Letter

Effective electroweak Hamiltonian in the gradient-flow formalism

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The effective electroweak Hamiltonian in the gradient-flow formalism is constructed for the current-current operators through next-to-next-to-leading-order QCD. The results are presented for two common choices of the operator basis. This allows for a consistent matching of perturbatively evaluated Wilson coefficients and nonperturbative matrix elements evaluated by lattice simulations on the basis of the gradient-flow formalism.

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I. INTRODUCTION

The gradient-flow formalism (GFF) [1] offers a promising solution to the matching of perturbative and nonperturbative calculations. A potential application is flavor physics, where nonperturbative matrix elements are typically evaluated using lattice regularization, while the Wilson coefficients are calculated perturbatively in dimensional regularization. The idea is to express the regular higher-dimensional operators of the effective electroweak Hamiltonian in terms of ultraviolet (UV)-finite flowed operators. The matching between the regular and the flowed operators is perturbative and can be absorbed into flow-time-dependent Wilson coefficients. The application of this approach to the energy-momentum tensor through next-to-next-to-leading-order (NNLO) QCD [2–4] has already been shown to give competitive results; see, e.g., Refs. [5–7]. More recently, the matching matrix has also been calculated for the quark dipole operators at next-to-leading-order (NLO) QCD [8,9] and for the hadronic vacuum polarization through NNLO QCD [10].

In Ref. [11], the matching matrix for the current-current operators of the effective electroweak Hamiltonian has been calculated at NLO QCD in the $\overline{\mathrm{DR}}$ scheme. Here, we present the NNLO expression for this quantity in the basis defined in Ref. [12] which allows us to adopt the $\overline{\mathrm{MS}}$ scheme with a fully anticommuting γ_5. We also provide the results for the nonmixing basis, though. The perturbative input for a consistent first-principles calculation of K- or B-mixing parameters on the basis of the GFF is thus available. Once the corresponding lattice input exists, it will be interesting to see how the GFF approach applied to flavor physics compares to results obtained with conventional approaches (see Ref. [13] for an overview).

II. OPERATOR BASIS

The effective electroweak Hamiltonian can be written schematically as

$$\mathcal{H}_{\mathrm{eff}} = -\frac{4G_{\mathrm{F}}}{\sqrt{2}} V_{\mathrm{CKM}} \sum_n C_n \mathcal{O}_n, \tag{1}$$

where G_{F} denotes the Fermi constant, V_{CKM} comprises the relevant elements of the Cabbibo-Kobayashi-Maskawa (CKM) matrix, and C_n are the Wilson coefficients. In this work, we focus on the current-current operators and choose

$$\begin{aligned} \mathcal{O}_1 &= -(\bar\psi_1 \gamma_\mu^{\mathrm{L}} T^a \psi_2)(\bar\psi_3 \gamma_\mu^{\mathrm{L}} T^a \psi_4), \\ \mathcal{O}_2 &= (\bar\psi_1 \gamma_\mu^{\mathrm{L}} \psi_2)(\bar\psi_3 \gamma_\mu^{\mathrm{L}} \psi_4) \end{aligned} \tag{2}$$

as our operator basis [12], where we adopt the Euclidean metric and use the short-hand notation

$$\gamma_\mu^{\mathrm{L}} = \gamma_\mu \frac{1-\gamma_5}{2}. \tag{3}$$

Our convention for the color generators is

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$$[T^a, T^b] = f^{abc}T^c, \quad \mathrm{Tr}(T^aT^b) = -T_\mathrm{R}\delta^{ab}, \tag{4}$$

with f^{abc} real and totally antisymmetric. Working in dimensional regularization with $D = 4 - 2\epsilon$, loop corrections lead to contributions which are not proportional to the operators in Eq. (2). They have to be attributed to so-called evanescent operators which vanish for $D = 4$ but mix with the physical operators at higher orders in perturbation theory [14]. Following Ref. [12], we choose

$$\begin{aligned}
\mathcal{O}_1^{(1)} &= -(\bar\psi_1\gamma^\mathrm{L}_{\mu\nu\rho}T^a\psi_2)(\bar\psi_3\gamma^\mathrm{L}_{\mu\nu\rho}T^a\psi_4) - 16\mathcal{O}_1,\\
\mathcal{O}_2^{(1)} &= (\bar\psi_1\gamma^\mathrm{L}_{\mu\nu\rho}\psi_2)(\bar\psi_3\gamma^\mathrm{L}_{\mu\nu\rho}\psi_4) - 16\mathcal{O}_2,\\
\mathcal{O}_1^{(2)} &= -(\bar\psi_1\gamma^\mathrm{L}_{\mu\nu\rho\sigma\tau}T^a\psi_2)(\bar\psi_3\gamma^\mathrm{L}_{\mu\nu\rho\sigma\tau}T^a\psi_4) - 20\mathcal{O}_1^{(1)} - 256\mathcal{O}_1,\\
\mathcal{O}_2^{(2)} &= (\bar\psi_1\gamma^\mathrm{L}_{\mu\nu\rho\sigma\tau}\psi_2)(\bar\psi_3\gamma^\mathrm{L}_{\mu\nu\rho\sigma\tau}\psi_4) - 20\mathcal{O}_2^{(1)} - 256\mathcal{O}_2
\end{aligned} \tag{5}$$

as evanescent operators, where $\gamma^\mathrm{L}_{\rho\mu_1\ldots\mu_n} \equiv \gamma^\mathrm{L}_\rho\gamma_{\mu_1}\ldots\gamma_{\mu_n}$. We will refer to the basis defined by Eqs. (2) and (5) as the Chetyrkin-Misiak-Münz (CMM) basis in what follows.

III. FLOWED OPERATORS

In the GFF, one defines flowed gluon and quark fields $B_\mu^a = B_\mu^a(t)$ and $\chi = \chi(t)$ as solutions of the flow equations [1,15]

$$\begin{aligned}
\partial_t B_\mu^a &= \mathcal{D}_\nu^{ab}G_{\nu\mu}^b + \kappa\mathcal{D}_\mu^{ab}\partial_\nu B_\nu^b,\\
\partial_t\chi &= \Delta\chi - \kappa\partial_\mu B_\mu^a T^a\chi,\\
\partial_t\bar\chi &= \bar\chi\overleftarrow{\Delta} + \kappa\bar\chi\partial_\mu B_\mu^a T^a,
\end{aligned} \tag{6}$$

with the initial conditions

$$B_\mu^a(t=0) = A_\mu^a, \quad \chi(t=0) = \psi, \tag{7}$$

where A_μ^a and ψ are the regular gluon and quark fields, respectively, and

$$\begin{aligned}
\mathcal{D}_\mu^{ab} &= \delta^{ab}\partial_\mu - f^{abc}B_\mu^c, \quad \Delta = (\partial_\mu + B_\mu^aT^a)^2,\\
G_{\mu\nu}^a &= \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + f^{abc}B_\mu^bB_\nu^c.
\end{aligned} \tag{8}$$

The parameter κ is arbitrary and drops out of physical quantities; we will set $\kappa = 1$ in our calculation, because this choice reduces the size of the intermediate algebraic expressions.

Our practical implementation of the GFF in perturbation theory follows the strategy developed in Ref. [16] and further detailed in Ref. [17]. On the one hand, it amounts to generalizing the regular QCD Feynman rules by adding flow-time-dependent exponentials to the propagators. The flow equations [Eq. (6)] are taken into account with the help of Lagrange multiplier fields which are represented by so-called "flow lines" in the Feynman diagrams. They couple to the (flowed) quark and gluon fields at "flowed vertices," which involve integrations over flow-time parameters.

While the flowed gluon field B_μ^a does not require renormalization [1,16], the flowed quark fields χ have to be renormalized [15]. The nonminimal renormalization constant $\mathring{Z}_\chi$ for the flowed quark fields χ is defined by the all-order condition [3]

$$\begin{aligned}
\mathring{Z}_\chi\langle\bar\chi\overleftrightarrow{\not{\mathcal{D}}}\chi\rangle_0\Big|_{m=0} &\equiv -\frac{2n_\mathrm{c}}{(4\pi t)^2},\\
\overleftrightarrow{\mathcal{D}}_\mu &= \partial_\mu - \overleftarrow{\partial}_\mu + 2B_\mu^aT^a,
\end{aligned} \tag{9}$$

where $\langle\cdot\rangle_0$ denotes the vacuum expectation value. The NNLO result for $\mathring{Z}_\chi$ can be found in Ref. [17].

The flowed operators are then defined by replacing the spinors ψ_i by renormalized flowed spinors $\mathring{Z}_\chi^{1/2}\chi_i$ in the regular operators, i.e.,

$$\begin{aligned}
\tilde{\mathcal{O}}_1 &= -\mathring{Z}_\chi^2(\bar\chi_1\gamma^\mathrm{L}_\mu T^a\chi_2)(\bar\chi_3\gamma^\mathrm{L}_\mu T^a\chi_4),\\
\tilde{\mathcal{O}}_2 &= \mathring{Z}_\chi^2(\bar\chi_1\gamma^\mathrm{L}_\mu\chi_2)(\bar\chi_3\gamma^\mathrm{L}_\mu\chi_4),
\end{aligned} \tag{10}$$

and analogously for the evanescent operators. Because of the damping character of the flow time $t > 0$, matrix elements of the flowed operators are UV finite after renormalization of the strong coupling and the quark masses. One can, thus, treat them in four space-time dimensions, which also means that flowed evanescent operators can be neglected. However, we prefer to keep them in our formalism, because it makes the equations more symmetric. Furthermore, the fact that they have to vanish provides a welcome consistency check on our results. The regular evanescent operators are still needed in our calculation, which will be described below.

IV. SMALL-FLOW-TIME EXPANSION

In the limit $t \to 0$, the flowed operators behave as [16]

$$\begin{pmatrix}\tilde{\mathcal{O}}(t)\\ \tilde{E}(t)\end{pmatrix} \asymp \zeta^\mathrm{B}(t)\begin{pmatrix}\mathcal{O}\\ E\end{pmatrix}, \tag{11}$$

where we use the notation

$$\begin{aligned}
\mathcal{O} &= (\mathcal{O}_1, \mathcal{O}_2)^\mathrm{T} \equiv (\mathcal{O}_1^{(0)}, \mathcal{O}_2^{(0)})^\mathrm{T},\\
E &= (\mathcal{O}_1^{(1)}, \mathcal{O}_2^{(1)}, \mathcal{O}_1^{(2)}, \mathcal{O}_2^{(2)})^\mathrm{T},
\end{aligned} \tag{12}$$

and analogously for the flowed operators. Here and in what follows, the superscript "B" marks a "bare" quantity which will undergo renormalization. The symbol $\asymp$ is used to indicate that terms of $O(t)$ are neglected. It will be convenient to adopt the block notation of Eq. (11) also for matrices. For example, for the renormalized matching matrix, we write

$$\zeta(t) = \begin{pmatrix} \zeta_{\mathcal{O}\mathcal{O}}(t) & \zeta_{\mathcal{O}E}(t) \\ \zeta_{E\mathcal{O}}(t) & \zeta_{EE}(t) \end{pmatrix}, \tag{13}$$

where the 2×2 submatrix $\zeta_{\mathcal{O}\mathcal{O}}$ concerns only the physical operators.

Since matrix elements of the bare operators are divergent, while those of flowed operators are finite, the bare matching matrix $\zeta^{\mathrm{B}}(t)$ is divergent as $D\to 4$. However, one may define renormalized operators whose matrix elements are finite:

$$\begin{pmatrix} \mathcal{O} \\ E \end{pmatrix}^{\mathrm{R}} = Z\begin{pmatrix} \mathcal{O} \\ E \end{pmatrix} \equiv \begin{pmatrix} Z_{\mathcal{O}\mathcal{O}} & Z_{\mathcal{O}E} \\ Z_{E\mathcal{O}} & Z_{EE} \end{pmatrix}\begin{pmatrix} \mathcal{O} \\ E \end{pmatrix}, \tag{14}$$

where Z is the corresponding renormalization matrix. It is common to define all its entries in the $\overline{\mathrm{MS}}$ scheme, except for the submatrix $Z_{E\mathcal{O}}$, whose finite part is chosen such that physical matrix elements $\langle\cdot\rangle$ of evanescent operators vanish to all orders in perturbation theory [14,18,19]:

$$\langle E^{\mathrm{R}}\rangle = Z_{E\mathcal{O}}\langle\mathcal{O}\rangle + Z_{EE}\langle E\rangle \stackrel{!}{=} O(\epsilon). \tag{15}$$

Inserting Eq. (14) into Eq. (11), it follows that

$$\zeta(t) = \zeta^{\mathrm{B}}(t)Z^{-1} = \begin{pmatrix} \zeta_{\mathcal{O}\mathcal{O}}(t) & \zeta_{\mathcal{O}E}(t) \\ \zeta_{E\mathcal{O}}(t) & \zeta_{EE}(t) \end{pmatrix} \tag{16}$$

is finite at $D=4$. Since $\langle\tilde{E}(t)\rangle = O(\epsilon)$, the renormalization condition in Eq. (15) is equivalent to

$$\zeta_{E\mathcal{O}}(t) = O(\epsilon). \tag{17}$$

V. CALCULATION OF THE MATCHING MATRIX

For the calculation of the matching matrix $\zeta(t)$, we use the method of projectors [20,21]. This means that we define a set of matrix elements

$$P_j^{(i)}[X] = \langle 0|X|i,j\rangle\Big|_{p=m=0}, \tag{18}$$

with $i\in\{0,1,2\}$ and $j\in\{1,2\}$, such that

$$P_j^{(i)}\left[\mathcal{O}_{j'}^{(i')}\right] = \delta_{ii'}\delta_{jj'}, \tag{19}$$

where we remind the reader of the unified notation for physical and evanescent operators defined in Eq. (12). In general, the projectors could also involve derivatives with respect to masses and/or external momenta, but this is not the case for the set of operators considered here. Since all external mass scales are set to zero in Eq. (18), it is sufficient to satisfy Eq. (19) at tree level, because all higher perturbative orders on the lhs vanish in dimensional regularization.

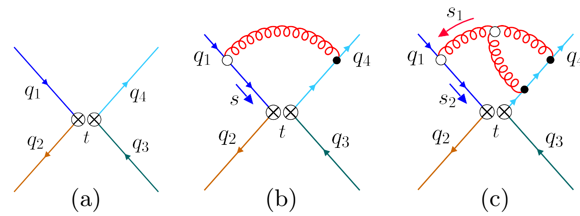

FIG. 1. Sample diagrams contributing to the determination of the matching matrix $\zeta(t)$ at LO, NLO, and NNLO QCD. The circles denote flowed vertices, lines with an arrow next to them denote flow lines, and the label next to the arrow is a flow-time integration variable (see Ref. [17] for details). The diagrams were produced with FeynGame [22].

The external states $|i,j\rangle$ are understood to project onto left-handed spinors only. Adopting an anticommuting γ_5 thus eliminates all γ_5's from the traces at any order in the calculation [12].

The bare matching matrix is obtained by applying the projectors to Eq. (11):

$$\zeta_{jj'}^{\mathrm{B},(ii')}(t) = P_{j'}^{(i')}\left[\tilde{\mathcal{O}}_j^{(i)}(t)\right], \tag{20}$$

where the index notation should be self-explanatory.[1] Because of the fact that we restrict ourselves to the case where all four quark flavors in the operator are different, the Feynman diagrams contributing to the rhs of this equation are obtained by dressing the generic tree-level diagram in Fig. 1(a) by virtual gluons and closed quark loops. Sample diagrams are shown in Figs. 1(b) and 1(c).

For the actual evaluation of the diagrams, we adopt the setup based on q2e/exp [23,24] described in Ref. [17]. Specifically, we generate the Feynman diagrams with QGRAF [25,26], apply the projectors, perform the traces, and simplify the algebraic expressions within FORM [27–29], and reduce the resulting Feynman integrals to master integrals with the help of Kira+FireFly [30–33]. The master integrals are the same as those found in Ref. [4].

VI. RESULTS

A. CMM basis

Performing the calculation and renormalization as described in the previous sections, we find for the physical components of the renormalized matching matrix through NNLO in QCD

[1]For the sake of clarity, let us point out that $\zeta_{jj'}^{(00)} \equiv (\zeta_{\mathcal{O}\mathcal{O}})_{jj'}$.

$$
\begin{aligned}
(\zeta^{-1})_{11}(t) &= 1 + a_s\left(4.212 + \frac{1}{2}L_{\mu t}\right) + a_s^2\left[22.72 - 0.7218n_\mathrm{f} + L_{\mu t}(16.45 - 0.7576n_\mathrm{f}) + L_{\mu t}^2\left(\frac{17}{16} - \frac{1}{24}n_\mathrm{f}\right)\right],\\
(\zeta^{-1})_{12}(t) &= a_s\left(-\frac{5}{6} - \frac{1}{3}L_{\mu t}\right) + a_s^2\left[-4.531 + 0.1576n_\mathrm{f} + L_{\mu t}\left(-3.133 + \frac{5}{54}n_\mathrm{f}\right) + L_{\mu t}^2\left(-\frac{13}{24} + \frac{1}{36}n_\mathrm{f}\right)\right],\\
(\zeta^{-1})_{21}(t) &= a_s\left(-\frac{15}{4} - \frac{3}{2}L_{\mu t}\right) + a_s^2\left[-23.20 + 0.7091n_\mathrm{f} + L_{\mu t}\left(-15.22 + \frac{5}{12}n_\mathrm{f}\right) + L_{\mu t}^2\left(-\frac{39}{16} + \frac{1}{8}n_\mathrm{f}\right)\right],\\
(\zeta^{-1})_{22}(t) &= 1 + 3.712a_s + a_s^2\left[19.47 - 0.4334n_\mathrm{f} + L_{\mu t}(11.75 - 0.6187n_\mathrm{f}) + \frac{1}{4}L_{\mu t}^2\right],
\end{aligned}
\tag{21}
$$

with $a_s = \alpha_s(\mu)/\pi$ and $L_{\mu t} = \ln 2\mu^2 t + \gamma_\mathrm{E}$, where α_s is the strong coupling renormalized in the $\overline{\mathrm{MS}}$ scheme with n_f quark flavors, μ the renormalization scale, and $\gamma_\mathrm{E} = 0.577\ldots$ Euler's constant. For the sake of compactness, we set $n_\mathrm{c} = 3$ and $T_\mathrm{R} = \frac{1}{2}$ and replaced transcendental coefficients by floating-point numbers. Analytical coefficients for a general SU(n_c) gauge group are included in the Supplemental Material [34].

Several observations support the correctness of this result. First of all, the literature expression for the renormalization matrix Z defined through Eqs. (14) and (15) [12,35,36] not only eliminates all UV divergences from the matching matrix, but also nullifies its $E\mathcal{O}$ component; see Eq. (17). Furthermore, we performed the calculation in R_ξ gauge and found the result to be independent of the gauge parameter ξ. Yet another check concerns the switch to a different basis as described in the following.

B. Nonmixing basis

It may be useful in physical applications to transform our result into the so-called nonmixing basis, defined such that the anomalous dimension matrix for the operators is diagonal. The physical operators in that basis read

$$
\mathcal{O}_\pm = \frac{1}{2}\left[(\bar\psi_1^\alpha\gamma_\mu^\mathrm{L}\psi_2^\alpha)(\bar\psi_3^\beta\gamma_\mu^\mathrm{L}\psi_4^\beta) \pm (\bar\psi_1^\alpha\gamma_\mu^\mathrm{L}\psi_2^\beta)(\bar\psi_3^\beta\gamma_\mu^\mathrm{L}\psi_4^\alpha)\right] \tag{22}
$$

with the color indices α and β. The definition of the evanescent operators as well as the transformation matrices with respect to the CMM basis are provided in Ref. [37] through NNLO.[2] We can easily evaluate the results in that basis by applying the corresponding transformation to the bare results for the projections obtained through Eq. (18) and then performing the renormalization in complete analogy to the calculation for the CMM basis. Alternatively, the transformation can be done at the level of the renormalized results by taking into account the required finite renormalization given in Ref. [37] to restore the renormalization scheme in the new operator basis [12]. The fact that both ways lead to the same result and that the physical matching matrix $\zeta(t)$ between the $\overline{\mathrm{MS}}$ renormalized and the flowed operators turns out to be diagonal in this basis is another strong check on our results. We find

$$
\begin{aligned}
\zeta_{++}^{-1}(t) &= 1 + a_s\left(2.796 - \frac{1}{2}L_{\mu t}\right) + a_s^2\left[14.15 - 0.1739n_\mathrm{f} + L_{\mu t}(6.509 - 0.4798n_\mathrm{f}) + L_{\mu t}^2\left(-\frac{9}{16} + \frac{1}{24}n_\mathrm{f}\right)\right],\\
\zeta_{--}^{-1}(t) &= 1 + a_s(5.546 + L_{\mu t}) + a_s^2\left[32.01 - 0.9524n_\mathrm{f} + L_{\mu t}(21.23 - 0.8965n_\mathrm{f}) + L_{\mu t}^2\left(\frac{15}{8} - \frac{1}{12}n_\mathrm{f}\right)\right],
\end{aligned}
\tag{23}
$$

where the same notation as in Eq. (21) is adopted.[3] Again, analytical results are provided in the Supplemental Material [34].[4]

We note in passing that the matching matrix also determines the small-t behavior of the flowed operators through the equation [10]

$$
t\partial_t\tilde{\mathcal{O}}(t) = \tilde\gamma(t)\tilde{\mathcal{O}}(t), \quad \tilde\gamma(t) = (t\partial_t\zeta(t))\zeta^{-1}(t). \tag{24}
$$

These equations hold in any basis, of course.

VII. THE EFFECTIVE HAMILTONIAN IN THE GRADIENT-FLOW FORMALISM

Inverting the small-flow-time expansion in Eq. (11), one can write the Hamiltonian as

[2]Note that the entry $\frac{8032}{75}$ in the matrix $\hat V$ in Eq. (B.5) of Ref. [37] [Eq. (A.8) in the arXiv version] should read $\frac{8032}{25}$.

[3]An immediate comparison of this result to the NLO expression of Ref. [11] is not possible, because the latter is obtained in the $\overline{\mathrm{DR}}$ scheme.

[4]Since the nonmixing basis in Ref. [37] was constructed for $n_\mathrm{c} = 3$, we also insert this value for ζ_{++}^{-1} and ζ_{--}^{-1} in the Supplemental Material [34] and, in addition, set $T_\mathrm{R} = \frac{1}{2}$. A nonmixing basis for general n_c could be easily constructed from our results, though.

$$\mathcal{H}_{\text{eff}} \asymp -\frac{4G_{\text{F}}}{\sqrt{2}} V_{\text{CKM}} \sum_n \tilde{C}_n(t) \tilde{\mathcal{O}}_n(t), \tag{25}$$

where the flowed Wilson coefficients are given by

$$\tilde{C}_n(t) = \sum_m C_m^{\text{R}} \zeta_{mn}^{-1}(t), \tag{26}$$

with $\zeta(t) \equiv \zeta_{\mathcal{O}\mathcal{O}}(t)$ the physical part of the matching matrix and $C_n^{\text{R}} = \sum_m C_m (Z^{-1})_{mn}$ the renormalized regular Wilson coefficients. It is important to evaluate C^{R} and $\zeta^{-1}(t)$ in the same renormalization scheme, including the treatment of γ_5 and the choice of (regular) evanescent operators. The flowed coefficients $\tilde{C}(t)$, on the other hand, are scheme and renormalization scale independent (up to higher orders in perturbation theory). Since also the flowed operators $\tilde{\mathcal{O}}(t)$ are scheme and renormalization scale independent, Eq. (25) allows one to combine perturbatively calculated Wilson coefficients with nonperturbative matrix elements without scheme transformation.

In order to avoid large logarithms, after matching the $C_m^{\text{R}} = C_m^{\text{R}}(M_W, \mu)$ to the Standard Model (SM) at $\mu \sim M_W$, they should be evolved down to $\mu \sim \sqrt{1/t}$ using the standard renormalization group equation [36,38], where t is sufficiently large to warrant small uncertainties in the lattice calculation. Alternatively, one may choose to perform the evolution to large t at the level of the flowed coefficients, using

$$t\partial_t \tilde{C}_m(t) = -\sum_n \tilde{C}_n(t) \tilde{\gamma}_{nm}(t) \tag{27}$$

with $\tilde{\gamma}(t)$ defined in Eq. (24). The compatibility of both approaches is left for future investigation.

For $|\Delta F| = 1$ processes, the Wilson coefficients C_m^{R} in the CMM basis for the SM can be found in Refs. [36,39] through NNLO. Thus, when neglecting penguin contributions, reexpanding the rhs of Eq. (26) through NNLO using the results for $\zeta^{-1}(t)$ above, directly gives the flowed Wilson coefficients to the same order. For $|\Delta F| = 2$ processes, the physical basis reduces to just one operator due to a Fierz identity. In this case, the SM Wilson coefficient is known through NLO [38], with two contributions for kaon mixing known through NNLO [40,41].

VIII. CONCLUSIONS AND OUTLOOK

We calculated the matching matrix of the current-current operators in the electroweak effective Hamiltonian to their flowed counterparts through NNLO QCD. We presented the results in the CMM and the nonmixing bases and performed a number of checks on their correctness. Our results can directly be applied to K- or B-meson mixing, for example. Their generalization, in particular, the inclusion of penguin operators, is work in progress. It remains to be seen how the GFF approach to flavor physics compares to conventional calculations.

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