j,t} - \frac{\pi_{i,t} \pi_{j,t}}{\pi_{i,t} + \pi_{j,t}} \right) \quad (42)$$

where  $L_{i-j,t}$  is constrained as [25]:

$$L_{i-j,t}^{min} \leq L_{i-j,t} \leq L_{i-j,t}^{max} \quad (43)$$

## 7) PENALTY-BASED FLEXIBLE LOAD

In some cases, the system operator has no choice but to interrupt the flexible load in order to prevent black-out in the system. This feature can be achieved from the penalty function which is considered in the objective function. The operator makes a contract with flexible consumers to reduce or interrupt their demand in peak times. When this program is running, the participating consumers should follow the instructions. If a consumer exceeds the contract, he should pay the penalty. This possibility was discussed in [37] which presents a rolling penalty function for real-time pricing of microgrids (see [37], [38] for more detail).

To this end, the heat sources in residential buildings can regulate their energy consumption as long as their properties do not violate existing constraints, thereby being considered

as flexible loads. Obviously, heating appliances should have the lowest energy consumption (i.e., the minimum temperature inside buildings) to minimize the costs, but on the other hand this matter could have negative impacts on the customers' satisfaction. In this condition, the operator must pay a penalty to compensate this dissatisfaction. The penalty paid to  $h^{th}$  house is shown as  $\vartheta_h$ , in which if the house temperature is less than the threshold value, the value of  $\vartheta_h$  will be greater than zero. It should be mentioned that the average value of minimum and maximum temperatures is considered as the threshold value. Hence, the penalty function could be rewritten as below:

$$\vartheta_h \geq \sum_{t \in \Psi_T} \left( \frac{T_{h,t}^{in,min} + T_{h,t}^{in,max}}{2} - T_{h,t}^{in} \right), \vartheta_h \geq 0; \forall h \in \Psi_H \quad (44)$$

$$\vartheta = \sum_{h \in \Psi_H} \lambda_h^{pen} \vartheta_h \quad (45)$$

where (44) calculates the amount of penalty to be paid to the  $h^{th}$  house, and (45) determines the total amount of operator payment to houses.

### III. CHANCE-CONSTRAINED BASED FORMULATION

The proposed MRES scheduling model is affected by the uncertainties caused by renewable energy sources and loads. To capture these uncertainties, stochastic programming based on chance-constrained approach is widely employed in the literature. Thus, the proposed scheduling is formulated using the chance-constrained method in this section to model the existing uncertainties. This technique is appropriate for solving stochastic optimization problems containing random variables in constraints and sometimes in the objective function. The mainstay of this approach is the fact that the constraints with random variables should be fulfilled with a distinctive probability or confidence level [39]. The details of this approach are provided in [39].

The hourly production of renewable resources and load demand impose the load balance equations. In other words, the uncertainties in power production of renewable energy sources and loads will affect directly the injected active power at each bus of the electricity distribution network and also the constraints of AC power flow, i.e., equations (29)-(32). Therefore, these constraints are reformulated using the chance-constrained method discussed in [39] is extended to capture the these uncertainties to satisfy a predefined confidence-level  $\forall n, m \in \Psi_N$  and  $t \in \Psi_T$  as follows.

$$prob\{P_t^{min} \leq P_t^{up} \leq P_t^{max}\} \geq 1 - \alpha_P \quad (46)$$

$$prob\{Q_t^{min} \leq Q_t^{up} \leq Q_t^{max}\} \geq 1 - \alpha_Q \quad (47)$$

$$prob\{(V_n^{min})^2 \leq (V_{n,t})^2 \leq (V_n^{max})^2\} \geq 1 - \alpha_V \quad (48)$$

$$prob\{(P_{n-m,t})^2 + (Q_{n-m,t})^2 \leq (S_{n-m}^{max})^2\} \geq 1 - \alpha_S \quad (49)$$

The above constraints are generally difficult to solve due to the need for calculating inverse cumulative distribution

function of uncertainties [39]. As a result, many different approximation methods were introduced in which among them the sample average approximation (SAA) method is the most commonly used one [40]. Indeed, the idea of SAA is to approximate the actual distribution of random variables with a set of sample scenarios and add binary variables to indicate whether the constraints are satisfied or not [40]. Accordingly, constraints (46)-(49) are reformulated as follows [40]:

$$P_{t,k}^{up} - z_k^P M \leq P_t^{max}; \forall t \in \Psi_T, k = 1, 2, \dots, N \quad (50)$$

$$P_{t,k}^{up} + y_k^P M \geq P_t^{min}; \forall t \in \Psi_T, k = 1, 2, \dots, N \quad (51)$$

$$\frac{1}{N} \sum_{k=1}^N z_k^P + y_k^P \leq \alpha_P \quad (52)$$

$$Q_{t,k}^{up} - z_k^Q M \leq Q_t^{max}; \forall t \in \Psi_T, k = 1, 2, \dots, N \quad (53)$$

$$Q_{t,k}^{up} + y_k^Q M \geq Q_t^{min}; \forall t \in \Psi_T, k = 1, 2, \dots, N \quad (54)$$

$$\frac{1}{N} \sum_{k=1}^N z_k^Q + y_k^Q \leq \alpha_Q \quad (55)$$

$$(V_{n,t,k})^2 - z_k^V M \leq (V_n^{max})^2; \forall n \in \Psi_N, t \in \Psi_T, k = 1, 2, \dots, N \quad (56)$$

$$(V_{n,t,k})^2 + y_k^V M \leq (V_n^{min})^2; \forall n \in \Psi_N, t \in \Psi_T, k = 1, 2, \dots, N \quad (57)$$

$$\frac{1}{N} \sum_{k=1}^N z_k^V + y_k^V \leq \alpha_V \quad (58)$$

$$(P_{n-m,t,k})^2 + (Q_{n-m,t,k})^2 - z_k^S M \leq (S_{n-m}^{max})^2; \forall n, m \in \Psi_N, t \in \Psi_T, k = 1, 2, \dots, N \quad (59)$$

$$\frac{1}{N} \sum_{k=1}^N z_k^S \leq \alpha_S \quad (60)$$

$$z_k^P, y_k^P, z_k^Q, y_k^Q, z_k^V, y_k^V, z_k^S \in \{0, 1\} \quad (61)$$

where the binary variables in (61) denotes whether associated constraints in (46)-(49) are satisfied and  $M$  denotes a acceptably large constant. Note that constraints (23)-(28) should be updated to consider index  $k = 1, 2, \dots, N$ , as well. To obtain more details on SAA method for solving chance-constrained optimization models, we refer to [40], [41]. Therefore, the proposed scheduling model of MRES based on the chance-constrained approach is a MINLP optimization problem which can be solved using available solvers.

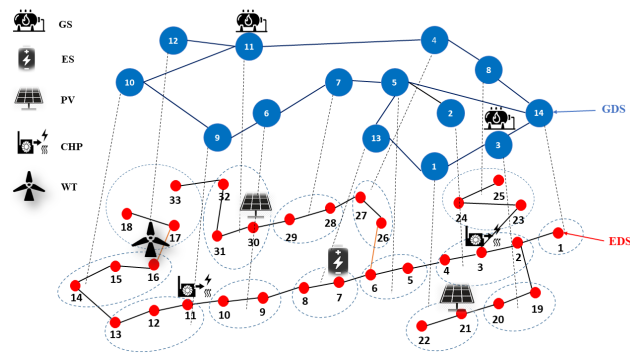
### IV. CASE STUDY

This Section presents the numerical results to investigate the performance of the proposed scheduling model. In this regard, after presenting the input data subsection IV-A, the numerical results and discussions are presented in subsections IV-B.



**A. DATA**

The proposed MRES scheduling model is applied on the IEEE 33-bus distribution network enhanced by the 14-node gas distribution network as shown in Fig. 4 [25]. The voltage level of the electrical grid is 10 kV and its capacity is 5 MVA. Each bus (node) of the electrical network has four residential buildings. Two CHP units are installed in nodes 3 and 11 of the electrical network, which are fed by nodes 2 and 9 of the gas network. The parameters related to different energy resources in both electric and gas network are shown in Table 2. Other important input parameters including gas network information, heat sources are given in Table 3.



**FIGURE 4.** The IEEE 33-bus EDS test system enhanced by 14-bus GDS.

**TABLE 2.** Resource information available in electrical and gas distribution networks (EDS and GDS).

Name	Location	Capacity	Nominal power
WT	EDS-16	1000 kVA	800 kW
PV1	EDS-21	800 kVA	600 kW
PV2	EDS-30	800 kVA	600 kW
Electric Storage	EDS-7	150 kWh	300 kW
Gas Storage 1	GDS-3	30 m <sup>3</sup>	10 m <sup>3</sup> /h
Gas Storage 2	GDS-11	30 m <sup>3</sup>	10 m <sup>3</sup> /h

**TABLE 3.** Required input data.

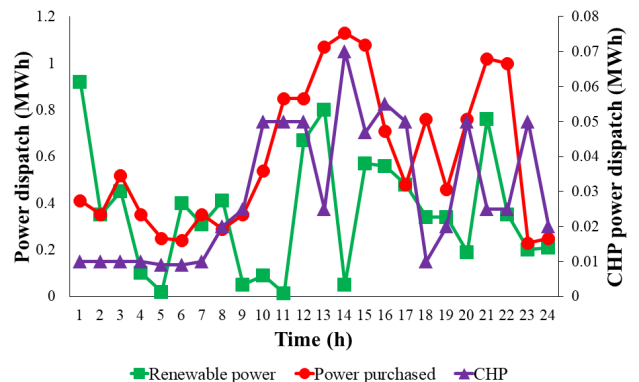
Parameter	Value	Parameter	Value
$T_{h,t}^{in,min}, T_{h,t}^{in,max}$ (°C)	20 and 24	$H_{h,t}^{D,max}$ (kW)	15.5
$E_{n,t}^{ES,min}, E_{n,t}^{ES,max}$ (%)	10 and 90	$H_{h,t}^{ext,max}$ (kW)	10
$E_{n,t}^{GS,min}, E_{n,t}^{GS,max}$ (%)	10 and 90	$\eta_h^{HP}, \eta_h^{GF}$ (%)	400 and 80
$V_n^{min}, V_n^{max}$ (p.u.)	0.9 and 1.1	$\eta^{ES,eh}, \eta^{ES,dis}$ (%)	90
		$\eta^{GS,in}, \eta^{GS,out}$ (%)	90

The proposed MINLP optimization problem is implemented in GAMS environment, is solved by the SBB solver, and validated by BARON. To calculate the uncertainty samples in SAA method, we assume that the random variables consist of deterministic forecast values plus random forecast errors. The forecast error of power production of renewable sources (PV and wind) and demand symmetrically follow the Standard Normal Distribution Function with zero Mean and Standard Deviation of 0.5 for PV and wind units, and 0.2 for the load. It should be noted that the load changes in each node are independent of the other nodes.

**B. NUMERICAL RESULTS AND ANALYSIS**

The proposed method is implemented with regard to the penalty coefficient  $\lambda_h^{pen}$  equal to 0.05 \$/h°C for the test system. Also, simulation results are provided for the confidence level equal to 5%, i.e.,  $\alpha = \alpha_P = \alpha_Q = \alpha_V = \alpha_S = 0.05$ . The optimal power and heat energies schedules are summarized in Table 4. According to Table 4, CHP units produce 0.71 MWh power to supply electrical loads. As previously discussed, the produced heat and electricity by CHP are coupled. Hence, as CHP generates electricity, its value of heating energy is 0.45 MWth which is utilized to supply heating loads. The heat pump and gas furnace produce 7.43 and 8.16 MWth, respectively, to meet the remaining thermal loads. Also, the system operator purchases 14.06 MWh electricity from the market. Moreover, according to Table 4, the total operation cost equals 3028.09 \$ where includes \$ 12.20 penalty cost, \$ 2562.31 for purchasing power from upstream electricity network, and \$ 453.58 for purchasing gas.

We also provide the optimal daily scheduling results of the electricity and heat resources as shown in Fig. 5 and Fig. 6. According to Fig. 5, CHP units operate with the maximum capacity during hours 10-22 (except hours 16 and 17). Also, the power purchased from the upstream network is increased at peak hours. Moreover, CHP units generate thermal energy based on the feasible region related to the power production. Also, during off-peak hours for electricity carriers, when CHP operates with the minimum capacity, the CHP’s set point is adjusted in the maximum heating capacity. Heat pump and gas furnace are committed during first and last hours of the day as can be seen from Fig. 6.



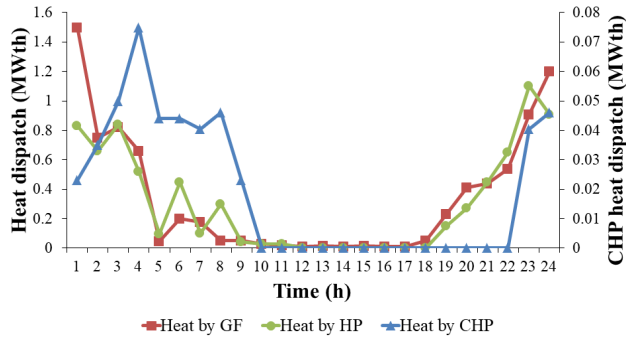
**FIGURE 5.** Optimal daily power dispatch of resources.

Furthermore, the proposed formulation is solved for the different values of the confidence coefficient ( $\alpha$ ). Figure 7 illustrates the cost of the whole system based on the  $1 - \alpha$  criteria for different amounts of confidence coefficients.

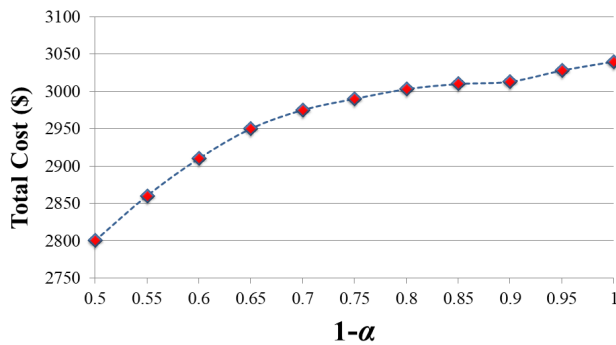
As can be seen from Fig. 7, the output power of the renewable energy sources that is injected to the network will be over conservative by increasing the value of  $1 - \alpha$ . As a result, the MRES operator has to make up for this conservatism by buying more energy from the market, and hence the daily

**TABLE 4.** The results of daily costs and the value of electrical and thermal power generated and purchased for  $\lambda_h^{pen} = 0.05\$/h^\circ C$ .

Daily operation cost (\$)			Power schedule (MWh)			Heat schedule (MWth)		
Penalty	EDS cost	GDS cost	Renewable resources	CHP units	Upstream network	CHP units	HP devices	GF systems
12.20	2562.31	453.58	8.50	0.71	14.06	0.45	7.43	8.16
<b>Total Cost: 3028.09 \$</b>								



**FIGURE 6.** Optimal daily heat dispatch of resources.



**FIGURE 7.** Price changes in terms of reliability.

cost will be much higher. By decreasing the value of  $\alpha$  or in other words increasing the level of confidence, the system operator (SO) obviously will incur more costs, but instead the SO would be able to supplies its electrical and thermal loads with more reliability.

The most important point with regard to MRES planning is the effectiveness of the cost of the thermal load penalty ( $\lambda_h^{pen}$ ). The amount of penalty will greatly impact the scheduling and also the costs of the system. Table 5 compares the total system costs for different values of  $\lambda_h^{pen}$ . From the parameters pertinent to the internal temperature of the house, given in Table 3, the comfort threshold temperature of the home will be equal to  $(20+24)/2=22$  degrees (as mentioned, the threshold temperature is considered equal to the average minimum and maximum internal temperature). Observe from the Table 5, when  $\lambda_h^{pen}$  is zero, there is a significant difference between the mean and the threshold house temperature (20.97-22). Although in this case the constraint (11) is not violated, this strategy cannot be favorable to the customers. As  $\lambda_h^{pen}$  increases, the average temperature of the house increases and reaches the threshold value, which

**TABLE 5.** The obtained results for the various values of  $\lambda_h^{pen}$ .

$\lambda_h^{pen}$	Amount of penalty	Average inside temperature	Total cost (\$)
0.00	0.00	20.97	2930.00
0.01	30.65	21.28	2944.05
0.02	26.50	21.55	3021.66
0.05	12.20	21.97	3028.09
0.10	6.86	21.99	3167.42

means that more heat supply sources must be committed. In other words, the amount of purchased gas and electricity ought to increase for their operation, which results in a growth in the total system cost. As can be seen, the penalty cost decreases when  $\lambda_h^{pen}$  increases.

Moreover, to investigate the effectiveness of the developed non-linear MRES scheduling formulation, the results of the proposed model are compared with the following cases and are shown in Table 6.

**TABLE 6.** Simulation outcomes of the proposed model and Cases I and II.

	Proposed model	Case I	Case II
Daily operation cost (\$)	3028.09	2723.45	2864.78
EDS cost (\$)	2562.31	2303.67	2408.57
GDS cost (\$)	453.58	411.35	444.51
Penalty (\$)	12.20	8.43	11.70
Running time (sec)	278	21.2	76.1

- Case I: Without considering electricity and gas networks. In this case, the constraints associated with electricity and gas distribution networks are relaxed in line with existing literature (e.g., [19], [20], [23], [27], [29], [31]), which do not model these networks.
- Case II: The original MRES model that the electricity and gas networks are linearized using piecewise linearization methods described in [25], [42].

As illustrated in Table 6, the daily energy cost of the proposed model is higher than that of Cases I and II. Also, the purchased electricity and gas energy from upstream networks to supply demand as well as the penalty cost are higher when we use the non-linear formulations for both grids. However, the running time of Cases I and II are significantly lower than that of the proposed model. Thus, a trade-off between running time and accuracy of the model is required.

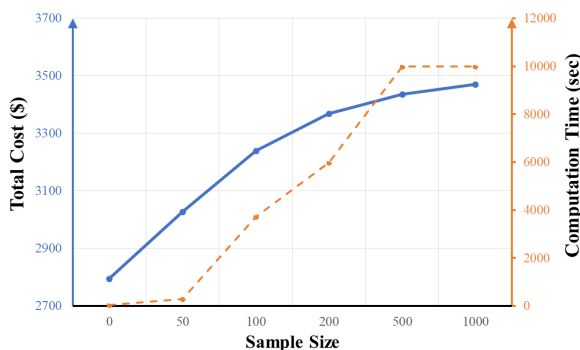
Finally, Table 7 shows the scale of the optimization problem for the deterministic and chance-constrained models of the presented test case. In this table,  $N$  is the number of samples for the SAA method. According to this table, the size of the optimization problem for the deterministic approach

**TABLE 7. Computational performance of the test case for the deterministic and chance-constrained models.**

	Deterministic	Chance-constrained
# of Constraints	19978+5568	19978+5568×N
# of Continuous Variables	20001+3168	20001+3168×N
# of Binary Variables	504	504+7×N

when the variation of random variables is not considered is significantly smaller than the chance-constrained model. The principal ground of this phenomenon is the number of scenarios to capture the uncertainties. Also, the chance-constrained ensures the reliable operation of the system which is very valuable for the operator.

Moreover, Fig. 8 illustrates the impact of the number of samples ( $N$ ) on the optimization performance and computation time. This figure confirms that increasing the sample size of SAA results in an increase in the total daily operation cost and computation time. In general, a larger number of scenarios can capture better the uncertainties in the proposed model. However, it increases the running time of the optimization problem. Hence, the number of scenarios must be chosen carefully to achieve a good trade-off between computation burden and accuracy.

**FIGURE 8. Impact of sample size on the total cost and computation time.**

## V. CONCLUSION

Interconnected multi-carrier electricity and natural gas-based energy systems have been given much more attention in recent years. The capability to convert and store energy from one type of carrier to another type by high-efficient technologies such as CHP, Heat Pumps, and electric and gas storage devices, not only help to integrate as many renewable sources as possible, also contributes to the system pliability. In this paper, day-ahead optimal scheduling for the multi-carrier residential energy systems equipped with CHP, GF, electric and gas storage devices, Heat Pumps, and wind and solar renewable sources was presented. The residential system operator as a price-taker entity attempts to minimize the cost of purchasing electricity and gas, in such a way the maximum heat demand of the system is provided. The proposed scheduling is modeled by the Chance-constrained programming method with regard to all governing constraints related

to the electrical and gas networks, and also the uncertainties stem from wind and solar sources, as well as load behavior. The simulation results have been discussed for the various amount of confidence factor. The outcomes demonstrated that the more the coefficient confidence with respect to supplying demand is, the higher price of purchasing electricity and gas will be.

The proposed MRES scheduling approach can be extended to consider an unbalanced distribution network and multiple contingencies such as line outages or load interruptions as well as equipment failures due to natural hazards. The model can also be adapted to include several demand response programs associated with both electric and thermal loads. These topics are left for future work.

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