Impartial Selection with Additive Guarantees via Iterated Deletion*

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In many situations where votes are cast or nominations are made there is an overlap between the set of voters and the set of candidates. When this is the case, voters may not reveal their true preferences about who should be selected in order to increase their own chance of winning. Incentive issues of this kind were first studied in a systematic way by Holzman and Moulin [3] and Alon et al. [1], who formalized the problem in terms of a directed graph in which vertices correspond to voters and a directed edge from one voter to another indicates that the former nominates the latter. A (deterministic) selection mechanism then takes such a nomination graph as input and returns one of its vertices. A mechanism is impartial if, for all nomination graphs, a change of the outgoing edges of some vertex v does not change whether v is selected or not. Holzman and Moulin have shown that impartial mechanisms are rather limited even in a setting where each voter casts exactly one vote, i.e., where each vertex has outdegree one: for every impartial mechanism there is a nomination graph where it selects a vertex with indegree zero, or a nomination graph where it fails to select a vertex with indegree $n-1$, where n is the number of voters. This shows in particular that the best multiplicative approximation guarantee for impartial mechanisms, i.e., the worst case over all nomination graphs of the ratio between the maximum indegree and the indegree of the selected vertex is at least $n-1$. On the other hand, a multiplicative guarantee of $n-1$ is easy to obtain by always following the outgoing edge of a fixed vertex. As multiplicative guarantees do not allow for a meaningful distinction among deterministic impartial mechanisms, Caragiannis et al. [2] proposed to instead consider an additive guarantee, i.e., the worst case over all nomination graphs of the difference between the maximum indegree and the indegree of the selected vertex. A mechanism due to Holzman and Moulin, majority with default, achieves an

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additive guarantee of $\lceil n/2 \rceil$. Caragiannis et al. [2] further proposed a randomized mechanism with an additive guarantee of $O(\sqrt{n})$ and gave a lower bound of 3 on the additive approximation of any deterministic impartial mechanism that always selects.

We develop a new deterministic mechanism for impartial selection that is parameterized by a pair of thresholds on the indegrees of vertices in the graph. The mechanism seeks to select a vertex with large indegree, and to achieve impartiality it iteratively deletes outgoing edges from vertices in decreasing order of their indegrees, until only the outgoing edges of vertices with indegrees below the lower threshold remain. It then selects a vertex with maximum remaining indegree if that indegree is above the higher threshold, and otherwise does not select. Any ties are broken according to a fixed ordering of the vertices. We give a sufficient condition for choices of thresholds that guarantee impartiality. The iterative nature of the deletions requires a fairly intricate analysis but is key to achieving impartiality. The additive guarantee is then obtained for a good choice of thresholds, and the worst case is the one where the mechanism does not select.

For instances with n vertices and maximum outdegree at most $O(n^{\kappa})$, where $\kappa \in [0,1]$, the mechanism provides an additive guarantee of $O(n^{\frac{1+\kappa}{2}})$. This is the first sublinear bound for a deterministic mechanism and any $\kappa \in [0,1]$, and is sublinear for all $\kappa \in [0,1)$. For settings with constant maximum outdegree, which includes the setting of Holzman and Moulin where all outdegrees are equal to one, our bound matches the best known bound of $O(\sqrt{n})$ for randomized mechanisms and outdegree one, due to Caragiannis et al. [2]. When the maximum outdegree is unbounded the bound becomes $O(n)$. This is of course trivial, as even a mechanism that never selects provides an additive guarantee of $n-1$. We show that it is also best possible by giving a matching lower bound. This improves on the only lower bound of 3 known prior to our work, and generalizes it to mechanisms that may not select. Just like the multiplicative lower bound for maximum outdegree one, our additive lower bound for arbitrary outdegrees is obtained through an axiomatic impossibility result. Holzman and Moulin have shown that in the outdegree-one case, impartiality is incompatible with positive and negative unanimity. Here, positive unanimity requires that a vertex with the maximum possible indegree of $n-1$ must be selected, and negative unanimity that a vertex with indegree zero cannot be selected. In the case of arbitrary outdegrees, this impossibility can be strengthened further: call a selection mechanism weakly unanimous if it selects a vertex with positive indegree whenever there exists a vertex with the maximum possible indegree of $n-1$; then weak unanimity and impartiality are incompatible. This can be shown by analyzing the behavior of deterministic and randomized impartial mechanisms on a restricted class of graphs with a high degree of symmetry among vertices. A suitable class of graphs for our purposes are those generated by partial orders on the set of ordered partitions. The impossibility is then obtained by combining counting results for ordered partitions and an argument similar to Farkas' Lemma.

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