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Scaling analysis of water retention curves: a multi-fractal approach

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Abstract

Water retention curve (WRC) is analyzed by means of the fractal geometry approach. Three models accounting for the fractal distribution of either the pore and solid phase of unsaturated porous media have been considered. By using data collected during a field scale internal drainage, we determine the functional relationship between the WRC, and the fractal dimension(s). In particular, it is shown that the fractal scaling of the WRC is feasible provided that a large enough set of measurements at the lowest water contents is available.

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Keywords: Scaling, fractal analysis, water retention curve.

1. Introduction and problem statement

The fractal geometry approach relating the water distribution in soils to the medium structure has been recently shown to be a useful tool to characterize soil water retention [1]. Indeed, the WRC can be determined consistently with the model of Brooks and Corey [2], and Campbell [3]. In the fractal approach to the WRC, the unsaturated porous medium is regarded as a Menger-sponge [4] with water filling a portion of it [5]. The main result is that properties of a porous medium (such as grain/pore size distribution, aggregates, etc) are described by a power law whose exponent depends upon the so called *fractal dimension* D_f . This power-law model relates, by a suitable variation of the Menger sponge geometry, the pore and solid phase of the porous medium to the volume as follows [6], [7]:

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$$\theta(h) = \theta_s - \frac{p}{p+s} \left[1 - \left(\frac{h}{h_{\min}} \right)^{D_f-3} \right] \quad (1)$$

where θ_s is the saturated water content, h the hydraulic head, h_{\min} the air-entry value, p and s the pore and solid phase fractions, respectively. Thus, by setting [8]:

$$A = \frac{p}{p+s}, \quad (2)$$

equation (1) is written as [6]:

$$\theta(h) = (\theta_s - A) + A \left(\frac{h}{h_{\min}} \right)^{D_f-3}. \quad (3)$$

Other Authors proposed different models by regarding the porous structure as a two-phase fractal object. Since these models will be also considered for comparison purposes, it is useful to briefly review them, herein. Thus, the first model is from Tyler and Wheatcraft [9], i.e.

$$\theta(h) = \theta_s \left(\frac{h}{h_{\min}} \right)^{D_f-3}, \quad (4)$$

whereas the second one is [10]:

$$\theta(h) = \theta_s - 1 + \left(\frac{h}{h_{\min}} \right)^{D_f-3}. \quad (5)$$

Models (4)-(5) can be shown (for details, see Fallico et al., [1]) to be particular cases of (3). Here, we focus on the fractal behavior of WRCs in the range of the low water contents, an aspect that in the past did not lead to definite conclusions.

2. Results

The estimate of the fractal dimension is achieved by optimizing (3)-(5) against measurements. Here, we refer to data collected during a field scale internal drainage. The experiment is described into details by Severino et al. [11]. For the sake of completeness, it is reviewed briefly in the sequel.

The field, located at the Ponticelli-site (Naples, Italy), is a sandy-soil. The soil texture in the upper 1m was characterized in detail by sampling at 0.20 m increments in several (randomly selected) locations. The main feature is that the soil is macroscopically homogeneous up to 0.80 m, with a layer of finer textured (loamy) soil at 0.80÷1.00 m. The soil resulted structureless in the sand component, and sub-angular blocky in the finer textured component. The plot (8 m width by 50 m long), equipped by a sprinkler irrigation-system, was set-up under a greenhouse. The sprinkler irrigation system supplied a flux of 10mm/day (with a uniformity-coefficient equal to 87%). The field was regularly irrigated for nine weeks. Along a transect mercury-water manometer-type tensiometers were installed at several depths (i.e. $z = 30, 60,$ and 90 cm) to measure the pressure head. At the same depth water contents were detected by means of TDR probes. After reaching steady state conditions irrigation was stopped, and in situ measurements of the pairs (h, θ) were taken for the whole duration of the drainage. In this way, WRCs at 4x50 locations were available. Here we shall limit to consider WRCs data collected in upper most soil

layer. A full analysis of all data is beyond the scope of the present paper, and it is deferred to future studies.

The models (3)-(5) can be cast in a compact form as follows:

$$\theta = \theta(A, h / h_{\min}, \theta_s, D_f) \tag{6}$$

where the constant A is dropped out for models (4)-(5). The calibration of the fractal dimension D_f was achieved by means of the linear scaling: $\log \theta^* - \log(h/h_{\min})$, in which $\log \theta^*$ assumes respectively the following expressions:

$$\log \theta^* = \log \frac{\theta + A - \theta_s}{A} \tag{7}$$

for the equation of Bird et al. [6];

$$\log \theta^* = \log \frac{\theta}{\theta_s} \tag{8}$$

for the equation of Tyler and Wheatcraft [9];

$$\log \theta^* = \log(\theta + 1 - \theta_s) \tag{9}$$

for the equation of Rieu and Sposito [10].

3. Discussion

The estimate of D_f was achieved in both the large and low water contents range, so that the whole range of the measured values was used. The main statistical parameters of the values of D_f are summarized in the Table 1. For illustration purposes, in the Figure 1 we have shown a typical fractal analysis performed by the model of Tyler & Wheatcraft [9]. It is clearly seen that the soil exhibits a bimodal fractal behavior represented by the slope of the two branches of the WRC.

For each location the cut-off limit, dividing the first from the second fractal range, was determined. In particular, for all the examined locations these cut-off values can be assumed equal to 0.01-50 cm for the first and 50-12000 cm for the second h -range. The results have clearly highlighted that the fractal behavior of the WRC is practically determined by h -values close to the saturation, being there $D_f \approx 3$, whereas within the second range (the one pertaining to the smaller water contents) the fractal dimension attains smaller values. Such a finding supports the previous analysis by Fallico et al. [1]. However, these latter could not reach definite conclusions due to the limited data-set which they have used

	TW Model		RS Model		PSF Model ($A = 0.60$)	
	1° Range	2° Range	1° Range	2° Range	1° Range	2° Range
	D_f	D_f	D_f	D_f	D_f	D_f
Min	2,981	2,689	2,990	2,906	2,982	2,765
Max	2,999	2,765	3,000	2,924	2,999	2,838
Mean	2,994	2,728	2,997	2,913	2,995	2,805
Var	1,85E-05	3,98E-04	4,85E-06	2,87E-05	1,58E-05	3,86E-04
SD	0,0043	0,0199	0,0022	0,0054	0,0040	0,0196
SE	0,0010	0,0045	0,0005	0,0012	0,0009	0,0044

Table 1. An example of a table.

4. Conclusions

We have carried out a fractal analysis of the WRC based on three (widely adopted) models. The assessment of the fractal dimension D_f was obtained by matching against real data measured during a field-scale drainage process. For all the experimental WRCs two characteristic fractal scaling have been clearly detected. It has been shown that the first (and more important) range is the one close to the saturation for which we have found that $D_f \approx 3$. In the remaining range the fractal dimension is lesser than 3 since it corresponds to pores that are practically empty. Finally, it is likely to assume that the reliability of such conclusions grows as one increases the number of measurements at the lowest water contents. This important issue is one topic of an ongoing research project.

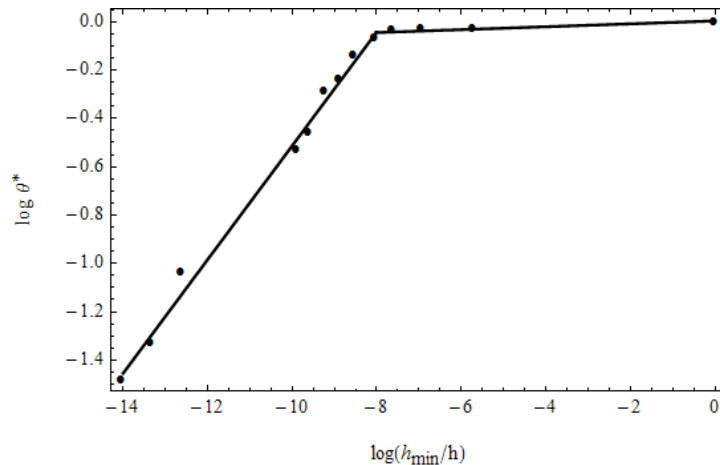


Fig. 1. Fractal scaling of the WRC for the location P24 (Ponticelli-Italy) obtained by the Tyler and Wheatcraft [9] model.

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References

- [1] Fallico C, Tarquis AM, De Bartolo S, Veltri M. Scaling analysis of water retention curve for unsaturated sandy-loam soils by using fractal geometry. *European Journal of Soil Science* 2010; **61**: 425-436.
- [2] Brooks RH, Corey AT. Hydraulic properties of porous media. *Hydrology Paper* n° 3, 1964; Colorado State University, Fort Collins.
- [3] Campbell GS. A simple method for determining unsaturated conductivity from moisture retention data. *Soil Science* 1974; **117**: 311-314.
- [4] Mandelbrot BB. *The fractal geometry of nature*. Freeman; 1982.
- [5] Pfeifer P, Avnir D, Farin D. Scaling behaviour of surface irregularity in the molecular domain: From adsorption studies to fractal catalysis. *Journal of Statistical Physics* 1984; **36**: 699-716.
- [6] Bird N, Perrier E, Rieu M. The water retention function for a model of soil structure with pore and solid fractal distributions. *European Journal of Soil Science* 2000; **55**: 55-63.
- [7] Bird N, Perrier E. The Pore-Solid Fractal model of soil density scaling. *European Journal of Soil Science* 2003; **54**: 467-476.

- [8] Wang K, Zhang R, Wang F. Testing the pore-solid fractal model for the soil water retention function. *Soil Science Society of America Journal* 2005; **69**: 776–782.
- [9] Tyler SW, Wheatcraft SW. Fractal processes in soil water retention. *Water Resources Research* 1990; **26**: 1047-1054.
- [10] Rieu M, Sposito G. Fractal fragmentation, soil porosity and soil water properties: I. Theory. *Soil Science Society of America Journal* 1991; **55**: 1231-1238.
- [11] Severino G, Comegna A, Coppola A, Sommella A, Santini A. Stochastic analysis of a field-scale unsaturated transport experiment. *Advances in Water Resources* 2010; **33**: 1188-1198.