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Abstract

We study soft scalar masses in comparison with gaugino masses in 4-dimensional string models. In general non-universal soft masses are derived in orbifold models. We give conditions on modular weights which lead to a large non-degeneracy in the soft scalar mass spectrum. This non-universality is applied to the unification of gauge coupling constants in the minimal string model.

1. Introduction

Supersymmetric models are very interesting as a unified theory. Local supersymmetry (SUSY) breaking induces soft SUSY breaking terms, such as gaugino masses, scalar masses and trilinear (A-terms) and bilinear (B-terms) couplings of scalar fields in global SUSY models [1]. The values of these soft terms determine the phenomenological properties of the models.

Superstring theories are the only known candidates for the unified theory when we take gravity into account. By now starting from 4-dim string models like Calabi-Yau models [2], orbifold models [3–6] and so on, we know a Kähler potential and a gauge kinetic function of supergravity as their effective theories. In recent papers [7–10] the soft SUSY breaking terms derived from superstring theories are studied assuming that nonvanishing F-terms of the dilaton field S and moduli fields cause the SUSY breaking. In these works the soft scalar masses are found to be non-universal at the string scale $M_{st} = 3.73 \times 10^{17}$GeV [11], although one usually assumes the universality of all the soft terms in the study of SUSY models. This non-universality affects the phenomenological features [12,13]. For example, in Ref. [12] it is shown that the unification scale of SU(3) and SU(2) gauge couplings in the minimal supersymmetric standard model (MSSM) is sensitive to a strong non-degeneracy among soft scalar masses. In general the non-universality raises the unification scale. This seems desirable for superstring unification.

However, the orders of magnitude of soft scalar masses derived in Refs. [8,10] do not seem to be so different from each other. In Ref. [10] it is assumed that only the dilation field S and an overall modulus field T contribute to the SUSY breaking. The unknown direction of the goldstino field is parametrized in the S–T space by the goldstino angle θ. In the general orbifold models there are three independent mod-

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uli fields. In this paper we study the general models in which three moduli fields T_i $(i = 1,2,3)$ as well as S can contribute to the SUSY breaking. We discuss the realization of the hierarchical soft scalar masses whose orders of magnitudes are different from one another. Such non-universal soft scalar masses are also applied to the study of gauge coupling unification in a minimal string model [7,14–16]. Here the minimal string model means a string vacuum which has the same massless spectrum as the MSSM.

This paper is organized as follows. In Section 2 we review the soft SUSY breaking terms obtained in Ref. [9] and also their parametrization following Ref. [10]. In Section 3 we reformulate the soft terms by taking account of the three moduli fields. Using them, it is shown that we can obtain scalar masses whose orders of magnitudes are different from one another. Conditions to yield such hierarchical soft masses are also given. In Section 4 such a non-universality is applied to the minimal string unification. Using threshold corrections of string massive modes [17,18], we investigate whether orbifold models can realize the observed low energy values of gauge couplings. Section five is devoted to conclusions and discussions.

2. Soft masses

In this section, we review the derivation of soft SUSY breaking terms in the superstring models following Ref. [9]. Here we assume that the SUSY breaking occurs only due to the non-vanishing F-terms of the dilaton S field and the moduli fields T^i $(i = 1–3)$, where T^i corresponds to the ith one among the three moduli of the 6-dim orbifolds [3–6]. We represent S and T^i by Φ^m $(m = 0–3)$, where Φ^0 is S and $\Phi^1 = T^1$, etc.. $N = 1$ supergravity theories are characterized by the Kähler potential K, the superpotential W and the gauge kinetic function f_a. The Kähler potential and the superpotential are expressed as follows,

$$K = \kappa^{-2}\hat{K}(\Phi,\bar{\Phi}) + K(\Phi,\bar{\Phi})_{I\bar{J}}Q^I\bar{Q}^J + (\tfrac{1}{2}H(\Phi,\bar{\Phi})_{IJ}Q^IQ^J + \text{h.c.}) + \cdots, \tag{2.1a}$$

$$W = \hat{W}(\Phi) + \tfrac{1}{2}\tilde{\mu}(\Phi)_{IJ}Q^IQ^J + \cdots, \tag{2.1b}$$

where $\kappa^2 = 8\pi/M_{\rm pl}^2$ and Q^I are chiral superfields. The ellipses stand for terms of higher orders in Q^I. Using these, we can write down the scalar potential V as follows,

$$V = \kappa^{-2}e^G[G_\alpha(G^{-1})^{\alpha\bar{\beta}}G_{\bar{\beta}} - 3\kappa^{-2}] + (\text{D-term}), \tag{2.2}$$

where $G = K + \kappa^{-2}\log\kappa^6|W|^2$ and the indices α and β denote Q^I as well as Φ^m. Hereafter we does not consider the D-term contribution to V. The gravitino mass $m_{3/2}$ is written as

$$m_{3/2} = \kappa^2 e^{\hat{K}/2}|\hat{W}|. \tag{2.3}$$

In (2.2) we take the flat limit $M_{\rm pl} \to \infty$ preserving $m_{3/2}$ fixed and then the soft scalar masses m_{IJ} for unnormalized fields Q_I are derived as

$$m_{I\bar{J}}^2 = m_{3/2}^2K_{I\bar{J}} - F^m\bar{F}^{\bar{n}}[\partial_m\partial_{\bar{n}}K_{I\bar{J}} - (\partial_{\bar{n}}K_{K\bar{J}})K^{K\bar{L}}(\partial_mK_{I\bar{L}})] + \kappa^2V_0K_{I\bar{J}}, \tag{2.4}$$

where the F^m are F-terms of Φ^m, the ∂_m denote $\partial/\partial\Phi^m$ and V_0 is the cosmological constant expressed as

$$V_0 = \kappa^{-2}(F^m\bar{F}^{\bar{n}}\partial_m\partial_{\bar{n}}\hat{K} - 3m_{3/2}^2). \tag{2.5}$$

Hereafter the gravitational coupling κ is set to one. The canonically normalized gaugino masses M_a are derived through the following equation,

$$M_a = \tfrac{1}{2}(\text{Re}\,f_a)^{-1}F^m\partial_mf_a, \tag{2.6}$$

where the subscript a represents a gauge group.

The orbifold models give the Kähler potential at the one-loop level as [18]

$$K = -\log Y - \sum_i \log(T^i + \bar{T}^i) + \prod_i (T^i + \bar{T}^i)^{n_I^i}Q^I\bar{Q}^{\bar{I}},$$

$$Y = S + \bar{S} - \sum_i \frac{\delta_{\rm GS}^i}{8\pi^2}\log(T^i + \bar{T}^i), \tag{2.7}$$

where $\delta_{\rm GS}^i$ are constants introduced through the Green-Schwarz mechanism to cancel duality anomalies [19,20,18] and n_I^i are modular weights corresponding to Q^I [21,7]. The untwisted sector associated with the jth plane has $n_I^i = -\delta_j^i$. The twisted sector with a twist v^i ($0 \leq v^i \leq 1, \sum_{i=1}^{3} v^i = 1$) has $n_I^i = v^i - 1$ for $v^i \neq 0$, and $n_I^i = 0$ for $v^i = 0$. Here the notation in Refs. [4,5] is used for the orbifolds and their twists except some permutation of elements v^i. The addition of oscillator modes changes the modular weights by one. We have also the following gauge kinetic function,

$$f_a = k_a S - \frac{1}{16\pi^2}\sum_i (b_a'^i - k_a \delta_{GS}^i)\log \eta(T^i)^4, \tag{2.8}$$

where the second term is a threshold correction due to string massive modes [17,18]. Here $b_a'^i$ is a duality anomaly coefficient and $\eta(T)$ is a Dedekind function. The level of each gauge group is denoted by k_a. In general the 4-dim string models lead to $k_a = 1$ for non-abelian gauge groups.

In Ref. [10] the investigation is done for the case of the overall modulus $T = T^i$. This case is characterized by $n = \sum_i n^i$, $b'_a = \sum_i b_a'^i$ and $\delta_{GS} = \sum_i \delta_{GS}^i$. Then the cosmological constant is given by

$$V_0 = -3m_{3/2}^2 + \frac{1}{Y^2}|F^0 - \frac{\delta_{GS}}{8\pi^2(T+\bar{T})}F^T|^2 + \frac{3}{(T+\bar{T})^2}(1 - \frac{\delta_{GS}}{24\pi^2 Y})|F^T|^2, \tag{2.9}$$

where F^T is the F-term corresponding to the overall modulus T. If $V_0 = 0$, one can parametrize the unknown F-terms by the goldstino angle θ as follows,

$$\frac{1}{Y}(F^0 - \frac{\delta_{GS}}{8\pi^2(T+\bar{T})}F^T) = \sqrt{3}m_{3/2}\sin\theta, \tag{2.10a}$$

$$\frac{\sqrt{3(1-a)}}{T+\bar{T}}F^T = \sqrt{3}m_{3/2}\cos\theta, \tag{2.10b}$$

where $a = \delta_{GS}/24\pi^2 Y$. To get the physical scalar mass from (2.3), we need to normalize the fields Q_I canonically. Here we assume the kinetic term to be diagonal. Then using eqs. (2.4) and (2.6)–(2.8), the masses of the scalar superpartners and the gauginos are expressed as

$$m^2 = m_{3/2}^2(1 + \frac{n}{1-a}\cos^2\theta), \tag{2.11a}$$

$$M_a = \frac{\sqrt{3}}{\mathrm{Re}\, f_a}m_{3/2}(k_a \mathrm{Re}\, S \sin\theta + \cos\theta \frac{b'_a - k_a\delta_{GS}}{32\pi^3\sqrt{3(1-a)}}(T+\bar{T})\hat{G}_2(T,\bar{T})), \tag{2.11b}$$

where $\hat{G}_2(T,\bar{T})$ is the Eisenstein function [5];

$$\hat{G}_2(T,\bar{T}) = -4\pi\eta(T)^{-1}\frac{\partial\eta(T)}{\partial T} - \frac{2\pi}{T+\bar{T}}.$$

The states with no oscillator modes have $n = -1$ or -2. The different modular weights lead to the non-universal soft masses in (2.11a). However, most of these masses are of $O(m_{3/2})$. Even if the orders of magnitude of scalar masses could differ from one another at M_{st}, loop effects due to the large gaugino mass might dilute the difference at M_Z and result in the same order scalar masses $O(m_{3/2})$. In fact the gaugino masses are of order $m_{3/2}$ unless $\sin\theta = 0$. Therefore in order to produce a fairly strong non-degeneracy in the soft scalar masses, we are interested in the case with $\sin\theta = 0$. Unfortunately, in this case, matter fields with $n \leq -2$ are not allowed, because these modular weights lead to imaginary masses in (2.11a). If we restrict ourselves to the states with no oscillator modes, the soft scalar masses are completely universal. They are of order of $\sqrt{|a|}m_{3/2}$, where a should be negative to make this mass real. Then the soft scalar masses are of the same order in the overall modulus case. This might become a very severe constraint on SUSY models inspired by superstring theories.

The $n = 0$ states with oscillator modes seem to yield a non-universality in the mass spectrum of the scalar fields. However the presence of such states is restricted in some cases, as shown in Refs. [15,16]. Moreover Yukawa couplings of such states are often forbidden as renormalizable couplings. Therefore we study another possibility to cause a strongly non-degenerate soft scalar spectrum in the following section.

3. Soft masses in case of three moduli

We study a more general case by taking account of three independent moduli fields T^i instead of the

[5] Several kinds of modular functions are shown in Ref. [22]

overall modulus. In this case the cosmological constant V_0 is written as

$$V_0 = -3m_{3/2}^2 + \frac{1}{Y^2}|F^0 - \sum_{i=1}^{3} \frac{\delta_{GS}^i}{8\pi^2(T^i+\bar{T}^i)} F^i|^2 + \sum_{i=1}^{3} \frac{1-a_i}{(T^i+\bar{T}^i)^2}|F^i|^2, \tag{3.1}$$

where $a_i = \delta_{GS}^i/8\pi^2 Y$, and which is estimated as $|a_i| \ll 1$. Here we parametrize the unknown F-terms as follows,

$$\frac{1}{Y}(F^0 - \sum_i \frac{\delta_{GS}^i}{8\pi^2(T^i+\bar{T}^i)} F^i) = \sqrt{3} m_{3/2} \sin\theta, \tag{3.2a}$$

$$\frac{\sqrt{1-a_i}}{T^i+\bar{T}^i} F^i = \sqrt{3} m_{3/2} \cos\theta \Theta_i, \tag{3.2b}$$

where $\Theta_1 = \sin\theta' \sin\theta''$, $\Theta_2 = \sin\theta' \cos\theta''$ and $\Theta_3 = \cos\theta'$. Unless $V_0 = 0$, in (3.2) $m_{3/2}$ should be replaced by $Cm_{3/2}$, where $C^2 = 1 + V_0/3m_{3/2}^2$. In Ref. [23] it is indicated that V_0 should be negative when taking account of loop effects of the observable sector. If V_0 is negative, C is less than one. We can then write down the canonically normalized soft masses as

$$m^2 = m_{3/2}^2 C^2 (1 + \cos^2\theta \sum_{i=1}^{3} \frac{3n^i}{1-a_i} \Theta_i^2) + 2m_{3/2}^2 (C^2 - 1), \tag{3.3a}$$

$$M_a = \frac{\sqrt{3}}{\mathrm{Re}\, f_a} C m_{3/2} [k_a \mathrm{Re}\, S \sin\theta + \cos\theta \sum_{i=1}^{3} \frac{b_a'^i - k_a \delta_{GS}^i}{\sqrt{1-a_i}} D_i(T^i, \bar{T}^i) \Theta_i], \tag{3.3b}$$

where $D_i(T^i, \bar{T}^i) = (T^i + \bar{T}^i)\hat{G}_2(T^i, \bar{T}^i)/32\pi^3$. It is remarkable that the summation in (3.3b) is taken on the moduli contributing to the threshold corrections.

We are interested in the case with $m > M_a$, where the non-universality of the soft scalar masses is not diluted by the loop effects of gauginos. To investigate such cases, it is necessary to see what modular weights guarantee real soft scalar masses when $\sin\theta = 0$ and $a_i < 0$. We show these conditions for Z_N and $Z_N \times Z_M$ orbifold models in Tables 1 and 2, respectively. The first column of these tables shows the modular weights of the orbifold model that contains no oscillator modes. Here $C = 1$ is assumed. In these tables only the modular weights corresponding to the matter sector [24,4,6] are shown and the modular weights for the antimatter sector are omitted. The underlining represent any permutation of the elements. The first three rows in Table 1 correspond to the untwisted sector, which is omitted in Table 2, and the others correspond to the twisted sector. For the twisted sector all the modular weights with $\sum_i n_i = -1$ are allowed under certain conditions as shown in the tables. It is remarkable that the modular weight $-(\underline{5,5,2})/6$ is also allowed under certain conditions, although in the case of the overall modulus all the modular weights with $\sum_i n_i = -2$ are forbidden. In the Z_3 and Z_7 orbifold models, no twisted matter fields are allowed. The Z_4 and Z_8-I orbifolds have only the modular weight

Table 1
Conditions for real soft scalar masses in Z_N orbifolds. The first column shows the modular weights obtained for the Z_N orbifold models of the second column. An asterisk (*) in the second column represents all the Z_N orbifold models except Z_3 and Z_7. Modular weights forbidden under any condition are indicated by a dash (–) in the third column

Modular Weight	Orbifold	Condition
$(-1,0,0)$		$\sin^2\theta' \sin^2\theta'' \le 1/3$
$(0,-1,0)$		$\sin^2\theta' \cos^2\theta'' \le 1/3$
$(0,0,-1)$		$\cos^2\theta' \le 1/3$
$-(2,2,2)/3$	Z_3, Z_6-I	–
$-(3,3,2)/4$	Z_4, Z_8-I	–
$-(2,2,0)/4$	*	$\sin^2\theta' \le 2/3$
$-(5,5,2)/6$	Z_6-I,Z_{12}-I	$\sin\theta' = 0$
$-(5,3,4)/6$	Z_6-II	–
$-(4,0,2)/6$	Z_6-II	$\sin^2\theta'' \le 1/2$
$-(2,0,4)/6$	Z_6-II	$\sin^2\theta'(2-\sin^2\theta'') \ge 1$
$-(\underline{6,5,3})/7$	Z_7	–
$-(\underline{7,3},6)/8$	Z_8-I	–
$-(\underline{7,5},4)/8$	Z_8-II	–
$-(6,2,0)/8$	Z_8-II,Z_{12}-I	$\sin^2\theta'(1+2\sin^2\theta'') \le 4/3$
$-(2,6,0)/8$	Z_8-II,Z_{12}-I	$\sin^2\theta'(3-2\sin^2\theta'') \le 4/3$
$-(\underline{11,5},8)/12$	Z_{12}-I	–
$-(8,8,8)/12$,	Z_{12}-I	–
$-(\underline{11,7},6)/12$	Z_{12}-II	–
$-(10,2,0)/12$	Z_{12}-II	$\sin^2\theta'(1+4\sin^2\theta'') \le 2$
$-(9,9,6)/12$	Z_{12}-II	–
$-(8,4,0)/12$	Z_{12}-II	$\sin^2\theta'(1+\sin^2\theta'') \le 1$
$-(4,8,0)/12$	Z_{12}-II	$\sin^2\theta'(2-\sin^2\theta'') \le 1$
$-(2,10,0)/12$	Z_{12}-II	$\sin^2\theta'(5-4\sin^2\theta'') \le 2$

Table 2
Conditions for real soft scalar masses in $Z_N \times Z_M$ orbifolds. In the second column $*1$ represents $Z_2\times Z_2, Z_4\times Z_4, Z_2\times Z_4, Z_2\times Z_6, Z_2\times Z_6', Z_6\times Z_6$ and $*2$ represents $Z_3\times Z_3, Z_6\times Z_6, Z_3\times Z_6$. Modular weights forbidden under any condition are indicated by a dash (-) in the third column

Modular Weight	Orbifold	Condition
$-(0,1,1)/2$	$*1, Z_3\times Z_6$	$\sin^2\theta'\sin^2\theta''\geq 1/3$
$-(1,0,1)/2$	$*1$	$\sin^2\theta'\cos^2\theta''\geq 1/3$
$-(1,1,0)/2$	$*1$	$\sin^2\theta'\leq 2/3$
$-(0,2,1)/3$	$*2, Z_2\times Z_6$	$\cos^2\theta''\leq 1/2$
$-(0,1,2)/3$	$*2, Z_2\times Z_6$	$\sin^2\theta'(1+\sin^2\theta'')\geq 1$
$-(2,0,1)/3$	$*2$	$\sin^2\theta''\leq 1/2$
$-(2,2,2)/3$	$*2, Z_2\times Z_6'$	-
$-(2,1,0)/3$	$*2$	$\sin^2\theta'(\sin^2\theta''+1)\leq 1$
$-(1,0,2)/3$	$*2$	$\sin^2\theta'(\sin^2\theta''-2)\leq 1$
$-(1,2,0)/3$	$*2$	$\sin^2\theta'(2-\sin^2\theta'')\leq 1$
$-(0,3,1)/4$	$Z_2\times Z_4, Z_4\times Z_4$	$\sin^2\theta'(2-\sin^2\theta'')\leq 1/3$
$-(0,1,3)/4$	$Z_2\times Z_4, Z_4\times Z_4$	$\sin^2\theta'(2+\sin^2\theta'')\leq 5/3$
$-(2,3,3)/4$	$Z_2\times Z_2, Z_4\times Z_4$	-
$-(5,5,2)/6$	$Z_2\times Z_6', Z_6\times Z_6$	$\sin^2\theta'=0$
$-(2,5,5)/6$	$Z_2\times Z_6', Z_3\times Z_6, Z_6\times Z_6$	$\sin^2\theta'\sin^2\theta''=1$
$-(5,2,5)/6$	$Z_2\times Z_6', Z_6\times Z_6$	$\sin^2\theta'\cos^2\theta''=1$
$-(0,5,1)/6$	$Z_2\times Z_6, Z_3\times Z_6, Z_6\times Z_6$	$\sin^2\theta'(4-5\sin^2\theta'')\leq 1$
$-(0,1,5)/6$	$Z_2\times Z_6, Z_3\times Z_6, Z_6\times Z_6$	$\sin^2\theta'(4+\sin^2\theta'')\geq 3$
$-(3,5,4)/6$	$Z_2\times Z_6, Z_6\times Z_6$	-
$-(3,0,1)/4$	$Z_4\times Z_4$	$\sin^2\theta'(3\sin^2\theta''-1)\leq 1/3$
$-(3,3,2)/4$	$Z_4\times Z_4$	-
$-(3,1,0)/4$	$Z_4\times Z_4$	$\sin^2\theta'(1+3\sin^2\theta'')\leq 4/3$
$-(1,0,3)/4$	$Z_4\times Z_4$	$\sin^2\theta'(3-\sin^2\theta'')\geq 5/3$
$-(1,3,0)/4$	$Z_4\times Z_4$	$\sin^2\theta'(3-2\sin^2\theta'')\leq 4/3$
$-(4,5,3)/6$	$Z_3\times Z_3, Z_6\times Z_6$	-
$-(5,1,0)/6$	$Z_6\times Z_6$	$\sin^2\theta'(1+4\sin^2\theta'')\leq 2$
$-(1,0,5)/6$	$Z_6\times Z_6$	$\sin^2\theta'(5-\sin^2\theta'')\geq 3$
$-(1,5,0)/6$	$Z_6\times Z_6$	$\sin^2\theta'(5-4\sin^2\theta'')\leq 2$

$-(1,1,0)/2$ in the twisted sector as the allowed one. Further, all considerable modular weights in each orbifold are not allowed simultaneously. For example, modular weights $(0,0,-1)$ and $-(5,5,2)/6$ cannot guarantee the scalar masses to be real at the same time. In the Z_N orbifold models, however, all the modular weights with $n=-1$ are allowed simultaneously under the conditions as $\sin^2\theta'=2/3$ and $\sin^2\theta''=1/2$.

We can also easily obtain the conditions in the case of $C\neq 1$. For example, the modular weight $(-1,0,0)$ can derive the real scalar masses under the following condition,

$$\sin^2\theta'\sin^2\theta''\leq 1-\frac{2}{3C^2}. \tag{3.4}$$

Thus the modular weight $(-1,0,0)$ is forbidden in the case of $C^2<2/3$ ($V_0<-m_{3/2}^2$). If V_0 is negative, and thus $C<1$, the modular weight $-(5,5,2)/6$ is not allowed. In contrast with the $C=1$ case all the modular weights with $n=-1$ cannot satisfy the conditions for the real scalar masses simultaneously.

Here we study in more detail the soft scalar masses in the case of $\sin\theta=0$ and $C=1$. Using (3.3a) we find that the representative orders of magnitude of the scalar masses are $O(m_{3/2})$ and $O(\sqrt{|a_i|}m_{3/2})$. We have $a_i\approx 5.0\times\delta^i_{GS}\times 10^{-3}$ if we use $\alpha_X^{-1}\approx 25$ as the unified gauge coupling. As an example, we consider the scalar fields with modular weights $(-1,0,0)$, $(0,-1,0)$ and $-(1,1,0)/2$. Most of the orbifold models have these modular weights. We take the angles as $\sin\theta=0$, $\sin^2\theta'=1/3$ and $\cos^2\theta''=1$. Then the scalar fields with $n_i=-(1,1,0)/2$ and $(-1,0,0)$ have the soft masses $m^2=m_{3/2}^2/2$ and $m_{3/2}^2$, respectively. On the other hand, the mass corresponding to $n_i = (0,-1,0)$ is obtained as $m^2 = |a_2| m_{3/2}^2$. Thus we can derive the different orders of the non-degeneracy among the soft scalar masses by taking account of the three moduli fields. In the case with other values of C we can also obtain the large non-universality under certain angles θ' and θ'' for other combinations of the modular weights. The largest mass is of order of $m_{3/2}$. Such a situation is impossible to be realized in the case of the overall modulus with $\sin\theta = 0$.

It is notable that the mass of order of $m_{3/2}$ cannot be obtained when some combinations of the modular weights constrain the angles θ' and θ'' severely. For example, we take the case where the matter fields with $n_i = (0,0,-1)$ are included in addition to the above combination of modular weights. This case is only allowed when the conditions $\sin^2\theta' = 2/3$ and $\sin^2\theta'' = 1/2$ are satisfied, as mentioned before. These angles result in scalar masses of order of $\sqrt{|a_i|}m_{3/2}$ at most. Similarly the scalar mass of $n_i = -(5,5,2)/6$ is less than $O(\sqrt{|a_i|}m_{3/2})$, because matter fields are allowed at $\sin\theta' = 0$. It is difficult to derive the soft scalar masses of order of $m_{3/2}$ when the angles θ' and θ'' are constrained severely.

Next we estimate the gaugino masses. They should be small enough not to dilute the non-degeneracy among the soft scalar masses by their loop effects. In (3.3b) $D_i(T^i,\bar{T}^i)$ takes following values: $D_i(T^i,\bar{T}^i) = 1.5\times10^{-3}, 2.7\times10^{-2}, 6.0\times10^{-2}$ and 6.6×10^{-1} for $T^i = 1.2$, 5.0, 10 and 100, respectively. In (3.3b) the first term proportional to $\sin\theta$ contributes mainly to the gaugino masses in the case of $\sin\theta > O(10^{-3})$ and $T \sim O(1)$. This is because $k_a = 1$ and $\alpha_X^{-1} \approx 25$ lead to $k_a \mathrm{Re}\, S \sim \mathrm{Re}\, f_a \sim 2$. The condition $\sin\theta < O(1/10)$ should be satisfied in order to avoid the above mentioned dilution due to the gaugino loop effects. A large value of T^i like $T_i > O(100)$ seems undesirable to preserve non-universality among the soft scalar masses at M_Z.

4. Minimal string unification

In Ref. [12] it is shown that the unification scale M_X of the SU(2) and SU(3) gauge coupling constants is sensitive to the non-universality of the soft masses in the MSSM. In that paper M_X is estimated using the measured gauge coupling constants at M_Z. The unification scale M_X is raising in most cases of the non-universal scalar masses. Especially the highest M_X is realized in the case where all the doublet scalar fields under SU(2) are heavier than the singlet ones. This type of non-universality corresponds to Case III in Ref. [12]. In this section we apply the result found in the previous section to the minimal string unification. Here we concentrate on the case where all the doublet scalar fields are heavier than the singlet ones. Note that the gauge coupling of $\mathrm{U}(1)_Y$ α_1^{-1} is not always unified, at M_X, with the other couplings, because the string theories can predict not only $k_1 = 5/3$ but also other values.

The running gauge coupling constants α_a^{-1} of the MSSM at μ are expressed as follows [17,18],

$$\alpha_a^{-1}(\mu) = \alpha_{\mathrm{st}}^{-1} + \frac{b_a}{4\pi k_a}\log\frac{M_{\mathrm{st}}^2}{\mu^2} - \sum_i \frac{b_a'^i - k_a\delta_{\mathrm{GS}}^i}{4\pi k_a}\log[(T^i+\bar{T}^i)|\eta(T^i)|^4], \quad (4.1)$$

where $\alpha_{\mathrm{st}}^{-1}$ is the universal string coupling at M_{st} and b_a is the one-loop β-fuction coefficient of the MSSM, i.e., $b_3 = -3$, $b_2 = 1$ and $b_1 = 11$. The last term in (4.1) represents the threshold correction due to the string massive modes and the duality anomaly coefficient $b_a'^i$ is written as

$$b_a'^i = -C(G_a) + \sum T(\underline{R_a})(1+2n^i), \quad (4.2)$$

where $C(G_a)$ is a quadratic Casimir of the adjoint representation and $T(\underline{R_a})$ is the Dynkin index of the $\underline{R_a}$ representation, i.e., $T(\underline{R_a}) = C(\underline{R_a})\dim(\underline{R_a})/\dim(G_a)$. Using Eq. (4.1) we can derive the relation between M_X and M_{st} as [7]

$$8\log\frac{M_X}{M_{\mathrm{st}}} = \sum_i (b_3'^i - b_2'^i)\log[(T^i+\bar{T}^i)|\eta(T^i)|^4]. \quad (4.3)$$

It is remarkable that $\log[(T^i+\bar{T}^i)|\eta(T^i)|^4]$ is negative for any value of T^i. The unification scale M_X is always less than M_{st} under the condition that the soft masses are less than 10 TeV even in the non-universal case [12].

The Z_3 and Z_7 orbifold models do not have the T-dependent threshold corrections and thus these orbifolds cannot yield the minimal string models consistent with observation. For the other Z_N orbifold models except Z_6-II, only the third modulus T^3 contributes to the threshold correction. In these orbifold models, the duality anomaly coefficients should satisfy $b_3'^3 > b_2'^3$ in order to result in $M_X < M_{st}$. First of all we consider the case where the matter fields have the modular weights $-(1,1,0)/2$, $(-1,0,0)$ and $(0,-1,0)$, as discussed in the previous section. The third element of these modular weights is zero and we obtain $b_3'^3 - b_2'^3 = -2$. In this case we cannot realize the measured gauge couplings at M_Z.

In order to avoid such a situation, we need the modular weights with a non-vanishing third element, which are $(0,0,-1)$ and $-(5,5,2)/6$. The former belongs to the untwisted sector and the latter exists only in the Z_6-I and Z_{12}-I orbifold models. If the SU(2) doublet scalar fields have such modular weights, the difference $b_3'^3 - b_2'^3$ increases. Now we are considering the case where the doublet fields are heavier than the singlet fields. It seems desirable that the scalar fields associated with $n_i = (0,0,-1)$ or $-(5,5,2)/6$ correspond to the doublets and that their masses are of order of $m_{3/2}$. However we cannot derive the soft scalar masses of order $m_{3/2}$ for the modular weight $-(5,5,2)/6$. It is not suitable for the above scenario. Moreover the simultaneous presence of the modular weights $(0,0,-1)$ and $-(1,1,0)/2$ forbids the scalar fields with $(0,0,-1)$ to have masses of order $m_{3/2}$, because the angle θ' is constrained as $\sin^2\theta' = 2/3$. Therefore we cannot realize the minimal string models with the non-universal soft scalar masses of Case III using the twisted sectors of the Z_4 and Z_8-I orbifold models, where only $n = -(1,1,0)/2$ is allowed among the twisted sectors. The Z_6-I orbifold models are not promising, either. Although we can use only the untwisted sectors, it seems unrealistic that the massless spectrum consists of the untwisted sector only. The scalar fields with $n = (0,0,-1)$ can obtain masses of order $m_{3/2}$ under the presence of some twisted matter fields in the Z_8-II and Z_{12}-I, II orbifold models, because these orbifolds allow several types of the modular weights.

Next, we consider the Z_6-II orbifold models, where T^2 and T^3 contribute to the threshold corrections. They have several types of modular weights which have non-vanishing elements on the second and third ones. For example we take the modular weights $-(1,1,0)/2$ and $-(2,0,4)/6$, and assign the former to the doublet fields and the latter to the singlet fields. We assume that for the modular weight $n_i = (-1,-1,0)/2$ the scalar mass takes the value of $m_{3/2}/2$ and the other scalar mass vanishes in the case with $\sin^2\theta' = 1/2$ and $\sin^2\theta'' = 0$. The latter scalar fields gain a mass of order of the gaugino mass at M_Z through loop effects. If $T^2 = T^3$, we obtain $b_3'^2 + b_3'^3 - b_2'^2 - b_2'^3 = -2$ under the above assignment of the modular weights to the matter fields. Therefore we cannot have gauge couplings consistent with observation. Then we consider the case where T^2 dominantly contributes to the threshold correction, i.e., $T^2 > T^3$. In this case we obtain $b_3'^2 - b_2'^2 = 2$. The results of Ref. [12] show that the unification scale of Case III is estimated as $\log_{10} M_X(\mathrm{GeV}) = 17.0$, 17.1, 17.2 and 17.3 in the case where the doublet superpartners have masses of 1.3, 2.0, 3.2 and 5.0 TeV, respectively, while the gauginos and the singlet superpartners have masses of 100 GeV. In this case the doublet scalar masses correspond to $m_{3/2}/2$ and then we can easily estimate $m_{3/2}$. Using (4.3) we obtain the desirable values of T^2 as $T^2 = 7.5$, 6.5, 5.5 and 4.5 for $\log_{10} M_X(\mathrm{GeV}) = 17.0$, 17.1, 17.2 and 17.3, respectively. Further, from these values of T^2 we derive $D_2(T^2,\bar{T}^2) = 0.043$, 0.037, 0.030 and 0.024, respectively. If $\sin\theta < 10^{-2}$, we can estimate the gaugino mass of SU(2) as

$$M_2 = \frac{\sqrt{3}}{\mathrm{Re} f_a} m_{3/2} D_2(T^2,\bar{T}^2) \times (b_2'^2 - \delta_{GS}^2) \sin\theta' \cos\theta''. \tag{4.4}$$

Suppose that $\delta_{GS}^2 = -1$, then the values of $T^2 = 7.5$, 6.5, 5.5 and 4.5 lead to $M_2 = 68$, 91, 120 and 150 GeV, respectively, in the case of $\sin\theta' = 1/\sqrt{2}$ and $\cos\theta'' = 1$. The masses of the singlet superpartners are of the same order as the gaugino masses. These spectra are consistent with the ones assumed initially.

It is remarkable that the gaugino mass of SU(3) is different from the one of SU(2) by a factor $(b_3'^2 - \delta_{GS}^2)/(b_2'^2 - \delta_{GS}^2)$, which is equal to -1 in the above example. The gaugino masses are in general non-universal when $\sin\theta = 0$. Actually we can obtain large values of $b_3'^2 - b_2'^2$, e.g., $b_3'^2 - b_2'^2 > O(10)$ [15,16] and these values could lead to a large non-universality of the gaugino masses.

We can eliminate δ_{GS} and the T-dependent term of (4.1) using α_3, α_2 and α_1, so that we have [15]

$$k_1 = \frac{12\Delta b' \log(M_{st}^2/\mu^2) - 4B' \log(M_X^2/M_{st}^2) - 4\pi\Delta b' \alpha_{em}^{-1}(\mu)}{\Delta b' \log(M_{st}^2/\mu^2) - 4b_2'^2 \log(M_X^2/M_{st}^2) - 4\pi\Delta b' \alpha_2^{-1}(\mu)} - 1, \quad (4.5)$$

where $\Delta b' = b_3'^2 - b_2'^2$ and $B' = b_1'^2 + b_2'^2 = -2$. Eq. (4.5) can be used at μ where the SUSY is preserved. We take the example where the masses of the doublet scalar fields are equal to 3 TeV. We have $\alpha_3^{-1}(3\text{ TeV}) = 10$, $\alpha_2^{-1}(3\text{ TeV}) = 31$ and $\alpha_{em}^{-1}(3\text{ TeV}) = 125$ in Case III. In addition to these values, we use $M_X = 10^{17.2}$ GeV and $\mu = 3$ TeV so as to obtain $k_1 = 1.4$. This seems reasonable compared with the results of Refs. [15,16]. The minimal string unification with non-universal soft masses can be realized for other assignments of the modular weights in Z_6-II orbifold models. In the above discussion, we do not take into account the duality anomaly cancellation condition [18,7], which is used as another constraint for the realistic models.

The $Z_N \times Z_M$ orbifold models have a rich structure of modular weights and the three moduli fields contribute to the threshold corrections. They can derive the minimal string models under several types of assignments of the modular weights to the matter fields.

5. Conclusions

We have studied the soft scalar masses in comparison with the gaugino masses in the case where the three independent moduli fields as well as the dilaton field contribute to the SUSY-breaking. We have showed that the superstring theories can derive a different order of non-universality in the scalar partner spectrum. For such non-universal cases, we have investigated the conditions under which the modular weights are allowed. In addition the superstring theories can also obtain the non-universal gaugino masses.

The non-universality of the soft terms affects the phenomenological properties of the models. As an example we have studied the gauge coupling unification of the minimal string models with a certain type of non-universal soft masses. We have shown that the minimal string unification with non-universal scalar masses is realized in restricted cases. It is very important to investigate all the possible models systematically, as in Refs. [7,15,16]. In a similar way, other cases of non-universality can be studied. If we detect non-universality of the superpartner spectrum in the future, we may constrain promising models in the minimal string models. It is easy to extend this analysis to the case of extended SUSY models.

Other phenomenological properties are influenced by the non-universality. For example, the electric dipole moment of the neutron is examined in Ref. [25]. It is very worthwhile to study which phenomenological features are sensitive to the non-universality of the soft terms. That might lead us to an indirect determination of the superpartner spectrum.

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