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PRODUCTION PLANNING IN THE PULP AND PAPER INDUSTRY[1]

This paper examines the short term production planning problem encountered in the fine-paper industry. The paper proposes a tight mixed-integer programming formulation of the problem It is showed that real size problem instances can be solved with commercial solvers. Furthermore, we show that by adding some simple valid inequalities to the proposed formulation, major improvements to the solution time can be achieved.

Introduction

The pulp and paper industry is one of the most important industries of Canada in terms of contribution to its balance of trade. In 2001, it represented 3% of Canada's GDP (FPAC 2002). The expertise of the Canadian pulp and paper industry is well renowned. Over the years, the industry has been confronted with different market pressures and competitiveness is growing very strong. To be able to compete, companies need to streamline their operation costs. In this paper, we tackle short term production planning problems encountered in the fine-paper industry. In this industry, the production process can be decomposed in four main stages. The first stage (the chips mill) transforms logs into chips. The second stage (the pulp mill) transforms chips and chemicals into pulp. The third stage (the paper mill) transforms pulp into paper rolls. The paper mill is usually composed of a set of parallel paper machines and these machines are often the bottleneck of the production system. This is why production plans are usually defined in terms of this bottleneck. Finally, the last stage (conversion mill) converts paper rolls into smaller rolls or sheets which are demanded by external customers. *Figure 1* illustrates the material flows within an integrated pulp and paper mill. As can be seen, some production stages can be partially or completely bypassed through external provisioning of intermediate products.

Paper machines can run 24 hours a day during the whole year, but they can also be stopped (or slowed down) from time to time to adapt to low market demand or for maintenance purposes. A small number of *intermediate products* (IP) is manufactured by each parallel paper machine and the processing sequence of these products on the machine is fixed by engineering constraints. Also,

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a changeover time is required to change products on a paper machine, which means that capacity is lost when there is a production switch. In the succeeding stages, the intermediate products are transformed into a large number of *finished products* (FP). It is assumed that conversion stages are not capacity constrained. This is realistic in the fine-paper industry because it is always possible to subcontract part of the finishing operations if additional capacity is required. The planning challenge is to synchronize the material flow as it moves through the paper making and conversion stages, to meet customer demand and to minimize operations costs. The aim of the paper is to propose a mathematical programming model to support this planning process.

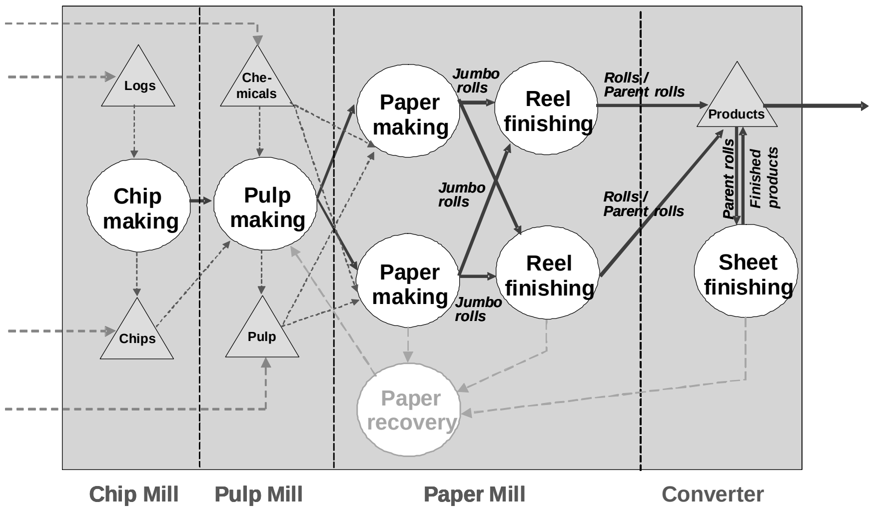

Figure 1: Processes and Material Flows in an Integrated Pulp and Paper Mill

The paper is organized as follows. In the next section, the problem is defined more precisely and the relevant literature on multi-item capacitated lot-sizing problems is reviewed. The mathematical model formulation is derived in section 3 and valid inequalities which can be added to the model to facilitate its solution are proposed in section 4. Finally, section 5 presents the results of our numerical tests.

Problem Definition

In this section, the problem studied is defined more precisely and the related literature is reviewed. The problem is essentially to plan the production and the inventory levels of multiple finished products and intermediate products over a finite planning horizon. Although the demand for products is partly planned and partly random, it is assumed, as is customary for ERP and APS systems, that it is deterministic and time-varying (dynamic). The demand is based on orders received and on forecasts, and it is assumes that the safety stocks required as protection against the randomness of demand are determined exogenously, prior to the solution of our problem. No inventory of intermediate products is kept, but the finished products can be stocked at the plant before they are shipped. In order to prepare adequate production plans, the relationships between the IP lot-sizes and the FP inventories and demands must be considered explicitly. Time periods represent days, shifts or other time buckets depending on production, inventory and customer service policies. The specific production context considered is illustrated in *Figure 2*.

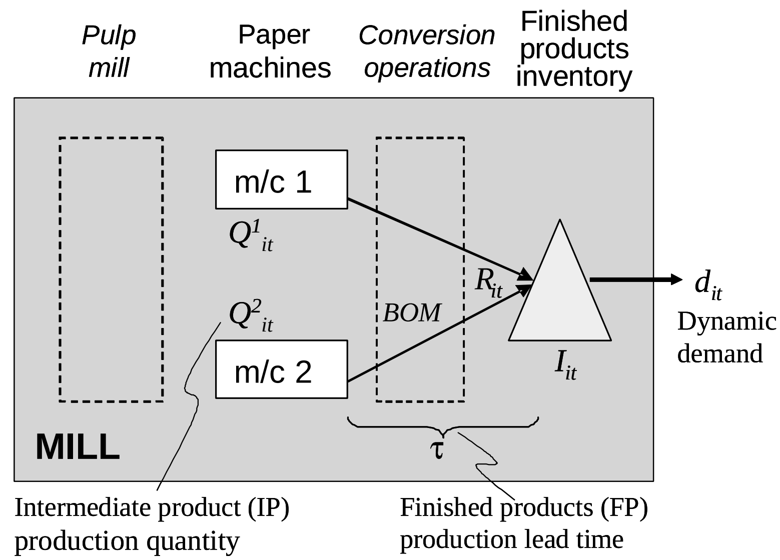

Figure 2: Flow Coordination Context for the Production-distribution of Paper

Without loss of generality, the finished product production lead time is assumed to be product independent. Production involves an intermediate bottleneck stage and an infinite capacity finishing stage. In the bottleneck stage, a small group of intermediate products (ex: rolls of various paper grades) are manufactured on multiple parallel machines with limited capacity. A predetermined production sequence must be maintained on each of the parallel machines. In the finishing stage, the intermediate products are converted into a possibly large number of finished products (ex: different sizes, quality and packaging of paper sheets), and it is assumed that any given finished product is made from a single intermediate product (divergent BOM). Moreover, it is assumed that at most one production changeover is allowed per paper machine per planning period. This is reasonable provided that the planning periods used are relatively short (a day or a shift). It is also assumed that it is not necessary to use the total capacity available in a given time period.

The production planning problem studied is related to the multi-item capacitated dynamic lot-sizing problem. A recent survey of the lot-sizing literature covering these problems is found in Rizk and Martel (2001). Under the assumptions that there is a single production stage, that set-up costs and times are sequence independent and that capacity is constrained by a single resource, three formulations of this problem have been studied extensively: the *Capacitated Lot-Sizing Problem* (CLSP), the *Continuous Setup Lot-Sizing Problem* (CSLP) and the *Discrete Lot-Sizing and Scheduling Problem* (DLSP). The CLSP involves the elaboration of a production schedule for multiple items on a single machine over a planning horizon, in order to minimize total set-up, production and inventory costs. The main differences between the CLSP and the CSLP are that in the latter, at most one product is produced in a period and a changeover cost is incurred only in the periods where the production of a new item starts. In the CLSP, several products can be produced in each period and, for a given product, a set-up is necessary in each period that production takes place. For this reason, CLSP is considered as a *large* time bucket model and CSLP as a *small* time bucket model. DLSP is similar to CSLP in that it also assumes at most one item to be produced per period. The difference is that in DLSP, the quantity produced in each period is either zero or the full production capacity.

Florian *et al.*, (1980) and Bitran and Yanasse (1982) showed that CLSP is NP-hard even when there is a single product and Trigeiro *et al.* (1989) proved that when set-up times are considered, even finding a feasible solution is NP-hard. Exact *mixed integer programming* solution procedures to solve different versions of the problem were proposed by Barany *et al.* (1984), Gelders *et al.* (1986), Leung *et al.* (1989) and Diaby *et al.* (1992). Heuristic methods based on mathematical programming were proposed by Thizy and Wassenhove (1985), Trigeiro *et al.* (1989), Lasdon and Terjung (1971) and Solomon e*t al.* (1993). Specialized heuristics were also proposed by Eisenhut (1975), Lambrecht and Vanderveken (1979), Dixon and Silver (1981), Dogramaci *et al.* (1981), Günther (1987), and Maes and Van Wassenhove (1988). When set-up costs are sequence dependent, the sequencing and lot-sizing problems must be considered simultaneously and the problem is more complex. This problem is known as *lot sizing and scheduling with sequence dependent set-up* and it has been studied by only a few authors (Haase, 1996; Haase and Kimms, 1996). Particular cases of the problem were also examined by Dilts and Ramsing (1989) and by Dobson (1992).

The problem studied in this paper can be considered as an extension of the CSLP to the case of several parallel machines with a predetermined production sequence, and with a two level (IP and FP) product structure. The multi-item CSLP has been studied by Karmarkar and Scharge (1985) who presented a Branch and Bound procedure based on Lagrangean relaxation to solve it. An extension to the basic CSLP that considers parallel machines was studied by De Matta and Guignard (1989) who proposed a heuristic solution method based on Lagrangean relaxation. The DLSP, which is also related to our problem, has been studied mainly by Solomon (1991).

Mathematical Model

In this section, we propose a mathematical programming model for the simultaneous planning of the lot-sizes of intermediate products on all the paper machines in a mill, as well as the production and inventory planning of its finished products. Let $g_{ii'}$ be the number of units of IP i required to produce one unit of FP i', taking any waste incurred in the transformation process into account. Since each FP is made form a single IP product, the set of FP can be partitioned according to the IP it is made of. In addition, recall that a standard production sequence of IP must be maintained for each machine $m = 1,..., M$, and that at most one product type can be produced in a given time period. Let e_m denote the index of the IP in the e^{th} position in machine m production sequence, so that $e_m = 1_m,..., f_m$, where f_m represents the product in the final position in machine m production sequence. Thus, when $e < f$, product $(e+1)_m$ can be produced on machine m only after product e_m has finished its production batch. The production resource consumption for intermediate products is assumed to be concave, that is, a fixed resource capacity consumption is incurred whenever production switches from one IP to another (changeover time), and linear resource consumption is incurred during the production of a batch of IP (see *Figure 3*). Inventory holding costs are assumed to be linear. The notation required to formulate the model is summarized in *Table 1* and *Table 2*.

The mathematical programming model required to plan the production of intermediate and finished products in a paper mill is the following:

$$Min \sum_{t=1}^{T}\left[\sum_{m=1}^{M}\sum_{i\in IP_m} K_{it}^{m}\ \rho_{it}^{m}\right] + \sum_{t=\tau+1}^{T+\tau}\left[\sum_{i=n+1}^{N} h_{it}I_{it}\right] \qquad \text{(P)}$$

subject to

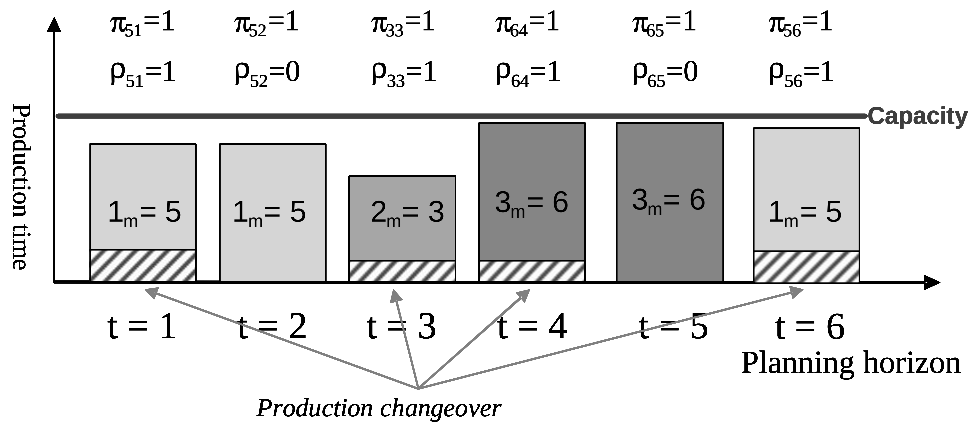

Figure 3: Example of a Production Plan for Paper Machine m

$$\sum_{m \in M_i} Q_{it}^m - \sum_{i' \in SC_i} g_{ii'} R_{i't} = 0 \qquad i = 1, \ldots n;\ t = 1, \ldots, T \qquad \text{(P.1)}$$

$$R_{it} + I_{i(t+\tau-1)} - I_{i(t+\tau)} = d_{i(t+\tau)} \qquad i = n+1, \ldots N;\ t = 1, \ldots, T \qquad (I_{i\tau} = 0) \qquad \text{(P.2)}$$

$$k_{it}^m \rho_{it}^m + a_{it}^m Q_{it}^m - C_t^m \pi_{it}^m \le 0 \qquad m = 1, \ldots, M;\ i \in IP_m;\ t = 1, \ldots, T \qquad \text{(P.3)}$$

$$\pi_{e_m t}^m - \sum_{u=1}^{t} \rho_{e_m u}^m + \sum_{u=1}^{t} \rho_{(e+1)_m u}^m = 0 \qquad m = 1, \ldots, M;\ e_m = 1_m, \ldots, (f-1)_m;\ t = 1, \ldots, T \qquad \text{(P.4)}$$

$$\pi_{f_m t}^m - \sum_{u=1}^{t} \rho_{f_m u}^m + \sum_{u=1}^{t} \rho_{1_m u}^m = 1 \qquad m = 1, \ldots, M;\ t = 1, \ldots, T \qquad \text{(P.5)}$$

$$\sum_{i \in IP_m} \pi_{it}^m \le 1 \qquad m = 1, \ldots, M;\ t = 1, \ldots, T \qquad \text{(P.6)}$$

$$\rho_{it}^m \le \pi_{it}^m,\ \pi_{it}^m \in \{0, 1\},\ \rho_{it}^m \in \{0, 1\},\ Q_{it}^m \ge 0 \qquad m = 1, \ldots, M;\ i \in IP_m;\ t = 1, \ldots, T \qquad \text{(P.7)}$$

$$I_{it} \ge 0 \qquad i = n+1, \ldots N;\ t = \tau+1, \ldots, \tau+T \qquad \text{(P.8)}$$

$$R_{it} \ge 0 \qquad i = n+1, \ldots, N;\ t = 1, \ldots, T \qquad \text{(P.9)}$$

R_{it}	Quantity of finished product $i \in FP$ added to the mill inventory for the beginning of period *t*
Q_{it}^m	Quantity of intermediate product $i \in IP$ produced with machine *m* during period *t*
I_{it}	Inventory level of finished product $i \in FP$ on hand in the mill at the end of period *t*
ρ_{it}^m	Binary variable equal to 1 if a new production batch of product *i* is started on machine *m* at the beginning of period *t* and to 0 otherwise
π_{it}^m	Binary variable equal to 1 if product *i* is made on machine *m* in period *t* and to 0 otherwise

Table 1: Decision Variables

T	Number of planning periods in the planning horizon
t	A planning period (t = 1,..., T)
IP	Set of intermediate products ($\{1, \ldots, n\}$)
FP	Set of finished products ($\{n+1, \ldots, N\}$)
i, i'	Product type indexes ($i, i' \in IP \cup FP$)
m	A paper machine for the production of IP ($m = 1,..., M$)
M_i	The set of machines m that manufactures product i ($i \in IP$)
IP_m	The set of intermediate products manufactured by machine m ($IP_m \subset IP$)
f_m	Number of intermediate products manufactured on machine m (i.e. $\lvert IP_m \rvert$).
e_m	The e^{th} item in the production sequence of machine m, $e_m = 1_m, \ldots, f_m$ ($f \leq n$)
τ	Planned production lead-time
d_{it}	Effective demand at the mill for product i during period t
C_t^m	Production capacity of machine m in period t (in time units)
k_{it}^m	Changeover time required at the beginning of period t to produce $i \in IP_m$ on machine m
K_{it}^m	Product i changeover cost on machine m in period t ($i \in IP_m$)
h_{it}	Inventory holding cost of product i at the mill in period t
$g_{ii'}$	Number of product i units required to produce one unit of product i'
a_{it}^m	Machine m capacity consumption rate of product $i \in IP_m$ in period t
SC_i	Set of finished products manufactured with intermediate product i ($SC_i = \{i' \mid g_{ii'} > 0\}$)

Table 2: Indexes, Parameters and Sets

In model P, (P.1) and (P.2) are the flow conservation constraints of IP and FP products at the manufacturing location. Constraints (P.3) ensure that production capacity is respected. Constraints (P.4) and (P.5) make sure that the production sequence is respected for each machine. For a given machine m, when $e < f$, constraint (P.4) enforces the number of product $(e+1)_m$ changeovers to be less than or equal to the number of product e_m changeovers for any given period of time. Hence, it forces product $(e+1)_m$ production to start only after the production batch of product e_m is completed. Constraints (P.5) do the same job for product f_m which has the particularity of being last in the machine m production sequence. Thus, after its production batch, machine m has to switch production to product 1_m and start another sequence. Constraints (P.6) make sure that at most one product is manufactured per period of time for each machine. Finally, constraints (P.7) restrict the changeovers on a machine to the periods in which there is some production.

Valid Cuts

In this section, valid inequalities to strengthen the proposed formulation (P) are proposed. The use of cuts to improve the solution of lot-sizing problems was first introduced by Barany, Van Roy and Wolsey (1984) and is gaining in popularity. In this paper, three valid inequalities are proposed for model (P). The first proposition is based on a general valid inequality proposed by Barany *et al.* (1984). The other two propositions, however, are derived from the specific properties of our problem.

Proposition 1

For a finished product $i \in IP$ and a period $1 \le t \le T$, the inequalities

$$I_{i(t+\tau-1)} \ge d_{i(t+\tau)}(1-\pi_{it}) \quad \text{(Cut 1)}$$

are valid for (P).

Proposition 2

For an intermediate product $i \in IP$ and a period $1 \le t \le T$, the inequalities

$$\sum_{t'=1}^{t}[\sum_{m \in M_i}\pi_{it'}^{m}] \ge \left\lceil [\sum_{t'=1}^{t}[\sum_{i' \in SC_i} g_{ii'}d_{i(t'+\tau)}]]/CAP_{it} \right\rceil \quad \text{(Cut 2)}$$

where $CAP_{it} = Max_{(m \in M_i),(1 \le t' \le t)}(C_{t'}^{m}/a_{it'}^{m})$,

are valid cuts for (P).

Proof: Since $I_{i'\tau} = 0$ and $I^1_{i'(\tau+\tau^{tr})} = 0$ for all $i' \in FP$, $\sum_{t'=1}^{t} Q_{i't'} \ge \sum_{t'=1}^{t}(d_{i'(t'+\tau)})$.

If we multiply both sides by $g_{ii'}$, we obtain $\sum_{t'=1}^{t} g_{ii'}Q_{i't'} \ge \sum_{t'=1}^{t} g_{ii'}d_{i'(t'+\tau)}$. A sum across all $i' \in SC_i$ leads to the aggregate inequality:

$$\sum_{t'=1}^{t}[\sum_{i' \in SC_i} g_{ii'}Q_{i't'}] \ge \sum_{t'=1}^{t}[\sum_{i' \in SC_i} g_{ii'}d_{i'(t'+\tau)}].$$

We know that $\sum_{i' \in SC_i} g_{ii'}Q_{i't'} = \sum_{m \in M_i} Q_{it'}^{m}$ and $Q_{it'}^{m} \le (C_{t'}^{m}/a_{it'}^{m})\pi_{it'}^{m}$. Thus,

$$\sum_{t'=1}^{t}[\sum_{m \in M_i}(C_{t'}^{m}/a_{it'}^{m})\pi_{it'}^{m}] \ge \sum_{t'=1}^{t}[\sum_{i' \in SC_i} g_{ii'}d_{i'(t'+\tau)}]$$

and hence:

$$\sum_{t'=1}^{t}[\sum_{m \in M_i}\pi_{it'}^{m}] \ge [\sum_{t'=1}^{t}[\sum_{i' \in SC_i} g_{ii'}d_{i'(t'+\tau)}]]/CAP_{it}$$

where $CAP_{it} = Max_{(m \in M_i),(1 \le t' \le t)}(C_{t'}^{m}/a_{it'}^{m})$.

Since the coefficients and variables of the last inequality left side are integer, it is seen that (Cut 2) are valid inequalities □.

Based on the total demand of a given product during a time interval and the available production capacity, (Cut 2) forces a tight lower bound on the number of set-ups needed to satisfy demand during the time interval.

Proposition 3

For each machine $m \in M$, each product $i \in IP$, and each period t, the following inequalities are valid cuts for (P):

$$\rho_{it}^{m} - \pi_{it}^{m} + \pi_{i(t-1)}^{m} \ge 0 \quad \text{(Cut 3)}$$

Proof: For a given $i \in IP$ there exists $1_m \le e_m \le f_m$ such that $i = e_m$. Based on constraints (P.4) and (P.5) π_{it}^{m} is given by $\pi_{e_m t}^{m} = \pi_{e_m(t-1)}^{m} + \rho_{e_m t}^{m} - \rho_{(e+1)_m t}^{m}$ (if $e_m = f_m$ then $(e+1)_m = 1_m$). Thus, $\rho^{m}_{e_m t} = \pi_{e_m t}^{m} - \pi_{e_m(t-1)}^{m} + \rho_{(e+1)_m t}^{m}$. Since $\rho_{(e+1)_m t}^{m} \ge 0$, (Cut. 2) are valid inequalities □.

(Cut.3) simply states that a product i changeover must occur at the beginning of period t if a new production process for this product is initiated ($\pi_{i(t-1)} = 0$ and $\pi_{it} = 1$) at the beginning of the same period.

Experimental Testing

Test Application

The models proposed in this paper were developed in the context of a project with Domtar Inc., a large pulp and paper company, and our numerical tests and comparisons are based on data obtained from that company. Domtar is the second largest producer of uncoated free sheet in North America and the third largest in the world. The Domtar Windsor plant has two paper machines. The two machines use different types of pulp to produce three different grades of papers ($IP = \{1, 2, 3\}$). Business grade papers, as its name indicates, are used in business offices and personal computers. Printing and publishing grade papers are used for publications and glossy publications. Specialty and technical grade papers are highly specialized papers used mainly in flexible packaging and in industrial applications. Every time a paper machine switches its production to a different grade, there are losses of paper sheets that vary depending on which grade was being produced before (i.e: changeover costs are sequence dependent). Changeover costs are estimated based on the paper sheet losses. Unit inventory holding costs are the same for all finished products and are time independent.

Experimental Design

This section describes our computational experiments on using the valid cuts introduced in the previous section for solving the production problem with a branch and cut algorithm. All experiments were performed on a 1.66 GHz Pentium III workstation with 1 GB of main memory, using the callable libraries of CPLEX 8.1, with two hours time limit. Based on the data obtained from Domtar, four different instances of the production planning problem were created by varying the length of the planning horizon (T) and the number of finished products (FP). The characteristics of the four instances are summarized in *Table 3.*

Adding valid inequalities to a problem formulation may have a positive or a negative impact on its solution time. Therefore, different experiments to measure the effectiveness of adding different combinations of the valid inequalities proposed were performed. The Cplex MIP solver also adds Gomory cuts, among several other classes of general cuts, to the formulation. To isolate the impact of the valid cuts proposed in this paper from those of Gomory cuts, two sets of experiments were performed. The first set is with Gomory cuts; the second is without Gomory cuts. The results of these experiments are summarized in *Table 3*, where the averages for the elapsed CPU time in seconds without Gomory cuts (w/o-Gcuts) and with Gomory cuts (w-Gcuts) are reported. Each entry in the table corresponds to the average solution time for the four instances described in *Table 3*.

Our results show that out of the three valid cuts proposed, Cut 2) is the most effective in improving problem (P) solution time. The marginal contribution of adding (Cut 1) and (Cut 3) on top of (Cut 2) doesn't seem to be significant. In *Table 4* we report the impact of adding all valid cuts to (P) without and with Gomory cuts respectively. The (Improv%) columns show the percentage by which the solution time is reduced when adding the proposed valid cuts. These results show that the contribution of the proposed valid inequalities is more considerable when added on top of Gomory cuts. The average solution time is reduced by more than a half.

Instance	T	FP
1	30	100
2	60	100
3	30	500
4	60	500

Test Instances

Formulation	Time(Sec)	
	w/o-Gcuts	w-Gcuts
P	2,586.75	1463.8
P+cuts1	2,155.87	952.23
P+cuts2	1,968.92	406.2
P+cuts3	2,738.00	777.3
P+cuts1+cuts2	1,517.80	681.28
P+cuts1+cuts3	2,149.98	754.43
P+cuts2+cuts3	2,148.64	436.86
P+All cuts	1,603.91	625.11

Table 3: Experiments with Different Combinations of Valid Cuts

	Without Gomory cuts			With Gomory cuts		
	Time(Sec)			**Time(Sec)**		
Instance	**P**	**P+allcuts**	**Improv%**	**P**	**P+allCuts**	**Improv%**
1	83.59	21.88	74%	58.06	16.11	72%
2	1,180.00	200.09	83%	1,166.92	110.27	91%
3	1,881.14	508.55	73%	1,935.30	216.48	89%
4	7,200.00	5,685.13	21%	2,694.92	2,157.56	20%
Average	2,586.18	1,603.91	38%	1,463.80	625.11	57%

Table 4: Experiments without Gomory Cuts

Conclusion

This paper proposes a mathematical programming formulation of a multi-stage production planning problem where a sequence of intermediate products has to be maintained. It also proposes valid inequalities that can be added to the original formulation to further improve its solution time, when solved with commercial mixed-integer programming libraries.

The computational experiments with realistic size problems suggest that the mathematical model proposed can be used to solve real cases in reasonable times. Furthermore, adding valid inequalities can be very effective in further improving the problem solution time. For future research directions, one may pursue the development of more effective valid inequalities, as well as the potential of exploiting binary variables special structures such as Special Ordered Sets (SOS) as defined by Beale and Tomlin [1970].

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