Downlink Multi-RIS Aided Transmission in Backhaul Limited Networks

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***Abstract*—This work proposes an analytical framework for multi-reconfigurable intelligent surface (RIS) aided networks with limited backhaul capacity, where a simple symmetric structure is considered to explore the characteristics of multi-RIS systems with coherent transmissions. By using the Gamma distribution to model the composite RIS-aided channels, closed-form expressions in terms of the outage probability and the ergodic rate are derived. The theoretical results provide two design guidelines: 1) the minimum outage can be achieved by deploying the assisted RISs at certain locations; 2) the rate performance is bounded, and an optimal number of assisted RISs exists for rate maximization. Simulations verify our analysis and demonstrate the backhaul capacity is the bottleneck for multi-RIS systems. Compared with single-RIS systems, multi-RIS systems require a minimal backhaul capacity to ensure the performance gain.**

***Index Terms*—Ergodic rate, limited backhaul, multi-RIS, optimal deployment, outage probability**

I. Introduction

The reconfigurable intelligent surface (RIS) is an emerging technology to realize the smart radio environment in next generation wireless networks [1]. Motivated by the capability of smartly steering the signal propagation, one potential application of RISs is to create additional line-of-sight (LoS) links between the base station (BS) and user equipment (UE) for improved achievable data rates.

For RIS-enabled communications, most existing performance analysis contributions focused on single-RIS aided systems. Since the RIS-aided link includes multiple channels, it is difficult to characterize its composite channel gain in exact expressions, and hence the central limit theorem (CLT) was leveraged to derive closed-form performance approximations [2, 3]. Recently, multi-RIS aided setups, which are more practical use cases for RIS-aided networks, were investigated to further boost the system capacity [4–6]. Similar to the idea of the cell-free massive MIMO [7], by tuning the phases of signals from different elements of distributedly deployed RISs according to channel state information (CSI), coherent transmissions can be realized to maximize the received power. In [4], the authors assumed that path losses of different RIS-aided links are the same. This work demonstrated that the theoretical outage performance derived relying on K_G distributions is more accurate than adopting CLT in the case of RISs with only several elements. In [5], the channels of different RISs were assumed to be independent but not identically distributed. In [6], the authors also investigated the RIS deployment strategy and concluded that equivalent performance is achieved when a RIS is located either near the BS or the UE. Although various accurate approximations of both the ergodic capacity as well as some interesting insights have been obtained for multi-RIS-aided networks in previous works, most of them consider ideal backhaul capacity links. In contrast to single-RIS aided systems, control signals for the synchronization among all assisted RISs are required for coherent transmissions in multi-RIS aided patterns. In networks with limited backhaul capacity, part of the backhaul capacity is for the control purpose, and hence the system capacity for data transmissions is degraded. Moreover, a more specific RIS deployment guideline is helpful to improve the received signal power.

In this letter, our goal is to analyse the outage and the rate performance of a multi-RIS aided system with limited backhaul capacity, where the control signal for coherent transmissions is considered. The main contributions of this work are: 1) considering multiple RISs distributed symmetrically around the BS, we derive a tight approximation of the outage probability, based on which we prove that for the minimum outage, one of the RISs needs to be deployed as close to the line between the BS and the UE as possible; 2) the closed-form expression of the ergodic rate is deduced. Theoretical results demonstrate the bounded rate performance and the existence of an optimal number of RISs due to the backhaul limitation; 3) numerical results show that the multi-RIS aided pattern improves the rate performance compared with the single-RIS counterpart under large backhaul capacity, but the conclusion is the opposite for extremely low backhaul capacity cases.

Notations: $(\cdot)^T$ and $(\cdot)^H$ denote the the transpose operation and the Hermitian transpose operation, respectively. $|x|$ denotes the amplitude of x. $\mathbb{E}[\cdot]$ is the expectation operator. $j=\sqrt{-1}$ is the imaginary unit. $\mathcal{N}(\mu,\sigma^2)$ is the Gaussian distribution. $\mathcal{CN}(\mu,\sigma^2)$ is the complex Gaussian distribution. $\mathrm{Nakagami}(m,\Omega)$ represents the Nakagami distribution and $\mathrm{Gamma}(k,\theta)$ stands for the Gamma distribution. $f'(x)$ is the first derivative of $f(x)$. $\Gamma(\cdot)$ is the Gamma function. $\gamma(\alpha,x)$ is the lower incomplete Gamma function [8, eq. (8.350.1)]. ${}_pF_q(\mathbf{a}_p;\mathbf{b}_q;x)$ is the generalized hypergeometric function [8,

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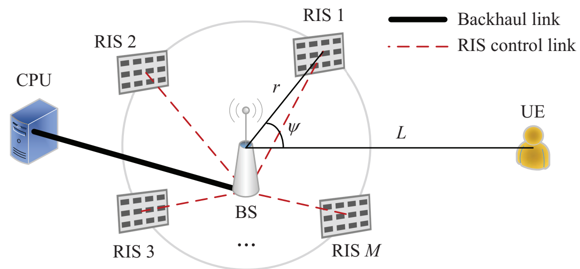

Fig. 1: Illustration of the spatial model for the considered multi-RIS aided transmission.

eq. (9.14.1)]. $G_{p,q}^{m,n}\left((\cdot)\middle|\begin{matrix}(\mathbf{a}_p)\\(\mathbf{b}_q)\end{matrix}\right)$ is the Meijer G-function [8, eq. (9.301)].

II. System Model

As shown in Fig. 1, we consider a downlink single-cell system, where the transmission between the BS and the UE is assisted by multiple RISs[1]. The distance between the BS and the UE is L. We fix the BS at the origin, and M RISs, i.e., $\{1,2,...,M\}$, are distributed symmetrically on a circle centered at the BS with radius r [2]. A central processing unit (CPU) is employed for the control purpose, to which the BS connects via a capacity-limited backhaul link. The maximum backhaul capacity is C_B^{max}. The coherent transmission is considered among M RISs and hence full CSI is required. The synchronization among RISs is controlled by the CPU and the control signal is delivered through wired RIS control links with ideal capacity. The rate of the control signal for each RIS is R_c.

We denote the angle of RIS 1-BS-UE as ψ_1, where $\psi_1 = \psi \in [0, \frac{2\pi}{M})$. For $\forall i \in \{1,2,...,M\}$, the angle of RIS i-BS-UE is $\psi_i = \psi + \frac{2\pi(i-1)}{M}$. Therefore, the distance between the RIS i and the UE can be expressed as

$$d_i(\psi) = \sqrt{r^2 + L^2 - 2rL\cos\left(\psi + \frac{2\pi(i-1)}{M}\right)}. \quad (1)$$

A. Channel Model

We assume that both the UE and the BS are equipped with a single antenna, and each RIS consists of N reflecting elements. We denote the phase-shifting matrix of the RIS i by $\mathbf{\Theta}_i = \mathrm{diag}\left(e^{j\phi_{i,1}}, ..., e^{j\phi_{i,N}}\right) \in \mathbb{C}^{N\times N}$, where $\phi_{i,n} \in [0, 2\pi)$ for $n \in \{1,2,...,N\}$. Let $\mathbf{H}_{I,i} \triangleq [h_{I,i,1},...,h_{I,i,N}]^T \in \mathbb{C}^{N\times 1}$ and $\mathbf{H}_{R,i} \triangleq [h_{R,i,1},...,h_{R,i,N}]^T \in \mathbb{C}^{N\times 1}$ denote the small-scale fading vectors of the BS-RIS link and the RIS-UE link, respectively. The $|h_{I,i,n}|$ and $|h_{R,i,n}|$ follow independent Nakagami-m fading[3], i.e., $|h_{I,i,n}| \sim \mathrm{Nakagami}(m_1, 1)$ and $|h_{R,i,n}| \sim \mathrm{Nakagami}(m_2, 1)$. Noticed the potential mobility of the UE, CSI of the RIS-UE link and the direct BS-UE link has the probability to be outdated. In this work, we consider a practical CSI model as in [6]. For the RIS-UE link, the received outdated channel vector $\hat{\mathbf{H}}_{R,i} = \kappa_r \mathbf{H}_{R,i} + \bar{\kappa}_r \mathbf{\Omega}_{R,i}$, where $0 \le \kappa_r \le 1$ is the correlation coefficient between the outdated channel $\hat{\mathbf{H}}_{R,i}$ and the actual channel $\mathbf{H}_{R,i}$. When $\kappa_r = 1$ the CSI is perfect. $\bar{\kappa}_r = \sqrt{1-{\kappa_r}^2}$ and $\mathbf{\Omega}_{R,i}$ is independently distributed from $\mathbf{H}_{R,i}$ with zero-mean and ${\sigma_{h_r}}^2$-variance complex Gaussian entries. Then the channel response of the cascaded RIS-aided link for the RIS i is expressed as

$$\hat{H}_i(\psi) = \kappa_r \sqrt{L_i(\psi)} {\mathbf{H}_{I,i}}^T \mathbf{\Theta}_i \mathbf{H}_{R,i} + \bar{\kappa}_r \omega_r, \quad (2)$$

where $\omega_r = \sqrt{L_i(\psi)} {\mathbf{H}_{I,i}}^T \mathbf{\Theta}_i \mathbf{\Omega}_{R,i}$ and $L_i(\psi) = (rd_i(\psi))^{-\alpha_r}$ is the path loss. The α_r is the path loss exponent.

Let $h_{r,i} = {\mathbf{H}_{I,i}}^T \mathbf{\Theta}_i \mathbf{H}_{R,i}$ denote the equivalent overall small-scale fading for the RIS-aided link which includes N channels. To maximize the received signal power from the RIS i, the phase shift on each element is adjusted so that signals from all channels are of the same phase at the UE [9]. Therefore, we have

$$h_{r,i} = e^{j\phi_{r,i}} \sum_{n=1}^{N} |h_{I,i,n}||h_{R,i,n}|. \quad (3)$$

Let μ_r and ${\sigma_r}^2$ denote the mean and variance of $|h_{r,i,n}| \triangleq |h_{I,i,n}||h_{R,i,n}|$, respectively. According to [10], the kth order moment of the double-Nakagami random variable is given by $\mathbb{E}\left[|h_{r,i,n}|^k\right] = \prod_{i=1}^{2} \frac{\Gamma(m_i+k/2)}{\Gamma(m_i)} \left(\frac{1}{m_i}\right)^{k/2}$. We calculate the mean by $\mu_r = \mathbb{E}\left[|h_{r,i,n}|\right]$ and the variance by ${\sigma_r}^2 = \mathbb{E}\left[|h_{r,i,n}|^2\right] - {\mu_r}^2$. Since N different channels of the RIS-aided link are independent and identically distributed, the CLT can be employed when N is large. The amplitude of the composite channel gain obeys Gaussian distribution $|h_{r,i}| \sim \mathcal{N}\left(N\mu_r, N{\sigma_r}^2\right)$. Using the method of moments, the distribution of the composite small-scale fading power gain of the RIS aided link can be fitted by a Gamma distribution [3]

$$|h_{r,i}|^2 \sim \mathrm{Gamma}\left(\frac{{M_r}^2}{V_r}, \frac{V_r}{M_r}\right) \triangleq \mathrm{Gamma}\left(k_r, \theta_r\right), \quad (4)$$

where $M_r = {\mu_r}^2 N^2 + {\sigma_r}^2 N$ and $V_r = 4{\mu_r}^2{\sigma_r}^2 N^3 + 2{\sigma_r}^4 N^2$.

Similarly, for the direct BS-UE link, the channel gain can be expressed as

$$\hat{H}_0 = \kappa_0 \sqrt{L^{-\alpha_d}} h_d + \bar{\kappa}_0 \sqrt{L^{-\alpha_d}} \omega_d, \quad (5)$$

where $0 \le \kappa_0 \le 1$, $\bar{\kappa}_0 = \sqrt{1-{\kappa_0}^2}$, and $\omega_d \sim \mathcal{CN}\left(0, {\sigma_{h_d}}^2\right)$. α_d is the path loss exponent. h_d denotes the small-scale fading variable, which obeys $\mathrm{Nakagami}(m_d, 1)$.

B. Performance Metrics

In this letter, we focus on the outage probability and the ergodic rate of the multi-RIS aided transmission. Both of the above metrics are related to the received SNR at the UE.

To maximize the overall received power, the coherent transmission is employed. Thus, the received SNR at the UE is given by

$$\mathrm{SNR}(\psi) = \frac{P_B \left|\kappa_0 \sqrt{L^{-\alpha_d}} h_d + \kappa_r \sum_{i=1}^{M} \sqrt{L_i(\psi)} h_{r,i}\right|^2}{{\sigma_0}^2 + {\sigma_{CSI}}^2}, \quad (6)$$

[1]To simplify the analysis, we consider the one-hop reflection model [4] in this work. The scattered signal among RISs is ignored.

[2]Studying this simple network structure aims to find useful design guidelines. These guidelines are validated in the simulation part by a general network structure based on randomly distributed RISs.

[3]The proposed analytical method in this letter is suitable for other fading channel models such as Rician fading.

where P_B represents the transmit power of the BS. ${\sigma_0}^2$ denotes the additive white Gaussian noise power. ${\sigma_{CSI}}^2 = \bar{\kappa}_0^2 P_B L^{-\alpha_d}\left(1-\mathbb{E}[h_d]^2\right) + \bar{\kappa}_r^2 N P_B \sum_{i=1}^{M} L_i(\psi)\left(1-\mathbb{E}[h_{r,i}]^2\right)$ is the noise due to the outdated CSI, [6]. Then the outage probability is defined as

$$P_{out} = \Pr\left(\mathrm{SNR}(\psi) < \tau\right), \tag{7}$$

where τ is the SNR threshold for the UE to successfully decode its message.

When ψ is fixed, the ergodic rate averaged over the fading distribution is given by

$$R(\psi) = \mathbb{E}\left[\log_2\left(1+\mathrm{SNR}(\psi)\right)\right]. \tag{8}$$

Finally, we denote $\bar{R} = \mathbb{E}\left[R(\psi)\right]$ as the average ergodic rate which is averaged over the RIS deployment angle ψ.

III. Outage Probability

When the UE is near the BS, the transmission between the UE and the BS has a high probability of being LoS. In this case, it is not necessary to further improve the channel condition by employing the multi-RIS assisted transmission. Therefore, this work mainly focuses on the scenario where the UE locates far from the BS. In this section, we first provide the outage probability of the UE.

Theorem 1. *Considering the backhaul limitation, when ($\tau \le \gamma_d$) the outage probability of the UE is*

$$P_{out}(\tau,\psi) \approx P_{approx}(\tau,\psi) = \frac{\gamma\left(Mk_r, \frac{\tau}{g(\psi)\theta_r}\right)}{\Gamma\left(Mk_r\right)}, \tag{9}$$

and when ($\tau > \gamma_d$) the outage probability of the UE is

$$P_{out}(\tau,\psi) = P_{approx}(\tau,\psi) = 1, \tag{10}$$

where $\gamma_d = 2^{C_d^{max}} - 1$, $g(\psi) = \frac{{\kappa_r}^2 P_B A(\psi)}{{\sigma_0}^2 + q_1 P_B + \bar{\kappa}_r^2 q_2 P_B A(\psi)}$, $q_1 = \bar{\kappa}_0^2 L^{-\alpha_d}\left(1-\mathbb{E}[h_d]^2\right)$, $q_2 = N\left(1-\mathbb{E}[h_{r,i}]^2\right)$, $A(\psi) = r^{-\alpha_r}\sum_{i=1}^{M} d_i(\psi)^{-\alpha}$, *and* $C_d^{max} = C_B^{max} - MR_c$.

Proof: See Appendix A. ■

The deployment of RISs also makes a difference to the performance of the UE. The ideal deployment strategy in terms of the outage probability is shown as follows.

Corollary 1. *When $\psi = 0$ and $\tau \le \gamma_d$, the outage probability of the UE is minimized and its value is*

$$P_{out}^{*}(\tau) \approx \frac{\gamma\left(Mk_r, \frac{\tau}{g(0)\theta_r}\right)}{\Gamma\left(Mk_r\right)}. \tag{11}$$

Proof: According to the monotonicity of the lower incomplete Gamma function, we have $\arg\min_{\psi} P_{out}(\tau,\psi) \equiv \arg\max_{\psi} g(\psi)$. To obtain the optimal ψ, we take the derivative of $g(\psi)$ and we have

$$g'(\psi) = \frac{{\kappa_r}^2 P_B({\sigma_0}^2 + q_1 P_B)A'(\psi)}{({\sigma_0}^2 + q_1 P_B + \bar{\kappa}_r^2 q_2 P_B A(\psi))^2}. \tag{12}$$

Note that $A'(\psi) = 0$ holds when RISs are distributed symmetrically about the line between the BS and the UE, we calculate that $\psi = 0$ and $\psi = \frac{\pi}{M}$ are the solutions of $g'(\psi) = 0$. Besides, $g'(\psi) < 0$ when $\psi \to 0^+$. Combining the property that $g(\psi)$ and $g'(\psi)$ have a period of $\frac{2\pi}{M}$, the outage probability can be minimized when $\psi = 0$. Similarly, we can find that the outage performance is the worst when $\psi = \frac{\pi}{M}$. ■

Remark 1. *Under symmetrically distributed RISs, the ideal deployment strategy is to keep the UE, one of the RISs, and the BS in a line. Considering the distribution of RISs is unlikely symmetrical and the ideal location of RIS is not always accessible in practical scenarios, we extend our conclusion for a general case: to reduce the outage, at least one RIS should be deployed as close as possible to the BS-UE line.*

IV. Rate Analysis

Then we are able to derive the ergodic rate with fixed ψ.

Theorem 2. *Based on the results in* **Theorem 1**, *the ergodic rate can be expressed as a closed form*

$$R(\psi) = C_d^{max} - \frac{\gamma_d}{\Gamma(K_{eq})\ln 2}\sum_{k=0}^{\infty}\frac{(-1)^k(\gamma_d/\theta_{eq})^{K_{eq}+k}}{k!(K_{eq}+k)(K_{eq}+k+1)} \times {}_2F_1\left(K_{eq}+k+1, 1; K_{eq}+k+2; -\gamma_d\right), \tag{13}$$

where $K_{eq} = Mk_r$ and $\theta_{eq} = g(\psi)\theta_r$.

Proof: When the backhaul capacity is limited, (8) can be rewritten as follows

$$\begin{aligned} R(\psi) &= C_d^{max} - \frac{1}{\ln 2}\int_0^{\gamma_d}\frac{P_{approx}(z,\psi)}{1+z}dz \\ &\overset{(a)}{=} C_d^{max} - \frac{1}{\ln 2}\int_0^{\gamma_d}\sum_{k=0}^{\infty}\frac{(-1)^k(z/\theta_{eq})^{K_{eq}+k}dz}{k!(K_{eq}+k)\Gamma(K_{eq})(1+z)}, \end{aligned} \tag{14}$$

where (a) is from some polynomial expansion manipulations of the lower incomplete Gamma function. Based on [8, eq. (3.194.1)], we calculate the integral and obtain (13). ■

Remark 2. *Since $R(\psi)$ can be expressed as the integral of the outage, we find that if $\tau \le \gamma_d$ holds, the ergodic rate can also be maximized when the UE, one of the RISs, and the BS located in a line.*

Remark 3. *When the transmit power $P_B \to \infty$, $g(\psi) \to \frac{{\kappa_r}^2 A(\psi)}{q1 + \bar{\kappa}_r^2 q2 A(\psi)} \triangleq C_0(\psi)$. Thus $R(\psi)$ is bounded by a deterministic value $C_{upper} = C_d^{max} - \frac{1}{\ln 2}\int_0^{\gamma_d}\frac{\gamma\left(Mk_r, \frac{z}{C_0(\psi)\theta_r}\right)}{1+z}dz \le C_d^{max}$. Particularly, if CSI is perfect, i.e., when $\kappa_r = \kappa_0 = 1$, we have $C_0(\psi) \to \infty$ and $C_{upper} = C_d^{max}$.*

Remark 4. *Note that $R(\psi) \equiv 0$ when the number of assisted RISs $M \ge C_B^{max}/R_c$ while for the case $M < C_B^{max}/R_c$ the C_d^{max} decreases monotonically with an increasing M, it can be concluded that with the increase of M, $R(\psi)$ increases to a maximum value first and then decreases to zero. Therefore, there exists an optimal $M \in \mathbb{N}^+$ to maximize the ergodic rate $R(\psi)$.*

Proposition 1. *When $C_B^{max} \to \infty$, i.e. considering ideal backhaul, the ergodic rate can be given by*

$$R_{ideal}(\psi) = \frac{\theta_{eq}}{\ln 2}{}_3F_1\left(MK_r+1, 1, 1; 2; -\theta_{eq}\right). \tag{15}$$

Proof: See Appendix B. ■

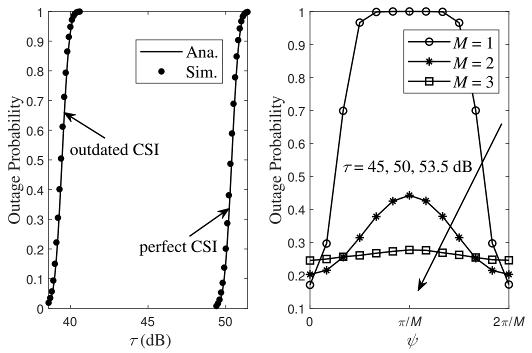

Fig. 2: Validating of the outage approximation. (a) Left: outage probability versus the threshold τ with $\psi = 0$; (b) Right: outage probability versus the angle ψ with $\kappa_0 = \kappa_r = 1$.

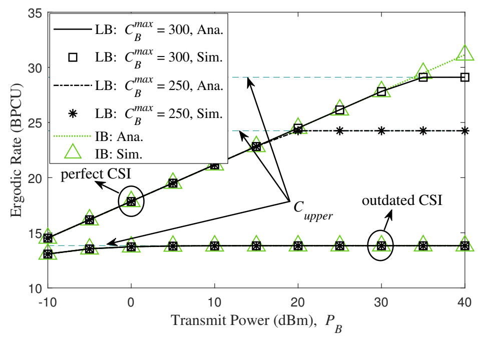

Fig. 3: Ergodic rate versus transmit power P_B with $M = 3$, where "LB" and "IB" represent limited backhaul and ideal backhaul, respectively.

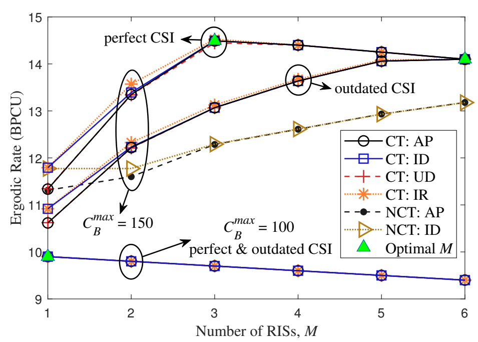

Fig. 4: Ergodic rate versus the number of RISs M, where "AP" refers to the average performance. "ID" means the ideal deployment. "UD" means that RISs are uniformly deployed on the circle with radius r, "IR" represents that one RIS is ideally deployed in UD. "CT" and "NCT" represent coherent transmission and non-coherent transmission, respectively.

V. Numerical Result

In this section, the theoretical performance of the considered backhaul-limited multi-RIS aided network is verified by Monte Carlo simulations. We consider both the perfect CSI and outdated CSI. The parameter setting is listed as follows: $r = 20$ m, $L = 150$ m, $\alpha_r = 2.2$, $\alpha_d = 4$, $N = 36$, $P_B = 0$ dBm, $m_1 = m_2 = 4$, $m_d = 1$, $N_f = 10$ dB. For the outdated CSI, $\kappa_0 = \kappa_r = 0.95$. The noise power ${\sigma_0}^2 = -170 + 10\log_{10} W + N_f$. The system bandwidth is $W = 10$ MHz and the maximum backhaul capacity is $C_B^{max} = 300$ Mbps. 1% of the overall rate is for control purposes of each RIS, i.e., $C_d^{max} = C_B^{max} - 0.01MC_B^{max}$.

Fig. 2 (a) plots the outage probability versus the threshold τ and validates the approximated expression of the outage probability in **Theorem 1**. Fig. 2 (b) shows the impact of RIS deployment angle ψ. As discussed in **Corollary 1**, the outage can be minimized when $\psi = 0$ while the maximum value occurs when $\psi = \frac{\pi}{M}$. Meanwhile, we can observe that the gap between the minimum outage and the maximum value is closed with the increase of the number of RISs M. It demonstrates that the multi-RIS aided pattern is able to help the UE located at a random azimuth of the BS to achieve similar performance while the UE only receives strong signals in limited regions under the single-RIS aided setup.

Fig. 3 compares the ergodic rate in bit per channel use (BPCU) versus transmit power P_B between systems with perfect CSI and outdated CSI. The average performance is considered. Both **Theorem 2** and **Proposition 1** are validated. As discussed in **Remark 3**, the ergodic rate in limited backhaul scenarios is bounded by C_{upper} for two categories of CSI. However, when the backhaul is ideal, the rate performance keeps increasing with the perfect CSI but is also bounded by a constant for outdated CSI cases. It can be explained that the noise from the outdated CSI seriously degrades the system performance.

Fig. 4 plots the ergodic rate versus the number of assisted RISs M. The curves for the ideal deployment are from **Remark 2**. Under "IR" one RIS is ideally deployed on the line BS-UE. The non-coherent transmission with perfect CSI is plotted for comparison. To observe the trend clearly, we set $P_B = -10$ dBm and $C_B^{max} = \{100, 150\}$ Mbps. It is observed that the rate performance is enhanced when one RIS of mutiple RISs is deployed on the line between the UE and the BS, and hence the deign guideline in **Remark 1** is verified. Besides, the existence of optimal M in backhaul-limited scenarios is validated as stated in **Remark 4**. Although the coherent transmission of multiple RISs significantly improves the signal power, it brings high signaling overheads which guarantee strict synchronization among different RISs. Since these overheads occupy part of the backhaul capacity, the available rate for data transmissions is reduced with the increase of M. As shown in the figure, the single-RIS aided transmission even outperforms the multi-RIS aided counterpart in some cases. Therefore, an appropriate number of assisted RISs should be selected for the practical system design according to channel conditions and the backhaul limitation.

VI. Conclusion

This letter has studied the outage performance and the ergodic rate of the multi-RIS aided single-cell network with limited backhaul capacity, where the control signal for coherent transmissions is considered. The analysis of this work has shown that: 1) the increase of assisted RISs finally leads to the degradation of rate performance due to the backhaul limitation; 2) the capacity is improved when one of RISs is deployed as close to the line between the BS and the UE as possible.

Appendix A: Proof of Lemma 1

Based on the backhaul limitation, the maximum capacity for data transmission is $C_d^{max} = C_B^{max} - MR_c$. Thus, if $\tau > 2^{C_d^{max}} - 1$, $P_{out} = 1$.

Then let us focus on the expression of the received SNR $\mathrm{SNR}(\psi)$. When the number of elements N is sufficiently large, the received SNR can be expressed as

$$\begin{aligned}\mathrm{SNR}(\psi) &= \frac{P_B r^{-\alpha_r}}{{\sigma_0}^2 + {\sigma_{CSI}}^2}\left(\tilde{H}_0 + \kappa_r \sum_{i=1}^{M} \sqrt{d_i(\psi)^{-\alpha}} |h_{r,i}|\right)^2 \\ &\overset{(a)}{\approx} \frac{{\kappa_r}^2 P_B r^{-\alpha_r}}{{\sigma_0}^2 + {\sigma_{CSI}}^2} |h_M|^2 \sum_{i=1}^{M} d_i(\psi)^{-\alpha}, \end{aligned} \tag{A.1}$$

where $\tilde{H}_0 = \kappa_0 h_d \sqrt{L^{-\alpha_d}} / \sqrt{r^{-\alpha_r}}$ and $|h_M|^2 \sim \mathrm{Gamma}\left(Mk_r, \theta_r\right)$. (a) follows from the Cauchy inequality and the fact that the term $\tilde{H}_0$ is negligible.

We denote $g(\psi) = \frac{{\kappa_r}^2 P_B r^{-\alpha_r}}{{\sigma_0}^2 + {\sigma_{CSI}}^2} \sum_{i=1}^{M} d_i(\psi)^{-\alpha}$ for simplicity. By leveraging the property of the Gamma variable, we have

$$\mathrm{SNR}(\psi) \sim \mathrm{Gamma}\left(Mk_r, g(\psi)\theta_r\right). \tag{A.2}$$

According to the cumulative distribution function of the Gamma distribution, we obtain (9). The proof is completed.

Appendix B: Proof of Corollary 1

When the ideal backhaul is considered, the ergodic rate can be derived as

$$\begin{aligned} R_{ideal}(\psi) &= \int_0^\infty \log_2\left(1+x\right) \Pr\left(\mathrm{SNR}(\psi) = x\right) dx \\ &\overset{(a)}{=} \frac{1}{\ln 2} \int_0^\infty \ln\left(1+x\right) \frac{x^{K_{eq}-1} e^{-\frac{x}{\theta_{eq}}}}{\Gamma(K_{eq}) {\theta_{eq}}^{K_{eq}}} dx, \end{aligned} \tag{B.1}$$

where (a) is from the approximation $f_{approx}(\tau, \psi)$ in **Lemma 1** and the probability distribution function of the Gamma random variable $\mathrm{SNR}(\psi)$.

Based on [11, eq. (8.4.6.5)], $\ln\left(1+x\right)$ can be expressed as the Meijer G-function, and (B.1) can be further expressed as

$$\begin{aligned} R_{ideal}(\psi) &= \frac{B_0}{\ln 2} \int_0^\infty G_{2,2}^{1,2}\left(x \middle| \begin{matrix} 1,\ 1 \\ 1,\ 0 \end{matrix}\right) x^{K_{eq}-1} e^{-\omega x} dx \\ &\overset{(b)}{=} \frac{B_0 \omega^{-K_{eq}}}{\ln 2} G_{1,3}^{3,0}\left(\frac{1}{\omega} \middle| \begin{matrix} 1-K_{eq},\ 1,\ 1 \\ 1,\ 0 \end{matrix}\right), \end{aligned} \tag{B.2}$$

where $\omega = \frac{1}{\theta_{eq}}$ and $B_0 = \frac{1}{\Gamma(K_{eq}) {\theta_{eq}}^{K_{eq}}}$. (b) follows from the Laplace transform of the Meijer G-function. By utilizing the relationship between the Meijer G-function and the hypergeometric function the corollary is proved.

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