Behavioural bumpiness: modelling the impact of behavioural heterogeneity in macro level systems

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Abstract

This thesis presents two examples of the impact of behavioural phenomena at macro scale within a social system. In the context of an asset market we study the impact of leverage and the impact that borrowing behaviour has for price dynamics. We find in turn that, the nature of that decision making whilst subtle, has significant consequences for the stability of leveraged asset markets. We also examine the role behaviour plays in the spread of a disease through a population using a very similar technique. In this instance we identify a parameter set that characterises a set of behaviours not previously identified in the literature, that characterises both substantially different behaviours and would appear to better align with field data. The existence of such alternative explanations for macro level dynamics has significant consequences for policy makers and researchers.

Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Copyright Statement

Preface

My own personal experience has been vital both in my motivation for undertaking this research and in informing many of the assumptions and conclusions that are drawn here. As an undergraduate I studied natural sciences, specialising in physics. During this time I authored a paper on decay curves and forecasting the estimated remaining yield of fracked shale oil and gas wells in the United States. This work formed both an introduction to economics and also to non linear estimation techniques. After my undergraduate degree I worked at the fund management firm M&G Investments as an analyst on the Real Estate Debt funds. This experience gave me significant insight into the operation of markets in practice. In particular the decision making process involved. Throughout this experience I was struck by the extent to which behavioural factors were factored into the investment process, an insight that has significantly informed this body of research.

Following my time at M&G I completed masters courses in Economics at the University of Manchester and the University of Amsterdam. As a consequence of my prior experiences I chose to focus my dissertations on the type of behaviourally non linear macro problems that had first interested me. This included estimating the level of behavioural heterogeneity for the FTSE 100 as well as investigating the impact of financial transaction taxes on behaviour in asset markets.

Finally, whilst this thesis concerns research undertaken at the University of Manchester as part of the PhD program, I have now progressed on to a position as a senior forecasting analyst at a major UK based distressed debt fund. I am currently applying the insights gained from this body of research to optimise capital allocation within an investment setup.

1 Introduction

Scientific approaches to investigating social systems have become increasingly important, and effective, throughout time. Across history, societies have identified and built systems (often self-fulfilling) on the basis of some set of expectations. This stands in contrast to the standard approach of much of economics, to describe homo economicus - the ideal person. He (or she) is a singular representative characterising the preferences, behaviour and decisions of an entire population. This is a natural assumption given that the computational requirements needed to represent even a crude distribution were out of the reach of researchers until very recently. However, recent advancements in computer technology have made estimations of human behaviour studying deviations from what would otherwise be considered rational very much possible.

This thesis focuses primarily on a particular aspect of human behaviour, namely a tendency for behavioural herding. We examine a form of herding behaviour within a more general behavioural framework specific real world contexts. In doing so we present evidence that behavioural factors in general, and behavioural herding with agents switching between beliefs in particular, are important to understand when considering the dynamics of macro level variables that describe social systems.

It is increasingly important to understand the role of human behaviour in the macro level dynamics of such systems, with the rise of interconnectedness within and between societies. To do so, we focus on the formation of beliefs in two narrow sets of circumstances. We will introduce two applications of behavioural adaptions to macro-level models and demonstrate that proper consideration needs to be given to behavioural factors when considering macro level emergent phenomena. We will see that such factors produce different predictions when compared with standard approaches. The results found here will also have policy implications. As we will discuss later in this thesis, we must be careful in assuming that behavioural equilibria (or at least sets of parameters that describe such an equilibrium) are stable or unique as assumed. What we will see is that assumptions regarding behaviour are crucial in our measurement of that behaviour and that this has consequences for the observed dynamics and therefore the appropriate policy response.

1.1 Motivation

The motivation for this thesis has its origins in the time I spent working as a debt analyst at M&G investments, where behavioural factors played a key element in investment decisions. In particular, rules of thumb were often used to simplify decision making when considering investment propositions.[1] Such rules of thumb have been studied extensively at a micro level, but the importance of such behaviours for macro level dynamics remains a subject of debate, but with leading economists such as Akerlof and Shiller arguing for their importance. For that reason I have chosen to use this thesis to focus on two specific examples of behaviour phenomena that emerge in different macro contexts.

[1] A case in point is the use of notching when assessing investments. As an example for how this works let us consider a senior loan secured against an office building. A fund manager might start by notching down from the sovereign credit rating to get a base line for a senior secured loan. The manager can then choose to adjust this rating further based on the difference in key metrics when compared to comparable investments, location, quality of security, tenancy arrangements etc. Whilst this process normally involves an element of quantitative analysis, the weight given to each factor is ultimately subjective and is assessed using a rule of thumb based on a managers experience.

1.2 Overview

This thesis is presented in a journal format. The work presented here is solely my own, except where indicated otherwise by references. This format is chosen rather than a more standard thesis format as the subsequent sections of this thesis from distinct contributions to the relevant literatures. Whilst in this thesis these section are closely related in conceptual origin, in application they are best understood separately given the differences in literature to which they are relevant. As such chapters II and IV are presented as self contained. In addition to this introduction, the thesis contains three subsequent chapters. Chapter II covers an investigation into the effects of leverage in a behavioural asset market. Chapter IV presents a study on the importance of behaviour in the spread of a pandemic. Finally, chapter V contains a summary of this body of research that draws together the various results and offers suggestions for future work.

2 Behavioural borrowing in the housing market

Abstract

We extend a behavioural model of boundedly rational investors by allowing agents to borrow to purchase assets. Such a model allows for a Minsky style interpretation of asset price movements. Agent strategies in our model are characterised as either speculative or non-speculative investors, with speculative investors borrowing to finance their investments. Agents evaluate strategies based on historic performance and switch between strategies using a logistic switching rule. Simulations of the model show that the use of a borrowing heuristic can leads to endogenous market cycles. We also estimate a such two-type behavioural switching model with leverage using data for The US housing market for the period Q1 1983 to Q1 2018. Our results indicate that a Minskyian interpretation for the housing bubble during this period is consistent with the observed data.

2.1 Background

The importance of behaviour on decision making has a long history in macroeconomics. Keynes General Theory dedicates an entire chapter to the role of 'animal spirits'. More recently, Akerlof (2002) in his Nobel acceptance lecture, highlighted six phenomena that he felt were insufficiently explained at the time within mainstream economic thinking and where behaviour may play a crucial role. As one of these six phenomena, Akerlof highlighted role of behaviour in asset markets, which is where this paper will focus.

Behavioural models have been proposed to account for some of the apparent inconsistencies in financial markets, providing a mechanism for amplification (Shiller, 1981) of fundamental movements in their effect on prices. As was observed by Shiller (1981, 2000), stock markets exhibit excess volatility. That is to say, fluctuations in stock prices are significantly larger than movements in the underlying fundamentals would suggest as rational. This is particularly true of bubbles and crashes in stock markets where, as noted by Shiller, market valuations can deviate wildly from that suggested by any fundamental assessment. Such behavioural interpretations for asset price movements have become increasing popular since the financial crisis of 2008 as a challenge to the more standard rational expectations explanation of asset pricing.

Following this trend of challenges to the economic orthodoxy on asset pricing, one development of note is that in the decade following the financial crisis of 2008 there has been a revival in the ideas of Hyman Minsky. Interestingly, this revival has occurred largely outside of the economic mainstream and is probably best summarised in The Economist (2016):

'Minsky's influence was, until recently, limited. Investors were faster than professors to latch onto his views. More than anyone else it was Paul McCulley of PIMCO, a fund-management group, who popularised his ideas. He coined the term "Minsky moment" to describe a situation when debt levels reach breaking-point and asset prices across the board start plunging. Mr McCulley initially used the term in explaining the Russian financial crisis of 1998. Since the global turmoil of 2008, it has become ubiquitous. For investment analysts and fund managers, a "Minsky moment" is now virtually synonymous with a financial crisis.'

Such an explanation raises the question of what in fact is a Minsky moment - and how should economists understand this concept in anything more than vague descriptive terms. Indeed, it is interesting to ask if this is a concept that can actually be applied in any useful analytical sense in markets, and if it can, does it really provide an explanation for the financial crisis of 2008 as many investors believe it does.

That an idea has gained such traction among investors whilst remaining relatively unstudied within the economic mainstream would appear unusual, but is perhaps less surprising when we examine Minsky more closely. In brief, Minsky's financial instability hypothesis (Minsky 1982, 1992) characterises system instability as the product of debt and describes three types of debt financing that economics units can engage in: hedge, speculative and Ponzi.

Within Minsky's framework, the hedge unit is one that can make debt repayments that cover both the principal and interest from its cash flows. Speculative units are able to cover interest payments with cash flows, but not pay down the principal, so will need to eventually refinance. Ponzi units are those cannot even make repayments on the principal with current cash flows, but borrow in anticipation that a rise is asset prices will allow them to make interest payments and hoping that they can refinance the debt in the future. In this case of Ponzi borrowers, falling prices, or even prices simply not rising fast enough is sufficient for them to become insolvent leading to involuntary sales of assets and decrease in demand, that puts additional downward pressure on prices. This decrease in prices in turn causes speculative units to become Ponzi units as they can no longer refinance their debt, so further exacerbating the problem. As prices collapse, eventually even hedge units may not be able to take out loans. Minsky states that as an economy enjoys a period of stable growth, units are increasing likely to move from hedge, to speculative and finally to Ponzi financing as the perceived risk of doing so decreases.

Unfortunately, such an explanation of financial crashes, is a little vauge. It is likely for this reason that such an interpretation of the US housing market crash and subsequent financial crisis of 2008 has found more traction with investors than economists. However, a notable exception to this trend has been in the Post Keynesian school of economic thought and a good overview of this literature is provided by Nikolaidi & Stockhammer (2017). Much of the older literature in this area focuses on firms as the relevant economic unit to study, whilst post 2008 there has been a renewed focus on asset prices, particularly notable is the work of Chiarella & Guilmi (2011) on equity prices and Ryoo (2016) focusing on real estate. A common trait in this literature is the development of very large models; models that because of their complexity do not then lend themselves to empirical analysis. This results in such models lack both convincing verification and utility beyond that of being able to tell a potentially interesting story.

However, one potentially promising avenue of research that has arisen out of the existing Minsky literature is the potential for using agent based models as tool to examine Minskyian dynamics. Indeed, this is the approach taken by Gatti et al (2010) and Chiarella & Guilmi (2011). Given that there is already a relatively large and growing literature using agent based models, and that it lies closer to the economic mainstream, such an approach would appear sensible.

Much work has already been done on the subject of heterogeneous agent's models with a good summary provided by Chen et al (2012). Heterogeneous agent models will typically, although not always, make use of adaptive expectations obtained through learning behaviour and evolutionary processes. This allows market participants to switch between beliefs, often referred to as shifting market fractions, depending on relative performance and therefore the market to become dominated by a particular belief type; sometimes leading to significant deviations of prices from fundamental values.

While ostensibly studying different mechanisms, given the similarities between heterogeneous agents models when applied to markets and Minsky's hypothesis, it is interesting to consider if the two views are compatible. The approach of Chiarella & Guilmi (2011) is to note that using an appropriate heterogeneous agent's framework it can be assumed that on average fundamentalist investors will tend to invest in hedge firms, and that chartists will prefer riskier investments. This offers a nice intuition, suggesting that if a market becomes increasingly dominated by chartists, we can also regard it is as likely becoming increasingly exposed to the risks of Ponzi finance under the Minskyian view, leading to market instability. Such an interpretation of Minsky's theory provides an explicit mechanism for the rapid collapse of a bubble, something often only speculated on or simply missing from the literature on heterogeneous agent's models. Given that previous work with heterogeneous agents models has empirically identified shifting market fractions and attributed bubble formation to them.

Perhaps the most attractive feature of the heterogeneous agent model literature lies in the fact it has been applied empirically to a number of different asset classes, with some success. For example, Chiarella et al (2014) and Lof (2014) all use such models to empirically study the price of stocks. Additionally, Westerhoff & Reitz (2003) use a similar type of model to study exchange rates and oil prices are analysed by Ter & Zwinkels (2010).

A different approach to studying asset markets and boom bust dynamics is found in experimental literature. Work following an experimental asset market approach, for example Smith et al (1988) has found mixed evidence for the presence of boom and bust asset market cycles, with some experiments producing boom and bust dynamics, but others failing to do so. The results obtained by Smith suggests that experience can reduce the probability of bubbles by eliciting the formation of common expectations (priors). Interestingly Lei et al. (2004) suggests that the departures from a fundamental value are not caused by the lack of common knowledge of rationality that leads to speculation, but that the subject behaviour can itself exhibit elements of irrationality, leading to the formation of bubbles. Work by Haruvy et al. (2007) attempts to study this directly by eliciting predictions from subjects, finding that beliefs about prices are informative in predicting future price movements and adapt based on past trends, and therefore play an important role in generating self fulfilling asset market cycles.

A possible explanation for this mixed evidence on bubble formation or lack thereof, is that it may be due to the complexity of the problem solving required during the experiment for the participants. This view is supported by Bao et al. (2021) in their review of the literature on Learning to Forecast Experiments. They observe that a there is a rapid convergence to the rational expectations equilibrium in negative feedback markets, but persistent bubbles and crashes in positive feedback markets. They also find that forecasting accuracy depends on the complexity of the task. This could be understood as subjects in less complex tasks being able to get sufficiently close to the 'correct answer' for their prediction that the asset price remains stable, whereas in more complex settings, subjects fall back on insufficiently precise rules of thumb, leading to asset price cycles. This will motivate one of the key contributions of this paper, where we examine a borrowing heuristic as a substitute for agents dynamically optimising their level of debt, given their beliefs over price changes. This is consistent with Lei et al. (2004) as such behaviour has an element of irrationality, whilst also reflecting the findings of Bao et al. (2021) with agents choosing to simplify a complex problem at the expense of accuracy.

For the purposes of this paper we will focus on the theoretical approach pioneered by Brock & Hommes (1997) and Brock & Hommes (1998). An important addition to the literature (for the purposes of this paper) was made by by the latter with the introduction of memory to heterogeneous agents models in Hommes and in't Veld (2018). This allowed agents over time to 'forget' past market history as well as learning from it. Such a mechanism provides for a progression of agents over time, from hedge to speculative to Ponzi financing, that Minsky suggests.

A futher advantage of this style of model is that it has previously been empirically applied to asset prices in Boswijk et al (2007), and more recently in Brock & Hommes (2007) and Hommes & in't Veld (2018). Given that a key motivation of this paper to examine the validity of a Minsky style interpretation of the 2008 housing market crash, this will allow us to test the model against US data for house prices in this period.

There is already a substantial literature on the housing market and the determinants of house prices. Particularly relevant for this paper is the work that emphasises the role of leverage on house prices and vice versa. Work such as that by Lamont & Stein (1997) finds that higher leverage ratios lead to a greater sensitivity of house prices to shocks. More recent work such as that of Cloyne et al. (2019) finds that there is also an effect of house prices on borrowing and that this is largely determined by collateral effects. A similar result from Miles & Munro (2021) emphasised the role of interest rates as a determinant of house prices, with falling rates making credit more affordable, in turn leading to an increase in borrowing and so pushing up prices. Together this suggests a feedback loop between borrowing and house prices. We contribute to this literature here by offering an approach that places the borrowing decision within a behavioural asset market framework that allows for feedback between house prices and borrowing.

In this paper we to embed some of the ideas of a Minsky style explanation for a leveraged asset market within a simple Heterogeneous agents model of an asset market provided by Hommes and int' Veld (2018). We use this model to study the price dynamics of an asset market using both simulations and empirics and examine if such a model can offer any advantages over a more standard model. Specifically, we are asking to what is the contribution of borrowing behaviour to the formation of asset market bubbles. This paper contributes to the existing literature in two significant ways. First, we extend a standard model of asset market pricing to include borrowing, where agents can change between mean reverting and trend following expectations for changes in future prices and borrow in line with their beliefs. This allows us to capture the difference between hedge and speculative/Ponzi investors in a manner analogous to that described by Minsky, allowing for a mainstream Minskyian interpretation of the housing market. We show that under this formulation, agents will switch between beliefs as they observe movements in house prices and that different borrowing behaviours result in very different price dynamics. In particular, we find that when agents borrow optimally in a simulated asset market, we see only a small amplification of asset price movements and do not observe any boom and bust price dynamics. However, the use by agents of a borrowing heuristic in a simulated asset market leads to the kind of boom and bust behaviour we would expect according to Minsky.

Second, we use empirical estimation to verify that the observations from real world data for the US housing market are consistent with our model assumptions. This is in line with the results obtained by Hommes and in' Veld (2018) for the S&P 500 using a similar framework. We find that the model produces results consistent with the results of Hommes and in' Veld (2018). We also discover that under this framework the estimated fraction of housing market speculators correlates with the observed delinquency rate on mortgages in the US, suggesting that this work supports a Minsky style narrative as a contributory factor leading to the 2008 financial crisis.

The remainder of this paper is organised in the following way. In section 2 we introduce the theory and model description. Section 3 contains simulated price dynamics. In section 4 we provide the estimation methodology for the model and the results obtained. Section 5 discusses these results and finally conclusions are presented in section 6.

2.2 Theory

As mentioned in our introduction, the model that we will use in this paper in based on that developed in Hommes & int Veld (2018) as this model allows us to characterise several key components of Minsky's financial instability hypothesis. The model uses two types of boundedly rational traders, whose expectations over prices will differ in the short run, but that agree on a long run fundamental value to which they expect prices to eventually return. These agents will then maximise expected returns each period based upon their expectations. In our model we will additionally allow our agents to borrow in order to boost their expected returns.

Our first contribution begins by augmenting the approach of Hommes & int Veld (2018) by allowing agents to borrow and so introducing debt into agent returns. Therefore, we start by calculating excess returns as follows:

$$R_{t+1} = \frac{P_t}{P_t - D_t}[P_{t+1} + Y_{t+1} - (1+r)P_t - iD_t] \tag{1}$$

Where R_t is the excess return, P_t is the price of a risky asset Y_t is the cash flow derived from the risky asset D_t is the amount borrowed against the risky asset, r is the return on a riskless asset and i is the interest rate on debt.

We assume that the risky asset is in zero net supply and that agents have demand for the asset such that:

$$z_{h,t} = E_{h,t}[R_{t+1}] \tag{2}$$

$$\sum_{h=1}^{H} z_{h,t} = 0 \tag{3}$$

Substituting $LTV_t = \frac{D_t}{P_t}$ we can the under these assumptions write the pricing equation as shown in equation (4), where with $n_{h,t}$ reflecting the proportion of the market that uses forecasting rule h at time t.

$$P_t = \sum_{h=1}^{H} \frac{n_{h,t}}{1 - LTV_{h,t}} \frac{E_{h,t}[P_{t+1} + Y_{t+1}]}{1 + r + iLTV_{h,t}} \tag{4}$$

Assuming the growth rate of cash flows follows an iid process the relationship between current and future cash flows can by described by equation (5) where g

represents long run growth rate of earnings

$$E_{h,t}[Y_{t+1}] = (1+g)Y_t \tag{5}$$

For simplicity we will from here on formulate the model in terms of the price to cash flow ratio where:

$$\delta_t = \frac{P_t}{Y_t} \tag{6}$$

$$x_t = \delta_t - \delta_t^* \tag{7}$$

$$R^* = \frac{1+r}{1+g} \tag{8}$$

The term δ_t^* is used here to describe the fundamental value for the price-yield ratio of the risky asset and x_t is therefore the deviation of the price-yield ratio from its fundamental value. Rewriting the pricing equation in terms of deviations from the fundamental we get:

$$x_t = \delta_t - \delta_t^* = \sum_{h=1}^{H} \frac{n_{h,t}}{1 - LTV_{h,t}} E_{h,t}[x_{t+1}] \frac{1+g}{1+r+iLTV_{h,t}} \tag{9}$$

Instead of focusing on the three agent types described by Minsky, we instead use two for the sake of simplicity and tractability. Whilst this may seem a significant simplification, Aoki (2002) uses a theoretical model to argue that two groups are sufficient to characterise the behaviour of many different market participants. We can then represent these two groups with representative agents. As in Hommes & int veld (2018) agents switch between beliefs using a logistic switching rule based upon past performance:

$$n_{h,t+1} = \frac{e^{\beta U_{h,t}}}{\sum_{h=1}^{H} e^{\beta U_{h,t}}} \tag{10}$$

Where:

$$U_{h,t} = (1-\omega)\pi_{h,t} + \omega U_{h,t-1} \tag{11}$$

The ω that appears in equation (11) represents the memory of the agents, and must lie between zero and one, with one being perfect recall and zero: not remembering anything. The parameter β controls how fast agents switch between forecasting rules in the model. We expect that our estimated value for this will be insignificant, as the model does not have sufficient power to determine this using least squares estimation. The parameter is however still necessary for the rest of the model to function.

Agents evaluate the performance of the beliefs by comparing expected returns with the realised returns as follows:

$$\pi_{h,t+1} = \alpha \frac{(E_{h,t}[x_{t+1}] - R^* x_t - \frac{iLTV_t}{r-g})(x_{t+1} - R^* x_t - \frac{iLTV_t}{r-g})}{(1 - LTV_t)^2} \tag{12}$$

Equation (12) allows agents to evaluate how well each belief performs by comparing the expected performance of each belief type, shown in the first set of parentheses, with the observed performance, shown in the second set of parentheses. This is analogous to the form derived in Hommes and in't Veld, but now adjusted for the introduction of leverage. This means that agents now need to account not only for the opportunity cost of the investment represented by $R^* x_t$, but also the amount that they borrow as well.[2] The introduction of leverage into the model here also leads to a $(1 - LTV_t)^2$ term being present as a denominator. This means that the payoff function becomes increasingly sensitive as leverage increases and so should act as an additional amplification factor in the model that is not present in Hommes and in't Veld (2018).

We assume that Agents form beliefs regarding future price-cashflow ratios that are

[2] As an example: if there is a belief that $E_{h,t}[x_{t+1}] > R^* x_t + \frac{iLTV_t}{r-g}$ and the agent then observed that $x_{t+1} > R^* x_t + \frac{iLTV_t}{r-g}$) the payoff function for holding this belief will be positive. The agent will then be more likely to hold this belief in the subsequent periods. Conversely, if the belief is such that $E_{h,t}[x_{t+1}] > R^* x_t + \frac{iLTV_t}{r-g}$ and then the agent observed that $x_{t+1} < R^* x_t + \frac{iLTV_t}{r-g}$) the payoff function for holding this belief will be negative and the agent will be less likely to hold this belief in the subsequent periods.

linear in the last observation as shown in equation (13). The parameter ϕ_h is the forecasting rule for agent h. We expect that we will have one agent that borrows to buy assets and will have ϕ_h greater than one. This represents trend following behaviour and these agents are the speculators in our market. As such they are an analogue to the Ponzi investors of the Minsky interpretation. We also have a second type of agent that we expect will have ϕ_h of less than or equal to one and who may or may not borrow. This agent is analogous to the hedge/speculative borrower.

$$E_{h,t}[x_{t+1}] = f_h(x_{t-1}) = \phi_h x_{t-1} \tag{13}$$

Finally we will assume that agents choose between the two types of beliefs such that:

$$x_t = \frac{1+g}{1+r+iLTV_{1,t}} \frac{n_{1,t}}{1-LTV_{1,t}} \phi_1[x_{t-1}] + \frac{1+g}{1+r+iLTV_{2,t}} \frac{n_{2,t}}{1-LTV_{2,t}} \phi_2[x_{t-1}] \tag{14}$$

2.3 Simulations

Using the theory developed in section 2, we simulate the price dynamics for the model. Doing so allows us to verify that the price dynamics are consistent with the intuition that motivates the model. As we have seen in the previous section, the model contains a number of normalisation parameters. For simplicity, we will ignore these for the purposes of the simulation and use normalised variables instead. This will mean that some of the parameters that we use here will not be directly comparable with those we find in our estimation but this will not affect the price dynamics of the model. For reference the parameters we use are shown in Table 1 and are selected to be broadly in line with those used in Hommes and in't Veld (2018). This is to illustrate the possible dynamics of the model, rather than to suggest these parameters are reflective of what would be achieved in an estimation.

Parameters	**Values**
ϕ_1	1.1
ϕ_2	0.7
β	10
ω	0.8
r	0.1
g	0.03

Table 1: Representative parameters for the simulation of model dynamics.

Setting $\phi_1 = 1.1$ implies that agent holding belief 1 expect that following a positive price shock prices will continue to rise relative to the fundamental value and that $\phi_2 = 0.7$ implies that agent holding belief 2 expect that following a positive price shock prices will trend back towards the fundamental value. As discussed in relation to equation (11), β controls how fast agents switch between forecasting rules in the model. Values for β are only weakly identified in the literature, beyond a requirement that it is large. Following Hommes and in't Veld, we set $\beta = 10$, as this should be sufficiently large. We will also $\omega = 0.8$, roughly consistent with the results found in Hommes and in't Veld and would suggest that agents in the model derive around 60% of their information from price observations made in the preceding year. We also choose g and r to approximate the for the owner equivalent rental value of housing and the capital growth derived from home ownership respectively.

First, we examine the case where no borrowing is allowed, equivalent to Hommes & int Veld (2018) and compare this with case where agents optimise borrowing for their given beliefs. We impose a single positive shock of 5% to price deviations at time $t = 0$, to approximately the higher end of historic quarterly price moves for the US housing market. In order to examine the case where borrowing is allowed, our agents choose a level of borrowing each period in order to maximise expected returns given their price forecasting rule. This would be akin to the behaviour expected by professional investors in an asset market. Agents are not credit constrained and switch between beliefs using the logistic switching model described in the theory. It seems unrealistic that agents borrow at the same interest rate for low and high levels of leverage so we model interest rates on debt as being proportional to the square of the leverage employed such that:

$$i = \gamma LTV_{h,t}^2 \tag{15}$$

We set γ such that at 70% LTV the interest rate is approximately equal to the observed mortgage rate in the United States. As a consequence of equation (15), the interest rate on agent borrowing i is state dependant when agents are free to choose their level of borrowing, However, we note that given this setup agents who believe prices will fall choose not to borrow at all, whilst only trend followers will employ leverage.

We see from Figure 1. that the dynamics shown are similar to that of the base model without leverage. The difference between the two is that deviations in the price from the fundamental in this case are slightly more persistent when leverage is employed. In other words, debt has the effect of amplifying deviations from the fundamental which is consistent with our intuition from the literature, and the theory presented in equation (12) that borrowing should amplify the effect of a price shock. However, interestingly, we do not see the boom and bust dynamics of Hommes and in't Veld (2018). This is due to the fact that agents are free to select a level of borrowing that optimises returns given any interest payments, so that potential gains from increasing leverage are offset by increased borrowing costs providing a stabilising mechanism within the model.

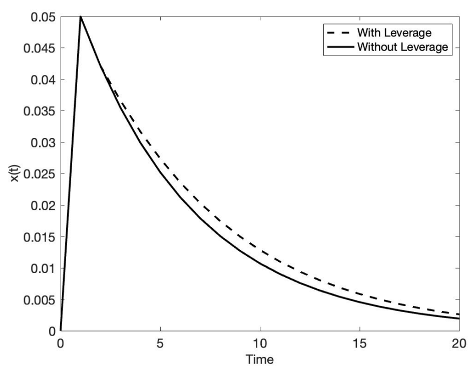

Figure 1: Plot of the price deviations from the fundamental value over time comparing price deviations in the case of no borrowing and optimised leverage in response to a positive shock

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To investigate this further, we examine the case where agents who are trend followers will respond to a positive shock by borrowing at a fixed loan to value. Based on the experimental literature on asset markets (Bao et al., 2021) we suggest that this kind of behaviour may be more consistent with the behaviour of home buyers borrowing from a bank in order to buy a house given that the calculation needed in previous example is a complex one. In this sense such a decision rule can be understood as a heuristic for optimising returns. We see in Figure 2.

that this setup, whilst still reflecting the underlying assumptions for the model, leads to markedly different price dynamics than in the previous cases. Whereas in

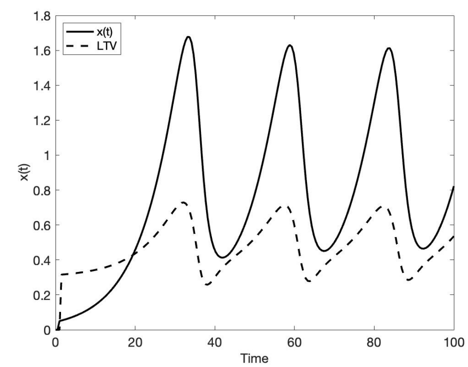

Figure 2: Plot of the simulated price dynamics for an asset market subjected to a positive shock and where speculators have a fixed leverage level.

Figure 1. we see deviations from the fundamental being attenuated over time and borrowing playing only a small amplifying role, in this case a single shock causes a persistent deviation of the asset price from the fundamental value for the parameters we examine. Perhaps most notably the model also produces endogenous cycles in price and debt deviations that are driven by shifting market fractions of agents between beliefs. This is due to agents observing the initial price increase, and then some of the agents that previously believed that prices would revert to the fundamental shifting to the belief that the trend of deviating from the fundamental will continue. This leads these agents to increasing their demand for the asset and so driving up the price even further. As an increasingly large share of agents transition to this trend following belief, there is a smaller pool of agents left to transition in future periods. This reduces the demand impulse in future periods, leading to a decline in the growth rate of the price deviation. As the price growth rate falls trend following becomes relatively less attractive and some agents transition back to a fundamental reversion belief. This continues until the price peaks and then declines, where this process then begins to repeat.

The difference in the borrowing behaviour between the simulation shown in Figures 1. and 2. is that agents in Figure 2. borrow according to a heuristic rather than dynamically optimising their leverage. In Figure 1., the agent will decrease their leverage ratio as prices deviate from the fundamental, providing a stabilisation mechanism to the dynamics, whereas in Figure 2., the agent leverage ratio is fixed as a function of their price belief. This is again amplification driven by debt, but very different to what we observed seen in Figure 1. [3]

Memory ω plays an important role in this framework by allowing agents to 'remember' how well each belief type did in the past. Based on the past performance of each belief type agents will choose which strategy to follow each period with more successful beliefs being relatively more popular and followed by a larger fraction of agents. This is in line with the findings of the experimental literature (Smith et al, 1988) that emphasises the importance of learning on asset market dynamics. Using ω here, allows for the agents to have imperfect memory, so that they place more emphasis on recent price moves[4]. In the model this prevents, agents converging to a single belief and provides for persistent boom and bust cycles. This is again consistent with the experimental literature (Bao, 2021), where cycles can persist even in the absence of shocks.

[3]As an example consider an asset that generates a fixed cash flow such as a bond. If the price of the bond increases, the coupon payments remain unchanged. We can borrow some money to purchase that bond, but if the increase in interest on the borrowed money is super-linear then there is an optimal amount that should be borrowed against the asset in order to maximise our return. So if the bond increases in price, we will look to borrow more, but reduce our overall leverage ratio as the cash flow from the bond remains stays the same.

[4]This can also be thought of as agents forgetting past events over time.

To study this mechanism further we test to see how the fraction of speculators changes over time, shown in Figure 3.

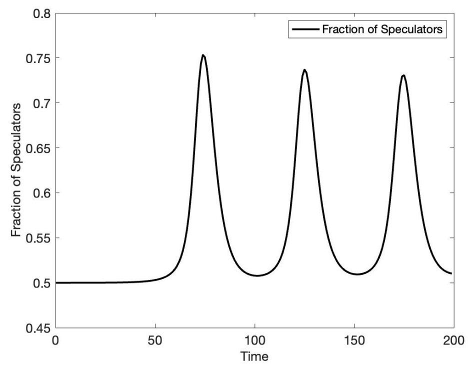

Figure 3: Plot of the fraction of speculators for an asset market subjected to a positive shock and where speculators have a fixed leverage level.

We can see from Figure 3. that the proportion of speculators varies over time as agents shift between beliefs. This is as we expected and is the mechanism that allows us to have a Minsky style interpretation of the asset market dynamics. To verify that it is this switching behaviour that leads to bubble formations we reexamine the dynamics with a fixed leverage ratio, but holding the fractions of the population that can hold each belief type constant.

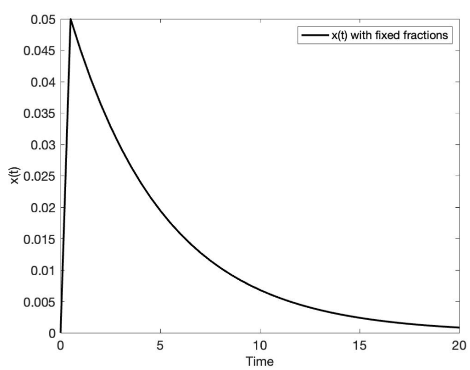

Figure 4: Plot of the simulated price dynamics for an asset market subjected to a positive shock and where speculators have a fixed leverage level and the proportion of speculators is also fixed.

Figure 4. Shows that where the fraction of speculators is not permitted to vary over time, the prices in the market remain stable. This confirms our expectation that this is the key mechanism driving the asset market cycles generated in Figure

2. Leverage in this model then provides an amplification factor by allowing speculators to inflate their demand for an asset by borrowing money. It also makes them more sensitive to slowdowns in the price increase for the same reason and because they now have to cover interest payments.

In summary, we can identify two different borrowing behaviours for agents in the model that are both consistent with a Minskyian motivation, but lead to very different price dynamics. This leads us to our first conclusion in this paper - that the motivation for borrowing, and therefore agents borrowing behaviour may play a significant role in markets. For this reason, careful consideration must be given to how borrowing is implemented within Minsky style models.

2.4 Estimations

In order to examine the usefulness of our model in more detail we examine how it performs in estimating behaviour in the US housing market. The estimation procedure is conducted in two stages. First, we construct the dataset of deviations from a fundamental as described by the theory shown in the previous section. Then we estimate parameters for beliefs, memory and switching using non-linear least squares regression. Estimation results are then compared with benchmark models.

2.4.1 Data

To construct the time series of deviations we use quarterly data obtained from the St Louis Fed for the period Q2 1983 to Q1 2020.[5] We construct a time series for the estimated total value of owner occupied homes in the united states by multiplying the number of owner occupied homes by the All-Transactions House Price Index for the United States and then fitting the data to match the observed mean sales price for houses in Q1 2010. Because we use mean sales price to fit the

[5] We are limited by the availability of data needed to calculate r. This is shorter than Hommes and in't Veld (2018), but should still be sufficient to obtain reasonable estimates for the behavioural parameters.

data, it is likely that our dataset provides a small overestimate for the total value of owner occupied housing, but this effect is likely to be small in comparison to the overall values.[6]

We combine this calculated data for prices with the dataset for the imputed rental rate of owner-occupied housing to get an estimate for the implied earnings yield of owner occupied housing. For the sake of simplicity we use the solution to the standard Gordon model (Gordon, 1962), as shown below, to estimate the fundamental value of house prices.

$$\delta_t^* = \frac{1+g}{r-g} \tag{16}$$

The growth rate of implied rents g, and the implied cost of equity $r = d/p + g$ are shown below in Table 2 alongside the calculated static Gordon fundamental.

r	g	i	LTV	δ^*
10.51	3.28	6.9	90	9.80

Table 2: Calculated parameters for the US housing market.

The values for r and g calculated here for owner occupied housing in the United States are somewhat higher than those found by Hommes and in't Velt (2018) for the S&P 500, resulting in a significantly lower Gordon fundamental. This is due to the time period over which the model is calculated and would likely be somewhat more similar if a longer time series of data were available.

Using the calculated value for the fundamental we can the construct the time series of price-earnings deviations by inverting the yield, as shown in equation (6) and then applying equation (7). The St Louis Fed also provides a data se-

[6] This is due to changes in the composition of the housing stock over time. As newly built houses are added to the housing stock the value of these houses will be recorded at the point of sale. However, given we might expect that new houses are more likely to be built in areas of high housing demand, we may also expect that these houses should have relatively higher prices. However, we expect this effect to be negligible as the number of newly completed and sold housing units each quarter will be small relative to the overall size of the housing stock.

ries for the total value of home mortgage obligations belonging to households and non-profits as a proxy for the total value of owner occupier mortgages. This will be a small overestimate for the total value of mortgages secured against owner occupied properties, but given that the vast majority of mortgages are held by households rather than non-profits this effect should negligible. We can make a useful comparison with the calculated prices to verify our intuition that the level of mortgage debt outstanding should be correlated with the total value of the owner occupied housing stock as seen below.

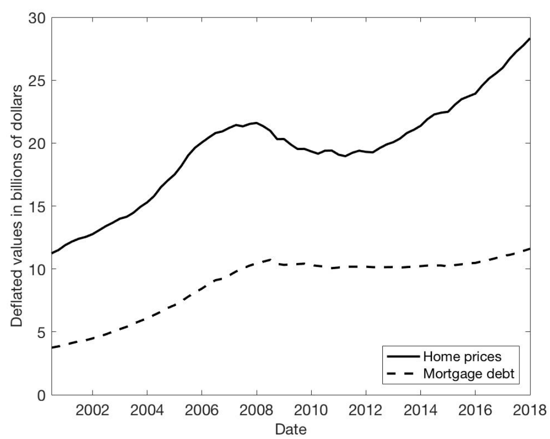

Figure 5: Comparison of the total value of the owner occupied housing stock, and the total value of residential mortgage debt belonging to households and non-profits. Both datasets are calculated reflect 2010 prices. We see that the two data series appear to co-move.

Comparing the time series for the estimated value of owner occupied homes and the total value of mortgages of owner occupied homes around the 2008 financial crisis, as shown in Figure 4., we see that there seems to be significant correlation between the two. This supports our motivating intuition behind the link between house prices and mortgage debt. Our model cannot however explain the upwards trend in both prices and debt.

2.4.2 Model Estimation

For the model estimation we follow a similar approach of Hommes and in't Veld (2018). First we calculate the fundamental value for housing using the static Gordon model. Then, following Hommes and in't Veld we estimate the model parameters using non-linear least squares. This is necessary given the non-linearities present in the model. Finally, we also introduce a benchmark model against which to compare our results for robustness.

In order to estimate the model we will assume a constant leverage ratio and interest rate for borrowers. These are significant simplifying assumptions, but necessary in order to properly identify the parameters in the model given the data available and model non-linearity. We assume a constant leverage ratio of 90% for our speculators and 0% for our non-speculators. This is in an attempt to ensure we capture the extreme ends of the expected behavioural distribution. Additionally use the average interest rate of 30 year fixed mortgages, calculated to be 6.9%, as the interest rate.

We compare our version of the model with leverage to the case where the model is estimated with no borrowing as a benchmark with the results shown in Table 3 below.

Parameters	**1983-2018 Without debt**	**1983-2018 With debt**
ϕ_1	1.382	1.168
	(0.063)	(0.055)
ϕ_2	0.53	0.932
	(0.071)	(0.080)
β	0.573	0.009
	(0.317)	(0.002)
ω	0.429	0.694
	(0.053)	(0.105)

Table 3: Estimation results using data for the US housing market for the period Q1 1983 to Q1 2018.

We notice that the difference between ϕ_1 and ϕ_2 for the estimation without borrowing is significantly greater than for the estimation with leverage. This is interesting as the difference between ϕ_1 and ϕ_2 provides the main amplification mechanism in the model without leverage. The theory presented in equations (12) and (14) suggests that when borrowing is included, then it should act as an additional amplification factor in the model. These results support this prediction, as with leverage included the estimated level of behavioural heterogeneity needed to fit the data is reduced, implying that leverage is indeed providing an extra amplification factor.

For the model with leverage, estimations for the agent price expectation rules give results of 1.168 and 0.932 for agents employing leverage and those that do not respectively, similar values to those found in Hommes and in't Veld (2018). We find a standard error for ϕ_1 of 0.055, meaning that we reject the hypothesis that $\phi_1 = 1$ at a 95% significance level. This implies that ϕ_1 is consistent with speculative behaviour as defined in our theory. We further find that ϕ_2 has a standard error of 0.080. As such we fail to reject the hypothesis that $\phi_2 = 1$ at a 95% significance level, which is consistent with naïve expectations. We also reject the hypothesis that $\phi_2 = \phi_1$, implying that the behaviours of the two agents types are statistically significantly different. We also find an R^2 value of 0.970, suggesting the model is able to explain 97% of the variation in the data. This is similar to the R^2 value of 0.985 for the model without any borrowing. Our estimate for ω for the model with borrowing is found to be 0.694, suggesting that agents gain 77% of their information from observations made in the last year. This is slightly smaller than the values obtained in Hommes & int Veld (2018), but still larger than the 0.429 found for the model estimated without borrowing.

We also estimate an AR(1) model for the deviations, as a benchmark for our model. This is done by re-estimating the model with the restriction that $\phi_2 = \phi_1 = \phi$, which collapses the model to an AR(1) process. We find a regression coefficient for ϕ_2 of 0.975 with a standard error of 0.009., which is statistically significantly different from one. The model obtains an R^2 of 0.961, which is slightly lower than that for the estimation with leverage, suggesting that using leverage might allow for a slight improvement in fit over an AR(1) process, but the difference between the two is small.

As we are specifically interested in the period around the 2008 financial crisis, and to test the robustness of this approach we re-estimate the model on data for the period Q1 2000 to Q1 2018. To do this we first have to recalculate values for r and g, along with the static Gordon fundamental as shown below in Table 4.

r	g	i	LTV	δ^*
8.59	2.60	5.3	90	11.91

Table 4: Calculated parameters for the US housing market.

The values we calculate for this shortened period, shown in Table 5, are slightly lower than those calculated previously.

Parameters	**2000-2018 Without debt**	**2000-2018 With debt**
ϕ_1	1.130	1.132
	(0.034)	(0.052)
ϕ_2	0.992	1.097
	(0.038)	(0.056)
β	2.509	0.063
	(3.695)	(0.049)
ω	-0.555	0.869
	(0.147)	(0.167)

Table 5: Estimation results using data for the US housing market for the period Q1 2000 to Q1 2018. Note that the estimation fails for the case without debt as can be identified by the negative memory parameter ω.

The most notable result we observe in the table above is the sign difference in the memory parameter between the two estimations. As defined in our theory, the memory parameter must fall between zero and one inclusive. With the memory parameter falling outside of this range we conclude that the estimation cannot properly identify parameters in the model without leverage. This means that leverage is a component necessary to successfully estimate of this kind of heterogeneous agents model for the US housing market.

Our estimations for our agents price expectation rules give results of 1.132 and 1.097 for agents employing leverage and those that do not respectively. We find a standard error for ϕ_1 of 0.052, meaning that we reject the hypothesis that $\phi_1 = 1$ at a 95% significance level. This implies that ϕ_1 is consistent with speculative behaviour as defined in our theory. We further find that ϕ_2 has a standard error of 0.056. As such we fail to reject the hypothesis that $\phi_2 = 1$ at a 95% significance level. Such a result is consistent with naïve expectations. Conversely, cannot reject the hypothesis that $\phi_2 = \phi_1$. We also find an R^2 value of 0.853, suggesting the model is able to explain 85% of the variation in the data. This is similar to the R^2 value of 0.870 for the model without any borrowing, albeit this is only achieved with the memory parameter ω falling outside the possible range as previously discussed. Our estimate for ω for the model with borrowing is found to be 0.869. This is similar to the values obtained in Hommes & int Veld (2018) as we would again expect. All together this suggests that our formulation of the model is supported by the empirical data.

We also estimate an AR(1) model for the deviations. This is done by re-estimating the model with the restriction that $\phi_2 = \phi_1$, which collapses the model to an AR(1) process. We find a regression coefficient of 1.0076 with a standard error of 0.014. We cannot reject the hypothesis that this is statistically significantly different from one. The model obtains an R^2 of 0.851, which is slightly lower than that for the estimation with leverage, but the difference between the two is small.

Using the parameter estimates we obtained from our estimation for the period Q1 2000 to Q1 2018 we construct the time series for the market fraction of speculators, by substituting the behavioural parameters we obtained in the estimation back into our simulation. We observe from Figure 5. that the fraction of speculators remains relatively constant until the around Q4 2005 at which point it begins to rise until it reaches a peak in Q2 2009. After this point the fraction of speculators falls away again until it returns to levels close to those observed at the beginning of the period. For comparison we have also plotted the delinquency rates for home mortgages.

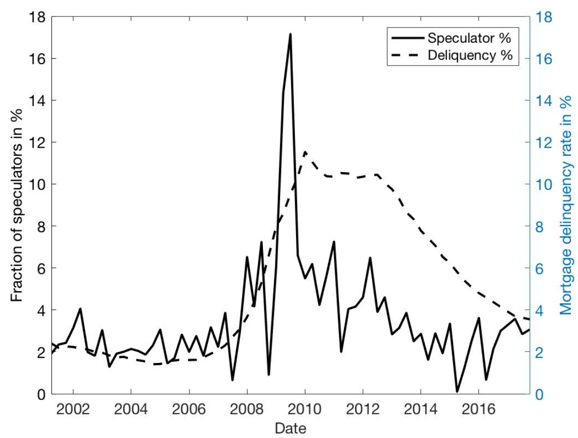

Figure 6: Plot of the market fraction of speculators.

We notice that up to Q2 2009 the proportion of speculators in the market closely tracks the delinquency rate. This makes sense in our model, given we expect that our speculators rely on rising asset prices to make repayments, so in other words would likely be struggling to otherwise repay their debts. From this point onwards however, the proportion of speculators and the delinquency rate begin to deviate. This is likely a hangover effect from the collapse of the housing market, wherein the proportion of speculators drops, but homeowners are unable to deleverage as quickly as they would like to and so remain delinquent on their mortgages for an extended period of time. This something for which our model does not account. We also notice that the estimated proportion of speculators is a relatively noisy time series. This is due to the fraction of speculators being sensitive to changes in the price and amplifying them.

For completeness, and to check the robustness of the estimation procedure, we also attempted to estimate the model for the period Q1 1983 to Q1 1999. We found that in this instance the estimation would not converge. This highlights a specific weakness of this approach, that the estimation procedure often struggles to identify parameters where the length of data is relatively short. In this specific instance this issue is potentially be further compounded by the changing macroeconomic environment of the 1980s and 1990s, with a large decrease in interest rates, inflation and an expansion of credit provision that accompanied the increase in house prices that took place in this period. To test this we attempted to reestimate the model for leverage ratios set at 60, 70 and 80%. A potential improvement here could be to use a time varying fundamental value or a time, to account for changing rates of interest and returns.

2.5 Discussion

Our motivation for the work we do in this paper is to place a Minskyian interpretation of asset price movements in a leveraged asset market, within the mainstream economic literature. To do this we have extended a standard behavioural asset market model developed by Hommes & int Veld (2018) to include leverage. We have seen from our data that there is co-movement between net borrowing and house prices in the USA. This serves to reinforce the intuition that house prices and mortgage debt are linked, as would be expected under a Minskyian interpretation.

We further explore the Minsky story through our simulations. The model we study in this paper is a two-type behavioural switching model and we construct so that our two representative agents act as rough analogues for Minsky style agents. On examining two different cases of borrowing, our first key result is that we find very different results depending on the borrowing behaviour. Specifically, agents that use a heuristic of maintaining a fixed leverage ratio to manage their borrowing, cause endogenous boom bust cycles to occur in response to only a single small shock to valuations, but this is not true when agents optimise their borrowing, given their beliefs. Whilst surprising, this is consistent with the results found in some asset market experiments. If such a result can be verified empirically it would be significant for our understanding of the role of debt plays in determining house prices, and possibly markets more generally.

In section 4 of this paper we attempt to estimate the model using data for the US housing market to test if the results are consistent with a Minsky style interpretation of the the 2008 financial crash. First we estimate the model on a dataset constructed for the period from 1983 to 2018 for the cases with and without borrowing. We find that excluding borrowing leads to the model predicting a significantly greater level of belief heterogeneity. As we examined through our simulations, it is agents switching between beliefs that leads to boom and bust asset market cycles in this model. The smaller difference in belief heterogeneity we find when including leverage suggests that this must provide an additional amplification factor in the model.

We further attempt to estimate the model for the periods 1983 to 2000 and 2000 to 2018, to check the robustness of this result. When doing so we find that the model fails to differentiate between the different agent's prediction strategies in the estimation procedure for 2000 to 2018, and to even converge for the period 1983 to 2000 highlighting a significant limitation in our methodology. However, for the period 2000 to 2018 we obtain results that are broadly in line with our expectations. Agents that employ leverage in the model are estimated to have a behavioural coefficient of 1.132 which is significantly different from 1 at a 95% confidence level. This is consistent with our assumption that such agents are behaving speculators. Agents that do not employ leverage in out estimation of the model are found to have a behavioural parameter of 1.097. This is not statistically significantly different from one at a 95% confidence level (albeit only just), meaning that they are best interpreted as behaving as if they have naive expectations. While this is not what we initially assume when we develop our theory, it may be explained by our study of a time interval wherein prices remain persistently elevated above fundamentals. However, we cannot reject the possibility in this paper that our agents follow the same prediction strategies due to the size of our standard errors and further work is needed to resolve this issue.

Comparing the results from the full estimation to the one restricted to 2000 to 2018, we find that the the behavioural coefficient for our speculators are similar in both periods, but that non speculators are found to have a smaller belief coefficient in the full estimation. This would suggest that there is a level of convergence towards speculative behaviour over time. We also find that the estimated memory parameter in the full estimation is smaller. This would imply that agents in the period 2000 onwards placed greater emphasis on historic house prices than measured over the full dataset. Given the greater access to historic house price data after 2000 when compared with before, due to the internet, it makes sense that agents would place greater emphasis on historic information. We note that for a majority of the sample period we have available, price-income deviations are positive. In other words, the price-yield we calculate for the US housing market remains persistently above the fundamental value for housing. This is therefore consistent with an assumption that housing has been overvalued in recent decades. Such a market characterisation is supported by the price dynamics we observe in our simulations; however, it also raises the possibility that the fundamental value we use here may not be the most appropriate for use in our estimation procedure.

In this paper we use the Gordon fundamental to reflect a typical method used by investors to value assets, including real estate. However, the appropriateness of the measure is a limitation that could be further studied. Given that we assume that homeowners use a heuristic to borrow, it is also possible that homeowners use alternative heuristics to evaluate house prices, leading to different results. Further investigation would be needed to identify if this is indeed the case. Using more granular data would allow us to identify if the Gordon fundamental more accurately reflects valuations. Additionally, examining different valuation methodologies for real estate that may more realistically reflect home owner behaviour would be another avenue for further work.

A significant issue arising for the methodology is that our non-linear estimation has some difficulty identifying the model parameters when estimating the model over shorter time periods. That we find a challenge in estimating non-linear models is a problem by no means unique to this paper. Indeed, this issue is encountered by Hommes & int Veld (2018) in their similar estimation for the S&P 500. As in their paper, we find that the model has insufficient power to identify the switching parameter in the model - albeit our estimate for the parameter is significantly smaller as expected. We also find standard errors for our parameters that are larger than those of Hommes & int Veld (2018). This is due to the relatively smaller sample of data we use in our estimations.

It is primarily because of identification issues that arise in estimating a non-linear model that we choose static values for leverage and interest rates in our regression. While we have quarterly data available that we could use in our estimation procedure, we find if we do so, there is not sufficient convergence power to estimate the models' behavioural parameters. It is likely that this problem could again be overcome by using a longer time series. We suggest that a possible better alternative to the datasets that we use here in this paper could be to use data such as that available in the Consumer Expenditure Survey. Another potential avenue for exploring this problem could be to elicit priors from homeowners, in a manner similar to that done in the experimental literature. This would allow the micro data to pin down the behavioural parameters. Whilst this would preclude the kind of approach taken in this paper, given that the beliefs and fractions are codetermined, it could be possible under an alternative framework. This could allow for an estimation procedure over a that has both a greater level of precision and covers a longer period. Further work on this is necessary but may resolve many of the issues that we have identified here.

Our final result in this paper is obtained by taking the values from our estimation and comparing the predicted proportion of speculators with the level of mortgage delinquencies observed in the US around the 2008 housing crisis. We expect that these should be correlated, as following a Minsky style telling of this period, we would expect the proportion of speculators to increase in the run up to 2008 and decrease afterwards, with the fall in house prices causing them to deleverage and being associated with a rise in delinquency rates. This is indeed what we observe, adding weight to the idea of a Minskian interpretation of the 2008 financial crisis.

2.6 Conclusion

The collapse of the US housing market was a watershed event in recent economic history, a moment that led to the implosion of the global financial sector and drove the world economy into recession. As such, understanding housing market dynamics and the role of leverage is an important step in helping to understand and anticipate for a future reoccurrence. The research we undertake here is of relevance to both policy makers and investors who wish to be better prepared for future crises.

In this paper we study asset price movement in a leveraged asset market using a model with behaviourally heterogeneous agents. Motivated by a Minsky style interpretation of the global financial crisis that is increasingly favoured by professional investors, we develop a simple Minsky inspired model for a leveraged asset market, building on the work of Hommes and in't Veld (2018) by allow investors to make use of leverage. Simulating the model, we see that agent borrowing behaviour plays an important role in determining price dynamics. Specifically, our first key result is that under this framework borrowing alone does not lead to any significant amplification in price movements provided that agents borrow optimally given their beliefs. However, where agents fall back on a simple heuristic to determine their level of borrowing we find that it can lead to boom and bust asset price cycles.

We then use a variation of the model with a borrowing heuristic to perform an empirical analysis of time series macro data the US housing market and find that our estimation results are consistent both with our theory and the empirical results obtained for other markets. Comparing our model with several benchmarks we see that leverage is an important component of analysis within a heterogeneous agents model framework. We also see that the model provides a small improvement in performance over an AR(1) model, whilst also offering an addition to our behavioural and economic intuition.

Looking specifically at the period around the 2008 financial crisis, we find our second key result: that using the model to estimate the fraction of speculators in the market allows us to match the mortgage delinquency rates observed in the US over this period. Interpreting speculators in this model as analogous to speculative/ponzi investors lends support to a Minskyan interpretation of the 2008 housing market crash.

Finally, we note that the estimations in this paper are limited by our use of macro data in estimating behavioural rules for market participants. Whilst this is convenient due to its availability and ease of use, the short time period available and aggregation of behaviour makes these estimations imprecise, and sometimes impossible. Combined with the significant non-linearities of the model, this makes identification of the behavioural parameters a particular challenge. A significant improvement to this methodology could be achieved using micro level data for longer time spans and more sophisticated valuation techniques which to attempt to address many of the identification issues.

To summarise, our analysis supports the idea of the Minsky style interpretation favoured by many investors regarding house price dynamics. Specifically, that a significant driving factor behind the US housing bubble was the increasing dominance of speculative investors using leverage to purchase assets. We do this by extending an asset market model proposed by Hommes and in't Veld (2018) to allow investors to make use of leverage. Simulations conducted in this paper show that, contrary to expectations, borrowing alone was not sufficient to lead to Minskyian dynamics under a price shock. However, with the introduction of a simple borrowing heuristic we recover the kind of boom and bust behaviour described by Minsky.

We test this framework against data for the US housing market and find that we can produce results that are broadly in line with what has been observed in the literature for other markets. However, we note that the weak convergence power of the estimation procedure places a limit on the robustness of the results and as such, work to improve on this such as incorporate fluctuations in fundamental asset values, or the use of micro level data, would be a valuable extension to this paper. We also more closely examine the period around the 2008 financial crisis, and find that the fraction of speculators predicted in the model correlates with the delinquency rates observed both leading up to and in the aftermath of the US housing market collapse. This is as expected under a Minsky style framework, where the proportion of speculative/ponzi investors are predicted to increase during a bubble as they take on more risk, before getting caught out when the bubble bursts.

This paper offers an alternative to much of the Minsky style modelling that has been done to date in attempting to place it within a mainstream body of work and supported by a measure of empirical verification. Given the findings presented here, policy makers and researchers may find a Minsky style interpretation of asset market dynamics a useful addition to their toolbox when assessing risks and opportunities in the housing market with a potential to be applied to financial markets more generally.

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3 Interlude

We have seen in the previous chapter how borrowing behaviour can affect the dynamics of an asset market. Specifically, its supports the argument that Minsky style borrowing behaviour influences the prices of assets, as supported by evidence for the US housing market. At the core of this results is the interaction of herding with individual level behaviours, an interaction that is often neglected from many economic models but in this instance has a significant impact of the market dynamics.

Whilst the preceding chapter focuses on the importance of behaviour in the context of asset markets, an area in which this type of model has been applied with some success previously, it is interesting to consider the possibility that such models could have relevance outside of such settings. Indeed, given that human behaviour is limited only by the extent of human activity, then we could expect to find such behavioural results in all areas of the social sciences. However, simply the existence of such an effect does not make it useful to understand it in such a way. Given the disciplining forces of markets in many contexts the added complexity of behavioural factors, may prove a false economy. For that reason I again choose in the next chapter to focus on an example in which the existing literature, in the absence of behavioural factors has smuggled to explain the macro level dynamics of a social system.

The following chapter will now use a similar technique to describe the behaviour of a population during a pandemic. We will again use a combination of individual level behaviour and macro herding but in this instance to examine the dynamic spread of a pandemic through a population. We will also introduce a second behavioural factor, namely attrition, as this will allow us to estimate the model from real world data, and see that behavioural factors are important for understanding the spread of a disease at a population level.

4 The behaviour of pandemics

Abstract

This paper introduces a novel adaption to a standard pandemic model that endogenizes agent behaviour using a discrete choice framework. In this model agents choose their preferred level of social interaction based upon observations of the current death rate, with this level of interaction then determining the future spread of the pandemic. Behavioural parameter estimates are obtained for the model in the case of the current Covid-19 pandemic using data for a several European countries, with a specific focus on the behaviour influencing the rate of transmission. Model estimates for these countries suggest a transmission period that is shorter than generally assumed in Covid-19 forecasts, but more consistent with emerging evidence from lab based studies, with implications for the continued spread of the pandemic and policy response. This paper also examines the role of behavioural attrition in determining the magnitude of outbreaks in pandemics with multiple waves. Finally, we use the parameters estimates to demonstrate a forecast for deaths due to Covid-19.

4.1 Background

Disease outbreaks and epidemics within human society are likely as old as human society itself. Indeed, one proposed explanation for the existence of religious dietary and hygiene rules is that they help inhibit the spread of disease amongst followers. It is unsurprising that epidemics throughout history have been recorded both in folk memory and by scholars, but it is interesting that they can also be associated with behavioural changes within a population long before disease itself were scientifically understood.

This paper investigates the spread of a disease through a population, where the population is aware of the spread of the disease and responds according to a simple decision rule. To do this we introduce a novel adaption of a standard SIRD model that allows for endogenous change in population behaviour in response to an epidemic. We use two types of agents that respond to changes in observed deaths and using that information according to a discrete choice function choose whether or not to engage in social distancing. We then simulate this model based on estimated parameters for suitable populations during the current COVID-19 pandemic to obtain projections of possible future outcomes.

This paper aims to empirically identify and investigate the importance of behaviour in the spread of Covid-19 using widely available macro data. Using this framework we can examine the validity of several behavioural assumptions that are commonly made in pandemic forecasting models. Additionally, we forecast based on these estimated parameters, future growth paths of the epidemic. As the estimated behavioural parameters in this model are structural, the model serves a further purpose as being able to identify the impact of changes in the spread of the virus due to the emergence of new strains of the disease.

As with much of the current mathematical modelling of epidemics, this paper finds its origins in the work of Kermack and McKendrick (1927). This body of work uses a compartmental approach to split population into multiple groups, and then tracks how agents move across these groups over time based upon a number of specified parameters. The unprecedented economic impact of the current COVID-19 epidemic combined with the widespread failure of standard epidemiological models has given rise to significant interest by economists into compartmental models and what contributions can be made stemming from the economics literature to improve future projections of the disease and its likely impact.

Many models have been produced by competing research groups to predict the future path of the pandemic. As an illustration of this, the data journalism specialists at FiveThirtyEight (538)[7] had ten different forecast models for the United States alone listed on their website as of May 28, 2021 when they stopped updating the list. As we can see from Table 6 on the following page summarising this list, these models differ across a variety of key assumptions.

These models are concerned specifically, with informing and influencing policy and as such choose to model policy interventions explicitly requiring large numbers of assumed fixed behaviours, meaning that the behaviour underpinning them is inherently non-structural. This allows for models that can produce reasonable short-run predictions for the impact of government interventions, but are consequently less well able to predict over longer time horizons as the underlying behaviour changes in response to both policy and the progress of the virus.

[7] FiveThirtyEight is a news outlet that specialises in data driven analytics and news reporting

Covid 19 Models and key assumptions	
Johns Hopkins University	Incorporates information about stay-at-home orders and assumes that the effectiveness of social distancing measures in a given state decreases by roughly 25 percent after those orders are lifted.
Iowa State	Does not make specific assumptions about the interventions in effect.
Colombia University	Assumes that contact between people will increase by 5 percent each week for the next two weeks.
University of Massachusetts	Factors affecting transmission will remain similar over the forecast horizon.
Northeastern University	Current social distancing policies will continue indefinitely
University of Arizona	Assumes interventions will remain in effect for at least four weeks after the forecasts were made
Los Alamos	Assumes that there will continue to be interventions, such as stay-at-home orders, but it does not specifically assume what those interventions will be. Instead, it considers various possible interventions to arrive at its forecast
Georgia Tech	Assumes that the effects of interventions are reflected in the observed data and will continue.
MIT	Accounts for state reopenings, and assumes that interventions would be reenacted if cases continue to increase.
UCLA	Incorporates state reopenings and assumes contact rates will increase after states are reopened.

Table 6: List of Covid 19 forecasting models and key assumptions taken from the list curated by FiveThirtyEight.

Given the non-linearities present in pandemic forecasting model, they are highly sensitive to assumptions. As such, work to determine the key parameters of these forecasting models is crucial. Meta-analyses by Alimohamadi et al (2020), Locatelli et al. (2021) and Billah et al. (2020) find values for R_0 to be 3.3, 2.2 and 2.87 respectively. Suggesting a wide range of uncertainty around these key assumptions. A notable contribution is made by here by Feretti et al. (2020) studying matching transmission pairs China. $R_0 = 2$, with almost half of this being attributed to presymptomatic transmission. Additionally, a very recent human challenge trial conducted in the UK by Killingley et al. (2022) established the average length of time from first exposure to the virus to viral detection and symptoms onset (incubation period) to be just 42 hours. This is much shorter than the time period that has typically been assumed in forecasting models. We attempt to contribute to this debate here by using a behavioural adaption of a standard pandemic model to estimate these parameters using data for observed death rates.

In addition to more traditional epidemiological studies, a new body of literature has recently emerged within economics that is concerned with endogenizing the dynamics of epidemiological models, inspired by the Covid-19 pandemic but with origins dating to the HIV outbreak of the 1990s. McAdams (2020b) provides a comprehensive review of economic epidemiology with a focus on recent developments. The key mechanism we will use in this paper is to endogenize social distancing by embedding an individual trade-off decision within an agent based behavioural framework, in order to study the implications of behaviour on the spread of Covid 19. As such this paper is similar to the work of Toxvaerd (2020) studying endogenously arising equilibrium social distancing and McAdams (2020a) thats endogenizes individual decisions to see how the epidemic trajectory can be shaped through the coordination of expectations. However, we will differ in approach by identifying the endogenous behaviour directly from the observed death rate. This paper also relates to the work of Farboodi et al. (2020) and

Bethune & Korinek (2020) and Garibaldi et al. (2020) on incentives and the tradeoffs arising from catching Covid and the implications this has for spread of the disease. This paper builds on that idea by considering an individual's tradeoff between catching Covid 19 and potentially dying that is embedded in the model.

Alternative approaches to pandemic forecasting have been proposed, such as that by Youyang Gu (2021) with his Covid-19-projections model, as well as in work by economists such as Cochrane (2020), Jones (2020). These models differ from those used in the aforementioned studies by focusing on characterising underlying behaviour using only minimal assumptions. In particular this approach differs from the vast majority of the literature in the sense that most other work assumes fixed behaviours and then allows for modellers to vary policy choices. Instead, this approach assumes that policy is fixed but that behaviour will change endogenously. This has a distinct advantage by allowing the model parameters to be estimated directly from the data rather than fixed by assumption.

The model presented in this paper lies in spirit between the approaches taken by Gu (2021) and that proposed by Jones (2020). In particular we follow the SIRD framework of Jones, adding behavioural switching in a discrete choice model as has previously been applied by Brock and Hommes (1998, 2000), Hommes and in't Veld (2018) in the context of financial markets to characterise herding within a population. This allows us to test a different set of assumptions regarding the underlying behaviours when compared with Gu, whilst allowing the more complex endogenous behaviour when compared to Jones that is necessary to match longer run observations on the path of the epidemic.

However, we would expect that herding of the style employed here could be insufficient to explain the large second waves of infections observed in many countries, given that in the long run it forces convergence towards an equilibrium. As such I consider a form of behavioural fatigue as a second factor that could influence individual decision making. Such a phenomenon is somewhat controversial with several prominent behavioural scientists asserting that no such behavioural fatigue exists, particularly early on in the pandemic. However, recently many more models have included such a phenomenon in models, so I will allow for the possibility in this setup that behavioural fatigue may occur within the population.

In order to build our model, we abstract away from several phenomena. This is necessary in order to have a model that is tractable but introduces several limitations. First, we assume that once an individual has been infected and recovered, they cannot then be reinfected. There is evidence that whilst contracting Covid-19 confers a level of immunity from reinfection, this immunity wanes over time and that individuals can be reinfected. In addition to this, we further assume in this paper that there is no widespread and effective vaccination program, or emergent variants of the virus that might previously infected individuals subject to reinfection. In all these cases this would substantially change the future growth path of the pandemic in way that cannot be described with this current model. We attempt to mitigate the effect of these assumptions by focusing the estimations on the period between March and November 2020, when the alpha variant emerged in the UK, before the rollout of effective vaccines and whilst and those previously infected will still have a level of immunity. Whilst these events are beyond the scope of the current model to handle they provide an interesting avenue for future research.

We also make a further assumption that government does not play substantial role in influencing the spread dynamics of the pandemic by following a 'Zero Covid' policy as seen in countries such as China. This is done for two reasons. First, we focus on the spread of Covid-19 in European countries that are liberal democracies. We can imagine that any restrictions on individual liberty that members of that population do not wish to comply with would be circumvented by failing to obey restrictions or challenging them through the legal system[8].

Equally any measure that granted more freedom than preferred would not be exercised. Furthermore, trying to impose a policy that differed substantially from the societal preferences would undermine confidence in the government, leading to increasing levels of non compliance, disincentivising government from doing so. As such, for liberal democracies we could might expect government interventions that align with the social equilibrium for preferred interventions. This does not necessarily mean that government exactly follows the preferences of the 'average' voter, but we might expect that democratic governments not to deviate too far provided they wish to be reelected[9].

In this setting the government plays the role of a coordinator taking decisions that individuals cannot make on their own, such as by closing schools or introducing furlough schemes to allow employees to stay away from work when these are broadly popular. This paper attempts to model the social equilibrium directly using a population with heterogenous beliefs, such that some people can ignore even very strict lockdowns, and others stay home even when there are very few Covid-19 cases. By allowing agents in the model to switch between beliefs we can allow the aggregate behaviour of the population to vary over time and in response to news about the pandemic.

The second motivation for eliminating the government from the model is that it allows us to avoid modelling policy interventions directly. This significantly reduces the number of additional assumptions required. Given the model is non-linear and so estimation will be prone to overfitting, the ability to eliminate any extra assumptions allows us to have greater confidence in any parameter estimates and forecasts.

[8]Whilst not a country studied here, this was particularly true for the US where many Covid restrictions were overturned by courts.

[9]This is in essence an appeal to the Median Voter Theorem (Black,1948).

The remained of this paper is presented as follows. In the following section I present and discuss some of the data available regarding the Covid-19 pandemic. Data quality has been on particular concern to many researchers, and examination of the data allows us to identify the appropriate data to use as well as sanity check our behavioural structure for the model. Using our assumptions regarding behaviour, in section 3. I then present the theoretical details of the model. Section 4 contains the simulated dynamics for a representative set of behavioural parameters where we can examine the effects of changes to these values. In section 5 we empirically determine the relevant parameter values for several different countries. The results are discussed in section 6 and the paper then closes with some final conclusions.

4.2 Data

In this section I will briefly discuss some of the data available to researchers, both in the context of its utility for modelling, as well as how it has informed the behavioural structure I have chosen to employ in this paper. A particularly significant difficulty during the COVID-19 outbreak has been the quality of available data. During the initial outbreak researchers understandably focused on case data from reported positive tests as a result of it being the only available data set. This proved to be wildly misleading. As was observed in many countries the demand for testing far exceeded capacity to supply tests, leading to significant underreporting of case numbers. Estimates suggest that this may have been by as much as a factor of 20 at the peak of the epidemic in some cases. A good contemporary analysis of the limitations of this approach is provided by the researchers at FiveThirtyEight.

An alternative to positive case numbers as a measure for the spread of the epidemic within the population is hospitalisations. Indeed in a number of countries modellers have moved towards this metric as an alternative. This is again not without its limitations. The use of hospitalisations data once again relies on the timely and complete reporting of admissions. In many countries where health care systems do not operate within a single unifying framework but rather independently either as sole hospitals or within healthcare groups. This has led to differences in reporting standards with data being significantly incomplete in some instances. Most importantly, if a country changes their testing methodology part way through the sample period, the data points before and after that change are no longer comparable.

Given the limitations of the previously discussed datasets, I will follow Gu (2021) and Jones (2020) in this paper focus on raw death tallies for COVID related cases. Death data is assumed to be harder to manipulate by governments that wish to make it appear as if their Coronavirus response is performing better than actuality, and is also significantly less prone to underreporting due to lack of testing capability, a significant problem observed in the first wave case data for many countries.

It is also important for us to consider country choice when identifying appropriate sets of data to use for the estimation procedure we will perform later. For countries that are too small we will find any trend in the data is swamped by statistical noise, this is equally true of countries that have not had significant outbreaks. Additionally, for countries that are too large geographic spread becomes a significant concern. This is a less important factor for a medium sized European country, or even a larger one where the population highly geographically concentrated, so for that reason we will focus on several European countries for comparisons. However, this would need to be considered for a country like the USA or India, which is a limitation of this approach. Finally we cannot consider countries where the government exerts significant control over the actions of its population, as this would violate our initial assumptions regarding the role of government. For that reason, we also exclude a number of countries, in particular China.

As we can see from the plot in Figure 7., the 7 day moving average for Covid-19 deaths in the United Kingdom, Italy and the Netherlands we observe multiple waves of infection, with a large gap between the first and second waves. We choose to use a 7 day moving average to adjust for the reduction in reporting that occurs over the weekend. Moreover, it is worth observing that there appears to be a third wave on infections that occurs towards the end of the period, this is due to the emergence of a new and more transmissible variant of the virus and is discussed in greater detail later in this paper.

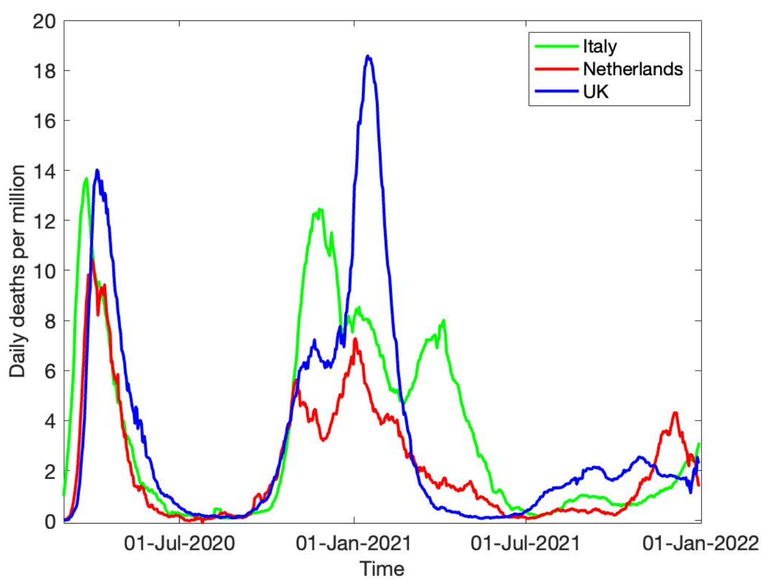

Figure 7: Plot of 7 day moving average of deaths per million for Italy (green), the Netherlands (red) and the UK (blue)

We see from Figure 7. that initially all the countries suffered very similar initial waves of infections, with the correlation between countries dissipating over time. There are several explanations for this. First is that this is due to the spread of variants to different countries at different times. This is likely true for the large second wave in the UK that coincided with the emergence of the alpha variant in that country. A second, is that this difference is partly due to the speed of vaccination programs in each country. As there is no easy way of handling these factors within our model, we will restrict our empirical work to the period before these happened and study the period from the start of the pandemic to the beginning of November. During this period we see that the countries all have very similar patterns for deaths, so we expect that the behavioural parameters we estimate should also be similar.

4.2.1 Population Behaviour

It is also useful for us to examine what data there is on the behaviour of populations, both to check our initial assumption that there should be an endogenous behavioural response to the state of the pandemic and also to inform our modelling intuition. Figures 8. and 9. show data on population mobility and lockdowns for the UK and Italy respectively.

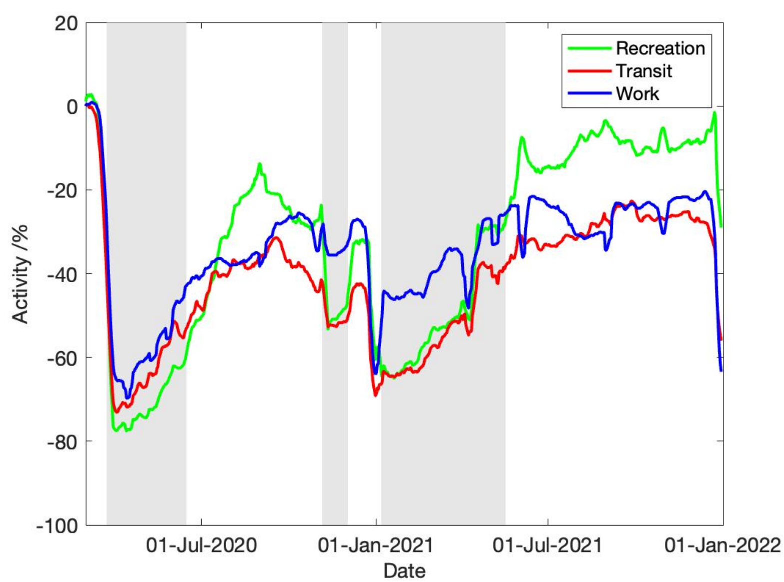

Figure 8: Percentage change in time spent at recreation venues (green), public transit usage (red) and at work (blue) in the UK since the start of the Covid-19 pandemic

Figure 8. shows that in the UK the amount of time spent in work and using public transport significantly decreased at the beginning of the pandemic, and that much of this decrease took place in advance of any implementation of a national lockdown. This is suggestive that the behaviour of the population changes endogenously, and in response to new information, rather than to strict government rules. We also observe that not only does behaviour change in advance of the lockdown order, the population behaviour continues to change during the lockdown, with mobility trending upwards throughout the lockdown periods. This again supports our reasoning that we expect behaviour to change endogenously, but also that some element of behavioural attrition may be at work, as agents tire of maintaining altered behaviours.

It is particularly interesting to examine second UK lockdown, when the UK government introduced a short lockdown with the intention of reducing infections rates before the Christmas period. This was unusual in that most lockdowns involved slowly relaxing restrictions, but in this case the lockdown restrictions that were imposed were removed all at once. We see that the deviation in behaviour is very small relative to both the initial drop and subsequent changes in behaviour. To confirm this we examine the same data for Italy, shown in Figure 9.

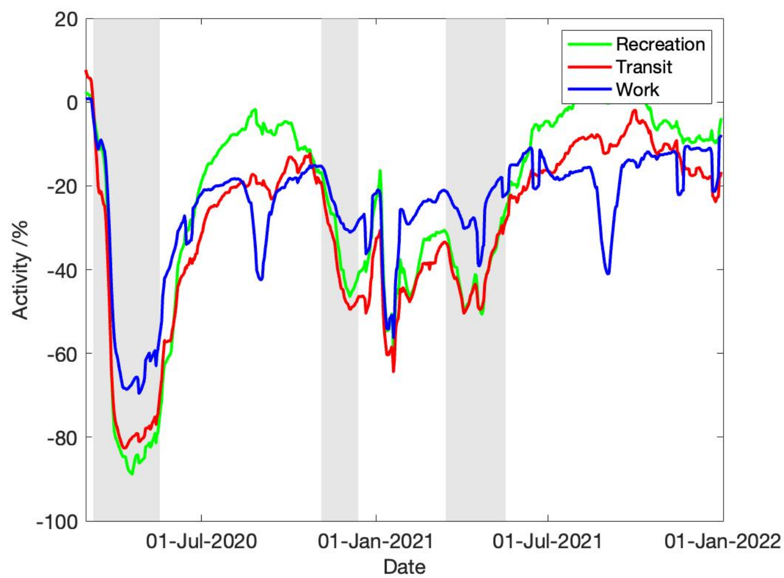

Figure 9: Percentage change in time spent at recreation venues (green), public transit usage (red) and at work (blue) in the UK since the start of the Covid-19 pandemic

Figure 9. shows that in Italy the amount of time spent in work and using public transport significantly also decreased at the beginning of the pandemic, but not before the imposition of lockdown. This may be due to Italy being the first European country to suffer a major outbreak. However, like the UK we see that behaviour changes throughout the period, not only in response to lockdowns. This supports our argument that government interventions such as lockdowns are not the primary factor in determining population behaviour.

4.3 Theory

In this section we outline the theoretical setup of the model. First we introduce the basic SIRD model as used in Cochrane (2020) and Jones (2020). Then we will adapt that setup to allow for endogenously changing behaviour with the addition a discrete choice framework. Finally we will discuss the integration of behavioural attrition into the model.

4.3.1 SIRD model

We begin by following standard notation in the literature and assume a constant population of N individuals, each of whom may be in one of five states:

$$S_t = \text{Susceptible}$$
$$I_t = \text{Infectious}$$
$$R_t = \text{Resolving}$$
$$D_t = \text{Dead}$$
$$C_t = \text{ReCovered}$$

Where the sum of all the states is equal to the total number of agents in the population and t is time. For the purposes of this paper t will be in days and the pandemic begins at $t = 0$.

$$S_t + I_t + R_t + D_t + C_t = N \tag{17}$$

We choose to make use of a SIRD rather than a SIR framework as one of the significant encouraged behaviours is that of self-isolation when individuals know that they are infected. This would result in having many infected individuals within the population who are not infectious to others due to their temporary withdrawal from interacting with the rest of the population. For that reason it is important for us to draw a distinction between those individuals that are both infected and infectious and others that are infected but not infectious.

The transitions between the different states of the compartmental model are described below. We start with the evolution of the number of susceptible individuals in our population. For simplicity I assume here that there is no time lag between exposure and infectiousness. This strictly decreases with the rate depending on β, which can be roughly interpreted as the number of significant contacts per day and the proportion of infectious individuals in our population. Notably in this paper, we will allow β to vary over time, allowing for multiple waves of infections.

$$\Delta S_{t+1} = -\beta_t S_t I_t / N \tag{18}$$

Next, we have the number of individuals in the population that are infectious. The first term describes the number of individuals that become infectious each period, and the second gives us the number that leave the infectious group (such as because the self isolate). The parameter $1/\gamma$ can be interpreted as the average length of time that an individual remains infectious to others within the population. This differs from a standard SIR model in that we assume agents are not infectious for the entire period that they are infected. In doing so we isolate the average length of time that agents actually spend infecting other agents, which has significant relevance for policies such as contact tracing. The size of γ also has implications for the proportion of transmission that is asymptomatic, with higher γ implying more asymptomatic transmission for a given period of infection for a given length of time before the onset of symptoms.

$$\Delta I_{t+1} = \beta_t S_t I_t / N - \gamma I_t \tag{19}$$

Once individuals transition out of the infectious stage, they enter a recovery period. This is necessary to reflect the time lag observed in the data between individuals being initially exposed to the disease and deaths being recorded, which on average can be anywhere from 2 to 4 (Linton et al., 2020, Ward & Johnsen, 2021) and is determined by parameter $1/\theta$.

$$\Delta R_{t+1} = \gamma I_t - \theta R_t \tag{20}$$

Finally individuals exit the recovery stage either by dying or making a full recovery and becoming immune to the disease. The proportion of individuals that make a full recovery is given by the parameter δ as shown below.

$$\Delta C_{t+1} = \delta \theta R_t \tag{21}$$

Conversely the proportion that die from the disease is given as $(1 - \delta)$.

$$\Delta D_{t+1} = (1 - \delta)\theta R_t \tag{22}$$

We assume that agents are aware of the number of deaths that occur and use this number to determine their level of social interaction as shown in the next subsection.

4.3.2 Discrete Choice

Our innovation to the standard SIRD model introduced in this paper is to allow for β to vary over time using the type of discrete choice framework that has previously been employed by Brock and Hommes (1998, 2000) to model asset market bubbles. In order to have a time varying level of social interaction β we will use a discrete choice model where individuals can choose one of two levels of interaction, either high or low. This model has been used extensively in the literature on asset pricing to characterise belief heterogeneity amongst agents, with two types of agents, as I use here, have been shown by Aoki (2002) to be sufficient to characterise the majority of behavioural heterogeneity in a population. To do this we further divide the population N into two fractions, those with a high beta and those with a low beta, such that: low

$$\beta_t = n_{low,t}\beta_{low,t} + n_{high,t}\beta_{high,t} \tag{23}$$

The sum of the factions again adds to the total number of agents in the population.

$$N = n_{low,t} + n_{high,t} \tag{24}$$

We assume that these fractions vary over time using a discrete choice rule such that for agent type h (high or low):

$$n_{h,t+1} = \frac{e^{U_{h,t}}}{\sum_{h=1}^{H} e^{U_{h,t}}} \tag{25}$$

Agents in each fraction evaluate the current prevalence of the disease within the population by comparing the number of deaths D_t with some constant value placed on life, w. α_t is a scaling parameter that determines the sensitivity of agents to their tradeoff, and is allowed to vary over time. Equations (26) and (27) mean that when the observed death rate is high relative to the value agents place on their lives, $U_{low,t}$ is positive and so more agents switch to the low level of interaction $\beta_{low,t}$. Conversely when the death rate is low relative to the value agents place on their lives, $U_{high,t}$ is positive and so more agents switch to the high level of interaction $\beta_{high,t}$.

$$U_{low,t} = \alpha_t D_t - w \tag{26}$$

$$U_{high,t} = -(\alpha_t D_t - w) \tag{27}$$

We choose this functional form for agents to use to evaluate their decisions rather than a more standard utility function in order to avoid having to make additional assumptions over parameters, in keeping with Anderson (1993) and the preceding literature. It could be possible to construct a utility based objective function, as has been done in the economic literature for similar problems. However, in this instance, given that we will estimate w directly from the data and that the evaluation rule has the same structure as a simple utility function, using a full utility function would offer little improvement in performance at the expense of additional complexity. Using these evaluations agents can then switch between fractions depending on the difference in their relative values.

Finally, we introduce behavioural attrition into the model. As has been seen from the data on avoiding public places in the previous section, we see that over time the population as a whole returns slowly towards normal behaviour. In order to account for this we allow (but do not assume) for an element of behavioural attrition in the actions of our agents.

$$\alpha_t = \alpha e^{-\kappa t} \tag{28}$$

Given that we will obtain a value for w empirically later in the paper and ascribe in no special meaning, we can allow α here to serve both as a normalisation factor on the level of behavioural attrition and a sensitivity parameter which would be typical of a discrete choice model. This is necessary as otherwise α is codetermined with the switching sensitivity and therefore not possible to disentangle empirically using this setup.

4.4 Simulations

It is instructive for us to examine the behaviour of the model using some simulations using some representative parameters. We can broadly characterise the parameters as being either epidemiological or behavioural. It should be emphasised that for some of these parameters, an arguments can be made that they could fit into both categories. However for the purposes of the simulations here we will use values consistent with those identified in the epidemiological literature where they are available, either by assumption or estimation (although we will see later in our estimations that some of these parameters may not be entirely reflective of the parameters used in this simulation, this is discussed further in the estimation section).

Additionally, because $\beta_{low,t}$ and $\beta_{high,t}$ are codetermined in the simulation procedure we need to set one of them as constant. We choose to set $\beta_{low,t}$ as constant because it is bounded between zero and $\beta_{high,t}$, whereas $\beta_{high,t}$ is only bounded below by $\beta_{low,t}$. For this reason we set $\beta_{low,t}$ to be 0.1, so that there is some minimal level of interaction for agents of this type. This accounts for the fact that even agents that are trying to avoid social interaction will still engage at some level with the rest of the population, for example by purchasing food. We can then generate $\beta_{high,t}$ and the evolving population fractions $n_{low,t}$ and $n_{high,t}$

endogenously in the simulation. We set the remaining parameters in line with the literature as shown in Table 4.

Representative Parameters	
Symbol	Value
R_0	3
β_{low}	0.1
γ	0.15
θ	0.05
δ	0.008
α	1
w	1

Table 7: Representative parameters given in the Covid-19 literature

Using these values we construct simulations of the path of deaths and cases that could occur during the Covid-19 pandemic. We will first examine the case without behavioural attrition. The key idea here is that the spread of the pandemic is driven by agents evaluating the behaviour according to equations 26 and 27 and as they observe deaths rising some choose to switch to a lower level of interaction to reduce their risk of dying. In doing so the overall level of interaction and therefore transmission in the population falls. Eventually, this leads to the wave of infections peaking and then decreasing, with this process continuing in reverse. Provided the disease is not fully eliminated in the population this process repeats until there is an insufficiently large proportion of susceptible people left despite everyone having eventually switched to a high level of interaction, i.e achieving complete herd immunity.

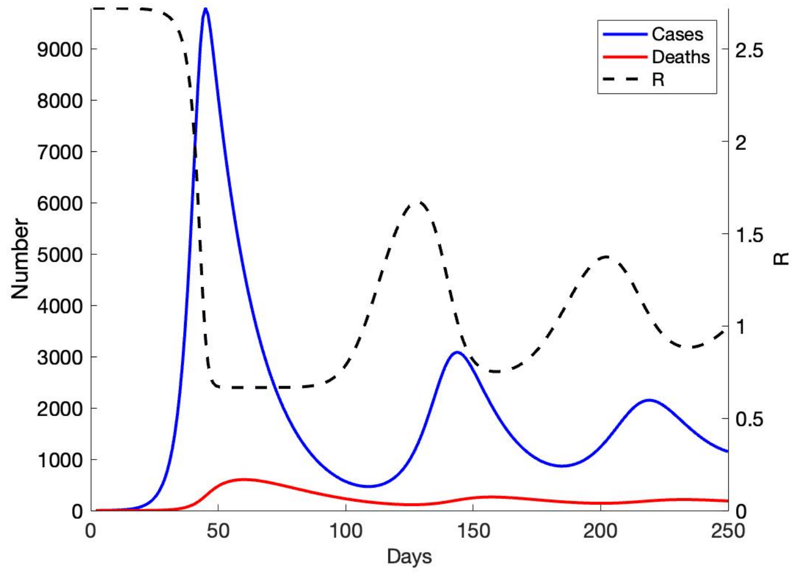

Figure 10: Short run pandemic simulation without behavioural attrition

We can see from the simulation in Figure 10. that with no behavioural attrition we can generate multiple waves of infection that occur as a result of agents responding endogenously to the number of deaths, as we would expect. However, we also observe that each subsequent wave is smaller than the previous wave. We have seen from the data in Figure 7. that this is not necessarily the case in practice.

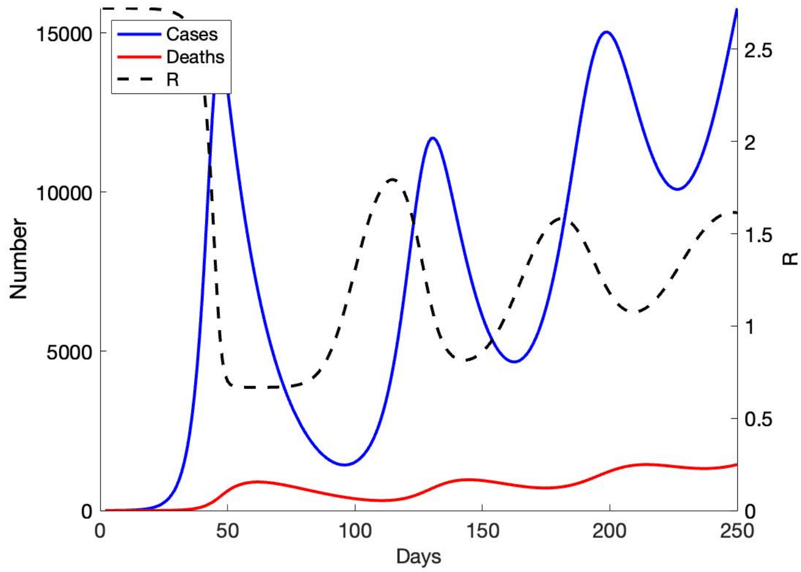

Figure 11: Short run simulation of a pandemic with behavioural attrition

Therefore in Figure 11. we simulate the model with behavioural attrition, setting $\kappa = 0.001$. We see that by allowing for behavioural attrition, we can now generate waves of increasing size over time, matching what we see in the data. This would appear to more closely correlate with the fact that some countries appear to have had second waves larger than the first.

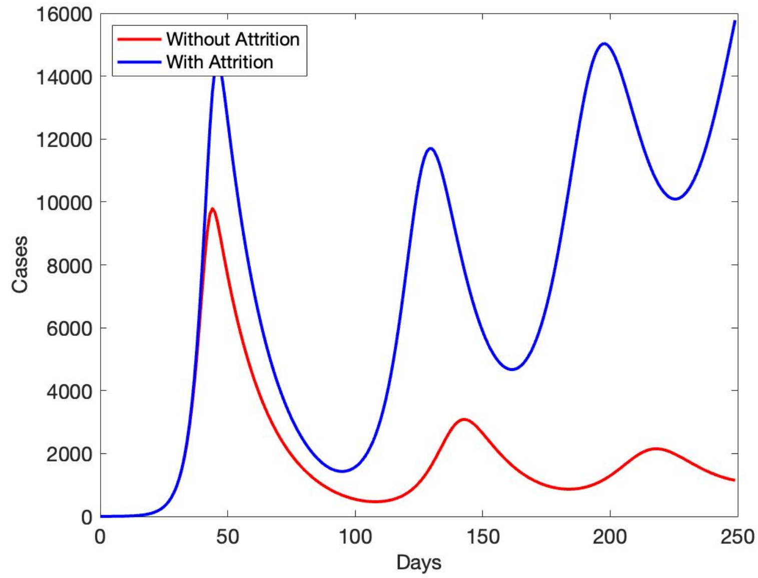

Figure 12: Short run simulation of a pandemic with behavioural attrition

Comparing the case rates from Figures 11. and 12., Figure 13. also shows us that the addition of behavioural attrition leads to more agents being infected more quickly. As a result of this, higher rates of behavioural attrition will lead to a population achieves complete herd immunity sooner.

It is also interesting to examine the long run behaviour of the model. Many of the alternative formulations available in the literature that are interested in using time varying interactions of agents rely on a mechanistic formulation such as Jones (2020) that results in a permanent decline in the rate of interaction. In this model, the rate of interaction is able to begin returning to its original pre-pandemic level once a degree of herd immunity is reached. We can see from the Figure below that the reproduction rate R, as shown by the dashed line, returns to its initial value, as it should in the event of the entirety of the population acquiring immunity

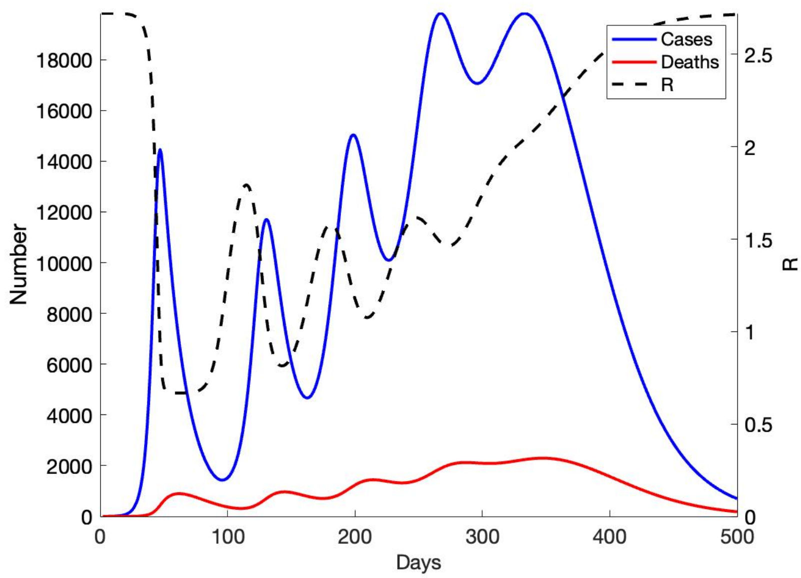

Figure 13: Long run simulation of a pandemic with behavioural attrition

Finally we examine the impact of the infectious period. It is commonly assumed in the literature that the infectious period of an individual that has been exposed to Covid-19 is around 5-7 days. This assumption is normally derived from clinical observations that an average individual can infect others for this length of time, but does not take into account the change in behaviour that could occur once individuals know they are infected. This is perhaps one of the most important parameters in the model given it is the focus of many policy interventions, such as the implementation of test and trace and the encouragement by governments of self isolation for symptomatic individuals and their close contacts.

Whilst the simulations we have seen above, are able to produce waves of infections similar to those seen in the data, given that model is non-linear we should be concerned that there may be other parameter sets that are also capable of fitting the data. An example of this is shown in Figure 14.

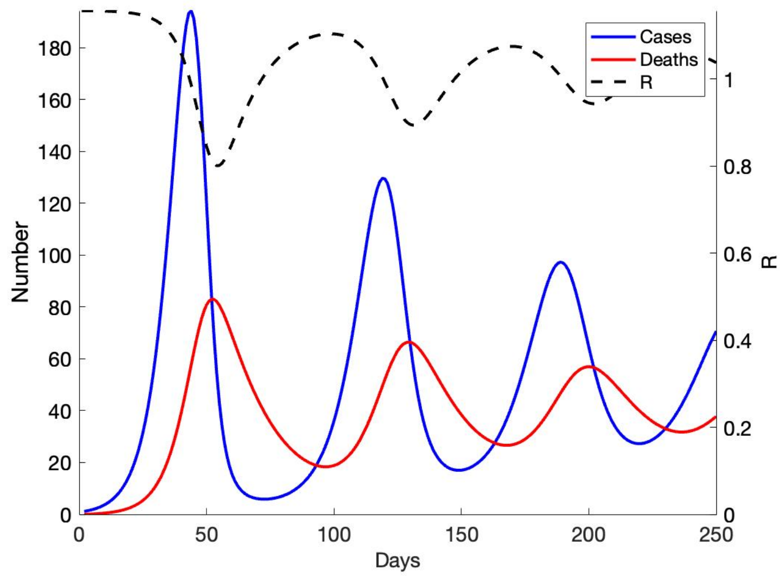

Figure 14: Short Run simulation of a pandemic without behavioural attrition using alternative parameter set

In the Figure 14 we shorten the infectious period to one day and decrease the R number to 1.3. Whilst the simulations shown here are just examples, we can see that two different sets of parameters can produce broadly similar dynamics. This is a phenomenon not found in other pandemic models, given that the infectious period is assumed fixed. This has the effect of pinning any estimation to a particular parameter set before an estimation is performed. Given that a high value of γ implies a low value of R identifying the appropriate parameter set has substantial implications for the spread of a pandemic and potential policy interventions such as the effectiveness of test and trace systems.

4.5 Estimation and Forecasting

This paper departs from the standard SIRD setup of Cochrane (2020) and Jones (2020) by utilising a discrete choice function that allows for heterogeneity over social distancing within the population. More specifically we assume that agents within the model can be broadly characterised as high or low social distancers. Agent choice over social distancing is then informed by observations of the daily death rate. The fractions of agents that choose each option is calculated using a discrete choice function in the style of Brock and Hommes (1998, 2000). This approach has the advantage of allowing the level of social interaction, and therefore transmission rates, to be determined endogenously within the model and as a function of data that is observable by the population. This means that unlike alternative formulations we do not have to rely on an exogenously determined fall in the level of social interaction over time and that over long periods the population level behaviour will return to its pre-outbreak level. As discussed in the previous section, there is a concern that there are multiple parameter sets that allow the model to fit the data. A further advantage of the approach taken in this model is that by endogenizing social interaction we can estimate the behavioural parameters directly from the data, allowing us to identify the most appropriate parameter set.

4.5.1 Parameter Estimates

Due to the nonlinear nature of both the model and the data most standard regression techniques are either inappropriate or fail due to insufficient convergence power. In particular, we might suspect that the distribution of parameters will not be normal. This is a well known problem for this kind of estimation problem and can often be mitigated by further assumptions on the shape of the underlying distribution. In this case however, given the level of uncertainty around the transmission of Covid-19 and the relevant data, we would like to avoid making these assumptions. For that reason we will estimate our parameters with a Montecarlo method using 10 million simulations for each country, comprising 10,000 simulations per pass to estimate an initial best fit, with this process repeated 1000 times to obtain a distribution for the fit. This technique is in keeping with Hommes & int Veld (2018) and has the advantage of being agnostic to the shape of the underlying probability distribution for our parameters, at the expense of being slow and computationally expensive.

There are several other drawbacks to this approach. First, the use of macro data, which whilst easy to obtain, is less optimal for estimating than micro level data sets that contain more detail. Second, this model does not allow for vaccines or variants that might affect transmission, in order to isolate the population behaviour. Given that there have been a number of Covid 19 variants discovered and vaccinations widely available, we are required to limit the data used for our estimation to the period before this happened. Finally, it requires assuming that government intervention does not significantly impact the level of social interaction beyond what the population would choose. We have some confidence that this should be the case for the UK from the mobility data presented earlier in Figure 8, but this remains a limitation to the approach. To address these issues in our estimation we will use a time series of death data for the UK, truncated at the beginning of November.

We assume as fixed $\theta = 0.03$ and $\delta = 0.008$. The parameter θ differs from the previous section for as we found that a smaller value produces a better fit, and it is not possible to estimate both θ and γ simultaneously[10]. We choose to fix these particular parameters for two reasons. First, the estimation procedure is too computationally expensive to estimate all the parameters. Given that we are particularly interested in the transmission of Covid-19 and the relevant behaviours

[10] We are limited in a practical sense here by the length of time it would take to run the Montecarlo estimations. The current model requires around 24 hours using a reasonably powerful computer. Including an additional estimation parameter would result in an estimation time on the order of multiple weeks. This could potentially be overcome with access to a distributed computing facility and this would offer an interesting avenue for future research.

we need to include parameters such as γ in the estimation, whereas θ and δ, are the least relevant for what we are looking to study. Second, we have relatively more certainty that their values are reasonable based upon the wider literature, as they are at least partially observable. In the case of θ, the parameter value corresponds with an infected (but not infectious period) of around one month rather than the 20 days implied by a value of 0.05 used previously. This is not too far outside the range found Linton et al. (2020) for cases in China of 15.1-29.5 days and Ward & Johnsen (2021) using clinical data for the UK of 17.4-24.7 days from infection determined by symptom onset to death. The discrepancy may be explained in this model by the significantly increased size of γ that we estimate, as shown in the table below for our parameter estimates.

Parameter estimates for a selection of countries			
Parameter	UK	Italy	Netherlands
R_0	1.2	1.2	1.1
γ	1.6	1.7	1.8
α	0.6	0.7	1.4
w	4.9	4.8	4.4
κ	0.02	0.02	0.06

Table 8: Behavioural parameter estimates across countries for Covid-19

From our parameter estimates we see that in contrast to much of the literature we find that R_0 is somewhat lower than typically assumed in forecasting models. This is unsurprising however given that our estimates for γ is around 1.6. That would imply that whilst a single individual in our population could expect to infect only 1.2 people they do so in the space of less than one day on average. This is in contrast to our original assumption, with a high R and low γ where an individual would infect more people, but over a longer period of time. The parameter estimates, whilst substantially different to those used in our initial simulations, do not significantly alter the appearance of the simulation plot as seen in Figure 14. This is due to the of multiple parameter sets that can fit the observed macro data, as is common with non-linear models.

An advantage of using Monte Carlo methods is that we can examine the sum of squared errors for each parameter set that is drawn to identify which parameter set provides a better fit in the event we have multiple possible fits. Doing so reveals that this model can select from two possible sets of parameters, from within the estimation space that fit the data. One of those sets is already identified widely in the literature. Namely a parameter set with a large R value and a small γ. Whilst we also find this fit, the estimation here finds a better fit with a small R value and a large γ.

Unfortunately, the existence of multiple fits means that the tools we would ordinarily use to check the robustness of our results are not applicable in this case. However, we can compare our results with studies conducted that do not rely on fixing γ. Crucially, the first human challenge trial conducted by Killingley et al (2022). found that the time between initial exposure and becoming symptomatic was less than two days. Given that it is unlikely that individuals become infectious immediately after exposure and in the countries such as the UK were required to self isolate on displaying symptoms and so could not infect anyone. Therefore, it seems likely that this study places an upper bound for the value of transmission period for the data we study that is substantially shorter than that 5-7 days that is commonly assumed, but is consistent with our larger value of γ.

Our result is further supported by analysis produced by the UK Government UK Health Security Agency (2022) for the R value in the UK over the relevant time period to fall within a of range 0.7-0.9 in June 2020, and peaking at 1.3-1.6 in October. This is slightly higher than predicted by this estimation, but we are much closer than the values of around 3-5 for R_0 that are required to obtain a small value for γ. This result might also go some way to explaining why many of the forecasting models have also proven to perform badly when used for forecasting beyond the very short term despite claiming high levels of confidence. We will test the forecasting ability of this model with the estimated parameters in the following section.

4.5.2 Behavioural Forecasts

Using the parameter estimates that we have obtained we can generate forecasts for the future growth path of deaths that occur as a result of the pandemic. We test this here, by using an estimate of the behavioural parameters obtained from data up to November 1^{st} 2020, and then simulating the expected deaths for the next 3 months. We choose November 1st as a cutoff here as this is when a new variant of Covid-19 began to emerge in the UK.

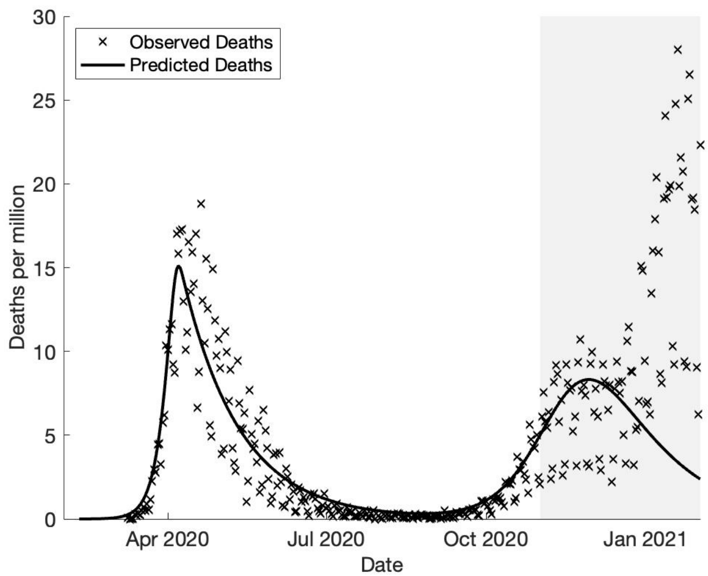

Figure 15: Forecast from November 1^{st} 2020 for Covid deaths in the UK (shown in the shaded area)

We see that projecting the forecast forwards from November 1^{st} 2020, the model is able to closely predict the number of observed cases up to the beginning of January. After this point the forecasts diverge, however this is expected as this was the alpha variant established itself of the dominant strain in the UK. This model performs well compared with alternative pandemic models, which often struggle to forecast more than a few weeks ahead with any accuracy due to their reliance on large numbers of fixed assumptions.

A significant contribution of the approach taken in this paper is that because the behavioural parameters are structural in nature the behaviour in the model changes endogenously and in a predictable manner. In the event of a new variant emerging this allows us to form a counterfactual forecast and gain a baseline for what would have happened in the absence of the new variant. Such forecasting could be of use to policy makers for the purposes allocating resources to best combat surges in cases and deaths as the pandemic progresses through multiple waves. In principle, it may be possible to use this model to identify the some of the key parameters of an emergent strain, such as the reproduction number R. However, this exercise would be highly computationally expensive given the need to disentangle the impact of the new variant from the original, and not possible without access to significant computing power[11]. Whilst this would put such a tool out of reach for individuals, for countries that do not have the ability to conduct surveillance of new variants in other ways, a technique that allows for the identification of more transmissible variants from data already being collected may be useful tool to have.

4.6 Discussion

We have seen that a relatively simple compartmental model with behavioural heterogeneity can be used to simulate the dynamics and spread of a pandemic within a population, and forecast its future growth path. Importantly the approach presented here relies on agents reacting endogenously to changes in the information available to them rather than this behaviour being mechanically determined by assumption.

An important result of our estimation procedure is that the average period of time in which one agent transmits the infection to another is far shorter in the

[11] We would have liked to do this as an extension to this paper, but on attempting this, discovered we do not have access to the computing power required

UK than is typically assumed in many forecasting models. We are able to find this result by allowing behaviour to respond endogenously within the model and our estimation procedure. Moreover, it is interesting to note that the results suggest that the average infectious period for the UK is less than 18 hours. This coincides with medical evidence that most individuals are asymptomatic in the first 24-48 hours post exposure, and are not infectious for all of that period (Killingley, 2022). In turn this would suggest that a majority of infections in the UK are actually transmitted by presymptomatic carriers, with many of the those only later developing symptoms and self isolating when they do so. This is consistent with the results of Feretti et al (2020) where 45% of transmission could be attributed to presymptomatic carriers whilst only 40% of Covid-19 transmission originated from carriers displaying symptoms.[12]

We also estimate a value for the behavioural attrition κ and find that behavioural attrition and behavioural responsiveness to deaths being similar to across European countries, along with the mobility data presented for the UK and Italy, this suggest that behavioural attrition plays a role in the spread of Covid-19. Finally we would expect the R_0 number to also be very similar in across these countries as well, given that in this setup it is determined by the biological fundamentals of the virus and not human behaviour. This is indeed the case with R_0 being calculated as 1.2 for the UK and Italy and 1.1 for the Netherlands. As with the estimated values for γ, this value is significantly smaller than those found in other pandemic models of Covid-19. A limitation of estimating parameters from the data in the way the we have done here is that R_0 may be underestimated. This is because observations begin only once Covid 19 is spreading in the population and therefore behaviour has begun to adjust. This should not substantially alter the ultimate dynamics, as using the time varying social interaction employ, we can actually permit R in the model to exceed R_0 at points in time. Despite this,

[12]For completeness: the remaining 15% of transmission was attributed to asymptomic carriers (5%) and environmental transmission (10%).

the value for R_0 found here does appear to match fairly closely with empirical fieldwork done on the spread of the virus where R is often measured to be close to one such as that produced by UK Health Security Agency (2022). They estimated R to be in the range 0.7-0.9 in June 2020 and only reached a range of 1.3-1.6 in October, when there was substantial relaxation of any government interventions and mobility data showing a return to close to normal behaviours. Whilst this is slightly higher than our estimate, we are substantially closer than pandemic forecasting models that assumed R_0 to be around 3.

One very significant point to highlight is potential for the existence of multiple equilibria for the estimation procedure, as demonstrated by comparing Figures 10 and 14, with two different parameter sets being able to produce simulations of infection waves. Specifically, the estimation procedure identifies two different equilibria as possible candidates, both the equilibria that is commonly identified in the literature, as well as a second with a lower R number and higher γ, preferring the latter. This is of significant interest as the assumptions we allow to vary here and which allow us to identify this new equilibrium are fixed in most pandemic models in such a way that the first equilibrium is found by construction. Moreover, the equilibrium we identify here appears to more closely match much of the estimates from fieldwork conducted on the spread of the virus, as well as the behaviour we would expect from individuals. This highlights the importance of developing models that can accommodate testing the behavioural assumptions that underpin them.

We have deliberately chosen to study three countries in this paper that are relatively similar on the expectation that they should produce relatively similar results. The fact that we are able to obtain very similar parameter for all the countries we look at gives us a degree of confidence in our results, However, as with any non-linear model that is estimated from data, there is always a concern with overfitting. This is a significant issue for all coronavirus models, and the model presented here is no exception. In this paper however we attempt to address this problem my reducing the number of variables to a minimal level required to generate the observed dynamics, within a structural framework. Given that parameters are still to some extent co-determined in our estimation procedure it is likely that this model is still overfitted, but to a lesser extent than alternative models.

In addition, we do still have to assume values for certain parameters in this model, such as δ controlling the Incidence fatality rate,, the length of time for which individuals are infected θ, and β_{low} the minimal level of social distancing that it is possible to achieve. Whilst the assumptions we make for these parameter values are consistent with the literature, as we have seen in this paper, such assumptions are not necessarily accurate. We have good reason to suspect that δ in particular should change over time as doctors gain greater knowledge in how to treat the disease. However, without sufficient data to form a strong assumption of how δ has changed over time then for the sake of simplicity we assume it as constant.

We do not consider in this model the elimination of the virus, either by government action or by the rollout of an effective vaccine. In the case of government intervention, we have seen from outbreaks in China that it is possible to eradicate the virus through the use of lockdown measures combined with other methods of controlling a population. We do not consider this here, as this paper concerns the type of society where such measures would be impossible to implement, and where when weaker measures are used - they are not fully complied with. In the case of a successful vaccination program, we would expect to begin to see significantly altered dynamics with regards the spread of the virus within a matter of weeks or months. Whilst this would certainly form an interesting extension to this paper, it would require accurate data on the specifics of a given vaccination program as well as good estimates of a given vaccines ability to both reduce death rates and viral transmission. We also do not consider the impact of variants that emerge. For the purposes of our estimation we select a time interval that does not include any variants as we expect this would alter the dynamics, but the model could in principle be extended to accommodate this in future work.

An important point to note is that in the setup presented here we are agnostic regarding the presence of a government. This does not imply that the presence and actions of a government do not matter. The function of a government in within this framework is to provide a coordination mechanism that allows for the collective action of a population. As such the government serves to communicate information centrally to the population and impose measures such as lockdowns that are generally preferred but require central coordination in order to obtain compliance. This remains a significant abstraction from reality and a limitation of this approach.

An area where this assumption could potentially fail is in the instance where government behaviour is not consistently reflective of the population over time, either as the result of a change in policy or in response to a change in social attitudes. As we have seen from a number of countries, governments have chosen to alter their approach to the pandemic around the holiday period and this will likely lead to an increase in infections that would not be predicted by this model. In that case, the model serves as a useful counterfactual to assess what would have happened if the government had continued to behave in a consistent manner and will allow in the future an assessment of costs of such a change in approach that can weigh the benefits to the economy with the cost of increased infections and therefore additional lives.

Finally, it is worth discussing the implications of these results for the notion of herd immunity. Under a standard SIR model herd immunity is commonly understood as being achieved when a sufficiently large proportion of the population has obtained immunity to a disease. In a typical epidemiological setting this state occurs through the infection and recovery or vaccination of a proportion of the population. However, it is also possible to at least temporarily depress transmission rates to significantly reduce the number of infections within a population. The model in this paper suggests that a population will do this through endogenously changing behaviour in response to information regarding an epidemic. We can see from the results of both the simulations and the estimations performed here that herd immunity exists not simply as a raw function of viral transmission within a population, but as the interaction of that transmission rate with the endogenously changing behaviour of that population.

4.7 Conclusions

It is speculated that as the world population grows and societies across the globe become more interconnected that pandemics are an increasingly likely threat. In this context it is important to understand not just the biological mechanisms of transmission, but also the social and behavioural components. This paper identifies a novel adaption of a standard SIRD model that allows for endogenous change in population behaviour in response to an epidemic. I find that changing behaviour within a population has a significant impact on the spread of the disease within a population. This leads to a reconsideration of the concept of herd immunity as being a function of population behaviour as well as the fundamentals of transmission for a given disease.

in particular, this paper highlights the importance of the infectious period in determining the transmission dynamics of a pandemic. Whilst most pandemic models assume this value is fixed at 5-7 days, we estimate that it in fact may be far shorter for some countries. This is important both for the production of accurate pandemic models, and for informing policy interventions. Estimates for the UK suggest that encouraging self isolation could have a significant impact on the pandemic dynamics that has been previously overlooked in the forecasting literature. Furthermore, this paper adds to the body of evidence that behavioural attrition is an important factor in understanding the spread of a virus within a population. This is contrary to some assertions in the literature that such an effect does not exist or is unimportant. Here we show that it both exists and is important for determining the size of later outbreaks in a pandemic that has multiple waves.

Whilst this model provides useful insights into the Covid-19 pandemic, it does so based on the assumptions that individuals cannot be reinfected and that there is no change in the infectious rate due to the emergence of new strains. As we have seen, new more transmissible strains of Covid-19 are emerging and that re-infection is possible, providing both a challenge to the techniques employed here and an avenue for future work.

Finally, whilst this model has been applied to the specific instance of Covid-19, unlike most other models it contains no assumptions that are unique to this pandemic beyond the fundamental nature of the disease itself. As such the model may prove a useful starting point for the modelling of future pandemics if we can assume population level economic and social behaviours to be relatively consistent over time.

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5 Summary

In this thesis we present two distinct instances of behavioural phenomena that influence the dynamics of macro level systems. In chapter II we have seen that the borrowing behaviour of agents in a leveraged asset market model can lead to boom bust cycles in asset prices. This is framed in the context of the housing market where we see that the expansion and contraction of borrowing combined the behaviour as a driving factor. Chapter IV in contrast focuses on the behavioural factors that underpin the spread of a pandemic within a population. We see here that behaviour is a key factor in understanding the dynamics of the Covid-19 epidemic and that understanding such behaviours is of significance for policy makers. What ties these papers together is the use of a behavioural framework that allows for the encapsulating of behavioural herding in macro models in such a way that these models can then still be estimated from data.

5.1 Discussion

A significant challenge to the work that presented here is the necessity of utilising non linear models. As discussed in the previous chapters, nonlinear estimation techniques are challenging to implement and have weak convergence power. Furthermore, it is often the case that there are multiple sets of values within the parameter space that can fit the model to data. As such significant discretion is required on the part of the researcher in performing estimations. This inevitably leads to situations where results are misidentified and poses two key problems, as we have seen in chapter IV of this thesis. First, it is entirely possible to miss a parameter set if it is excluded from the regression window by assumption. In the case of the Covid-19 pandemic this is the result of an entirely understandable modelling assumption that is made widely in the literature. Secondly, the existence of multiple sets of parameters that provide a reasonable fit for the data introduces significant uncertainty over estimation results. For this reason I have attempted to identify instances where such instances occur, what choices are taken and the rationale for doing so. Whilst it is generally accepted that behavioural factors play a role in social and economic decision making, it is still a matter of uncertainty the extent to which such behaviours manifest at a macro level and how best to describe them. In this thesis we take the position that it is primarily behaviour that can be used to explain the variation observed in the data we study, but recognise that other alternative explanations are possible.

5.2 Further work

As we have seen from the work contained in this thesis, the macro-level effects of individual behaviour are important for understanding emergent dynamics of complex systems. We have seen that in the context of a leveraged asset market, understanding the borrowing behaviour of individuals is necessary to predict the market dynamics. Equally, in a pandemic setting, understanding the behaviour of the population is important to properly inform any policy response. This thesis is limited to only two examples of the macro consequences of behaviour in very the specific contexts. In both cases, the choice to examine the relevant phenomena from a behavioural perspective resulted from the lack of convincing explanation in the existing literature obtained using more standard techniques. This raises a question regarding what other phenomena can also be investigated in this way.

An interesting extension of chapter II would be to incorporate a similar framework into a real business cycle model. This would be in keeping with the Minsky style conceptualisation whilst also tying in neatly with more recent thinking on short run debt cycles from industry. Such an extension would provide improved behavioural foundations for the boom and bust debt cycles that characterise the business cycle that whilst a common focus of interest in the investment industry, is often overlooked in academic economics. In addition, chapter IV has some obvious extensions such as examining the impact of disease variants in a pandemic. It may also has some less obvious applications such as predicting the transmission dynamics internet memes.[13]

A significant limitation in this field is in the absence of effective estimation techniques. Many of these questions we have managed to answer would not have been possible to answer even a few years ago, and it is only the improvements in computer processing power that allows us to investigate them now. However, advances in machine learning and artificial intelligence have the potential to significantly improve our ability to investigate the types of non linear behavioural relationships studied here and several machine learning techniques are used in this thesis, although these are not without their own drawbacks. From an economists point of view, there is utility to be gained from observing not just the result, but the mechanism as well and AI systems do not yet offer that capability. That being said, in the near future advancements in computer technology may enable us to investigate the types of problems we study here with far greater accuracy, and with any luck understand them in far greater detail.

[13]They are called viral for a reason. This was the context in which I originally developed the model before repurposing it for the Covid-19 pandemic.