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Phenomenological features in a model with non-universal gaugino CP phases

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Abstract

We study phenomenological features in an extended gauge mediation SUSY breaking model which has non-universal gaugino masses and CP phases. We show that large CP phases in soft SUSY breaking parameters can be consistent with the constraints coming from the electric dipole moment (EDM) of an electron, a neutron, and also a mercury atom. Masses of the superpartners are not necessarily required to be larger than 1 TeV but allowed to be $O(100)$ GeV. We also investigate the mass spectrum of Higgs scalars and their couplings to gauge bosons in that case. Compatibility of this model with the present experimental data on the Higgs sector is discussed.

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1 Introduction

In supersymmetric extensions of the standard model, new CP phases are generally introduced through supersymmetry (SUSY) breaking. Although these CP phases could play an interesting phenomenological role related to the cosmological baryon number asymmetry, for example, it is well known that the electric dipole moment (EDM) of an electron and a neutron [1] imposes severe constraints on such CP phases of soft SUSY breaking parameters in the minimal supersymmetric standard model (MSSM) [2, 3]. It seems to be very important to examine these constraints because of their phenomenological consequences.

Some possibilities to overcome these constraints have been proposed by now. In the first type solution, the soft SUSY breaking parameters are taken to be $O(100)$ GeV by assuming that the soft CP phases are smaller than 10^{-2} [2]. Since such small phases are not protected by any symmetry, it is usually considered to be unnatural, and regarded as a CP problem in the MSSM. In the second one, the soft CP phases are supposed to be $O(1)$ while a part of the relevant soft SUSY breaking parameters are assumed to be $O(1)$ TeV or larger.¹ However, considering the SUSY breaking larger than $O(1)$ TeV seems to be unattractive from a viewpoint of the weak scale SUSY. It may also be difficult to expect any phenomenological effects through the present and near future experiments in this case.

As the third possibility, we can expect the cancellation among various contributions to the EDMs [6, 7, 8, 9]. If such a cancellation occurs and both the CP phases of $O(1)$ and the soft SUSY breaking parameters of $O(100)$ GeV can be consistent with the EDM constraints, we might have a lot of interesting phenomenology at the weak scale [9, 10, 11, 12, 13, 14]. If we consider the origin of the baryon number asymmetry in the universe due to electroweak baryogenesis, for example, it will be necessary to introduce some new sources of CP violation. It is known that the Cabibbo-Kobayashi-Maskawa (CKM) phase in the standard model (SM) is insufficient to explain the baryon number asymmetry because of a suppression due to the smallness of the quark flavor

¹Various possibilities have been suggested. In one possibility, it is assumed that the sfermions in the first and second generation have heavy masses of $O(1)$ TeV [3]. In another one, the A parameters are assumed to be non-universal and those related to the first and second generation are supposed to be very small such as $A_f = (0, 0, A)$ [5]. In this case, one needs to assume $\arg(\mu) < 10^{-2}$ and then the smallness of the CP phase is partially required as in the first solution [4].

mixing [15]. If there exist large CP phases in the soft SUSY breaking parameters, the requirement for the electroweak phase transition to be strongly first order might be relaxed and the required Higgs mass bound could be larger [16]. Various SUSY leptogenesis scenarios also seem to require the large CP phases in the soft SUSY breaking parameters [17, 18]. Thus, the existence of such CP phases is a fascinating possibility from a viewpoint that they present us promising sources for the CP violation required in baryogenesis and leptogenesis. Moreover, such CP phases might be checked through the LHC experiments.

Various works on this third possibility have suggested that the constraints on the EDMs of an electron and a neutron could be satisfied even in the case that the CP phases in the soft SUSY breakings are $O(1)$ and the superpartners are rather light. It is based on the effective cancellation among various contributions to the EDMs.² On the other hand, there is another claim that if we add the constraint from the EDM of the mercury atom, the allowed parameter regions disappear. It suggests that the parameter region for their cancellation are different between the electron and the mercury [19]. However, it is useful to note that the usual analyses of the EDMs are based on the assumption for the universal gaugino masses as stressed in [9]. If we do not take this assumption, we may find the way out of this difficulty.

Since the gaugino masses are universal in the usual SUSY breaking scenario, it may be considered that such an assumption is unrealistic. However, non-universal gaugino masses can be realized naturally, if we consider, for example, the intersecting D-brane model [9, 21], the extended gauge mediation SUSY breaking [22], and the SUSY breaking mediated by the Abelian gaugino kinetic term mixing [23]. In the previous paper [24], we examined the possibility of the reconciliation between the CP phases of $O(1)$ and the experimental EDM constraints in a model with non-universal gaugino masses. In that study we showed that the EDM constraints could be satisfied in rather large regions of the SUSY breaking parameter space under the existence of the large CP phases, as long as there are physical CP phases in the gaugino masses. However, since the allowed parameter

²In the case of the EDM of the electron, the cancellation between the chargino contribution and the neutralino contribution has been shown to occur [6, 7, 9]. On the other hand, the EDM of the neutron (EDMN) it has been known that there are several types of cancellation, that is, the cancellation between the diagrams of the gluino exchange and the chargino exchange diagrams and also the cancellation among the gluino exchange diagrams themselves *etc* [6, 8]. In the case of the EDMN, the combined effect of these cancellations allows the large soft CP phases [6, 7, 9].

regions tend to be obtained for the small $\tan\beta$ [24], Higgs phenomenology might constrain the model strongly through the present Higgs search [11, 12, 13, 25].

In this paper we extend the study to the Higgs sector using the parameter regions allowed by the constraints from the EDMs of the electron, the neutron, and the mercury atom. We discuss the consistency of the scenario with the Higgs phenomenology. The paper is organized as follows. In section 2 we introduce the model for the soft SUSY breaking with the non-universal gaugino masses. In section 3 we briefly describe the EDM of the mercury atom as an example of the EDM calculation. The numerical analysis of the EDM constraints is carried out by using the renormalization group study. We apply this result to the estimation of the masses of the neutral Higgs scalar and the couplings between the Higgs scalars and the gauge bosons. We also discuss predicted values of $g-2$ of the muon and the electron. Section 4 is devoted to the summary.

2 A model with non-universal gaugino CP phases

We briefly introduce the model with non-universal gaugino masses studied in this paper and fix the notation. We consider an extension of the well known minimal gauge mediation SUSY breaking (GMSB) scenario, which is defined by the following superpotential for the messenger fields [22]:

$$W_m = \lambda_q \hat{S}_1 \hat{q} \hat{q} + \lambda_\ell \hat{S}_2 \hat{\ell} \hat{\ell}, \quad (1)$$

where \hat{q} , \hat{q} are $\mathbf{3}$, $\mathbf{3}^*$ of $SU(3)_c$ and $\hat{\ell}$, $\hat{\ell}$ are the doublets of $SU(2)_L$. If both the singlet fields \hat{S}_1 and \hat{S}_2 couple with the hidden sector where the SUSY breaks down, \hat{q} , \hat{q} and $\hat{\ell}$, $\hat{\ell}$ play the role of messenger fields as in the case of the ordinary scenario [26, 27, 28]. Only difference from the ordinary minimal GMSB scenario is that \hat{q} , \hat{q} and $\hat{\ell}$, $\hat{\ell}$ couple with the different singlet chiral superfields \hat{S}_1 and \hat{S}_2 in the superpotential W_m . It is realized if we impose a suitable discrete symmetry on the model [22]. If both their scalar components S_α and their auxiliary components F_{S_α} obtain vacuum expectation values (VEVs) due to the couplings with the SUSY breaking sector, the masses of the gauginos and the scalars in the MSSM are generated at one-loop and two-loop level, respectively. They are represented as functions of $\Lambda_\alpha = \langle F_{S_\alpha} \rangle / \langle S_\alpha \rangle$ in a similar way as the ordinary scenario. However, the mass formulas are somewhat modified from the usual ones since the messenger fields (\hat{q}, \hat{q}) and $(\hat{\ell}, \hat{\ell})$ couple with the different singlets.

In this kind of model, the gaugino masses can be written in the form as [22]

$$M_3 = \frac{\alpha_3}{4\pi} \Lambda_1, \quad M_2 = \frac{\alpha_2}{4\pi} \Lambda_2, \quad M_1 = \frac{\alpha_1}{4\pi} \left(\frac{2}{3} \Lambda_1 + \Lambda_2 \right), \quad (2)$$

where $\alpha_r = g_r^2/4\pi$ and g_r stands for the coupling constant for the standard model gauge group. These formulas show that M_3 can be smaller than $M_{1,2}$ in the case of $\Lambda_2 > \Lambda_1$. Since Λ_α is generally independent, the phases contained in the gaugino masses are non-universal even in the case of $|\Lambda_1| = |\Lambda_2|$. In that case, we cannot remove them completely by using the R -transformation unlike in the case of universal gaugino masses. In fact, if we define the phases as $\Lambda_\alpha \equiv |\Lambda_\alpha| e^{i\theta_\alpha}$ and make M_2 real by the R -transformation, the phases of the gaugino masses M_r can be written as [22]

$$\begin{aligned} \phi_3 &\equiv \arg(M_3) = \theta_1 - \theta_2, & \phi_2 &\equiv \arg(M_2) = 0, \\ \phi_1 &\equiv \arg(M_1) = \arctan \left(\frac{2|\Lambda_1| \sin(\theta_1 - \theta_2)}{3|\Lambda_2| + 2|\Lambda_1| \cos(\theta_1 - \theta_2)} \right). \end{aligned} \quad (3)$$

These formulas show that the phases of the gaugino masses can be parameterized three parameters, that is, $|\Lambda_1|$, $|\Lambda_2|$ and $\theta_1 - \theta_2$.

The scalar masses are induced through the two-loop diagrams as in the ordinary case. Their formulas can be given as [22]

$$\tilde{m}_f^2 = 2|\Lambda_1|^2 \left[C_3 \left(\frac{\alpha_3}{4\pi} \right)^2 + \frac{2}{3} \left(\frac{Y}{2} \right)^2 \left(\frac{\alpha_1}{4\pi} \right)^2 \right] + 2|\Lambda_2|^2 \left[C_2 \left(\frac{\alpha_2}{4\pi} \right)^2 + \left(\frac{Y}{2} \right)^2 \left(\frac{\alpha_1}{4\pi} \right)^2 \right], \quad (4)$$

where $C_3 = 4/3$ and 0 for the SU(3) triplet and singlet fields, and $C_2 = 3/4$ and 0 for the SU(2) doublet and singlet fields, respectively. The hypercharge Y is expressed as $Y = 2(Q - T_3)$ by using both the electric charge Q and the diagonal SU(2) generator T_3 . As it is clear from this formula for the masses of the scalar superpartners, we have no FCNC problem induced by these soft scalar masses as in the ordinary case. This is the case even if we take account of the renormalization group effects since the running due to the renormalization groups occurs only for the narrow range.

We apply this soft SUSY breaking scenario to the MSSM framework. The MSSM superpotential contains the terms

$$W = \sum_j \left(h_j^U \hat{H}_2 \hat{Q}_j \hat{U}_j + h_j^D \hat{Q}_j \hat{H}_1 \hat{D}_j + h_j^E \hat{L}_j \hat{H}_1 \hat{E}_j \right) + \mu \hat{H}_1 \hat{H}_2, \quad (5)$$

where we take the Yukawa coupling diagonal basis for the quarks and the leptons. All Yukawa couplings h_j^f are supposed to be real. The Higgsino mass parameter μ is generally

complex. The soft SUSY breaking terms corresponding to the superpotential (5) are introduced as³

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} &= \sum_{\alpha} \tilde{m}_{\alpha}^2 |\phi_{\alpha}|^2 - \left[\sum_j \left(A_j^U h_j^U H_2 \tilde{Q}_j \tilde{U}_j + A_j^D h_j^D \tilde{Q}_j H_1 \tilde{D}_j + A_j^E h_j^E \tilde{L}_j H_1 \tilde{E}_j \right) \right. \\
&\quad \left. - B\mu H_1 H_2 - \frac{1}{2} \sum_r M_r \lambda_r \lambda_r + \text{h.c.} \right], \tag{6}
\end{aligned}$$

where we put a tilde for the superpartners of the chiral superfields corresponding to the standard model contents. The first term represents the soft SUSY breaking masses for all scalar components of the MSSM chiral superfields. They are assumed to be given by eq. (4). The third term in the brackets represents the gaugino mass terms, which are supposed to be given by eq. (2). The soft SUSY breaking parameters B and A_j^f are the coefficients of the bilinear and trilinear scalar couplings with a mass dimension.

In the minimal GMSB model, as discussed in [29], the soft SUSY breaking parameters A_f and B can be induced through the radiative correction. In the case that $A_f(\Lambda) = B(\Lambda) = 0$ is satisfied at the SUSY breaking scale Λ which is expected in many GMSB scenario, A_f and B are proportional to M_2 at the low energy regions as a result of the renormalization group effect. Thus, all of the CP phases in the soft SUSY breaking parameters are rotated away as long as the gaugino masses are universal [27, 29]. However, in the present case this situation is broken and there remain the CP phases in the gaugino masses even in the case of $A_f(\Lambda) = B(\Lambda) = 0$ since the phases in the gaugino masses are not universal. The generation of the bare A_f and B is completely model dependent in this model as in the ordinary GMSB scenario. In the following study, we do not fix their origin and treat them as free parameters.

Here we make an additional assumption for the trilinear scalar couplings such that they are proportional to the Yukawa couplings so as to satisfy the FCNC constraints. Although the soft SUSY breaking parameters A_j^f , B and M_r can generally include the CP phases, all of these are not independent physical phases. If we use the R -symmetry and redefine the fields appropriately, we can select out the physical CP phases among them. We take them as

$$A_j = |A_j| e^{i\phi_{A_j}}, \quad \mu = |\mu| e^{i\phi_{\mu}}, \quad M_r = |M_r| e^{i\phi_r} \quad (r = 1, 3), \tag{7}$$

³We adopt the sign convention for μ , B and A_f to make the mass eigenvalues of quarks and leptons to be positive by a suitable field redefinition.

where $B\mu$ and M_2 are assumed to be real. Although the VEVs of the doublet Higgs scalars H_1 and H_2 are taken to be real in this definition at the tree level, the radiative correction could generally introduce the CP phases to them. Taking account of this aspect and following [12], we define the VEVs of the doublet Higgs scalars H_1 and H_2 as

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}. \quad (8)$$

Finally we summarize the model parameters related to the SUSY breaking. In the present framework, the free parameters related to the masses of the gauginos and the scalar superpartners are confined into Λ_1 and Λ_2 . Their phases are related to the physical phases ϕ_3 and ϕ_1 in eq. (3). Since we assume the universality for the A parameter such as $A_j^f(\Lambda) = A$ at the SUSY breaking scale Λ , there remain five independent real parameters ϕ_μ , ϕ_A , $|\mu|$, $|B|$, and $|A|$ in the sector of A_j^f and B . Thus, the model parameters relevant to the soft SUSY breaking are composed of eight real parameters,

$$|\Lambda_1|, \quad |\Lambda_2|, \quad |A|, \quad |B|, \quad |\mu|, \quad \phi_3, \quad \phi_A, \quad \phi_\mu. \quad (9)$$

The phase ξ in eq. (8) will be determined by these parameters through minimizing the CP-violating Higgs potential [12].

3 Phenomenological effects of gaugino CP phases

3.1 Constraints from EDMs

In order to explain the constraints on the SUSY breaking parameters from the EDM, we at first take a mercury case as an example to give a brief discussion. The detailed discussion on the EDMs of the electron and the neutron in the present model can be found in [24].

An effective interaction term representing the color EDM of the quark can be written as

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \mathcal{G} \bar{q} \frac{\lambda^\alpha}{2} \sigma_{\mu\nu} q F_\alpha^{\mu\nu}. \quad (10)$$

In the estimation of the mercury EDM, we use the formula expressed by

$$d_{\text{Hg}}/e = - \left(\tilde{d}_d - \tilde{d}_u - 0.012\tilde{d}_s \right) \times 3.2 \times 10^{-2}, \quad (11)$$

where \tilde{d}_f is the color EDM of an f -quark [19]. It is related to the effective coupling \mathcal{G} in eq. (10) through the formula

$$\tilde{d}_f = \text{Im}(\mathcal{G}). \quad (12)$$

The effective coupling \mathcal{G} is composed of the contributions from the one-loop diagrams containing one of a gluino, a chargino or a neutralino in the internal line. The experimental data for the mercury EDM d_{Hg} gives the constraint on the color EDM of the quarks such as [20]

$$|\tilde{d}_d - \tilde{d}_u - 0.012\tilde{d}_s| < 0.66 \times 10^{-26} \text{ cm}. \quad (13)$$

For the preparation to estimate the color EDM of the quarks, we need to fix a relevant part of the MSSM to give their analytic formulas. As in the case of the EDM of the electron and the neutron, the mixing matrices of the charginos, the neutralinos and the squarks are important elements to write down them at the one-loop approximation.

In the basis of the superpotential (5) and the soft SUSY breaking (6), the mass terms of the charginos can be written as

$$-\left(\tilde{H}_2^+, -i\lambda^+\right) \begin{pmatrix} |\mu|e^{i\phi_\mu} & \sqrt{2}m_Zc_W \sin\beta \\ \sqrt{2}m_Zc_W \cos\beta & M_2 \end{pmatrix} \begin{pmatrix} \tilde{H}_1^- \\ -i\lambda^- \end{pmatrix}, \quad (14)$$

where $\tan\beta = v_2/v_1$ and the abbreviations $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$ are used. The mass eigenstates χ_i^\pm are defined in terms of the weak interaction eigenstates in eq. (14) through the unitary transformations in such a way as

$$\begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix} \equiv W^{(+)\dagger} \begin{pmatrix} \tilde{H}_2^+ \\ -i\lambda^+ \end{pmatrix}, \quad \begin{pmatrix} \chi_1^- \\ \chi_2^- \end{pmatrix} \equiv W^{(-)\dagger} \begin{pmatrix} \tilde{H}_1^- \\ -i\lambda^- \end{pmatrix}. \quad (15)$$

The canonically normalized neutralino basis is taken as $\mathcal{N}^T = (-i\lambda_1, -i\lambda_2, \tilde{H}_1^0, \tilde{H}_2^0)$ and their mass terms are defined in such a form as $\mathcal{L}_{\text{mass}}^n = -\frac{1}{2}\mathcal{N}^T\mathcal{M}\mathcal{N} + \text{h.c.}$. The 4×4 neutralino mass matrix \mathcal{M} can be expressed as

$$\begin{pmatrix} |M_1|e^{i\phi_1} & 0 & -m_Zs_W \cos\beta & m_Zs_W \sin\beta \\ 0 & M_2 & m_Zc_W \cos\beta & -m_Zc_W \sin\beta \\ -m_Zs_W \cos\beta & m_Zc_W \cos\beta & 0 & -|\mu|e^{i\phi_\mu} \\ m_Zs_W \sin\beta & -m_Zc_W \sin\beta & -|\mu|e^{i\phi_\mu} & 0 \end{pmatrix}. \quad (16)$$

Mass eigenstates χ^0 of this mass matrix are related to the weak interaction eigenstates \mathcal{N} as

$$\chi^0 \equiv U^T\mathcal{N}, \quad (17)$$

where the mass eigenvalues are defined to be real and positive so that the mixing matrix U is considered to include the Majorana phases.

Since we do not have the flavor mixing in the sfermion sector in the present model, the sfermion mass matrices can be reduced into the 2×2 form for each flavor. This 2×2 sfermion mass matrix can be written in terms of the basis $(\tilde{f}_{L_\alpha}, \tilde{f}_{R_\alpha})$ as

$$\begin{pmatrix} |m_\alpha|^2 + \tilde{m}_{L_\alpha}^2 + D_{L_\alpha}^2 & m_\alpha(|A_\alpha|e^{i\phi_{A_\alpha}} - |\mu|e^{-i\phi_\mu} R_f) \\ m_\alpha(|A_\alpha|e^{-i\phi_{A_\alpha}} - |\mu|e^{i\phi_\mu} R_f) & |m_\alpha|^2 + \tilde{m}_{R_\alpha}^2 + D_{R_\alpha}^2 \end{pmatrix}, \quad (18)$$

where m_α and $\tilde{m}_{L_\alpha, R_\alpha}$ are the masses of the ordinary fermion f_α and its superpartners $\tilde{f}_{L_\alpha, R_\alpha}$, respectively.⁴ R_f is $\cot \beta$ for the up component of the SU(2) fundamental representation and $\tan \beta$ for the down component. $D_{L_\alpha}^2$ and $D_{R_\alpha}^2$ represent the D -term contributions, which are expressed as

$$D_{L_\alpha}^2 = m_Z^2 \cos 2\beta (T_3^f - Q_f s_W^2), \quad D_{R_\alpha}^2 = m_Z^2 s_W^2 Q_f \cos 2\beta, \quad (19)$$

where T_3^f takes $1/2$ for the sfermions in the up sector and $-1/2$ for those in the down sector. Q_f is the electric charge of the field f . We define the mass eigenstates $(\tilde{f}_1, \tilde{f}_2)$ by the unitary transformation

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} \equiv V^{f\dagger} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}. \quad (20)$$

In the MSSM, there are various contributions to the quark color EDM \tilde{d}_f , which come from the one-loop diagram with the superpartners of the standard model fields in the internal lines and can be expressed as $\tilde{d}_f \equiv \tilde{d}_f^g + \tilde{d}_f^\chi$. The contribution \tilde{d}_f^g including the gluinos in the internal lines can be written as

$$\begin{aligned} \tilde{d}_f^g &= \frac{\alpha_s}{8\pi} \frac{1}{|M_3|} \sum_{a=1}^2 \text{Im}(\mathcal{A}_g^{f_a}) \left(\frac{1}{3} G(x_a) + 3F(x_a) \right), \\ \mathcal{A}_g^{f_a} &= V_{2a}^f V_{1a}^{f*} e^{i\phi_3}, \end{aligned} \quad (21)$$

where $x_a = \tilde{m}_a^2/|M_3|^2$. This formula shows that the gluino phase ϕ_3 can bring the drastic changes in the gluino contribution to the quark color EDM. It is remarkable that a suitable value of ϕ_3 can change even the sign of the gluino contribution compared with the $\phi_3 = 0$ case.

⁴In this sfermion mass matrix, the sign convention of A_α is changed from the one in the previous work [24].

On the chargino and neutralino contributions \tilde{d}_f^X to the quark color EDM, we can calculate it in the same way as the ordinary EDM of the electron [24]. We find that it can be written as

$$\begin{aligned}\tilde{d}_u^X &= \frac{\alpha m_u}{8\pi s_W^2} \left[\frac{1}{m_u} \sum_{j,a} \frac{-2}{3m_j} G(x_{aj}) \text{Im}(\mathcal{A}_{\chi_j^0}^{u_a}) + \frac{1}{m_W \sin \beta} \sum_{j,a} \frac{1}{3m_j} G(x_{aj}) \text{Im}(\mathcal{A}_{\chi_j^\pm}^{u_a}) \right], \\ \tilde{d}_d^X &= \frac{\alpha m_d}{8\pi s_W^2} \left[\frac{1}{m_d} \sum_{j,a} \frac{1}{3m_j} G(x_{aj}) \text{Im}(\mathcal{A}_{\chi_j^0}^{d_a}) - \frac{1}{m_W \cos \beta} \sum_{j,a} \frac{2}{3m_j} G(x_{aj}) \text{Im}(\mathcal{A}_{\chi_j^\pm}^{d_a}) \right].\end{aligned}\quad (22)$$

In these formulas the mixing factors $\mathcal{A}_{\chi^\pm}^f$ and $\mathcal{A}_{\chi^0}^f$ are defined by

$$\begin{aligned}\mathcal{A}_{\chi_j^\pm}^{u_a} &= \frac{1}{\sqrt{2}} W_{1j}^{(+)} W_{2j}^{(-)} |V_{1a}^d|^2 + \frac{1}{2 \cos \beta} \frac{m_d}{m_W} W_{1j}^{(+)} W_{1j}^{(-)} V_{2a}^{d*} V_{1a}^d, \\ \mathcal{A}_{\chi_j^\pm}^{d_a} &= \frac{1}{\sqrt{2}} W_{1j}^{(-)} W_{2j}^{(+)} |V_{1a}^u|^2 + \frac{1}{2 \sin \beta} \frac{m_u}{m_W} W_{1j}^{(+)} W_{1j}^{(-)} V_{2a}^{u*} V_{1a}^u, \\ \mathcal{A}_{\chi_j^0}^{u_a} &= - \left[\left(\frac{2}{9} t_W^2 U_{1j}^2 + \frac{2}{3} t_W U_{1j} U_{2j} \right) V_{1a}^{u*} V_{2a}^u \right. \\ &\quad \left. - \frac{m_u}{2m_W \sin \beta} \left\{ \left(\frac{1}{3} t_W U_{1j} U_{4j} + U_{2j} U_{4j} \right) |V_{1a}^u|^2 - \frac{2}{3} t_W U_{1j} U_{4j} |V_{2a}^u|^2 \right\} \right] \\ \mathcal{A}_{\chi_j^0}^{d_a} &= - \left[- \left(\frac{1}{9} t_W^2 U_{1j}^2 + \frac{1}{3} t_W U_{1j} U_{2j} \right) V_{1a}^{d*} V_{2a}^d \right. \\ &\quad \left. - \frac{m_d}{2m_W \cos \beta} \left\{ \left(\frac{1}{3} t_W U_{1j} U_{3j} - U_{2j} U_{3j} \right) |V_{1a}^d|^2 + \frac{1}{3} t_W U_{1j} U_{3j} |V_{2a}^d|^2 \right\} \right],\end{aligned}\quad (23)$$

where $t_W = \sin \theta_W / \cos \theta_W$. We neglect the higher order terms of the quark mass in the expression of $\mathcal{A}_{\chi^0}^f$. Since the fermions in the external lines are very light compared with the fields in the internal lines, $F(x)$ and $G(x)$ are approximately written as

$$F(x) = \frac{1-3x}{(1-x)^2} - \frac{2x^2}{(1-x)^3} \ln x, \quad G(x) = \frac{1+x}{(1-x)^2} + \frac{2x}{(1-x)^3} \ln x. \quad (24)$$

The gluino contribution is expected to be larger than other contributions because of the strong coupling constant. If we expect the cancellation among these contributions, \tilde{d}_f^g should be suppressed to have the similar magnitude to others. In order to find the condition for it, we may estimate a factor $\text{Im}(\mathcal{A}_g^{f_a})$ in the case of $|A| \gg |\mu|$, for example. In that case it can be found to be approximated as

$$\text{Im}(\mathcal{A}_g^{f_a}) = O \left(\frac{m_{f_a} |A|}{M_2^2} \sin(\phi_3 - \phi_A) \right). \quad (25)$$

This shows that the existence of the gluino phase ϕ_3 may make it possible to suppress the gluino contribution to the level of others. If it happens, the experimental bounds can be satisfied.

Both contributions of the charginos and the neutralinos are crucially affected by the relative magnitude of μ and $M_{1,2}$. If $|\mu| < |M_{1,2}|$ is satisfied, Higgsino components dominate both the lightest neutralino and the lightest chargino. Although they are expected to yield the largest contribution to the EDM, Higgsino exchange effects can be suppressed due to the smallness of Yukawa couplings. On the other hand, in the case of $|M_1| < |\mu| < |M_2|$, the lightest neutralino and the lightest chargino seem to be dominated by the bino and the Higgsinos, respectively. Since the gauge coupling g_1 is larger than the relevant Yukawa couplings which determine the magnitude of their contribution, the chargino contribution can be suppressed in comparison with the neutralino contribution. As a result, they can yield the similar order contributions. If the latter situation for μ and $M_{1,2}$ is realized, the EDM constraint may be satisfied even in the case that the large CP phases exist in the soft SUSY breaking parameters. In the next part, we mainly focus our attention on such situations and carry out the numerical calculation.

3.2 Numerical results of the EDM constraints

At first we explain the procedure for the calculation. We evolve the soft SUSY breaking parameters from a certain SUSY breaking scale Λ to the weak scale by using the one-loop renormalization group equations (RGEs). There is an ambiguity on the scale where the soft SUSY breaking parameters are introduced and start their running. In the present analysis, we adopt $\Lambda = \min(|\Lambda_1|, |\Lambda_2|)$ as such a scale, for simplicity. Since we mainly study the region where $|\Lambda_2|/|\Lambda_1|$ is not so large, this prescription is not considered to affect the results largely. For the gauge and Yukawa coupling constants we use the two-loop RGEs. The RGEs from the unification scale M_U to Λ are composed of the SUSY ones for both the gauge and Yukawa coupling constants. The β -functions are calculated for the MSSM contents and the messenger fields. We solve these RGEs for various initial values of the Yukawa couplings at M_U and examine whether the masses of the top and bottom quarks and also the tau lepton are obtained at the weak scale. The messenger fields are supposed to decouple and the soft SUSY breaking parameters are introduced at Λ . Thus, the RGEs become the same as those of the MSSM below this scale.

In order to determine the phenomenologically interesting parameter regions, we impose several conditions on the parameters at the weak scale obtained by the RGEs. As such conditions, we adopt the followings additionally to the above mentioned ones:

(1) Various experimental mass bounds for the superpartners, such as gluinos, charginos, stops, staus, and charged Higgs scalars, should be satisfied. The color and the electromagnetic charge also should not be broken;

(2) The physical true vacuum should be radiatively realized as the minimum of the scalar potential and satisfy $\sin 2\beta = 2B\mu/(m_1^2 + m_2^2 + 2|\mu|^2)$. As another true vacuum condition, moreover, we impose the consistency between this $\sin 2\beta$ and the value of $\tan \beta$ predicted from the Yukawa coupling and the top quark mass.⁵ Only if the difference between them is sufficiently small, the parameters are accepted.

After restricting the parameter space at the high energy scale by imposing these conditions on the weak scale values, we finally calculate the EDMs of the electron, the neutron and the mercury atom. We compare these results with the present experimental bounds [30, 31]

$$|d_e/e| < 1.6 \times 10^{-27} \text{ cm}, \quad |d_n/e| < 0.3 \times 10^{-25} \text{ cm} \quad (26)$$

for the electron and the neutron and also (13) for the mercury.

We present the results of the numerical analysis, in which we fix some parameters to the typical values such as $|\Lambda_1| = 50 \text{ TeV}$, $|\mu| = 100 \text{ GeV}$, and $\phi_\mu = -1.65$, for simplicity. It seems hard to have consistent solutions for $|\Lambda_1| \leq 35 \text{ TeV}$ and $|\Lambda_1| \geq 55 \text{ TeV}$. We adopt the value of ϕ_μ to introduce a seed for the large CP violation in the model. Since the one-loop RGEs do not make the phase run largely, this input value is equal to the weak scale one. We also tune the initial value of $|B|$ so as to realize $\tan \beta = 3.85$, since only the very restricted values of $\tan \beta$ like $3.5 - 4$ seem to be consistent with the EDM constraints. Under these settings, we search the parameter regions which satisfy the above mentioned phenomenological conditions by scanning the remaining parameters through the following ranges at the scale Λ :

$$\begin{aligned} 50 \text{ TeV} \leq |\Lambda_2| \leq 150 \text{ TeV}, \quad 80 \text{ GeV} \leq |A| \leq 500 \text{ GeV}, \\ 0 \leq \phi_3 \leq \pi, \quad -\pi \leq \phi_A \leq 0. \end{aligned} \quad (27)$$

Solutions are found for rather small values of $|A|$ such as $190 - 250 \text{ GeV}$, which satisfy $|A| > |\mu|$. The desired relation $|M_1| < |\mu| < |M_2|$ is also satisfied.

In Fig. 1 we show the allowed regions in the (ϕ_3, ϕ_A) plane for various values of $x(\equiv |\Lambda_2|/|\Lambda_1|)$, which satisfy all the EDM constraints of the electron, the neutron and the

⁵We use $m_t = 174.3 \text{ GeV}$ in this analysis.

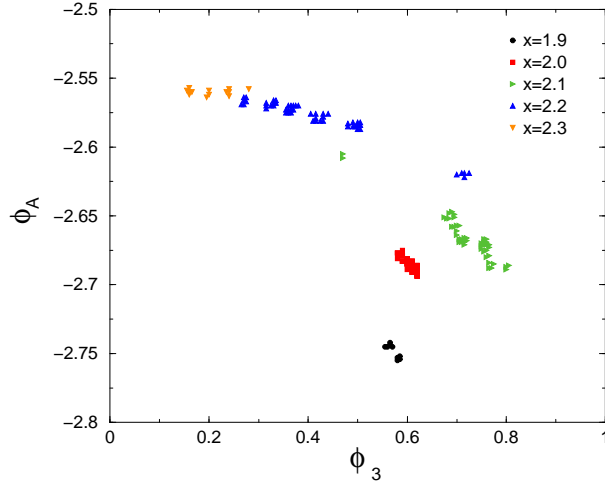


Fig. 1 Allowed regions in the (ϕ_3, ϕ_A) plane which satisfies the imposed conditions including the EDM constraints.

mercury atom. The imposed constraints restrict the regions of x to $1.9 \lesssim x \lesssim 2.3$. Since the obtained values of ϕ_3 yield small values for ϕ_1 as found from eq. (3), both sectors of the chargino and the neutralino seem to have no large influence of the phases in the gaugino masses. The EDM constraint of the electron is considered to be satisfied without its help. As long as the charginos are heavier than the neutralinos, the cancellation between them can occur. In fact, this is satisfied in the present solutions. On the other hand, the phase ϕ_3 of the gluino mass affects the EDMs of the neutron and the mercury atom through the gluino contribution. It happens to cause the cancellation for the EDM of the neutron and the mercury atom. In fact, the values of $\phi_3 - \phi_A$ obtained here can bring the suppression for the gluino contribution as found in eq. (25). This seems to suggest that the CP phases in the gaugino sector play the crucial role to satisfy the EDM constraints even in the case of the large ϕ_A and ϕ_μ .

In order to show the features of the SUSY breaking for these solutions, we show the mass spectrum of some superpartners as a function of x in Fig. 2. They are determined through the values of $|\Lambda_1|$ and $|\Lambda_2|$ as found in eqs. (2) and (4). For the sfermion masses \tilde{m}_t , \tilde{m}_b and \tilde{m}_τ , we plot smaller mass eigenvalues. The mass ratio of the chargino to the neutralino can be much larger than that in the ordinary GMSB case ($x = 1$). It is also remarkable that the gluino can be lighter than the squarks. The neutralino is the lightest superpartners except for the gravitino. The mass of the charged Higgs scalar takes its

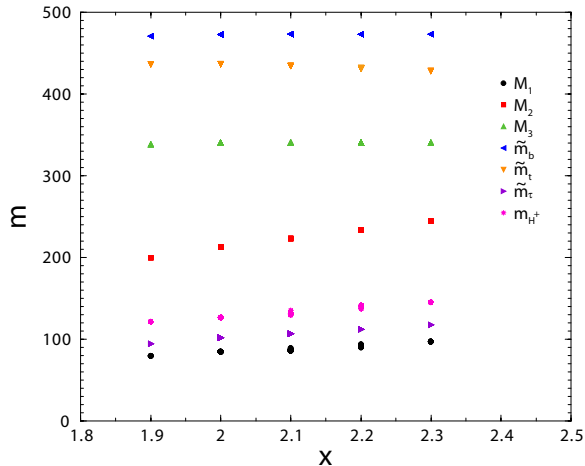


Fig. 2 The mass spectrum of superpartners at the weak scale as the functions of x .

value in the range of 120 – 150 GeV.

3.3 Phenomenology in the Higgs sector

The allowed parameter regions obtained from the EDM constraints generally require small values for $\tan\beta$. However, as is well known in the CP conserving case, the small $\tan\beta$ predicts the small value of the lightest neutral Higgs mass in the MSSM. Then it can be a serious obstacle to the present solutions for the EDM constraints. It is an important issue to check whether our model can be consistent with the constraints from the present Higgs search [25]. In various works [11, 12, 13], it has been shown that the CP phases in the SUSY breaking parameters could largely change both the Higgs mass eigenvalues and their couplings to the gauge bosons and the fermions. It happens due to the mixings among the CP-even and CP-odd Higgs scalars. In the recent analysis of the CP violating benchmark model CPX with a certain top quark mass, the combined LEP data seem to give no universal lower bound for the lightest neutral Higgs mass, although they can restrict the $\tan\beta$ to be larger than 2.6 [25]. In the present model, the similar feature may also be found for the parameter region derived from the EDM constraints, and it can be consistent with the present experimental data for the Higgs sector.

In order to study this aspect, we follow the one-loop effective potential method discussed in [12], in which the one-loop effective potential is expanded by the operators up to the fourth order and the effective Higgs quartic couplings are analytically determined.

Our EDM study suggests that the small $\tan\beta$ is favorable and also both $|A|$ and $|\mu|$ tend to be smaller than the soft scalar masses of the left- and right-handed stops. These features seem to make the usage of this method validate for the present analysis. In our model the gaugino masses are non-universal and then there can be physical CP phases in the gaugino masses in addition to those in the μ and A -parameters. This is the different situation from that in [12]. The gaugino phases could contribute to the one-loop effective potential mainly through the neutralino and chargino loops. However, since these CP violating corrections to the effective potential are considered to be smaller than the one coming from the stop contribution, we neglect them in this study, and we directly apply the formulas in [12] to this analysis.

In the following part, we focus our study on the mass eigenvalues of the Higgs scalars and the couplings between the Higgs scalars and the gauge bosons. They can be represented by using the Higgs quartic effective couplings λ_{1-7} . These definitions and their analytical formulas [12] are presented in the appendix. If we impose the potential minimum conditions, we can write the neutral Higgs mass matrix in the form as

$$\mathcal{M}_0^2 = \begin{pmatrix} \mathcal{M}_S^2 & (\mathcal{M}_{PS}^2)^T \\ \mathcal{M}_{PS}^2 & M_a^2 \end{pmatrix}, \quad (28)$$

where \mathcal{M}_S^2 is a 2×2 mass matrix for the CP-even Higgs scalars and \mathcal{M}_{PS}^2 is a 1×2 matrix representing the mixing among the CP-odd and CP-even Higgs scalars. These sub-matrices can be expressed as

$$\begin{aligned} \mathcal{M}_S^2 &= M_a^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + 2v^2 \times \\ &\begin{pmatrix} -2(\lambda_1 c_\beta^2 + \text{Re}(\lambda_5 e^{2i\xi}) s_\beta^2 + \text{Re}(\lambda_6 e^{i\xi}) s_\beta c_\beta) & \lambda_{34} s_\beta c_\beta + \text{Re}(\lambda_6 e^{i\xi}) c_\beta^2 + \text{Re}(\lambda_7 e^{i\xi}) s_\beta^2 \\ \lambda_{34} s_\beta c_\beta + \text{Re}(\lambda_6 e^{i\xi}) c_\beta^2 + \text{Re}(\lambda_7 e^{i\xi}) s_\beta^2 & -2(\lambda_2 s_\beta^2 + \text{Re}(\lambda_5 e^{2i\xi}) c_\beta^2 + \text{Re}(\lambda_7 e^{i\xi}) s_\beta c_\beta) \end{pmatrix}, \\ \mathcal{M}_{PS}^2 &= 2v^2 \begin{pmatrix} \text{Im}(\lambda_5 e^{2i\xi}) s_\beta + \text{Im}(\lambda_6 e^{i\xi}) c_\beta & \text{Im}(\lambda_5 e^{2i\xi}) c_\beta + \text{Im}(\lambda_7 e^{i\xi}) s_\beta \end{pmatrix}, \end{aligned} \quad (29)$$

where $\lambda_{34} = \lambda_3 + \lambda_4$, $s_\beta = \sin\beta$ and $c_\beta = \cos\beta$. M_a^2 corresponds to the physical mass of the CP-odd Higgs scalar in the CP conserving MSSM, and it can be written as

$$M_a^2 = \frac{1}{s_\beta c_\beta} \left[\text{Re}(m_{12}^2 e^{i\xi}) + 2v^2 \left\{ 2\text{Re}(\lambda_5 e^{2i\xi}) s_\beta c_\beta + \frac{1}{2}\text{Re}(\lambda_6 e^{i\xi}) c_\beta^2 + \frac{1}{2}\text{Re}(\lambda_7 e^{i\xi}) s_\beta^2 \right\} \right]. \quad (30)$$

The mass of the charged Higgs scalars can be expressed as

$$M_{H^\pm}^2 = \frac{1}{s_\beta c_\beta} \left[\text{Re}(m_{12}^2 e^{i\xi}) \right]$$

$$+ 2v^2 \left\{ \frac{1}{2} \lambda_4 s_\beta c_\beta + \text{Re}(\lambda_5 e^{2i\xi}) s_\beta c_\beta + \frac{1}{2} \text{Re}(\lambda_6 e^{i\xi}) c_\beta^2 + \frac{1}{2} \text{Re}(\lambda_7 e^{i\xi}) s_\beta^2 \right\}. \quad (31)$$

The Higgs couplings to the gauge bosons are also changed from those in the CP conserving case. This occurs due to the mixing among the CP-even and CP-odd Higgs scalars, which is induced by \mathcal{M}_{PS}^2 . The interaction Lagrangian for the mass eigenstates of the Higgs scalars H_i is found to be expressed as

$$\begin{aligned} \mathcal{L}_{HVV} &= g_2 M_W \sum_{i=1}^3 g_{H_i VV} \left(H_i W_\mu^+ W^{-\mu} + \frac{1}{2c_W^2} H_i Z_\mu Z^\mu \right), \\ \mathcal{L}_{HHZ} &= \frac{g_2}{2c_W} \sum_{j>i=1}^3 g_{H_i H_j Z} \left(H_i \overleftrightarrow{\partial}_\mu H_j \right) Z^\mu, \\ \mathcal{L}_{HH^\pm W^\mp} &= \frac{g_2}{2} \sum_{i=1}^3 \left[g_{H_i H^- W^+} \left(H_i i \overleftrightarrow{\partial}_\mu H^- \right) W^{+\mu} + h.c. \right]. \end{aligned} \quad (32)$$

In these interaction Lagrangians for the Higgs scalars, each coupling normalized to the value in the standard model can be written as

$$\begin{aligned} g_{H_i VV} &= c_\beta O_{1i} + s_\beta O_{2i}, \quad (V = W^\pm, Z) \\ g_{H_i H_j Z} &= O_{3i}(c_\beta O_{2j} - s_\beta O_{1j}) - O_{3j}(c_\beta O_{2i} - s_\beta O_{1i}), \\ g_{H_i H^- W^+} &= c_\beta O_{2i} - s_\beta O_{1i} + i O_{3i}, \end{aligned} \quad (33)$$

where O_{ij} is the element of the orthogonal matrix which relates the mass eigenstates H_i to the weak eigenstates. It is defined as the diagonalization matrix for \mathcal{M}_0^2 in such a way as

$$O^T \mathcal{M}_0^2 O = \text{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2), \quad (34)$$

where the mass eigenvalues $m_{H_i}^2$ for the eigenstates H_i satisfy the relation such as $m_{H_1}^2 \leq m_{H_2}^2 \leq m_{H_3}^2$. Since there are the following relations among these neutral Higgs couplings:

$$g_{H_k VV} = \varepsilon_{ijk} g_{H_i H_j Z}, \quad \sum_{i=1}^3 g_{H_i VV}^2 = 1, \quad (35)$$

all of the couplings of the neutral Higgs scalars to the gauge bosons can be completely determined by the two values of $g_{H_i ZZ}$, for example [32].

As mentioned already, the CP violating effect in the Higgs sector appears through the mixing \mathcal{M}_{PS}^2 between the CP-even and CP-odd Higgs scalars. If we use the analytic formulas for the quartic couplings $\lambda_{5,6,7}$ in the appendix, we find that the order of these

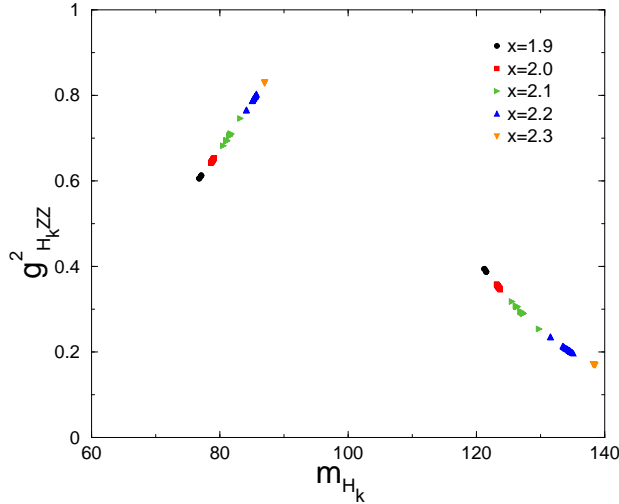


Fig. 3 The mass eigenvalues and the coupling constants with the gauge bosons of the neutral Higgs scalars H_1 and H_3 .

off-diagonal elements are estimated as

$$\mathcal{M}_{SP}^2 \simeq O\left(\frac{m_t^4 \text{Im}(A\mu)}{v^2 64\pi^2 M_S^2}\right) = O\left(\frac{v^2 |A| |\mu| \sin \phi_{CP}}{64\pi^2 M_S^2} \left(\frac{\tan^2 \beta}{1 + \tan^2 \beta}\right)^2\right), \quad (36)$$

where $\phi_{CP} = \phi_A + \phi_\mu$ which is the measure for the CP violation in the Higgs sector. If $\text{Im}(A\mu)$ can have large values, they can be so large as to have crucial effects on the composition of the mass eigenstates of the neutral Higgs scalars. Thus, the larger values of $|\mu|$, $|A|$ and ϕ_{CP} constitute the interesting parameter regions, in which the CP violating effects on the Higgs sector are substantial. On the other hand, as discussed in [12], the CP violating effects on the Higgs sector also tend to be enhanced in the case that the charged Higgs mass M_{H^\pm} takes a small value.⁶ In the present model, A , B and μ are free parameters. Since they are not directly related to other SUSY breaking parameters such as the gaugino masses and the sfermion masses, we can study their interesting regions without making large influence on the mass spectrum of the gauginos and the sfermions as long as $\tilde{m}_f > |A|, |\mu|$ is satisfied. However, we should note that the EDM constraints tend to favor the small values of $\tan \beta$, $|A|$ and $|\mu|$ as partially seen in eq. (25), for example. Thus, the EDM constraints may make the CP violating effects in the Higgs sector small

⁶This is expected to be realized for the case of small value of $m_{12}^2 (= B\mu)$. However, if the top quark mass is larger, the CP violating effect seems to appear independently of the charged Higgs mass [25].

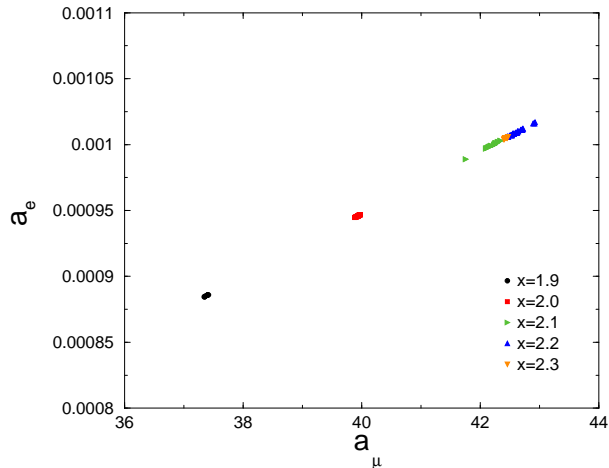


Fig. 4 The SUSY contributions to the $g - 2$ of the muon and the electron expected for the obtained solutions.

even in the case with the large ϕ_{CP} . Although the cases where $|A|$ and $|\mu|$ are not large but ϕ_{CP} is $O(1)$ seem to be promising in the present context, the situation is subtle and the detailed numerical study is required to clarify this point.

We calculate both the Higgs mass eigenvalues and the Higgs couplings for the parameter sets obtained in the previous part. In Fig. 3 we plot the mass eigenvalues of the neutral Higgs scalars H_k and their coupling constants $g_{H_k ZZ}^2$ with the gauge bosons. Both the lightest neutral Higgs scalar H_1 and the heaviest one H_3 are plotted in the same figure for each value of x . Since eq. (35) is satisfied among the couplings, $g_{H_2 ZZ}^2$ is negligible in the present case. The mass eigenvalues m_{H_k} are the increasing functions of x . If we combine this figure with Fig. 1, we can see that they are affected largely by the phase ϕ_{CP} . Since these Higgs mass eigenvalues take rather small values, the model might be considered to have been already excluded by the Higgs search at the LEP. However, the Higgs couplings are also influenced largely by this ϕ_{CP} . as observed in Fig. 3. Figure 3 shows that the H_1 coupling $g_{H_1 ZZ}^2$ can be much smaller than the MSSM one. The values of m_{H_1} and $g_{H_1 ZZ}^2$ shown in Fig. 3 seem to be marginal against the LEP2 data [25].

We could only say that our solutions for the EDM constraints might be consistent with the present Higgs phenomenology on the basis of our analysis. However, our results suggest that the validity of the model can be checked if the new experiments start at the LHC, anyway. Our analysis can also give several predictions for the relevant physical quantities. As a good example, we estimate the SUSY contributions a_μ and a_e to the

anomalous magnetic moment of the muon and the electron. The results are shown in Fig. 4. Both of a_μ and a_e are plotted in the unit of 10^{-11} . This predicted values of the muon $g - 2$ seem to be in the interesting regions for the present experimental data.

4 Summary

Non-universality of the gaugino masses can potentially cause various interesting phenomenology at the weak scale. We have considered the extended gauge mediation SUSY breaking scenario as a concrete example which could realize the non-universal gaugino masses. In this model the CP phases can remain in the gaugino sector as the physical phases after the R -transformation. In addition to this aspect, the model has several features different from the usual MSSM or the ordinary gauge mediation SUSY breaking. For example, the $SU(2)_L$ non-singlet superpartners tend to be heavier than the $SU(2)_L$ singlet ones whether they are colored or not. The right-handed stop becomes rather light and the neutralino can be lighter than the stau. These features can affect various phenomenology to give the different results from the ordinary MSSM.

We have calculated the effect of the CP phases on the EDM of the mercury atom, the electron and the neutron by solving the RGEs for the soft SUSY breaking parameters. As a result of this analysis, we have found that the experimental bounds for these EDMs could be simultaneously satisfied without assuming the heavy superpartners with the mass of $O(1)$ TeV even in the case that the soft SUSY breaking parameters have the large CP phases. The effective cancellation among the contributions from the gluino, the neutralino, and the chargino makes them possible to satisfy the experimental constraints. In this cancellation, the CP phases in the gaugino sector seem to play the crucial role. Although this kind of phenomena have already been suggested in several works, we have shown this in the concrete model with the definite spectrum of the superpartners.

The Higgs sector could also be affected by the the existence of the large CP phases in the soft SUSY breaking parameters. Since the CP-even Higgs scalars mix with the CP-odd Higgs scalar, the lightest neutral Higgs mass and its couplings to the gauge bosons could be largely modified from those in the CP invariant case. We have studied these aspects in the parameter regions where the EDM constraints are satisfied. From this study, we have found that our model might be consistent with the present data obtained from the

Higgs search at the LEP2. The validity of the model will be checked at the LHC.

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Appendix

The effective Lagrangian which describes the most general CP-violating Higgs potential of the MSSM is given by

$$\begin{aligned}
\mathcal{L} = & \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) + m_{12}^2(\Phi_1^\dagger\Phi_2) + m_{12}^{*2}(\Phi_2^\dagger\Phi_1) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 \\
& + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_5^*(\Phi_2^\dagger\Phi_1)^2 \\
& + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1), \quad (37)
\end{aligned}$$

where $\Phi_{1,2}$ are related to the scalar components $H_{1,2}$ of the Higgs superfields $\hat{H}_{1,2}$ through $H_1 = i\tau_2\Phi_1^*$ and $H_2 = \Phi_2$. At the tree level, coefficients in eq. (37) are represented as

$$\begin{aligned}
\mu_1^2 = -m_1^2 - |\mu|^2, \quad \mu_2^2 = -m_2^2 - |\mu|^2, \quad m_{12}^2 = B\mu, \\
\lambda_1 = \lambda_2 = -\frac{1}{8}(g_2^2 + g_1^2), \quad \lambda_3 = -\frac{1}{4}(g_2^2 - g_1^2), \quad \lambda_4 = \frac{1}{2}g_2^2, \\
\lambda_5 = \lambda_6 = \lambda_7 = 0. \quad (38)
\end{aligned}$$

Taking account of radiative corrections due to the trilinear Yukawa couplings between the Higgs scalars and stops/sbottoms, the quartic couplings $\lambda_{5,6,7}$ generally have complex nonzero values. If we assume that M_S is a SUSY breaking scale, analytic expressions of these quartic couplings are given by [12],

$$\begin{aligned}
\lambda_1 = & -\frac{g_2^2 + g_1^2}{8} \left(1 - \frac{3}{8\pi^2} h_b^2 t \right) \\
& - \frac{3}{16\pi^2} h_b^4 \left[t + \frac{1}{2} X_b + \frac{1}{16\pi^2} \left(\frac{3}{2} h_b^2 + \frac{1}{2} h_t^2 - 8g_3^2 \right) (X_b t + t^2) \right] \\
& + \frac{3}{192\pi^2} h_t^2 \frac{|\mu|^4}{M_S^4} \left[1 + \frac{1}{16\pi^2} (9h_t^2 - 5h_b^2 - 16g_3^2) t \right], \\
\lambda_2 = & -\frac{g_2^2 + g_1^2}{8} \left(1 - \frac{3}{8\pi^2} h_t^2 t \right) \\
& - \frac{3}{16\pi^2} h_t^4 \left[t + \frac{1}{2} X_t + \frac{1}{16\pi^2} \left(\frac{3}{2} h_t^2 + \frac{1}{2} h_b^2 - 8g_3^2 \right) (X_t t + t^2) \right] \\
& + \frac{3}{192\pi^2} h_b^2 \frac{|\mu|^4}{M_S^4} \left[1 + \frac{1}{16\pi^2} (9h_b^2 - 5h_t^2 - 16g_3^2) t \right], \\
\lambda_3 = & -\frac{g_2^2 - g_1^2}{8} \left[1 - \frac{3}{16\pi^2} (h_t^2 + h_b^2) t \right] \\
& - \frac{3}{8\pi^2} h_t^2 h_b^2 \left[t + \frac{1}{2} X_{tb} + \frac{1}{16\pi^2} (h_t^2 + h_b^2 - 8g_3^2) (X_{tb} t + t^2) \right] \\
& - \frac{3}{96\pi^2} h_t^4 \left(\frac{3|\mu|^2}{M_S^2} - \frac{|\mu|^2 |A_t|^2}{M_S^4} \right) \left[1 + \frac{1}{16\pi^2} (6h_t^2 - 2h_b^2 - 16g_3^2) t \right], \\
& - \frac{3}{96\pi^2} h_b^4 \left(\frac{3|\mu|^2}{M_S^2} - \frac{|\mu|^2 |A_b|^2}{M_S^4} \right) \left[1 + \frac{1}{16\pi^2} (6h_b^2 - 2h_t^2 - 16g_3^2) t \right],
\end{aligned}$$

$$\begin{aligned}
\lambda_4 &= \frac{g_2^2}{2} \left[1 - \frac{3}{16\pi^2} (h_t^2 + h_b^2) t \right] \\
&\quad + \frac{3}{8\pi^2} h_t^2 h_b^2 \left[t + \frac{1}{2} X_{tb} + \frac{1}{16\pi^2} (h_t^2 + h_b^2 - 8g_3^2) (X_{tb}t + t^2) \right] \\
&\quad - \frac{3}{96\pi^2} h_t^4 \left(\frac{3|\mu|^2}{M_S^2} - \frac{|\mu|^2 |A_t|^2}{M_S^4} \right) \left[1 + \frac{1}{16\pi^2} (6h_t^2 - 2h_b^2 - 16g_3^2) t \right], \\
&\quad - \frac{3}{96\pi^2} h_b^4 \left(\frac{3|\mu|^2}{M_S^2} - \frac{|\mu|^2 |A_b|^2}{M_S^4} \right) \left[1 + \frac{1}{16\pi^2} (6h_b^2 - 2h_t^2 - 16g_3^2) t \right], \\
\lambda_5 &= \frac{3}{192\pi^2} h_t^4 \frac{\mu^2 A_t^2}{M_S^4} \left[1 + \frac{1}{16\pi^2} (6h_t^2 - 2h_b^2 - 16g_3^2) t \right], \\
&\quad + \frac{3}{192\pi^2} h_b^4 \frac{\mu^2 A_b^2}{M_S^4} \left[1 + \frac{1}{16\pi^2} (6h_b^2 - 2h_t^2 - 16g_3^2) t \right], \\
\lambda_6 &= -\frac{3}{96\pi^2} h_t^4 \frac{|\mu|^2 \mu A_t}{M_S^4} \left[1 + \frac{1}{16\pi^2} \left(\frac{15}{2} h_t^2 - \frac{7}{2} h_b^2 - 16g_3^2 \right) t \right], \\
&\quad + \frac{3}{96\pi^2} h_b^4 \frac{\mu}{M_S} \left(\frac{6A_b}{M_S} - \frac{|A_b|^2 A_b}{M_S^3} \right) \left[1 + \frac{1}{16\pi^2} \left(\frac{9}{2} h_b^2 - \frac{1}{2} h_t^2 - 16g_3^2 \right) t \right], \\
\lambda_7 &= -\frac{3}{96\pi^2} h_b^4 \frac{|\mu|^2 \mu A_b}{M_S^4} \left[1 + \frac{1}{16\pi^2} \left(\frac{15}{2} h_b^2 - \frac{7}{2} h_t^2 - 16g_3^2 \right) t \right], \\
&\quad + \frac{3}{96\pi^2} h_t^4 \frac{\mu}{M_S} \left(\frac{6A_t}{M_S} - \frac{|A_t|^2 A_t}{M_S^3} \right) \left[1 + \frac{1}{16\pi^2} \left(\frac{9}{2} h_t^2 - \frac{1}{2} h_b^2 - 16g_3^2 \right) t \right], \quad (39)
\end{aligned}$$

In these formulas, the following definitions are used:

$$\begin{aligned}
t &= \ln \left(\frac{M_S^2}{\bar{m}_t^2} \right), \quad h_t = \frac{m_t(\bar{m}_t)}{v \sin \beta}, \quad h_b = \frac{m_b(\bar{m}_t)}{v \cos \beta}, \\
X_t &= \frac{2|A_t|^2}{M_S^2} \left(1 - \frac{|A_t|^2}{12M_S^2} \right), \quad X_b = \frac{2|A_b|^2}{M_S^2} \left(1 - \frac{|A_b|^2}{12M_S^2} \right), \\
X_{tb} &= \frac{|A_t|^2 + |A_b|^2 + 2\text{Re}(A_b^* A_t)}{2M_S^2} - \frac{|\mu|^2}{M_S^2} - \frac{||\mu|^2 - A_b^* A_t|^2}{6M_S^4}, \quad (40)
\end{aligned}$$

where \bar{m}_t is the pole mass of the top quark, which can be related to the running mass m_t as

$$m_t(\bar{m}_t) = \frac{\bar{m}_t}{1 + \frac{4}{3\pi} \alpha_3(\bar{m}_t)}. \quad (41)$$

We assume that the SUSY breaking scale M_S^2 is defined as the arithmetic average of the squared stop mass eigenvalues in the numerical calculation of the Higgs sector.

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