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Measuring relative volatility in high-frequency data under the directional change approach

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Summary

We introduce a new approach in measuring relative volatility between two markets based on the directional change (DC) method. DC is a data-driven approach for sampling financial market data such that the data are recorded when the price changes have reached a significant amplitude rather than recording data under a pre-determined timescale. Under the DC framework, we propose a new concept of DC micro-market relative volatility to evaluate relative volatility between two markets. Unlike the time-series method, micro-market relative volatility redefines the time-scale based on the frequency of the observed DC data between the two markets. We show that it is useful for measuring the relative volatility in micro-market activities (high-frequency data).

KEYWORDS

directional change, events, high-frequency data in FX markets, relative volatility

1 | INTRODUCTION

High-frequency data have been of interest since the late 1980s, when the ability to collect data with the aid of new and improved technology arose (Dacorogna et al., 2001). With the breakdown of the Bretton Woods system in 1971, researchers were attracted to the study of floating exchange rates using time-series (TS) data (weekly and monthly), especially in the statistical analysis of the foreign exchange (FX) price changes. Boothe and Glassman (1987) stressed that the distribution of the exchange rate changes is essential for examining the uncertainty of the price movements (referred to as volatility). The earlier studies focused on finding a proper distribution to summarize the exchange rate changes in low-frequency data (i.e., weekly and daily). Westerfield (1977) indicated that the exchange rate changes were Paretian stable.¹ Rogalski and Vinso (1978) used the same data as Westerfield, and they suggested that the floating exchange rates were better described by the Student distribution. McFarland et al. (1982) examined the logarithmic daily exchange rates

and concluded that the logarithmic daily exchange rates followed a stable Paretian distribution (also called a stable distribution). Boothe and Glassman proved that the exchange rate changes were not following a normal distribution and noted that the data were sharp leptokurtic and more fat-tailed than the normal distribution. Glassman (1987) compared the bid-ask spreads with the volatility and concluded that the size of the spread is related to the exchange rate volatility. The latest methods use recurrent (long short-term memory) neural networks to estimate the volatility after training on TS data; see Petneházi and Gáll (2019) or Verma (2021). Artificial neural network methods have even been used to predict the direction of change of crude oil futures (Galeshchuk & Mukherjee, 2017). This leads neatly into the related concept of “directional change” (DC), where the changes are significant relative to a statistical threshold.

Market transactions are not uniformly distributed over time. Unexpected economic data, different geographical time zones, natural disasters, or even comments from politicians may cause transaction volumes to fluctuate. These complex and uncertain events drive

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significant price changes, which are difficult to sample on a regular timescale. The DC concept was first published by Guillaume et al. (1997), who presented an algorithm to sample the market data. Guillaume et al. elucidated the advantages of the timescale varying between trends but with a fixed threshold. Tsang (2010) formally defined the concept of DC: The price movements are defined by a series of DC uptrends and downtrends (the formal DC definition will be introduced in Section 2). Glattfelder et al. (2011) illustrated the statistical discovery of 12 scale laws based on DC in high-frequency FX data. Tsang et al. (2015) defined the reversal points as extreme points, which are confirmed when the cumulative price changes reach a threshold. A threshold defines the size of the significant price changes. Tsang et al. (2017) presented a set of DC indicators capturing market information. In the study of high-frequency data, Chen and Tsang (2021) showed that DC can be built upon to track the market tick by tick. Chronologically, DC records the extreme points, and these are then converted into a DC sequence. The previous studies in the DC method mainly focus on analyzing single price sequences of one major market, which includes forecasting the price trend reversals, trading algorithm design, stock index trading strategies, using the DC scaling laws to build trading models, DC agent-based models, and measuring regime changes under the DC approach (Bakhach et al., 2016, 2018; Dupuis & Olsen, 2012; Golub et al., 2017; Ma et al., 2017; Petrov et al., 2018; Tsang & Chen, 2018).

Under the DC framework, this paper focuses on a new path in measuring relative volatility between two markets. Evaluating volatilities between different financial instruments is a primary idea in the application of risk management and trading strategy. The classical approach of measuring relative volatility is through comparing the variance of the price return on the regular timescale. It is capable of evaluating relative volatility if the objective dataset could better coincide with a period of relatively high homogeneity (like a daily or weekly time interval). However, in high-frequency data, the general approach might not present an accurate result for evaluating relative volatility, and there are two main reasons: (1) As already discussed, on the predetermined timescale it is hard to summarize the real behavior in terms of micro-market activity because, for instance, the volume of the participants' transactions are not equal during the regular timescale. (2) The markets' reactions to a sudden event might not be synchronously recorded in the prices; in other words, there might be a time delay between the markets. For instance, in measuring the consistency of the co-jumps between two markets, one price jump of market A may be followed by a price jump from market B with a short time delay. Under the DC framework, we propose a new concept of DC micro-market relative volatility (mRV) in evaluating relative volatility. In mRV, measuring relative volatility does not require a predefined timescale since the mRV approach determines the timescale based on a data-driven process. Specifically, we build the DC relative sequence (RS), which combines the DC sequences of two markets into a single sequence. In a DC RS, the timescale is passively defined by the observation of the DC data.

The remainder of this paper is organized as follows. Section 2 introduces the DC concept and the volatility measurement in the DC

method. Section 3 presents the measure of DC relative volatility under a predetermined period. Section 4 introduces the concept of DC micro-market relative volatility mRV with its measurement method. Section 5.1 contrasts the classical method (TS approach) with the DC method from the perspective of measuring relative volatility. Section 5.2 illustrates the back-testing of measuring relative volatility between EURUSD and GBPUSD over 7 years from 2012 to 2018. Particularly, mRV detected that sterling was extremely volatile in comparison with the euro in the week of the Brexit referendum. Inter alia, mRV detected that GBPUSD was extremely volatile compared with EURUSD after the voting time of the Brexit referendum. In Section 5.3, we discuss the benefits of measuring mRV compared with the classical method. In addition, Section 5.4 proposes a scaling law to evaluate the relationship between the average period of subsequence and threshold (definitions to follow in Sections 4.2 and 5.4). In Section 6 we give a conclusion.

2 | DIRECTIONAL CHANGE

2.1 | Introduction

DC is a new framework in the data sampling of financial market transactions for the analysis of the market behaviors. The process of DC data sampling is based on the DC algorithm in Equations (1) and (2) (Guillaume et al., 1997; Tsang et al., 2015). In TS analysis, the market data are collected under a predetermined timescale. However, the mechanism of DC data sampling considers the significant price changes such that the market data are recorded when the price change has reached a certain threshold from the last peak/trough of the price. In practice, the analyst determines the threshold as a percentage. Hence, price changes are recorded as a series of alternative uptrends and downtrends, and the timestamp of each DC data point is determined dynamically. In an uptrend, a peak is determined as a DC extreme point (EP) when the current price P_t is lower than the last high price P_h by a fixed threshold (in percent) θ :

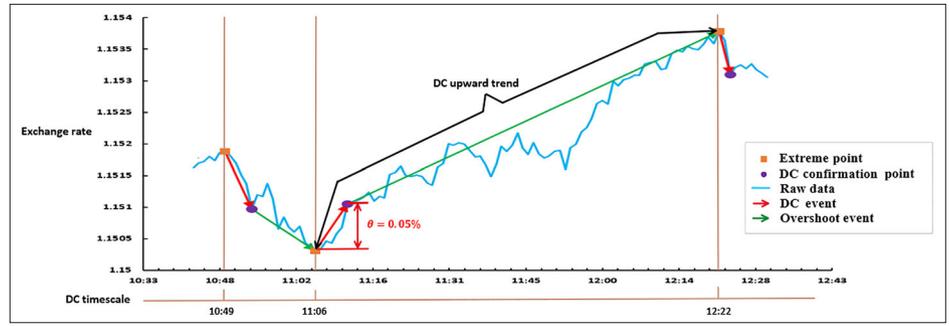
$$P_t = P_h(1 - \theta) \quad (1)$$

In contrast, a downtrend is terminated by a DC EP when the current price P_t is higher than the last low price P_l by a fixed threshold:

$$P_t \geq P_l(1 + \theta) \quad (2)$$

where the size of the threshold θ is given by the analyst. We define the current price P_t as the DC confirmation point when the DC EP is determined. Figure 1 is an example of a DC summary of the exchange rates of EURUSD as a sequence of extreme points. According to Tsang et al. (2017), a DC downtrend (uptrend) decomposes into two parts: a DC event and an overshoot event. The DC timescale, in Figure 1, illustrates a dynamic timescale that the end of the current interval is determined when the price has changed to a threshold from the last highest or lowest price.

FIGURE 1 The price curve of EURUSD on May 3, 2016. An example of directional change (DC) summaries with a threshold θ of 0.05%. The three vertical brown lines (determined by DC extreme points) separate the price curve into a downtrend and an uptrend. On a DC timescale, we have the timestamps of the three extreme points (10:49 a.m., 11:06 a.m., and 12:22 p.m.)



A DC EP is a couple that contains a timestamp $EP.t$ with a price $EP.p$:

$$EP = (EP.t, EP.p) \quad (3)$$

A DC sequence S_A^{θ} is a finite sequence that comprises the extreme points of the market A ordered by $EP.t$:

$$S_A^{\theta} = (EP_1, EP_2, \dots, EP_k, \dots, EP_n) \quad (4)$$

where EP_k is a DC extreme point.

It is worth reiterating that DC and TS sample data differently. Therefore, given the same raw tick data, DC and TS will sample different datasets. Although volatility measures under DC and TS both reflect the market, they cannot be compared directly. It is possible that one measure shows high volatility when the other shows low volatility (Tsang et al., 2017). As DC is data driven, we cannot do processing until we encounter the next EP. If no EP is observed, then nothing interesting happens in the market as far as DC (under the threshold employed) is concerned.

2.2 | DC volatility

DC measures the volatility of a single market based on the frequency of the observed EPs over a period (Guillaume et al., 1997). Tsang et al. (2017) discussed how the DC approach could measure market volatility. Given a period of T , the more observed DC trends found the more volatile the market is. As explained in Figure 1, a DC trend is defined by connecting two adjacent EPs. Hence, the number of DC trends are quantified by the number of observed extreme points N_{DC} . Over the period T , the higher value of N_{DC} indicates higher volatility. Petrov et al. (2019) presented the measure of instantaneous volatility where the equation is developed based on the theory of Brownian motion for the price returns:

$$\sigma_{DC} = \theta \sqrt{\frac{N_{DC}}{T}} \quad (5)$$

where N_{DC} is the number of extreme points from a market over the period T and θ is the threshold which is utilised to obtain the market's DC sequence.

3 | DC RELATIVE VOLATILITY

DC relative volatility (DCRV) is a concept for comparing the intensity of one market's volatility relative to another market in a period T . The general method of evaluating relative volatility is through comparing the variances of the price returns between the two markets in a period T , which requires the same timescale of the two markets' price returns. For instance, analysts compare the variances of hourly price returns between market A and market B in a particular month. In DCRV, the relative volatility is measured by differencing the values of two markets' DC volatilities σ_{DC} in a period T ; for example, the measure of DCRV between market A and market B, denoted $\sigma_{DC(A,B)}$, is given by

$$\sigma_{DC(A,B)} = \sigma_{DC,A} - \sigma_{DC,B} = \theta \frac{\sqrt{N_{DC,A}} - \sqrt{N_{DC,B}}}{T} \quad (6)$$

where $\sigma_{DC,A}$ and $\sigma_{DC,B}$ are the DC volatility of the market A and market B respectively, $N_{DC,A}$ and $N_{DC,B}$ are the number of extreme points of market A and market B, respectively, over the period T , and θ is the threshold that is applied to obtain the DC sequences of market A and market B.

Given the $\sigma_{DC(A,B)}$ over a period T :

1. If $\sigma_{DC(A,B)} > 0$, the volatility of market A is relatively higher than the volatility of market B.
2. If $\sigma_{DC(A,B)} = 0$, the volatility of market A and market B are at the same level.
3. If $\sigma_{DC(A,B)} < 0$, the volatility of market A is relatively lower than the volatility of market B.

4 | DC MICRO-MARKET RELATIVE VOLATILITY

Section 3 introduced the measure DCRV in a predetermined period T evaluating the relative volatility depending on the length of the period. However, given a set of data, the DCRV may indicate different results in measuring relative volatility when the length of T is selected randomly. In the following example, Figure 2 shows a segment of the DC sequences of market A and market B. Given the three different

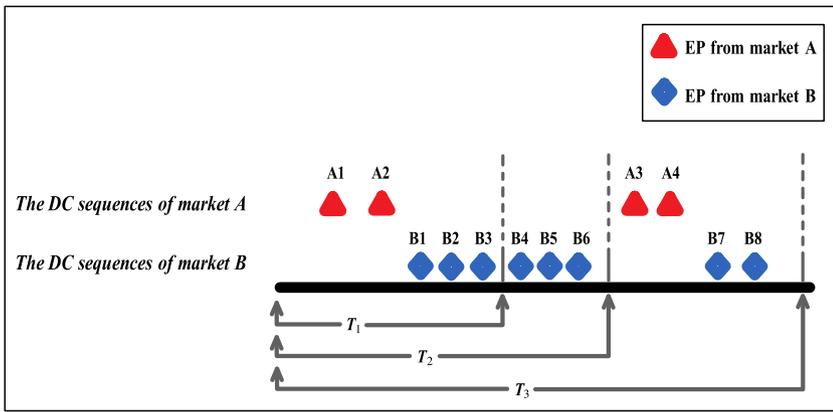


FIGURE 2 The directional change (DC) sequences of market A and market B with the periods T_1 , T_2 , and T_3 . Under the three different lengths of the periods, the DC relative volatility measurement shows the different conclusions in evaluating relative volatility between the two markets. EP: extreme point

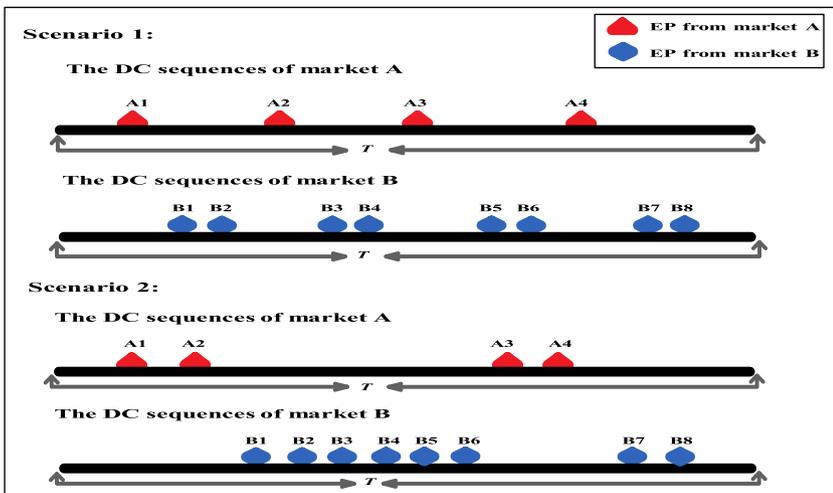


FIGURE 3 The same number of extreme points from market A and market B in the same period T . DC: directional change; EP: extreme point

lengths of the periods T_1 , T_2 , and T_3 , we obtain the different number of EPs from the two markets. According to Equation (6), the DCRV approach indicates three different results ($0.32\theta/\sqrt{T_1}$, $1.04\theta/\sqrt{T_2}$, and $0.83\theta/\sqrt{T_3}$) in measuring the relative volatility under the periods of T_1 , T_2 , and T_3 . Figure 2 raises the issue of how we should select the length of the T for measuring the relative volatility.

DC takes a data-driven approach to sampling. Based on the same principle, it may be better to let the data pick T . That motivates us to find a data-driven measure of relative volatility. Also, the DCRV approach might be incapable of evaluating the event-based collapse at the micro-level. Figure 3 shows two differently arranged frequencies of the EPs from market A and market B in the same period T . In scenario 1, there is a constant frequency of the observed EPs between the two markets that, every two EPs of market B follows one EP of market A. In scenario 2, there are the same number of total EPs as in scenario 1. However, the frequency of the observed EPs is entirely different (six consecutive EPs of market B follow two EPs of market A, then two EPs of market B follow two EPs of market A). Although the two scenarios show different arrangement of frequencies, the DCRV approach presents the same result because of the same number of EPs of the two scenarios (according to Equation (6)).

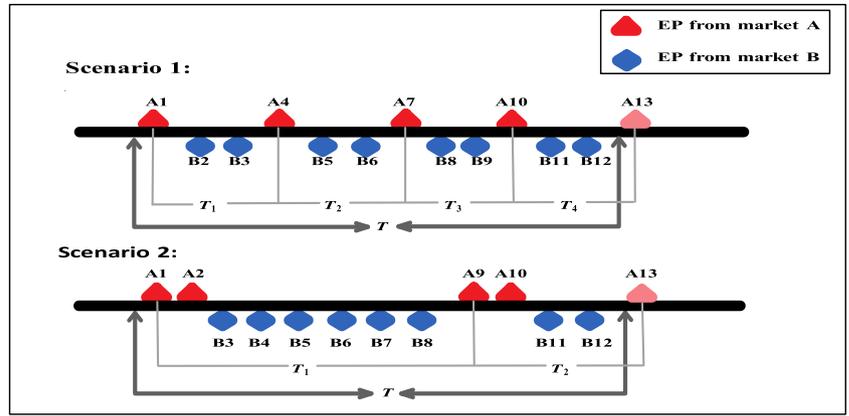
The shortcoming described in the previous paragraph is addressed with the concept of DC micro-market relative volatility (mRV). This is a

concept used to evaluate the relative volatility based on the data-driven process. In mRV, the period T is determined according to the observation of the EPs of the two markets. It is important to note time is passively defined in mRV. A formal definition of mRV and how it may be measured is given in the next sections.

4.1 | DC RS

As discussed at the beginning of Section 4, a progressive method is to dynamically determine the period T based on the observed EPs of the two markets. The DC RS combines the two DC sequences into a new sequence ordered chronologically. In an RS, the termination of the current period depends on the market identity between the current EP and the next EP. Figure 4 illustrates the DC RSs according to scenario 1 and scenario 2 in Figure 3. In scenario 1 of Figure 4, the T_1 is terminated when the identity of the EP.A4 is different from the identity of the EP.B3. In scenario 2 of Figure 4, the T_2 is terminated when the identity of the EP.A13 is different from the identity of the EP.B12 (we suppose that the EP.13 is from market A). Hence, in scenario 1 of Figure 4, the DC RS is decomposed into the four subsequences with periods T_1 , T_2 , T_3 , and T_4 . Likewise, the DC RS of scenario 2 is decomposed into the two subsequences T_1 and T_2 .

FIGURE 4 The decomposition into periods of Figure 3. We assume that the extreme point (EP). A13 is from market A for both scenario 1 and scenario 2



4.2 | Formal definition of DC RC

A DC combined sequence comprises all observed EPs from the two DC sequences of S_A^θ and S_B^θ ordered by the timestamp EP. t :

$$S_{S_A^\theta, S_B^\theta} = (EP_1, EP_2, \dots, EP_m) \quad (7)$$

where m equals the amount of the total number of EPs from both S_A^θ and S_B^θ , and EP_1, EP_2, \dots, EP_m are either from S_A^θ or S_B^θ . The examples of scenario 1 and scenario 2 from Figure 3 are summarized as follows: Scenario 1 from Figure 3:

$$S_{S_A^\theta, S_B^\theta} = (A1, B2, B3, A4, B5, B6, A7, B8, B9, A10, B11, B12) \quad (E.1)$$

Scenario 2 from Figure 3:

$$S_{S_A^\theta, S_B^\theta} = (A1, A2, B3, B4, B5, B6, B7, B8, A9, A10, B11, B12) \quad (E.2)$$

A DC RC is generated by a division process $\Gamma(S_{S_A^\theta, S_B^\theta}^\theta)$ that divides a DC RS into z subsequences according to the identity of the adjacent EPs:

$$RS_{S_A^\theta, S_B^\theta}^\theta = \Gamma(S_{S_A^\theta, S_B^\theta}^\theta) = (Y_1, Y_2, \dots, Y_j, \dots, Y_z) \quad (8)$$

where Y_j is a subsequence of RS. All Y_j contain at least two EPs, one EP from S_A^θ and another from S_B^θ ; thus, the maximum value of z is $m/2$. Otherwise, at least one Y_j contains more than two EPs, so $z < m/2$. For every Y_j we have that

$$\forall j: Y_j = (EP_{j,1}, EP_{j,2}, EP_{j,3}, \dots, EP_{j,k-1}, EP_{j,k}) \quad (9)$$

The termination of the current subsequence Y_j depends on the identity of the next EP. When the identity of the upcoming EP is not the same as the identity of the current EP, the length of the period of the current Y_j is determined by

$$T(Y_j) = EP.t_{j+1,1} - EP.t_{j,1} \quad (10)$$

Given the DC sequences S_A^θ and S_B^θ of scenario 1 and scenario 2 in Figure 4, we obtain the following DC RSs:

Scenario 1 from Figure 4:

$$RS_{S_A^\theta, S_B^\theta}^\theta = ((A1, B2, B3)_1, (A4, B5, B6)_2, (A7, B8, B9)_3, (A10, B11, B12)_4) \quad (E.3)$$

Scenario 2 from Figure 4:

$$RS_{S_A^\theta, S_B^\theta}^\theta = ((A1, A2, B3, B4, B5, B6, B8)_1, (A9, A10, B11, B12)_2) \quad (E.4)$$

4.3 | The measure of DC mRV

The approach of DC mRV is based on Equation (6), while the subject of the measurement is the subsequence Y of $RS_{S_A^\theta, S_B^\theta}^\theta$:

$$mRV_Y = \theta \frac{\sqrt{N_{DC,A}} - \sqrt{N_{DC,B}}}{\sqrt{T(Y)}} \quad (11)$$

where $T(Y)$ is defined in Equation (10). We shall abuse the notation by using mRV as a measure as well as an abbreviation of the concept. Given the measure mRV_Y of the subsequence Y :

If $mRV_Y > 0$, the volatility of market A is relatively higher than the volatility of market B.

If $mRV_Y = 0$, the volatility of market A and market B are at the same level.

If $mRV_Y < 0$, the volatility of market A is relatively lower than the volatility of market B.

In scenario 1 of Figure 4, we measure the mRV in the first subsequence Y_1 of $RS_{S_A^\theta, S_B^\theta}^\theta$ through Equation (11):

$$mRV_{T(Y_1)} = \frac{\theta(\sqrt{1} - \sqrt{2})}{\sqrt{T(Y_1)}} = \frac{-0.41\theta}{\sqrt{T(Y_1)}}$$

Given the DC RS of Equation (8), mRV can be evaluated on each subsequence through Equation (11):

$$\text{mRV}\left(\text{RS}_{S_A^{\theta} S_B^{\theta}}\right) = \left(\text{mRV}_{(Y_1)}, \text{mRV}_{(Y_2)}, \dots, \text{mRV}_{(Y_j)}, \dots, \text{mRV}_{(Y_i)}\right) \quad (12)$$

where $\text{mRV}\left(\text{RS}_{S_A^{\theta} S_B^{\theta}}\right)$ is a sequence and Y_j refers to the subsequence j .

In scenario 1 and scenario 2 of Figure 4, the mRV is given as follows.

Scenario 1 from Figure 4:

$$\text{mRV}\left(\text{RS}_{S_A^{\theta} S_B^{\theta}}\right) = \left(\text{mRV}_{(Y_1)}, \text{mRV}_{(Y_2)}, \text{mRV}_{(Y_3)}, \text{mRV}_{(Y_4)}\right)$$

Scenario 2 from Figure 4:

$$\text{mRV}\left(\text{RS}_{S_A^{\theta} S_B^{\theta}}\right) = \left(\text{mRV}_{(Y_1)}, \text{mRV}_{(Y_2)}\right)$$

4.4 | Discussion: The merits of using mRV in micro-markets

When measuring mRV, the subsequence Y_j is the primary object. $T(Y_j)$ is a secondary object defined by the subsequence. DC is a data-driven approach of sampling the market data such that the DC data are only recorded when significant price changes are observed (for details, see Section 2). Under the DC framework, the DC RS is a combined sequence of two markets' sequences. We then divide the DC RS into a number of subsequences based on the market identity of the adjacent EPs. At the end of Section 4.2, the example of scenario 1 (Equation E.3) illustrates that B3 is the last EP of the first subsequence because A4 (the next EP) is from a different market than B3 is. According to Equation (10), the period T of the first subsequence Y_1 is passively determined by $T(Y_1) = \text{EP}.t_{2,1} - \text{EP}.t_{1,1}$; in the example of Equation (E.3), we have $T(Y_1) = \text{A4}.t - \text{A1}.t$. Hence, the period T is intrinsically determined by the behavior of the two markets' price changes, rather than being a fixed time interval predetermined by the analyst. Based on the subsequence, we can precisely locate the timestamp when a significant mRV value is determined within the period T . For example, an unusual "flash event" may produce a series of EPs from market A compared with one EP from market B within a subsequence. We can then simply measure the relative volatility of this special event by calculating the mRV of the subsequence.

4.5 | Discussion: Regarding threshold selection

Fundamentally, the DC data summarize the original price movement based on a predetermined threshold. In practice, observers utilize the threshold to capture the significant price changes and filter out the unnecessary noise of the price movement. Hence, the magnitude of

the threshold directly impacts the frequency of the EPs over a period. An extremely small threshold will cause every tick data point to be determined as an EP. On the other hand, an extremely large threshold will give the result of recording no DC data. So, what is the "right" threshold for us to use? It is unlikely to find an "optimal" threshold for sampling DC data in this research. In fact, there are no "wrong" ways of determining the size of thresholds. It is actually the observer's prerogative to set the threshold to suit the individual observer's needs. High-frequency traders might prefer a smaller threshold to acquire the micro price changes, whereas institutions might be more focused on larger price movements. In addition, Glattfelder et al. (2011) showed that the same statistical measures can be observed under different thresholds.

5 | EXPERIMENT

The experiment is separated into four parts: (1) contrasting the measure of relative volatility between the classical method and mRV; (2) the back-testing of measuring mRV between sterling and the euro; (3) the observations about the benefits of measuring mRV; and (4) the relationship between the threshold and the average period of the subsequence.

5.1 | Contrasting relative volatility between TS and DC

In this section, we contrast the realized volatility (the classical TS method under the regular timescale) and mRV in the measure of relative volatility. It is worth reiterating that DC and TS work on different data series (although they are extracted from the same tick data). Therefore, volatility measures in them cannot be compared directly. The aim of this experiment is to examine the consistency of measuring the relative volatility between the two methods.

In TS, we select four groups of the data under the equalized time interval $\Delta t = \{10 \text{ s}, 1 \text{ min}, 5 \text{ min}, 15 \text{ min}\}$. The return at time t , R_t , is defined by

$$R_t = \ln P_t - \ln P_{t-\Delta t} \quad (13)$$

where $\ln P_t$ is the logarithmic price at the end of each time interval Δt . Given the sequence of the returns over a period τ (e.g., a trading day or a trading week), the realized volatility is defined by the standard deviation:

$$\sigma_t = \sqrt{\frac{\sum (R_t - \bar{R})^2}{n-1}} \quad (14)$$

where n is the number of returns over a period τ and \bar{R} is the mean of the sequence of the returns. Given the standard deviation of market A and market B, we calculate the difference of $\sigma_{\tau,A}$ and $\sigma_{\tau,B}$ to evaluate the relative volatility between market A and market B over a period τ :

$$\text{Dsd}_{r,A-B}^{\Delta t} = \sigma_{r,A} - \sigma_{r,B} \quad (15)$$

where Δt is the initially selected time interval to obtain the logarithmic price.

Glattfelder et al. (2011) discovered 12 DC scaling laws in the market. For instance, the analytical relationship between the size of threshold and the average percentage change of a DC trend. DC scaling law 10 gives the statistical property that the average period of a DC trend (T_{tmv}) is approximately equal to a function of the threshold θ :

$$\langle T_{tmv} \rangle = \left(\frac{\theta}{C_{t,tmv}} \right)^{E_{t,tmv}} \quad (16)$$

where $E_{t,tmv}$ and $C_{t,tmv}$ are the scaling law parameters, $\langle \cdot \rangle$ is the operator to calculate the mean, and θ is the threshold. Based on Equation (16), we can estimate the average period of a DC trend (T_{tmv}) given a threshold θ , and vice versa. Hence, we obtain the four corresponding thresholds given the time intervals $\Delta t = \{10 \text{ s}, 1 \text{ min}, 5 \text{ min}, 15 \text{ min}\}$. A DC total movement defines a trend of the price movement between two adjacent extreme points (see Figure 1 in Section 2). According to the DC definition, a trend is terminated when the price changes have reached a certain threshold θ from the last peak/trough of the price. In DC, the peak/trough defines the EP. Given a threshold and the scaling law 10 (Equation (16)), we can estimate the average period of the trend and vice versa. Glattfelder et al. (2011) estimated the average values of the parameters $C_{t,tm}$ and $E_{t,tm}$ across 13 pairs of exchange rates and obtained $C_{t,tm} = 0.00165$ and $E_{t,tm} = 2.02$. In this experiment, $\langle T_{tmv} \rangle$ is the Δt . Given Δt values, using Equation (16), we obtain the corresponding thresholds, $\theta = \{0.005\%, 0.013\%, 0.028\%, 0.048\%\}$. Based on the four thresholds, we calculate the DC sequences of market A and market B and generate the DC RS $RS_{S_A, S_B}^{\Delta t}$ through Equation (8). Then, we measure the mRV through Equation (12). According to Equation (15), we evaluate the relative volatility in the period τ of daily (D), weekly (W), and monthly (M) of $\text{Dsd}_{r,A-B}^{\Delta t}$. As introduced in Equation (12), $\text{mRV}_{(RS_{S_A, S_B}^{\Delta t})}$ is a sequence. Hence, we calculate the mean value of $\text{mRV}_{(RS_{S_A, S_B}^{\Delta t})}$ over the period τ to compare with the value of $\text{Dsd}_{r,A-B}^{\Delta t}$. In the back-testing, we calculate the mean of daily $\langle \text{mRV}_{(RS_{S_A, S_B}^{\Delta t})} \rangle_D$, the mean of weekly $\langle \text{mRV}_{(RS_{S_A, S_B}^{\Delta t})} \rangle_W$, and the mean of monthly $\langle \text{mRV}_{(RS_{S_A, S_B}^{\Delta t})} \rangle_M$. The data source is Tickstory,² which gives direct access to the database of Dukascopy.³ We select EURUSD as the major exchange rate to be compared with five exchange rates. Table 1 summarizes the two approaches in the measure of relative volatility.

In the 7 years' dataset, we obtain 1,825 results for $\text{Dsd}_{D,A-B}^{\Delta t}$, 366 results for $\text{Dsd}_{W,A-B}^{\Delta t}$, and 84 results for $\text{Dsd}_{M,A-B}^{\Delta t}$. $\langle \text{mRV}_{(RS_{S_A, S_B}^{\Delta t})} \rangle_{\tau}$ also has the same size in terms of the number of results. Given the results of the back-testing, we measure the correlation between the results of the two approaches. As the data do not fit a Gaussian distribution, we evaluate the correlation through the Spearman rank-order correlation coefficient. The Spearman correlation tests the association of the ordinal relationship between $\text{Dsd}_{r,A-B}^{\Delta t}$ and $\langle \text{mRV}_{(RS_{S_A, S_B}^{\Delta t})} \rangle_{\tau}$. Table 2 is summarizes of the results of the correlation coefficient.

Table 2 provides a summary the results of the correlation coefficients between $\text{Dsd}_{r,A-B}^{\Delta t}$ and $\langle \text{mRV}_{(RS_{S_A, S_B}^{\Delta t})} \rangle_{\tau}$. The statistical tests report strong positive correlation, in that all the correlation coefficients are over 0.6. The far-right column is the mean of each row, which indicates the average correlation coefficients crossing the five pairs of the exchange rates under the time intervals $\Delta t = \{10 \text{ s}, 1 \text{ min}, 5 \text{ min}, 15 \text{ min}\}$ (with the four corresponding threshold θ values). In Figure 5a, the three dot-lines illustrate the values of the right end column in the periods of daily, weekly, and monthly. Figure 5a indicates that the correlation coefficients are tight under the Δt of 10 s and 1 min, whereas the spreads are increasing in the 5 min and 15 min time intervals. Figure 5b shows the average correlation coefficients of each dot-line and that the average correlation coefficients are 0.795, 0.836, and 0.864 in the periods of daily, weekly, and monthly, respectively. Overall, the results of the correlation test lead to the conclusion that a positive correlation exists between $\text{Dsd}_{r,A-B}^{\Delta t}$ and $\langle \text{mRV}_{(RS_{S_A, S_B}^{\Delta t})} \rangle_{\tau}$ from 2012 to 2018.

5.2 | Back-testing of mRV between sterling and the euro

This section will discuss the application of measuring mRV between GBPUSD and EURUSD. The unexpected result of the Brexit referendum caused sterling to fall -8.016% against the US dollar on June 24, 2016, which was the most significant single-day drop since 2000.⁴ On the same day, the euro crashed -2.65% against the US dollar.

The goal of this experiment is to ask whether mRV is useful for measuring the relative volatility between the two markets. To answer that question, we have conducted two sets of experiments. First, we examine the average monthly mRV over a long historical period from 2012 to 2018 to view the relative volatility between sterling and euro over the long term. Second, we test the mRV at the micro-level, in that we monitor the mRV in each subsequence during the week of the Brexit referendum.

As discussed in Section 4.5, the magnitude of the threshold selection depends on the individual observer's needs. Thus, throughout the two experiments, we select two threshold θ values, 0.05% and 0.1%, to calculate the mRV. According to Equation (11), the value of mRV could be enormously small when we select a lower threshold. Hence, we normalize the values of mRV by the threshold, $\text{mRV} = \text{mRV}/\theta$. Moreover, we simplify the mean of monthly $\langle \text{mRV}_{(RS_{GBPUSD, EURUSD}^{\Delta t})} \rangle_M$ to $\langle \text{mRV}_{(RS)}^{0.05\%} \rangle_M$ in this section.

Figure 6 illustrates the mean of monthly $\langle \text{mRV}_{(RS)}^{0.05\%} \rangle_M$ under the threshold of 0.05% over 7 years. From January 2012 to September 2014, the volatility of EURUSD was relatively higher than GBPUSD, in that $\langle \text{mRV}_{(RS)}^{0.05\%} \rangle_M$ changed smoothly between -0.01 and 0 (except the months of August 2013, January 2014, and February 2014, for which the values of mRV were slightly positive). During the year 2015, EURUSD was highly volatile in comparison with

TABLE 1 The summaries of the two approaches in the measurement of relative volatility: micro-market relative volatility (mRV) and difference in standard deviation of market A and market B (Dsd)

	$Dsd_{\tau, A-B}^{\Delta t}$	$mRV \left(RS_{S_A^{\theta}, S_B^{\theta}} \right)_{\tau}$
Raw data sampling	The sequences of the returns under $\Delta t = \{10 \text{ s}, 1 \text{ min}, 5 \text{ min}, 15 \text{ min}\}$ over seven years from 2012 to 2018	The directional change relative sequence under $\theta = \{0.005\%, 0.013\%, 0.028\%, 0.048\%\}$ over seven years in tick data from 2012 to 2018
Periods of measurement	Daily: $Dsd_{D, A-B}^{\Delta t}$; weekly: $Dsd_{W, A-B}^{\Delta t}$; monthly: $Dsd_{M, A-B}^{\Delta t}$	Daily: $mRV \left(RS_{S_A^{\theta}, S_B^{\theta}} \right)_D$; weekly: $mRV \left(RS_{S_A^{\theta}, S_B^{\theta}} \right)_W$; monthly: $mRV \left(RS_{S_A^{\theta}, S_B^{\theta}} \right)_M$
Measure of the pairs of exchange rates	$Dsd_{\tau, GBPUSD-EURUSD}^{\Delta t}$, $Dsd_{\tau, USDJPY-EURUSD}^{\Delta t}$, $Dsd_{\tau, AUDUSD-EURUSD}^{\Delta t}$, $Dsd_{\tau, USDCAD-EURUSD}^{\Delta t}$, $Dsd_{\tau, GBPJPY-EURUSD}^{\Delta t}$	$mRV \left(RS_{S_{GBPUSD}^{\theta}, S_{EURUSD}^{\theta}} \right)_{\tau}$, $mRV \left(RS_{S_{USDJPY}^{\theta}, S_{EURUSD}^{\theta}} \right)_{\tau}$, $mRV \left(RS_{S_{AUDUSD}^{\theta}, S_{EURUSD}^{\theta}} \right)_{\tau}$, $mRV \left(RS_{S_{USDCAD}^{\theta}, S_{EURUSD}^{\theta}} \right)_{\tau}$, $mRV \left(RS_{S_{GBPJPY}^{\theta}, S_{EURUSD}^{\theta}} \right)_{\tau}$

For the classical method, Equations (13–15) provide the definitions for the measurement of the relative volatility in the daily, weekly, and monthly cases. The back-testing picked 24 hr tick-by-tick data in weekdays from midnight on Monday to 10:00:00.000 p.m. on Friday.

GBPUSD after the quantitative easing announcement from the European Central Bank.⁵ In the periods between January 2016 and June 2016, there was a sharp climb in the values from -0.011 to 0.0374 . After the month of the Brexit referendum (June 2016), sterling kept its higher volatility compared with the euro until the end of 2016.

Under the threshold of 0.1% , the $\langle mRV_{(RS)}^{0.05\%} \rangle_M$ shows a consistent result (see Figure A1 in Appendix A).

In the second application, we evaluate the mRV in each subsequence under the thresholds of 0.05% and 0.1% . We select the DC RSs $RS_{S_{GBPUSD}^{\theta}, S_{EURUSD}^{\theta}}^{<0.05\%}$ and $RS_{S_{GBPUSD}^{\theta}, S_{EURUSD}^{\theta}}^{<0.1\%}$ from June 16 to June 30, 2016, such that the periods cross the five working days before and after the Brexit referendum day on June 23, 2016. Given the DC RSs, we calculate $mRV_{(RS)}^{0.05\%}$. Figure 7 plots the $mRV_{(RS)}^{0.05\%}$ of the 2,200 subsequences under the threshold 0.05% . Note that the x-axis in Figure 7 is not physical time but indices of the subsequences; the y-axis is the mRV value. We highlight (in red) the subsequences in the period right after the voting of the Brexit referendum until the end of the next day from 10:00 p.m. (22:00 UTC) June 23 to 10:00 p.m. June 24, 2016⁶ (24 hr after the Brexit referendum vote). This corresponds to index 0 to 2,200 in Figure 7. Hence, the 2,200 subsequences are separated into three parts.

Part 1: from midnight (0:00:01 a.m.) on June 16 to 10:00 p.m. on June 23, 2016 (140 hr in total trading hours);

Part 2: from 10:00 p.m. on June 23 to 10:00 p.m. on June 24, 2016 (24 hr);

Part 3: from midnight (0:00:01 a.m.) on June 27 to midnight (11:59:59 p.m.) on June 30, 2016 (96 hr).

Two observations stand out from the results shown in Figure 7:

Observation 1. GBPUSD is highly relatively volatile compared with EURUSD in Part 2.

In the highlighted area of Figure 7 (the period of Part 2), there are enormous changes in mRV after the voting time. In Part 2, we observed the subsequence of the highest mRV reached 0.834 in the period T from 11:17:53 p.m. to 23:18:27 p.m. on June 23, 2016. In this subsequence, there are 35 EPs of GBPUSD and one EP of EURUSD in 34 s. In contrast, the lowest value of mRV is -0.633 , in that there is one EP of GBPUSD and four EPs of EURUSD in the period T of 3 s (from 3:59:28 a.m. to 3:59:31 a.m. on June 24, 2016). In Table 3, we present the mean and median of the $mRV_{(RS)}^{0.05\%}$ in the periods of the three parts (from the second column to the fourth column). Visibly, the values of $\langle mRV_{(RS)}^{0.05\%} \rangle$ and $\text{Median} \left(mRV_{(RS)}^{0.05\%} \right)$ of Part 2 are higher than the values in Part 1 and Part 3, which leads to the conclusion that there is significant volatility of GBPUSD relative to EURUSD after the voting. This conclusion is further supported by the ratio test, as shown in the last two columns of Table 3. In the Part 2/Part 1 column, the ratios reach 5.736 and 4.743 for the mean and median values of $mRV_{(RS)}^{0.05\%}$, respectively. In the Part 2/Part 3 column, the respective ratios reach 3.723 and 2.591 .

Observation 2. GBPUSD and EURUSD are much more volatile in Part 2 than in Part 1 or Part 3.

During the period of Part 2, we observed 949 subsequences out of the total of 2,200, which account for 43% of the total subsequences in 11 trading days. The period of Part 2 is 24 hr after the Brexit referendum, which means around 39 subsequences are determined in each hour. Also, we observed 1,251 subsequences in the

TABLE 2 The results of the correlation coefficient. The function $\text{Corr}(\cdot)$ is the correlation test given the two sequences obtained by the approaches of $\text{Dsd}_{\tau, A-B}^{\Delta t}$ and $\text{mRV} \left(\text{RS}_{A-B}^{\theta} \right)_{\tau}$

	Correlation coefficient					
	GU-EU	UJ-EU	AU-EU	UC-EU	GJ-EU	Average
<i>Daily</i>						
$\text{Corr} \left(\text{Dsd}_{D, A-B}^{10^s}, \text{mRV} \left(\text{RS}_{A-B}^{0.005\%, \theta=0.005\%} \right)_D \right)$	0.830	0.835	0.634	0.766	0.782	0.769
$\text{Corr} \left(\text{Dsd}_{D, A-B}^{1 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.013\%, \theta=0.013\%} \right)_D \right)$	0.869	0.921	0.713	0.794	0.862	0.832
$\text{Corr} \left(\text{Dsd}_{D, A-B}^{5 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.028\%, \theta=0.028\%} \right)_D \right)$	0.839	0.917	0.733	0.784	0.847	0.824
$\text{Corr} \left(\text{Dsd}_{D, A-B}^{15 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.048\%, \theta=0.048\%} \right)_D \right)$	0.780	0.858	0.657	0.703	0.778	0.755
<i>Weekly</i>						
$\text{Corr} \left(\text{Dsd}_{W, A-B}^{10^s}, \text{mRV} \left(\text{RS}_{A-B}^{0.005\%, \theta=0.005\%} \right)_W \right)$	0.855	0.839	0.621	0.783	0.791	0.778
$\text{Corr} \left(\text{Dsd}_{W, A-B}^{1 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.013\%, \theta=0.013\%} \right)_W \right)$	0.908	0.941	0.723	0.794	0.893	0.852
$\text{Corr} \left(\text{Dsd}_{W, A-B}^{5 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.028\%, \theta=0.028\%} \right)_W \right)$	0.898	0.950	0.786	0.830	0.905	0.874
$\text{Corr} \left(\text{Dsd}_{W, A-B}^{15 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.048\%, \theta=0.048\%} \right)_W \right)$	0.881	0.919	0.751	0.780	0.868	0.840
<i>Monthly</i>						
$\text{Corr} \left(\text{Dsd}_{M, A-B}^{10^s}, \text{mRV} \left(\text{RS}_{A-B}^{0.005\%, \theta=0.005\%} \right)_M \right)$	0.952	0.845	0.615	0.797	0.792	0.800
$\text{Corr} \left(\text{Dsd}_{M, A-B}^{1 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.013\%, \theta=0.013\%} \right)_M \right)$	0.938	0.955	0.757	0.789	0.898	0.867
$\text{Corr} \left(\text{Dsd}_{M, A-B}^{5 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.028\%, \theta=0.028\%} \right)_M \right)$	0.887	0.966	0.846	0.867	0.943	0.902
$\text{Corr} \left(\text{Dsd}_{M, A-B}^{15 \text{ min}}, \text{mRV} \left(\text{RS}_{A-B}^{0.048\%, \theta=0.048\%} \right)_M \right)$	0.839	0.949	0.833	0.886	0.925	0.886

EU: EURUSD; GU: GBPUSD; UJ: USDJPY; AU: AUDUSD; UC: USDCAD; GJ: GBPJPY.

The last column indicates the average value of each row crossing the five pairs of exchange rates under the parameters of Δt and θ . All the correlation coefficients satisfy the significance level of $P < 0.05$.

periods of Part 1 and Part 3 (236 hr in total). Thus, there are approximately five subsequences in each hour over 236 hr. According to the definition of DC volatility (Section 2.2), in a period T , the higher value of N_{DC} (the number of EPs) indicates higher volatility. Hence, we evaluated the instantaneous volatility σ_{DC} (Equation (5)) of GBPUSD and EURUSD in Part 2 and obtained the values of 0.00598 and 0.00374, respectively. We also measured the daily σ_{DC} of Part 1 and Part 3 to compare with the σ_{DC} of Part 2. Figure 8 illustrates the daily instantaneous volatility from June 16 to June 30, 2016. For both GBPUSD

and EURUSD, there is an ascent during June 23, 2016 and a peak on June 24, 2016 (the period of Part 2). The σ_{DC} of GBPUSD and EURUSD declined after June 24, 2016.

We summarize the testing results in Table 4: The third column and the fourth column present the mean and median of σ_{DC} in Part 1 and Part 3; Part 2. σ_{DC} is the σ_{DC} of Part 2; the last two columns are the ratios of Part 2. σ_{DC}/\langle Part 1. $\sigma_{DC}\rangle$ and Part 2. σ_{DC}/\langle Part 3. $\sigma_{DC}\rangle$. For both GBPUSD and EURUSD, the instantaneous volatility of Part 2 is much higher than for Part 1 or Part 3. For GBPUSD, the ratios of

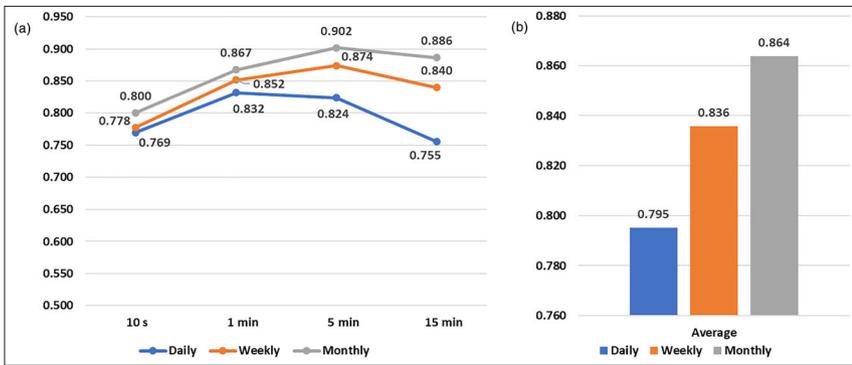


FIGURE 5 (a) The three dot-lines indicate the average values of the correlation under the pairs of parameters Δt and θ ; (b) the three columns show the average values of each line from (a), which indicate the average correlation coefficients of daily, weekly, and monthly periods

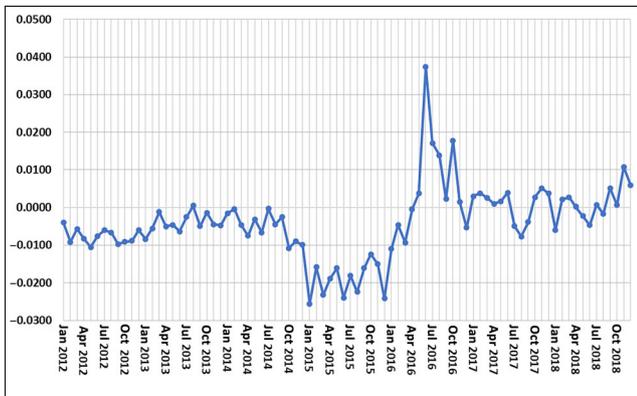


FIGURE 6 The mean of monthly $\langle mRV_{(RS)}^{0.05\%} \rangle_M$ (the y-axis) measures the monthly average micro-market relative volatility (mRV) under the threshold of 0.05%. From 2012 to 2018, there are 84 data points. The values of mRV are normalized by θ

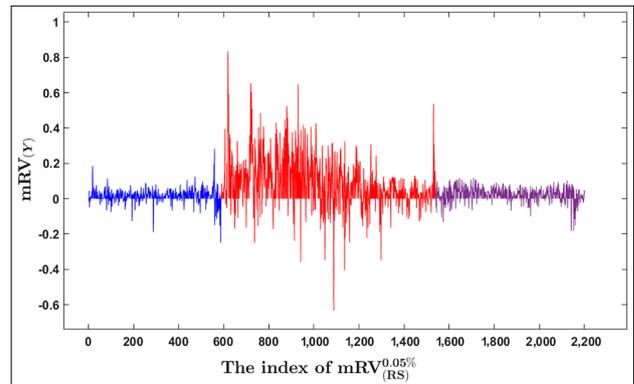


FIGURE 7 The sequence of $mRV_{(RS)}^{0.05\%}$ in the periods from June 16 to June 30, 2016. We select the tick-by-tick data of GBPUSD and EURUSD to calculate the micro-market relative volatility (mRV) of each subsequence. This figure plots 2,200 subsequences observed under the threshold of 0.05%. Note that the x-axis refers to the index to the subsequences. Part 1(blue line): from midnight on June16, 2016, to 10:00 p.m. on June 23, 2016 (140 hr); Part 2 (red line): from 10:00 p.m. on June 23, 2016 to 10:00 p.m. on June 24, 2016 (24 hr); Part 3 (purple line): from midnight on June 27, 2016, to midnight on June 30, 2016 (96 hr)

Part 2. $\sigma_{DC} / \langle \text{Part 1. } \sigma_{DC} \rangle$ and Part 2. $\sigma_{DC} / \langle \text{Part 3. } \sigma_{DC} \rangle$ are 3.4 and 2.61, respectively. For EURUSD, the ratios are 3.17 and 2.49, respectively. Obviously, in the period of Part 2, the volatility of GBPUSD and EURUSD was much higher than in the periods of Part 1 and Part 3. The results of evaluating instantaneous volatility prove the conclusion of observation 2.

We repeated the second application under the threshold of 0.1%. The results are consistent with what we found in the second application in Section 5.2 (for details, see Appendix B).

5.3 | Benefits of measuring mRV

As discussed in Section 4.4, the mRV measure is developed under the DC framework. DC is an alternative approach to record price movements. Instead of recording the transaction prices at fixed time intervals, as is done in TS, DC lets the data alone decide when to record the transaction. In practice, we measure the mRV of every observed subsequence. The subsequences are the result of the division process of a DC RS (see Equation (8)). The period of a subsequence is passively determined by the observed EPs of the two markets. Hence, we can precisely locate the time when we observe

a significant value of mRV (for details, see Observation 3 later herein). The precise time location of mRV allows the observation of significant values that may not be registered by Dsd (an example will be presented in Observation 4). Because the division process of an RS is not conducted using regular time intervals, the frequency of the subsequences varies over a given trading period (e.g., a trading day). The more observed EPs of the two markets there are, the more subsequences will be determined (we will discuss this point in Observation 5 later herein).

Observation 3. DC can precisely locate the exact times within which an extreme mRV occurred. This cannot be done under TS.

As mentioned at the beginning of Section 5.3, using mRV can give a precise time location when there is a significant value of the relative volatility. In micro-market analysis, it is beneficial for

TABLE 3 The mean and median of the $mRV_{(RS)}^{0.05\%}$. The operator Median(.) is used to calculate the median of a sequence

	Part 1	Part 2	Part 3	Part 2/Part 1	Part 2/Part 3
$mRV_{(RS)}^{0.05\%}$	0.014	0.082	0.022	5.736	3.723
Median($mRV_{(RS)}^{0.05\%}$)	0.011	0.053	0.02	4.743	2.591

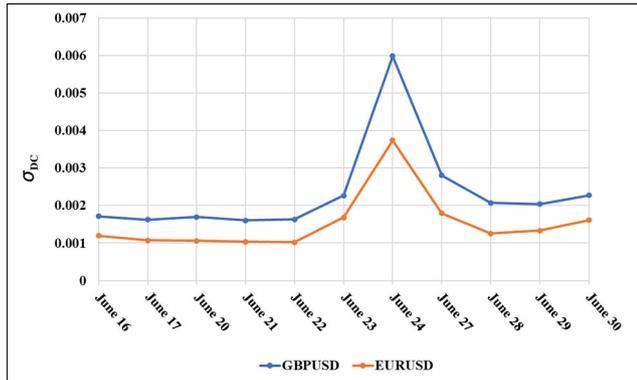


FIGURE 8 The daily instantaneous volatility σ_{DC} of GBPUSD and EURUSD in June 2016. On June 17 (Friday), the trading hours were terminated at 10:00 p.m. (22:00 UTC). On June 23, we select the period from midnight to 10:00 p.m. (the period before the end of the voting). On June 24, the period was selected from 10:00 p.m. June 23 to 10:00 p.m. on June 24 (the period of Part 2)

analysts who need to monitor the relative volatility in high-frequency data. In contrast, the classical method Dsd cannot give the same precise timing because the measure of Dsd is based on sampling at fixed time intervals. So, the presence of a significant value can only be narrowed down to the particular fixed time interval in which it occurred in this case.

Fundamentally, DC and TS are different frameworks for data sampling; there is no direct comparison between mRV and Dsd. To draw parallels with the mRV result in Figure 7, we calculated the Dsd between GBPUSD and EURUSD during the same time period (from midnight on June 16 to midnight on June 30, 2016). Based on Equation (15), we sampled the TS data at 10 s time intervals ($\Delta t = 10$ s) and calculated the value of Dsd for every period of 10 min ($\tau = 10$ min). Sampling at 10 s intervals allows the capture of patterns in high-frequency data, and then the period of 10 min for the calculation of Dsd permits the gathering of sufficient data points for an accurate calculated figure. In Figure 9b, we labelled the four significant Dsd values with their respective time intervals. Correspondingly, there were also four significant values of mRV. As shown in detail in Table 5, for mRV, the periods of the four significant values were located within the time intervals associated with the significant values of Dsd. Specifically, the periods of the four subsequences are distinct and each is less than 1 min.

For instance, as illustrated in Figure 10, the time interval of the highest Dsd (Dsd-2) was determined as being the 10 min interval from 11:10:00 p.m. to 11:20:00 p.m. In contrast, we observed

that the subsequence of the highest mRV (mRV-2) was contained within the 34 s time interval that ran from 11:17:53 p.m. to 11:18:27 p.m., which was located within a small subinterval of the time interval for Dsd.

Observation 4. Through mRV, DC enables us to observe change in relative volatility that is not observable under Dsd in TS.

We observed a subsequence (which we labelled as mRV-5 in Figure 9a) with the biggest negative mRV value from 3:59:28 a.m. to 3:59:31 a.m. on June 24, 2016. This subsequence only lasted for 3 s. The mRV-5 just mentioned records the lowest mRV value (-0.6331) in the whole period observed in Figure 9a. Notice that we do not observe significant negative values in Dsd in Figure 9b. There are two possibilities why the significant negative value might not be reflected in the Dsd that we can take away from this case. First, the 3 s of high relative volatility for EURUSD compared with GBPUSD (as indicated by mRV-5) tended to be diminished by the rest of the recordings within the 10 min. Second, with a sampling period of 10 s, a 3 s spike might well not be even sampled in the first place. Thus, mRV enables us to observe changes in relative volatility between markets that cannot be observed by other means.

Observation 5. The frequency of determining subsequences depends on the intrinsic behavior of the two markets' price changes.

As discussed at the beginning of this section, the period T of the subsequences obtained in order to calculate the values of mRV are passively determined by the observation of the EPs of the two markets. Hence, the period T is intrinsically determined by the behavior of the two markets' price changes, rather than being a fixed time interval predetermined by the analysts. In Figure 9a, the majority of the subsequences are determined within the period of Part 2 (from 10:00 p.m. on June 23 to 10:00 p.m. on June 24, 2016 [24 hr]) as both exchange rates were much more volatile in Part 2 (see Observation 2) than within the periods of Part 1 and Part 3. This illustrates how the approach facilitates the recording of more of the fine-grained behavior during periods of high flux. In contrast, we cannot observe such a quantity of data in TS as the data were collected using a fixed time interval. Specifically, in Table 6, there were 949 subsequences confirmed in Part 2, which accounted for 43% of the total subsequences. However, during the same period, 144 Dsd values were calculated under TS, which only accounted for 9% of the total observations.

Name	Part 1	Part 3	Part 2. σ_{DC} /Part 1. σ_{DC}	Part 2. σ_{DC} /Part 3. σ_{DC}
GBPUSD				
σ_{DC}	0.00176	0.00230	3.40	2.61
Median(σ_{DC})	0.00166	0.00217		
EURUSD				
σ_{DC}	0.00118	0.00150	3.17	2.49
Median(σ_{DC})	0.00107	0.00147		

TABLE 4 The measure of instantaneous volatility in the periods of three parts

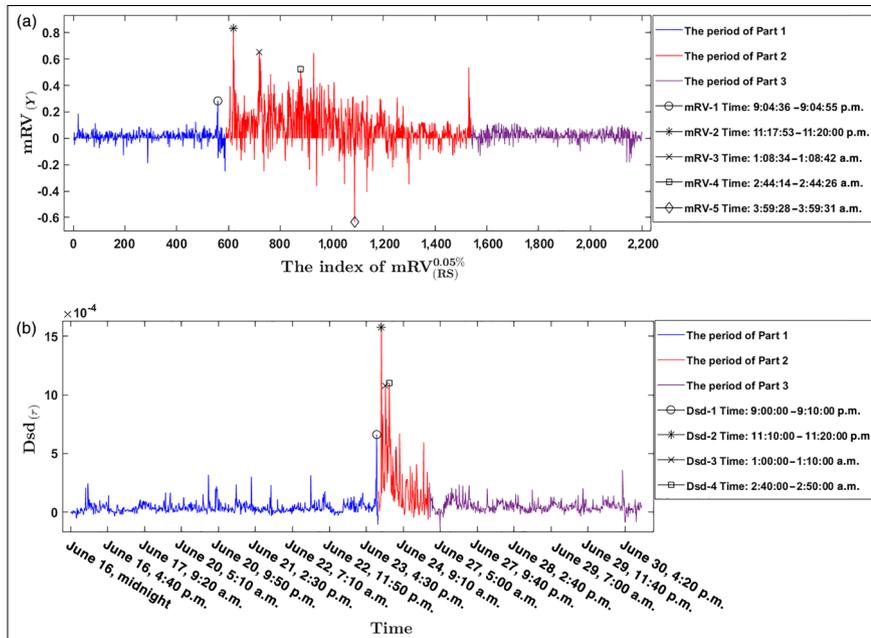


FIGURE 9 The measure of relative volatility in the period from June 16 to June 30, 2016. Part 1 (blue line): from midnight on June 16 to 10:00 p.m. on June 23 (140 hr); Part 2 (red line): from 10:00 p.m. on June 23 to 10:00 p.m. on June 24 (24 hr); Part 3 (purple line): from midnight on June 27 to midnight on June 30 (96 hr). (a) The sequence of $mRV_{(RS)}^{0.05\%}$ between GBPUSD and EURUSD; $\theta = 0.05\%$; the x-axis refers to the index of the subsequence; the y-axis refers to the value of micro-market relative volatility (mRV). (b) The series of $Dsd_{\tau=10 \text{ min}}^{\Delta t=10 \text{ s}}$ between GBPUSD and EURUSD; $\Delta t = 10 \text{ s}$, $\tau = 10 \text{ min}$; the x-axis refers to the timescale; the y-axis refers to the value of Dsd

mRV (directional change)		Dsd (time series)			
Period	Value	Period	Value		
mRV-1	9:04:36–9:04:55 p.m.	0.2831	Dsd-1	9:00:00–9:10:00 p.m.	0.0006624
mRV-2	11:17:53–11:18:27 p.m.	0.8343	Dsd-2	11:10:00–1:20:00 p.m.	0.001575
mRV-3	1:08:34–1:08:42 a.m.	0.6531	Dsd-3	1:00:00–1:10:00 a.m.	0.00108
mRV-4	2:44:14–2:44:26 a.m.	0.5236	Dsd-4	2:40:00–2:50:00 a.m.	0.001103
mRV-5	3:59:28–3:59:31 a.m.	-0.6331			

TABLE 5 The observations of relative volatility using the methods of micro-market relative volatility (mRV) and difference in standard deviation of market A and market B (Dsd)

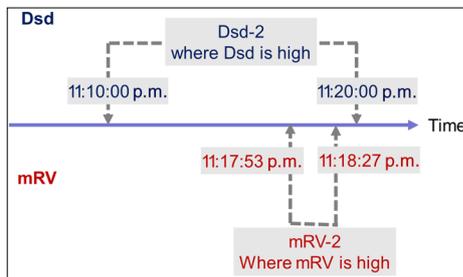


FIGURE 10 Micro-market relative volatility (mRV) shows a more precise period of high relative volatility between GBPUSD and EURUSD

5.4 | Relationship between the threshold and the average period of the subsequence

In Section 4.4 we discussed the merits of not requiring a pre-determined time interval for measuring mRV. The period T of the subsequence is passively determined by the observation of the EPs of the two markets. However, how long in practice is the period of a subsequence before being terminated? Is there a relationship between the threshold's magnitude and the period of the subsequence? Hence, can we obtain a degree of control over the period of a typical subsequence through intelligent selection of the threshold? We

TABLE 6 The number of observations in the periods of three parts under the methods of micro-market relative volatility (mRV) and difference in standard deviation of market A and market B (Dsd)

	Number of observations (percentage of total, %)	
	mRV	Dsd
Part 1	590 (27)	840 (54)
Part 2	949 (43)	144 (9)
Part 3	661 (30)	576 (37)
Total	2,200	1,560

TABLE 7 Specification of the back-testing

Data Type	Tick-by-tick
Periods	24 hr weekdays, from 2015 to 2018
Directional change relative sequences	$RS_{GBPUSD \rightarrow EURUSD}^{\theta}$, $RS_{USDJPY \rightarrow EURUSD}^{\theta}$, $RS_{AUDUSD \rightarrow EURUSD}^{\theta}$, $RS_{USDCAD \rightarrow EURUSD}^{\theta}$
Thresholds	100 thresholds from 0.005% to 0.104% with an increment of 0.001%

implemented back-testing to examine the relationship between the average period of the subsequence $\langle T(Y) \rangle$ and the size of the threshold θ . As discussed in Section 5.1, Glattfelder et al. (2011) developed 12 scaling laws under the DC framework. DC scaling law 10 gives an estimation of the average period of a DC trend given a DC threshold. Following their work, we discovered a scaling law between the average period of a subsequence $\langle T(Y) \rangle$ and the size of the threshold θ .

As shown in Table 7, we selected four pairs of exchange rates over four years (from 2015 to 2018). The experiment selected 100 thresholds to calculate the $\langle T(Y) \rangle$ over the four years, ranging from 0.005% to 0.104% with the values increasing in increments of 0.001%. The raw data type is tick-by-tick. Table 7 summarizes the details of the data sources for the back-testing.

Following Equation (16) in Section 5.1, we have a new “period-threshold” scaling law between the average period of a subsequence $\langle T(Y) \rangle$ and the size of threshold θ :

$$\langle T(Y) \rangle = \left(\frac{\theta}{C_{T,\theta}} \right)^{E_{T,\theta}} \quad (17)$$

where $\langle T(Y) \rangle$ indicates the average period of the subsequence related to a certain threshold θ , and $E_{T,\theta}$ and $C_{T,\theta}$ are the parameters of the scaling law. Figure 11 illustrates the log-log chart of the $\langle T(Y) \rangle$ versus the DC threshold θ in the four pairs of exchange rates. Under logarithmic scaling, there are apparent linear relationships between $\langle T(Y) \rangle$ and θ crossing the four pairs of exchange rates. For example, the blue dot-line indicates the scaling law of GBPUSD and EURUSD.

Under the DC framework, the data-driven approach passively determines the time interval of the subsequence based on observed EPs of the two markets. On the other hand, unlike a TS, which uses a

fixed time interval, there is no explicit timeline for the termination of a subsequence. In other words, if there is no upcoming DC data, we cannot terminate the current subsequence. The ‘period-threshold’ scaling law gives a relationship between $\langle T(Y) \rangle$ and θ . This gives us a basic estimate for the average period of the subsequence given the size of the threshold. However, in practice, there is no explicit guarantee between the average period and the actual period of a subsequence. For example, for the subsequence of $RS_{GBPUSD \rightarrow EURUSD}^{\theta}$, the $\langle T(Y) \rangle$ is approximately 1,493 s (or 25 min) if the threshold is specified as 0.05%; but using the same size of the threshold, the $\langle T(Y) \rangle$ was 40 s in the 24 hr after the Brexit referendum. By changing the threshold, the ‘period-threshold’ scaling law allows the analyst control of the typical time period when the market is behaving normally. In future work, we would like to investigate the effect of the threshold on the deviation of the time period from the average values given by the scaling law in order to obtain indications as to the accuracy of the results from the ‘period-threshold’ scaling law.

5.5 | Discussion of experiments

In Section 5.1 we calculated the relative volatility using the approaches of $Dsd_{r,A-B}^{dt}$ and $\langle \sigma_{DC}^{(RS_{X\%})} \rangle_{\tau}$. The Spearman correlation test indicated high correlation for the measure of relative volatility between the two approaches. The correlation coefficients reached average values of 0.795, 0.836, and 0.864 in the periods of daily, weekly and monthly windows, respectively. This means mRV agrees moderately with the relative volatility measure from the TS methodology. In Section 5.2, the results of monthly relative volatility indicated that EURUSD was more relatively volatile than GBPUSD from 2012 to 2015. Starting from 2016, GPBUSD was exceedingly more volatile than EURUSD after the unexpected result of the Brexit referendum. Throughout the long-term back-testing, we observed that the significant mRV changes corresponded to the major historical events during that period. The second application summarizes two observations in high-frequency data. The first observation concluded that GBPUSD was far more relatively volatile than EURUSD right after the time of the Brexit vote. For the second observation, we noted a substantial number of subsequences in Part 2, which accounted for 43% of the total subsequences in 11 trading days. This observation indicates that GBPUSD and EURUSD were both more volatile in Part 2 compared with in Part 1 and Part 3. In Section 5.3, compared with the TS method Dsd, we illustrated that DC can precisely locate the exact times within which an extreme mRV occurred (Observation 3). One weakness of the DC approach is that we do not know when the current subsequence will terminate. This is a disadvantage of the data-driven approach; if there is no upcoming DC data, we cannot terminate the current subsequence. This is only a problem during times with limited amounts of DC events. In Section 5.4 we proposed the ‘period-threshold’ scaling law to estimate the average period of a subsequence $\langle T(Y) \rangle$ given a certain threshold. In practice, the deviation between the average value $\langle T(Y) \rangle$ and the actual value $T(Y)$ could be significant, especially during

Directional change relative sequence	$C_{T,\theta}$	$E_{T,\theta}$	R^2
$RS_{GBPUSD,SEURUSD}^\theta$	6.80147E-06	1.711108	0.99665483
$RS_{USDJPY,SEURUSD}^\theta$	6.92514E-06	1.739012	0.997322782
$RS_{AUDUSD,SEURUSD}^\theta$	6.72848E-06	1.68019	0.996119037
$RS_{USDCAD,SEURUSD}^\theta$	6.20971E-06	1.698149	0.996568535

TABLE 8 Parameters of the “period-threshold” scaling law

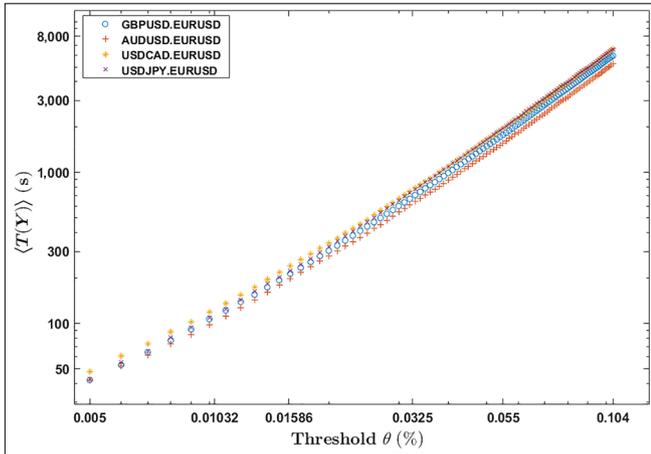


FIGURE 11 The scaling law of the average period of a subsequence related to the size of the directional change threshold. On the horizontal axis, the thresholds are chosen from 0.005% to 0.104% with an incremental step of 0.001%. On the vertical axis, the unit of $T(Y)$ is seconds. The estimated scaling law parameters are summarized in Table 8

major events. Nevertheless, this new scaling law gives observers a basic guide to inform their influence on the average period of the subsequence when they select the size of the threshold.

5.6 | Discussion: Contrasting DC and TS in measuring relative volatility

It is worth emphasizing that the relative volatility measures in DC and TS presented herein are not comparable directly. Fundamentally, this is because they sample data differently. Given the same tick-to-tick data, they record data at different times. DC measures volatility between Eps. TS measures volatility over fixed time intervals. Therefore, the period in which volatility is measured under DC does not normally coincide with a volatility measured in the same period under TS. For that reason, it is difficult to compare the volatility measures in DC and TS directly. Furthermore, we measure mRV based on the number of trends counted in the two markets within a subsequence (see formal definition in Section 4.2). The time period occupied by each subsequence is data driven. On the other hand, we measured relative volatility under TS over predetermined time intervals (see Table 1 for details). The subsequence intervals do not normally coincide with the selected time intervals under TS. Therefore, one cannot

directly compare an mRV measure under DC with a Dsd measure under TS.

DC is an event-driven method of observation with a real-time self-adjusting observation period and hence cannot be replicated in the TS setting except by crude approximation, which may or may not capture the most significant events. Any summary measure is effectively compression of a data stream into a statistic that will result in the loss of information from the data. Zhou (1992) indicated that one shortcoming of equally spaced time intervals is that information is insufficient in highly volatile time intervals and redundant at other times. DC suffers informational loss in terms of those events that do not meet the threshold and complete loss of those events that do not end within the measurement period. The informational losses of the two approaches are somewhat orthogonal to each other, indicating that they would be best considered as complementary measures rather than competing measures to be compared. To illustrate the aforementioned utility of the DC approach in relation to the TS approach for high-frequency data with the example in Table 6, we observed that the number of the observations in mRV is almost completely decoupled from the length of the three parts' periods. This is especially true in Part 2, where we observed 43% of the observations in mRV, compared with 9% of the observations in Dsd.

6 | CONCLUSIONS

DC is an alternative way of sampling the price changes to form a DC sequence based on a data-driven process. Under the DC framework, this paper opens a new path in studying the relative volatility between two markets. The DCRV approach evaluates the relative volatility based on the predetermined period T . We have shown (in Section 4) that the DCRV measure is sensitive to the size of T . In this paper, we introduce mRV, a data-driven measure of relative volatility. To develop mRV, we introduce the DC RS (Section 4.2). This is a sequence that chronologically combines two markets' DC sequences. In practice, the termination of the current subsequence depends on the identity of the upcoming EP. Hence, the period T is dynamically defined by the length of the subsequence. In Section 5.1, the correlation test proved that mRV has a similar conclusion to the TS method in measuring relative volatility. Also, the correlation test indicates a positive relationship between the correlation coefficient and the period τ , in that the longer the selected period τ is the higher the correlation coefficient obtained is. In Section 5.2, we executed back-testing in evaluating relative volatility between GBPUSD and

EURUSD. In the long historical period from 2012 to 2018, the significant changes in mRV corresponded to the major historical events. We also tested the relative volatility in high-frequency data from June 16 to June 30, 2016, such that the periods cross the five working days before and after the Brexit referendum day on June 23, 2016. Specifically, we separated the 11 trading days into three parts: (1) Part 1, from midnight on June 16 to midnight on June 23, 2016 (140 hr of total trading hours); (2) Part 2, from 10:00 p.m. on June 23 to 10:00 p.m. on June 24, 2016 (24 hr following the Brexit referendum vote); (3) Part 3, from midnight on June 27 to midnight on June 30, 2016 (96 hr). The advantage of the data-driven process is that it was possible to locate the subsequences that showed the highest and the lowest mRV. In Part 2, we observed significant changes in mRV, which indicated the extreme volatility of GBPUSD versus EURUSD. In Observation 2, by comparing the number of subsequences between the three parts, we concluded that GBPUSD and EURUSD were both more volatile in Part 2 than in Part 1 and Part 3.

In terms of potential directions for future work, factor models could provide a method of achieving both a more quantitative study of mRV and integration with existing research/frameworks. The two papers by Verdelhan (2018) and Mueller et al. (2017) analyzed, from the perspective of factor models, at least partially, the effect of shocks on the forex markets. In Verdelhan (2018), bilateral exchange rate data were analyzed in relation to the dollar factor and the carry factor as explanatory variables, and it was concluded that there must also be two sets of global shocks to explain the value of stochastic discount factors, and hence the movement of the exchange rates. The analysis indicated there must be two independent cross-sections of currency risk-premia, one of which relates to the shocks experienced by the dollar factor that are US-specific shocks and the other to world shocks priced locally. This suggests that it would be a potentially productive area of research to investigate the correlation of mRV in the appropriate exchange rates (which is related to the presence of shocks/jumps) with the unexpected US macroeconomic announcements (i.e., shocks to the US market). The carry factor depends only on global shocks priced globally. A global shock could be an event similar to the COVID pandemic, which, as with Brexit, we would expect to find to be correlated with mRV. Further research would be to look at the mRV for various bilateral exchange rates compared with these two shock terms from a similar model. Positive associations would both reinforce our thinking that mRV is a useful measure for different types of shocks and also possibly give some more insight into how our data-driven approach associates with the market structure. This could further increase its utility as an instrument for decisions regarding pricing investments. This is further highlighted by the factor models in Mueller et al. (2017) that also use global shocks priced locally, which suggests an association of exchange rate co-movement and the global and local shock terms. The real-time nature of the mRV as a measure indicates that it could be potentially very useful in the real-time analysis of events of major historical significance, such as Brexit, the COVID pandemic, and flash crashes.

To conclude, under the DC framework, we developed a new method to measure relative volatility by using mRV that can be

narrowed down to a precise time location in times of extreme values of relative volatility. This cannot be done under TS (Observation 3; for details, see Section 5.3). We believe, for instance, that mRV can give an alternative approach to monitor in near real-time the relative volatility in micro-market activities when analysts consider high-frequency data or tick data.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author.

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ENDNOTES

- ¹ Paretian stable refers to the fact that the exchange rates changes follow the stable distribution (Fama, 1963).
- ² Tickstory is a retailer of market data and their data source is from Dukascopy. <https://www.tickstory.com/>
- ³ Dukascopy Bank is a Swiss online bank that provides high-quality market data in different types. <https://www.dukascopy.com/swiss/english/home/>
- ⁴ According to the data source from Reuters Eikon.
- ⁵ For details, check https://www.ecb.europa.eu/press/pr/date/2015/html/pr150122_1.en.html
- ⁶ The voting ended at 10:00 p.m., which corresponds to index 590 (the period of the subsequence is from 9:53:58 p.m. to 10:00:06 p.m. on June 23, 2016) in Figure 7.

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APPENDIX A: The mean of monthly $\langle mRV_{(RS)}^{0.1\%} \rangle_M$

To show that the results in this paper are relatively insensitive to the choice of DC threshold, we repeated the same experiment (the first application in Section 5.2) under the threshold 0.1%. Figure A1 shows the mean of monthly $\langle mRV_{(RS)}^{0.1\%} \rangle_M$ over seven years from 2012 to 2018.

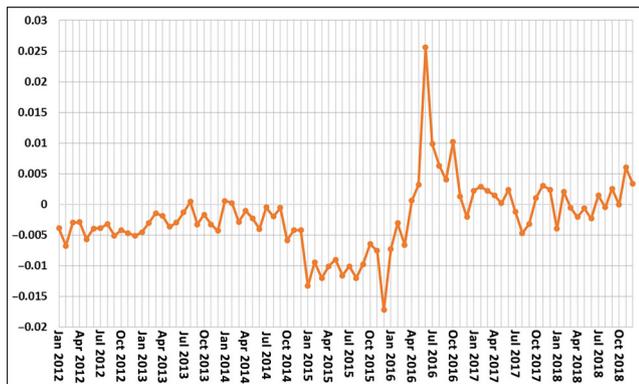


FIGURE A1 The mean of monthly $\langle mRV_{(RS)}^{0.1\%} \rangle_M$ (the y-axis) measures the monthly average micro-market relative volatility (mRV) under the threshold of 0.1%. From 2012 to 2018, there are 84 data points. The values of mRV are normalized by θ

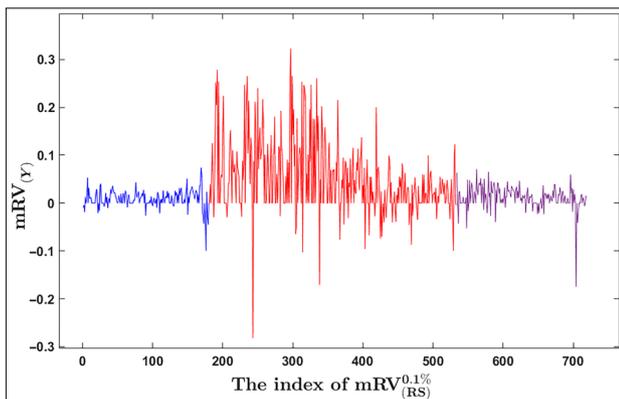


FIGURE B1 The sequence of $mRV_{(RS)}^{0.1\%}$ in the periods from June 16 to June 30, 2016. The figure plots 718 subsequences observed under the threshold of 0.1%. Part 1 (blue line): from midnight on June 16 to 10:00 p.m. on June 23, 2016 (140 hr); Part 2 (red line): from 10:00 p.m. on June 23 to 10:00 p.m. on June 24, 2016 (24 hr); Part 3 (purple line): from midnight on June 27 to midnight on June 30, 2016 (96 hr)

APPENDIX B: Evaluating mRV in the subsequences under the threshold 0.1%

Under the threshold 0.1%, there are a total of 718 subsequences observed. Figure B1 illustrates the values of mRV in the same time period (from June 16 to June 30, 2016) as shown in Figure 7. Over the periods of the three parts, we detected 180 (Part 1), 353 (Part 2), and 185 (Part 3) subsequences. Table B1 presents the mean and median of the $mRV_{(RS)}^{0.1\%}$ in the periods of the three parts.

TABLE B1 The mean and median of the $mRV_{(RS)}^{0.1\%}$. The operator Median(.) is used to calculate the median of a sequence

	Part 1	Part 2	Part 3	Part 2/ Part 1	Part 2/ Part 3
$mRV_{(RS)}^{0.1\%}$	0.009	0.051	0.011	5.979	4.475
Median($mRV_{(RS)}^{0.1\%}$)	0.008	0.032	0.011	3.959	2.818