Imperial College London

Department of AERONAUTICS

Frequency domain analysis of the mean and osculating trajectories of LAGEOS-1

Davide Amato,[1] Isaias A. Araya[1,2]

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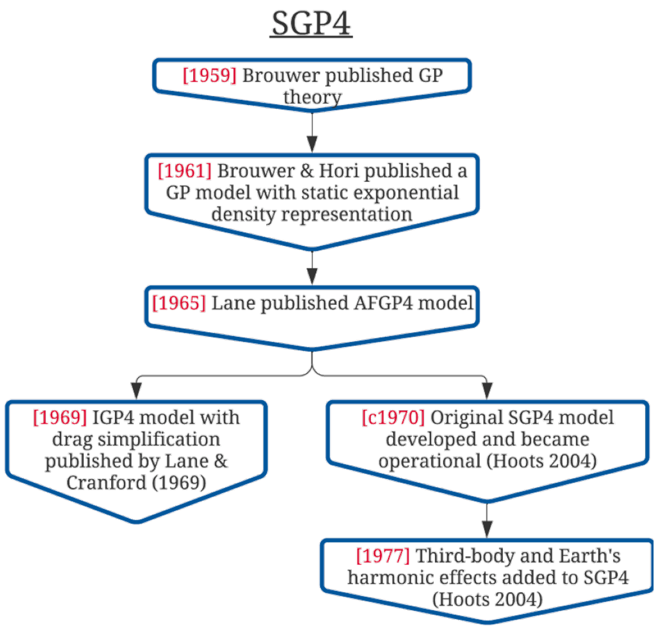
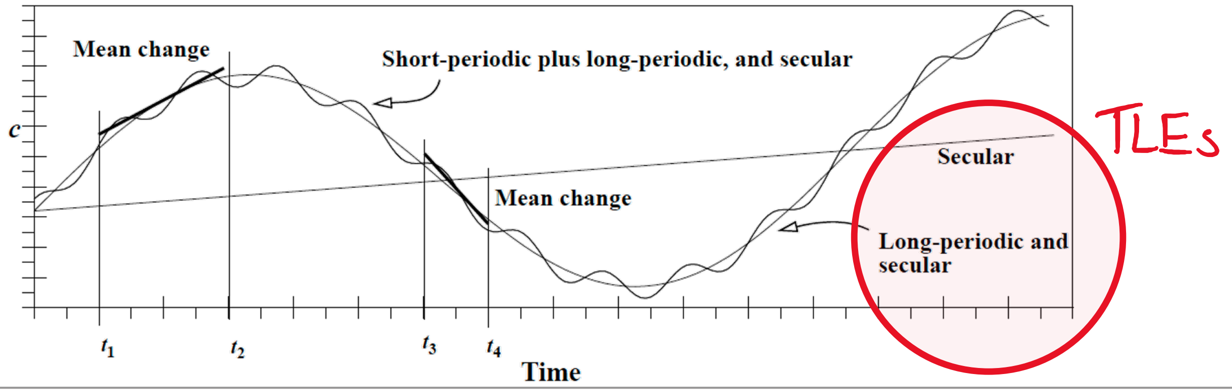
1. *Imperial College London, London, United Kingdom*
2. *Bank of America, London, United Kingdom*

Motivation

- US Space Catalogue of **Two-Line Elements (TLEs)** is still the most complete source of ephemerides for Resident Space Objects (RSOs)
- TLEs originally devised **singly-averaged elements** under the SGP4 theory
- Individual TLE accuracy limited due to lack of short-periodic terms





Credit: *Vallado et al. (2013)*

Singly-averaged elements

- **Perturbed two-body problem** in osculating elements:
  - Numerical solution timestep dictated by highest frequency, O(n)
  - No closed-form solutions

$$\frac{\mathrm{d}\hat{E}_i}{\mathrm{d}t} = n(\hat{a})\delta_{i,6} + \epsilon F_i(\hat{\boldsymbol{E}}, t)$$

- **Method of averaging:** take the average of the dynamics
  - Equivalent to *low-pass filtering*, or removing DoFs from Hamiltonian
  - Numerical solution generally faster
  - Simplification enables general perturbations

$$\langle f(\boldsymbol{E}, t)\rangle \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\boldsymbol{E}, t)\,\mathrm{d}\lambda$$

- Singly-averaged elements associated with a **theory**, i.e., definition of $f(\boldsymbol{E}, t)$
  - TLEs associated with **Simplified General Perturbations 4**

Osculating solutions from TLEs according to SGP4

- Mean-to-osculating transformations include short periodic terms $\mathcal{O}(J_2), \mathcal{O}(e^0)$ *(Hoots, 2004; Spacetrack Report #3)*:

$$u = \tan^{-1}\left(\frac{\sin u}{\cos u}\right), \qquad \Delta r = \frac{k_2}{2p_L}(1-\cos^2 i)\cos 2u$$

$$\Delta u = -\frac{k_2}{4p_L^2}(7\cos^2 i - 1)\sin 2u, \qquad \Delta\Omega = \frac{3k_2\cos i}{2p_L^2}\sin 2u$$

$$\Delta i = \frac{3k_2\cos i}{2p_L^2}\sin i\cos 2u, \qquad \Delta\dot{r} = -\frac{k_2 n}{p_L}(1-\cos^2 i)\sin 2u$$

$$\Delta r\dot{f} = \frac{k_2 n}{p_L}\left[(1-\cos^2 i)\cos 2u - \frac{3}{2}(1-3\cos^2 i)\right]$$

The short-period periodics are added to give the osculating quantities,

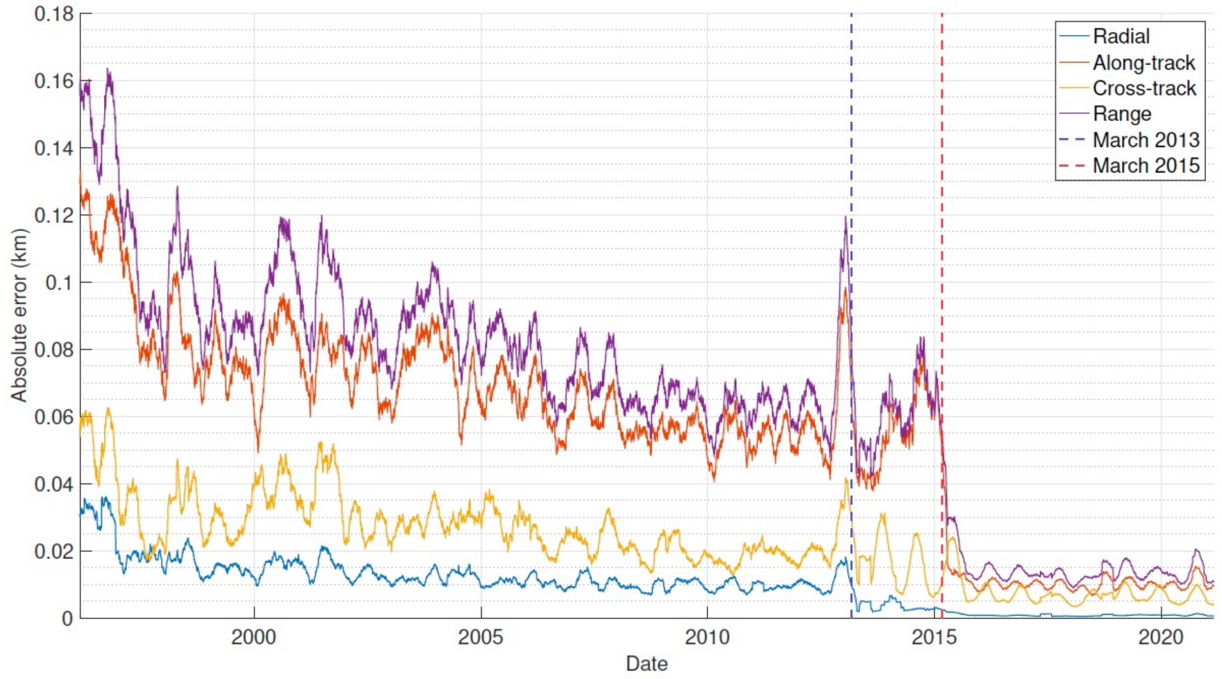
$$r_k = r\left[1 - \tfrac{3}{2}k_2\left(\sqrt{1-e^2}/p_L^2\right)(3\cos^2 i - 1)\right] + \Delta r$$

$$u_k = u + \Delta u, \qquad \Omega_k = \Omega + \Delta\Omega, \qquad i_k = i + \Delta i$$

$$\dot{r}_k = \dot{r} + \Delta\dot{r}, \qquad r\dot{f}_k = r\dot{f} + \Delta r\dot{f}$$

US Space Catalogue state-of-art

- *Currently* (2021), TLEs are obtained as a **numerical fit** on underlying Special Perturbations solutions *(Cappellucci 2006, Hejduk et al. 2013)*



Can we improve **individual** TLE accuracy by recovering **additional short-periodic** terms? J_2^2, third bodies…

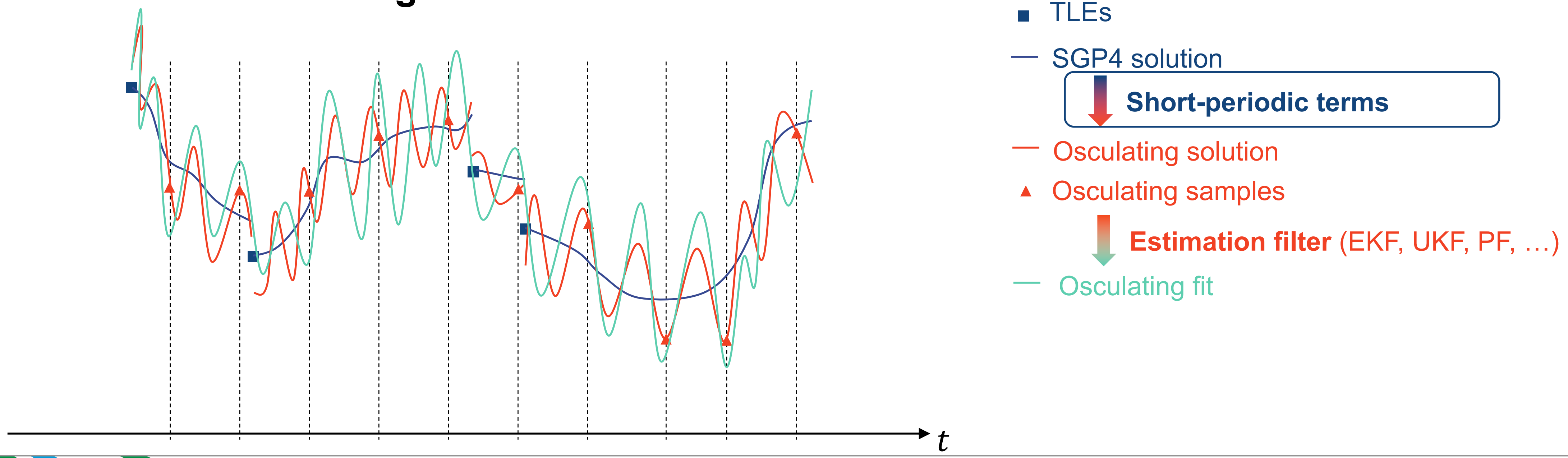
Use **Fourier transform** to assess TLE frequency content

Fourier transform

- Understand **energy distribution** as a function of frequency
- **Discrete Fourier transform:** $E(\omega_k) = \sum_{j=0}^{N-1} e(t_j) \exp(-i\, \omega_k j), \qquad \omega_k = \frac{2\pi}{N} k\,, \qquad k = 0, \dots, N-1$
  - $E(\omega_k) \in \mathbb{C}$
- **Amplitude spectrum** (one-sided): $A(\omega_k) = \sqrt{2}|E(\omega_k)|/\mathrm{N}$
- Classical DFT defined for **uniform sampling interval** $\Delta t,\ t_j = t_0 + j\Delta t$
  - **Nyquist critical frequency** $\omega_c = \frac{\pi}{\Delta t}$ limits accuracy of frequency reconstruction

Orbit determination from TLEs

- TLEs are 1) **not uniformly spaced** 2) **not updated frequently enough** to ascertain short-periodic terms directly from TLE series (Nyquist limit)
- TLEs are often used as pseudo-observations in an OD process
  - **Piecewise osculating solution**



LAGEOS-1 osculating trajectory

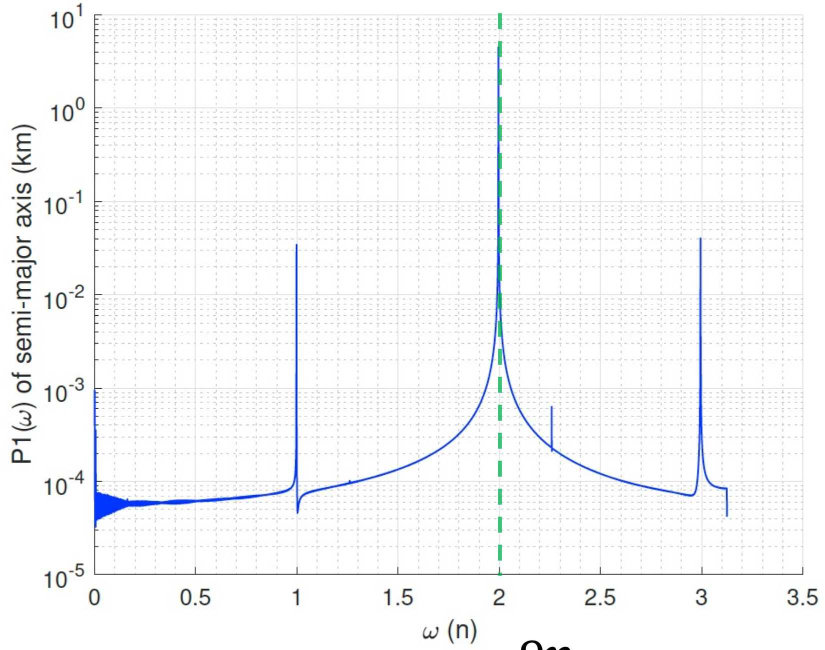
- **LAGEOS-1 orbit characteristics:**
  - Stable MEO, $a = 12270$ km. No drag, **constant mean period** of 225 minutes
  - Very low $\frac{A}{m} \sim 10^{-3}\ \mathrm{m^2/kg}$ implies small SRP perturbations
  - Precise Orbit Ephemerides easily available
- Piecewise osculating solution generated from TLEs for epochs from 24-Jul-2015 to 24-Feb-2021
- Sampling interval: 36 minutes, $f_c = 1/2\Delta \approx 3n$
  - f_c corresponding to highest frequency of *Brouwer (1959)* short-periodic terms

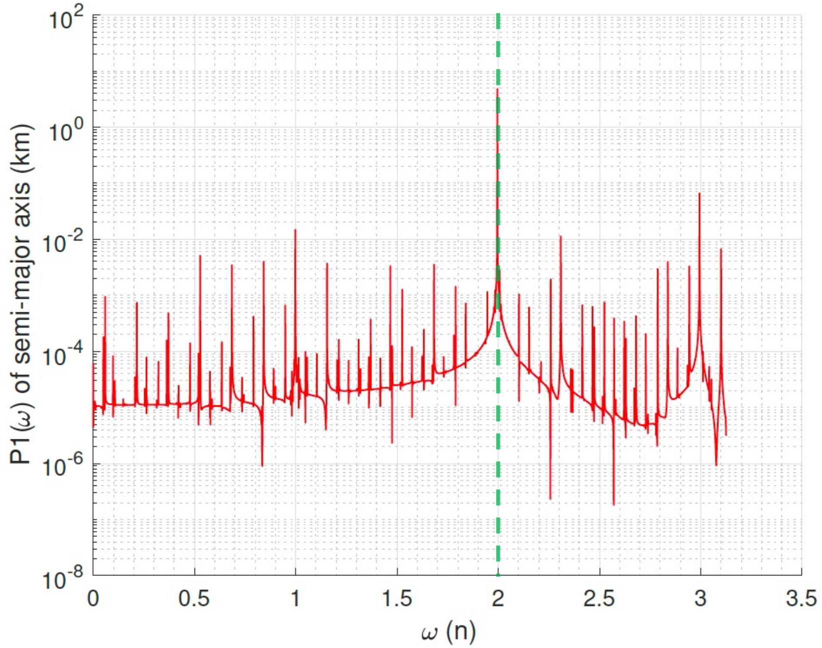
Credit: *NASA*

LAGEOS-1 spectral analysis setup

- Only short-periodic terms of frequency $2n$ from Brouwer (1959) are **retained in SGP4**
- If TLEs still **consistent with SGP4**, we should see only the retained terms in the amplitude spectrum
- Comparison with THALASSA orbit propagator (Amato et al, 2019)
  - Physical model includes 14x14 geopotential
- DFT window is entire 5.5-year timespan
- DFT performed for each element, with mean removed
  - Will only show a, e, i as secular trend in Ω, ω makes analysis slightly more involved

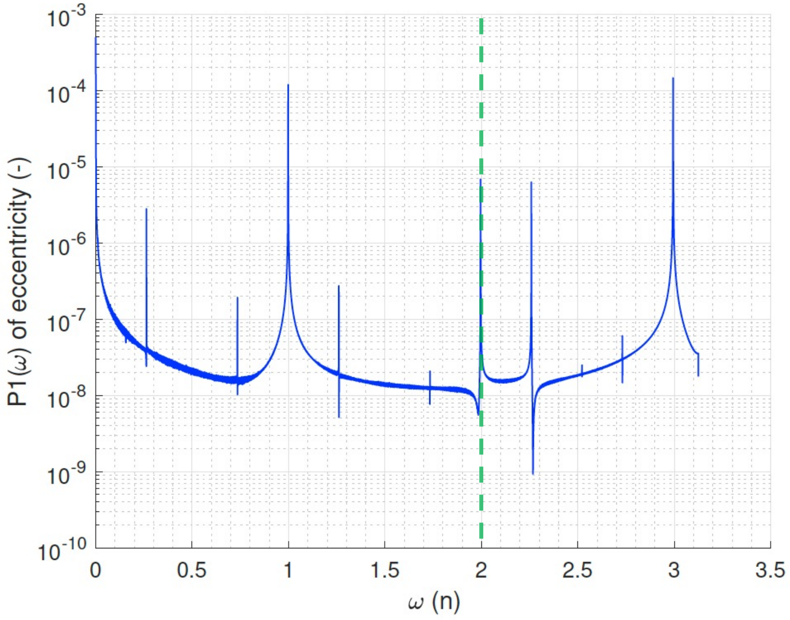
LAGEOS-1 semi-major axis spectrum

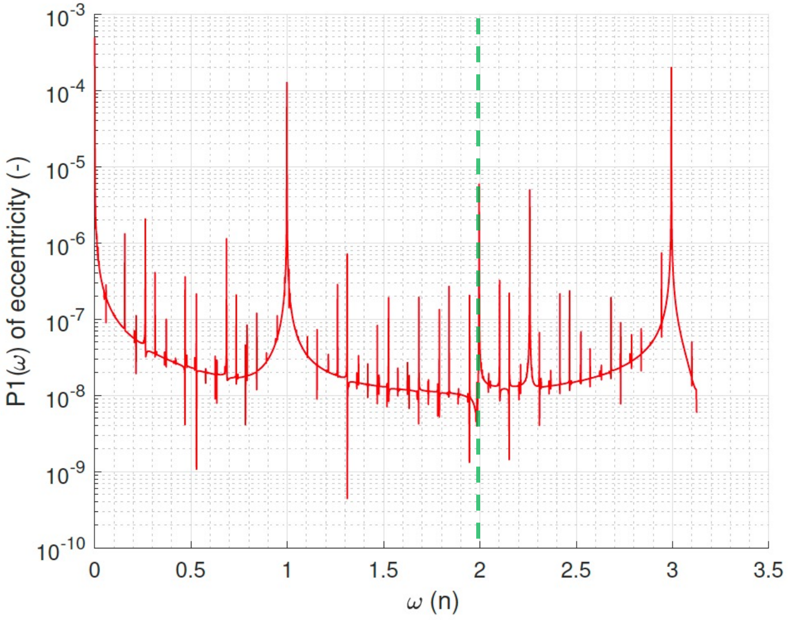




- **Additional harmonics at $n, \frac{9n}{4}, 3n$ present in TLE solution**
- Complex frequency response in THALASSA solution likely due to higher-order harmonics

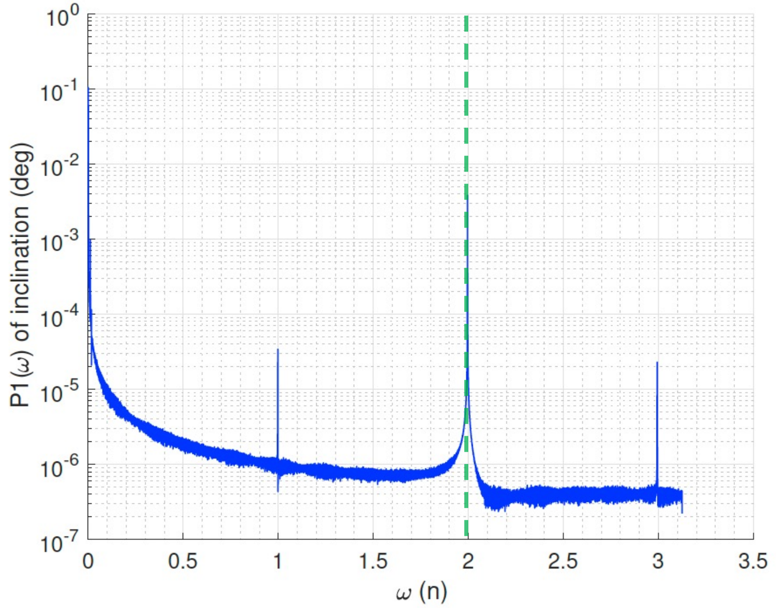
LAGEOS-1 eccentricity spectrum

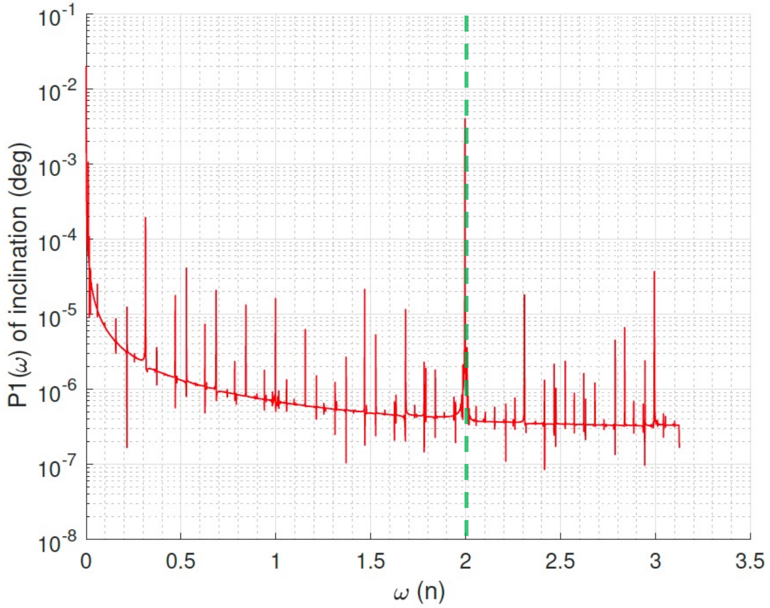




- **Additional harmonics at** $n, 2n, \frac{9n}{4}, 3n$ **present in TLE solution**

LAGEOS-1 inclination spectrum



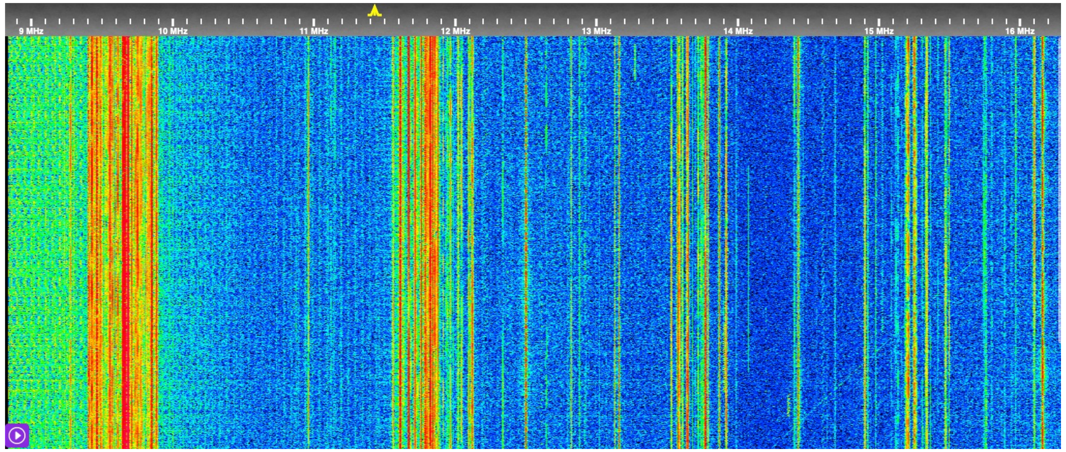


- **Additional harmonics at $n, 2n, 3n$ present in TLE solution**

Conclusion

- Analysis of mean and osculating TLE solutions by examining **Fourier amplitude spectrum**
- **Non-SGP4 short-periodic terms present in osculating trajectories recovered from TLEs**
  - Higher order terms in J_2, e?
- Short-periodic terms likely introduced through:
  - “Numerical extrapolation” of TLEs to match underlying special perturbations vectors performed by 18th SpCS
  - Observational updates → jumps in the osculating solution
- Additional short-periodic terms **should not be added to TLEs**
  - SGP4-XP ephemeris data (introduced from 2021) might include more short-periodic terms

Future work



- Time-frequency analysis methods for objects in dissipative regimes
  - Short-period Fourier transforms (for spectrograms)
  - Wavelet transforms
- Spectral integration methods
- “Synthetic proper elements” for RSOs
  - Talk by Vartolomei et al. (ESR4) this afternoon