A Policy for Managing Operational Assets to Minimize Deprivation Costs

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Every year, humanitarian organizations assign a sizable slice of their limited financial resources to procure, operate and maintain operational assets, without which service delivery would be impossible. In this paper, using vehicles as representative of operational assets, we identify policies for sizing and allocating operational capacity to minimize deprivation costs in a humanitarian development context. In the first stage, we develop a stochastic dynamic programming model and show that it is too complex to be solved for medium- and large-size problems. Then, we develop an efficient heuristic policy that considers the interaction of asset purchasing and operating decisions when budget is uncertain. Based on a data set provided by a large international organization, we estimate the parameters of our model to run numerical experiments. Results demonstrate the following: (i) although budget uncertainty increases the deprivation costs and decreases capacity utilization, but its negative impact is mitigated if budget saving between periods is allowed; (ii) a policy to minimize deprivation costs over time may avoid using all available assets in all the periods; (iii) in situations where the differences of criticality of missions are large, both deprivation costs and fleet utilization decrease; and (iv) in most situations, a centralized asset procurement model outperforms a decentralized model in minimizing deprivation costs.

Keywords: asset procurement; fleet management; humanitarian development programs; stochastic dynamic programming.

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1. Introduction

The World Health Organization (WHO) reports that in South Sudan, a country where 91% of citizens have no access to proper sanitation, essential medical and surgical equipment is outdated. WHO also reports that the lack of service-delivery capacity (i.e., inadequate fleet size) in Ethiopia is the main cause of the low utilization rate of health services, which is only 0.32 per capita.[1] Although humanitarian organizations (HOs) need to maintain an adequate level of relief items (such as food, blankets, medicines and vaccines) to satisfy their beneficiaries' primary needs, they also require sufficient operational assets (such as vehicles, generators and medical facilities) to deliver services. Operational assets are the supporting equipment used to deliver aid to regions of urgent need. Typically, they are durable items that supply services over multiple periods, are often expensive to acquire and require costly, regular maintenance. Yet their shortage poses critical challenges for effective service delivery in poor countries (McCoy and Lee 2014). Therefore, HOs assign a sizable slice of their limited financial resources to procure, operate and maintain operational assets. Due to the amount of spending relative to other expenses and due to its essential role in service delivery, asset management is arguably the most significant determinant of how effectively the HO delivers on its mandate. Nevertheless, the idiosyncratic characteristics of how humanitarian development programs make decisions to acquire and use operational assets present a challenge. In this paper, we propose policies about the purchase and allocation of operational assets. We also analyze whether and to what extent external factors affect the decisions of the proposed polices as well as the overall deprivation costs.

To develop our model, we focus on *vehicles* as representative of operational assets that also is the centerpiece of humanitarian service delivery (Eftekhar and Van Wassenhove 2016). For example, in countries like Afghanistan, where 40% of population lives more than one-hour of travel from a nearby health center,[2] the availability of vehicles enhances the quality of service delivery -in both frequency and consistency of visits- to outlying communities (McCoy and Lee 2014). Also, vehicle procurement has characteristics similar to the acquisition of many other types of assets, like power generators and water-purification systems (Besiou et al. 2014), which makes this practice a suitable choice for our study. Finally, fleet management is the second-largest expense to HOs (Pedraza Martinez and Van Wassenhove 2013) and is often cited as a major challenge for

[1] World Health Organization. Retrieved February 25, 2015. `http://www.who.int/countries/en/`

[2] World Health Organization. Retrieved July 17, 2016. `http://www.who.int/countries/en/`

humanitarian operations (de la Torre et al. 2012); a well-managed fleet saves millions of dollars for HOs every year (Eftekhar and Van Wassenhove 2016).

By and large, asset procurement in humanitarian settings is subject to many limitations, such as unpredictability in funding (Natarajan and Swaminathan 2014), earmarked supports (Besiou et al. 2014), and delays between receiving the pledged cash during the donation process. Although facing budget limitation, HOs' essential goal is to minimize *deprivation costs*, not necessarily the operating expenditures (Holguin-Veras et al. 2013). In addition to these multiple [conflicting] objectives, inherent challenges are attached to humanitarian operations, which further complicate fleet management (i.e., purchasing and operating vehicles). First, security problems, lack of reliable roads, and poor infrastructure depreciate vehicles less predictably than under normal conditions, and increase the chance of accidents (Eftekhar and Van Wassenhove 2016). Therefore, instead of following a preset policy based on age or mileage to replace used vehicles, delegations might get rid of used vehicles much later or earlier than planned. Second, due to the lack of budget for maintenance and fuel, the available vehicles may not always be usable (McCoy and Lee 2014). Third, a vehicle is a multiple-use asset assigned to missions with different levels of criticality (e.g., staff transportation and material distribution). Considering all these aspects, this paper proposes a heuristic model that determines how many vehicles to buy, and how many to operate in each period to minimize deprivation costs over multiple periods.

To derive general insights from the characteristics of our proposed policies under various operational conditions, we formulate the problem as an infinite-horizon Markov Decision Process (MDP). Inspired by Holguin-Verasa et al. (2013) and Vanajakumari et al. (2016), we set minimizing deprivation costs as the objective function. In addition to the most common cost functions, such as vehicle acquisition costs, operating costs, fixed costs and residual value (Vemuganti et al. 1989), we take three essential factors into account: (i) the probability of individual vehicle dismissal, (ii) the budget uncertainty and (iii) the variation of missions' criticality. Since driving the optimal policy is analytically and numerically complex, we develop a heuristic policy. To test the performance of our heuristic and to derive additional managerial insights, we run numerical experiments. To avoid using synthetic data as the input for our experiments, we use field data to estimate the model parameters empirically. Applying Bayesian analysis method, we obtain point estimation for variables based on a data set containing information on 1,074 Toyota Land Cruisers that the International Committee of the Red Cross (ICRC) had owned from 2000 to 2015 in five countries (Iraq, Kenya, Liberia, Sudan and Syria).

Our analysis show that an increase in budget uncertainty increases the deprivation costs and decreases utilization of fleet capacity. However, we find that the opportunity to budget savings between periods neutralizes the negative effect of budget uncertainty on deprivation costs. Also, the deprivation costs rise in harsh environments where the probability of vehicle dismissal is high. Nevertheless, an optimal policy that minimizes deprivation costs over time avoids operating *all* vehicles in *all* periods. This is mainly due to the uncertainty in the budget and vehicle replacements that suggests adopting *frugality* to a forward-looking HO. This pattern is different from commercial, profit-oriented fleet management in which there is no limited financial budget, implying that developing fleet management solutions specific to humanitarian sector is necessary. Furthermore, results demonstrate that when the most critical missions are much more important than the least critical ones, the deprivation costs are less than when the differences of criticality of missions are small. In other words, the average deprivation costs surges if there is less variation in the criticality of the missions. Likewise, we observe that in situations where the differences in mission criticality decrease, fleet utilization grows. Finally, our numerical results indicate that a centralized vehicle procurement model (with a longer procurement lead time but a cheaper purchase price) outperforms a decentralized model (with a shorter lead time but a more expensive purchase price) in minimizing deprivation costs over time.

Literature review- Given their importance, asset procurement and utilization in the commercial sector have been the subject of significant research (Rust 1985). Because we chose the vehicle as a representative of an operational asset, we focused on references that concentrate on fleet management. In particular, the OM/MS community paid substantial attention to fleet management -due to its pivotal role in order fulfillment and its cost magnitude- to find an optimal level of transportation capacity under different circumstances. For example, Turnquist and Jordan (1986) report that fleet sizing is a *several billion dollar* decision for asset-intensive companies like General Motors. Attention has been dedicated to vehicle replacement policies (Brosh et al. 1975), fleet sizing with deterministic demand but stochastic travel time (Turnquist and Jordan 1986), fleet sizing with stochastic demand (Papier and Thonemann 2008; Slaugh et al. 2016) and optimal fleet capacity allocation policies (Savin et al. 2005; Papier and Thonemann 2010). This body of literature studies a variety of novel problems. Yet its results are not tailored to a humanitarian setting.

In a humanitarian context, fleet management has received ample attention in relief operations. Scholars mostly consider aid distribution (Yi and Ozdamar 2007), vehicle routing (Campbell et al. 2008; Vanajakumari et al. 2016) and scheduling for victim evacuation (Barbarosoglu et al. 2002). Typical objectives of these studies are to minimize service delivery time or to maximize demand coverage. Compared to relief operations, fleet management in development programs is narrower in number of studies but wider in topics; it studies the vehicle supply chain (Besiou et al. 2014), the replacement policy and vehicle reliability (Pedraza Martinez and Van Wassenhove 2013; McCoy and Lee 2014), field-vehicle utilization (Eftekhar and Van Wassenhove 2016) and the fleet-sizing question (Eftekhar et al. 2014). The closest paper to the present study is that of Eftekhar et al. (2014) that develop a stylized quadratic control model to determine optimal fleet sizing over time. The objective of their study is to minimize the total operational costs (i.e., purchasing price, operating costs and vehicle-price decline) over a finite horizon, where demand is deterministic but varies frequently. In addition to the vehicle replacement policy, their model take a deterministic earmarked budget (lower-bound) and deterministic budget limitation (upper-bound) into account. Their model generate simple but interesting insights on fleet sizing in this context. Nevertheless, it does not take any budget uncertainty, inherent to a humanitarian setting, into consideration.

In this paper, we take one step forward and develop this setting so it reaches to a more sophisticated level. In particular, our model takes into account budget uncertainty, random vehicle dismissal and differences in mission criticality. Also, the objective function of our model is to minimize human suffering due to a deficit in fleet capacity. Furthermore, our model determines the right quantities of vehicles to acquire and to operate. To the best of our knowledge, such a setting has not been studied before.

The contribution of this paper to the existing literature has three components. First, it extends the existing literature by conceding limitations and characteristics specific to humanitarian operations, such as demand variation, differences in mission criticality, budget uncertainty and random vehicle dismissal. Second, it considers the joint decisions of purchasing and operating vehicles, and explicitly analyzes the interaction of the two decisions. We show that these decisions are highly interdependent when missions have different levels of criticality and the budget is uncertain. Finally, we construct a model that retains the dynamics enforced by all these limitations and determine a heuristic policy that achieves close-to-minimal deprivation costs over time.

2. Model Description

In this section, we describe a model for vehicle fleet sizing and allocation at a national (or field) delegation level with the objective of minimizing deprivation costs. Therefore, we consider a case where, at the beginning of each period, the delegation makes two critical decisions: (i) how many vehicles should be acquired and (ii) how many vehicles should be operated. These decisions are made once the delegation knows the total budget available and the existing fleet size, given the demand for humanitarian missions in the period. Next, we describe each component of our model. A table of symbols is available in Appendix A.

2.1. Asset demand and deprivation cost

Similar to Eftekhar et al. (2014), we assume that delegations periodically predict the demand for transportation services assigned to different missions and provide estimates of the demand for capacity (e.g., the number of required vehicles), which we denote by D_t. Demand in humanitarian development programs is usually predictable (Pedraza Martinez and Van Wassenhove 2013), though it may vary over time (Eftekhar et al. 2014). Therefore, inspired by Eftekhar et al. (2014), we consider a non-monotonic sinusoidal demand pattern

$$D_t = \left[D_m + d \sin\left(\frac{2\pi}{n}(t+\omega_0) \right) \right], \tag{1}$$

in which D_m is the average demand during a cycle, d is the demand oscillation magnitude, n is the number of seasons per cycle, ω_0 represents the initial phase, and [.] denotes the closest integer to (.). Our model and the obtained solution can easily be adapted to any type of seasonal demand pattern, though we use the assumption of sinusoidal demand for illustration.

If the delegation provides larger capacity (i.e., more vehicles) than D_t, it causes additional operating and fixed costs, which is not desirable for the HO facing budget limitations. However, due to the nature of humanitarian missions, it is reasonable to assume that failing to fulfill a demand results in a deprivation cost. Deprivation cost is *"valued as changes in human well-being"* and measures beneficiaries' suffering from not receiving humanitarian aid (Holguin-Veras et al. 2013).

While HOs' ultimate goal is to alleviate the deprivation costs by maximizing demand coverage, all missions are not equally critical; the impact of some missions to alleviate human suffering might be stronger than others. This is especially true for HOs like the ICRC that run development programs in conflict zones as well. Thus, given the resource limitations, the delegation has to prioritize the operations and assign vehicles to the most critical missions. Holguin-Veras et al. (2013) explain that deprivation cost should be *"monotonic, non-linear, and convex with respect to the deprivation time"*, and their results suggest an exponential function to capture it. An advantage of the exponential function is that it allows valuing the relative importance of each mission (or category of missions) with respect to the other missions. Inspired by this approach, we define the per-period cost function

$$R_t(a_t) = e^{b_t[D_t - a_t]^+} - 1, \tag{2}$$

where R_t is the deprivation cost in period t if the delegation decides to assign a_t vehicles ($a_t \geq 0$) to serve a proportion of demand (i.e., to operate a_t vehicles in period t). Parameter b_t is the deprivation cost function's convexity factor that specifies the relative importance of missions. To find b_t rigorously, we borrow a Pareto analysis developed by the Unite States Coast Guard (USCG 2003) that prioritizes missions in health-related risk management. In this analysis, fraction ν of the total deprivation costs is addressed by fulfilling the most critical $(1-\nu)$ fraction of all potential missions in a period. Consequently, b_t should satisfy the following equation:

$$e^{b_tD_t} - e^{\nu b_tD_t} = \nu\left(e^{b_tD_t} - 1\right), \qquad \nu > 0.5. \tag{3}$$

We assume that, regardless of the variations in D_t from one period to another, the proportion of missions belonging to each category of importance did not change. In other words, for a given ν, the total deprivation costs are the same in any period, that is

$$e^{b_tD_t} - 1 = e^{b_{t'}D_{t'}} - 1, \qquad \forall t, t'. \tag{4}$$

2.2. Purchasing and operating costs

At the beginning of period t, the delegation decides how many new vehicles to buy, denoted by u_t, to complement the existing asset capacity, denoted by x_t. Each purchased vehicle costs p, and vehicles bought during period t become effectively available in period $t+1$ (i.e., there is a procurement lead time of one period). Asset utilization imposes operating costs. For instance, assigning a vehicle to missions in a period incurs costs such as maintenance, fuel and driver's salary that are called operating costs and are captured by $c_o > 0$ in our model. However, even maintaining an idle vehicle in the field imposes fixed costs such as a refreshing cost, workshop and office expenses, and a monthly insurance fee, which we denote by $c_f \geq 0$.

2.3. Financial resources

The financial resources to manage and operate the fleet that are available at the beginning of the period is denoted by SB_t and consists of three components. First, at the beginning of each period, the delegation receives a budget, K_t, that is a random variable, which is independent and identically distributed among periods. We assume that it is sufficient to at least cover the fixed costs, i.e., $K_t \geq c_f x^{max}$. Second, the delegation has some (non-earmarked) budget saved from the previous periods that is available at the beginning of period t, S_{t-1}. The savings in each period depends on the initial budget, SB_t, and the amount spent during that period,

$$S_t(a_t, u_t) = SB_t - c_f x_t - c_o a_t - p u_t. \tag{5}$$

Third, the delegation earns revenue through selling used vehicles. Due to the environmental conditions in which HOs operate, vehicle replacement may not follow a preset policy (Eftekhar and

Van Wassenhove 2016). The literature indicates that many vehicles are replaced following unpredictable events such as accidents. To model this situation, we assume that at the end of each period, a random number of vehicles are dismissed, and an average residual value, r $(0 \leq r < p)$, for each sold vehicle is obtained. For the sake of simplicity, we use a fixed probability γ that a vehicle is dismissed in a given period. This implies that the available fleet size at the beginning of a period, $L(x_t)$, follow a binomial distribution with population x_t and success probability $1-\gamma$. We tested this assumption with our data set (see Section 4). A Kernel density estimation graphically supports our assumption that the fleet size follows a binomial distribution, and a chi-square goodness-of-fit test could not reject it.

For notational simplicity, we define $c_L = c_f + c_o + \gamma(p-r)$ as the average per-period cost of a single vehicle at full utilization.

2.4. Dynamic programming model

We write the state equation for the fleet size in period $t+1$ as

$$x_{t+1} = L(x_t) + u_t, \tag{6}$$

and the state equation for the available budget in period $t+1$ as

$$SB_{t+1} = S_t + r(x_t - L(x_t)) + K_{t+1}. \tag{7}$$

For the purpose of numerical analysis, we limit the fleet size to a maximum of x^{max} and the budget to SB^{max}, which seem reasonable, given the typical budget limitations in the humanitarian sector. This leads us to the following model:

$$J_0(x_0, SB_0) = \min \mathbf{E}\left(\lim_{N\to\infty} \frac{1}{N}\sum_{t=0}^{N} R_t(a_t)\right), \tag{8}$$

subject to

$$a_t \leq x_t, \tag{9}$$

$$S_t = SB_t - c_f x_t - c_o a_t - p u_t, \tag{10}$$

$$SB_{t+1} = \min\{S_t + r(x_t - L(x_t)) + K_{t+1}, SB^{max}\}, \tag{11}$$

$$x_{t+1} = \min\{L(x_t) + u_t, x^{max}\} \quad \text{and} \tag{12}$$

$$S_t \geq 0, \ \text{and} \ a_t, u_t, x_t \in Z^+. \tag{13}$$

The objective function minimizes the average deprivation costs over an infinite horizon by choosing the optimal values for purchasing, u_t, and operating vehicles, a_t, in each period. The first constraint implies that the number of vehicles to operate does not exceed the available number of vehicles. Constraints (10)-(12) represent the state equations introduced above. Constraint (13) ensures the non-negativity of the state and decision variables. We consider an infinite horizon, because humanitarian development programs are typically long-term programs without a prior known ending.

3. Heuristic Development

Our optimization model contains three state variables (t, x_t and SB_t), two decision variables (u_t and a_t) and two stochastic elements ($L(x_t)$ and K_t), all of which render it too complex to be solved for problems of realistic size. Furthermore, the constraints do not allow the optimal policy to have second-order properties, such as convexity, which implies that the optimal policy does not have the typical monotone structure[3]. Consequently, due to the limitations in both analytical and numerical methods, we propose a heuristic approach, the "Simultaneous Allocation Optimization (SAO)" heuristic, which generates *close-to-optimal* decisions in a reasonable time.

3.1. Benchmark policy

In this subsection, we describe a policy of commercial fleet management that we use as a benchmark for our solution. A key aspect of commercial fleet management (e.g., Papier and Thonemann 2008) is that serving demand directly leads to cash inflow that can be used to pay for the operating and fixed costs. Accordingly in this setting, it is optimal to serve as many demands as possible; It

[3] For instance, decision variable a_t is not necessarily monotone in any of the state variables t, x_t, or SB_t. Likewise, u_t is not necessarily monotone in t.

would make sense to operate all the capacity in the current period, as long as the revenues of the marginal mission exceed the operating costs. This yields a myopic policy

$$a_t^b(x_t, SB_t) = \left\lfloor \min\left\{x_t, \frac{SB_t - c_f x_t}{c_o}, D_t\right\}\right\rfloor, \tag{14}$$

where $a_t^b(x_t, SB_t)$ denotes the number of vehicles operated, which is constrained only by the fleet size, the available budget to cover the operating costs, and the demand.

Furthermore, as long as the average lifetime revenues from an asset item (e.g., a vehicle) exceeds its purchase and operating cost, it is reasonable to fulfill the demand completely. This implies an *order-up-to* purchasing policy to fill the gap between demand and capacity (i.e., the fleet size) in the next period. This leads to a purchasing decision of

$$u_t^b(x_t, SB_t) = \min\left\{\left\lfloor \frac{SB_t - c_f x_t - c_o a_t^b(x_t, SB_t)}{p}\right\rfloor, (D_{t+1} - \lfloor x_t(1-\gamma)\rfloor)^+\right\}, \tag{15}$$

where $u_t^b(x_t, SB_t)$ denotes the number of vehicles to be purchased in period t.

The aforementioned policy seems to correspond to the current practice not only in the commercial sector but also in a humanitarian setting. Based on interviews with practitioners[4], we learned that conventional wisdom in the humanitarian sector is to completely fulfill demand (100% service level) whenever sufficient capacity is available. Furthermore, the policy described previously also coincides with a special case of Eftekhar et al. (2014). Consequently, we use the policy described in equations (14) and (15) as a benchmark for our heuristic policy.

3.2. Development of the SAO heuristic

The SAO heuristic allocates the available budget among the decisions to purchase new vehicles, to operate a fraction or all of the available vehicles, and to save part of the available budget for future operations. The SAO heuristic estimates the marginal reduction in deprivation costs of every possible amount of operating, purchasing and saving. Then, it applies a portfolio optimization to allocate the available budget to these three allocation options such that the estimated social gain is maximized. First, we describe how we estimate the marginal value functions.

[4] Throughout this project, we discussed with executives of ICRC, Mercy Corps, American Red Cross, Catholic Relief Services, and several freelance consultants specializing in the humanitarian sector.

From equation (2), the social gain from *operating* i vehicles in period t is given by

$$V_t^a(i) = e^{b_t D_t} - e^{b_t [D_t - i]^+}. \tag{16}$$

Due to the uncertainty in budget and in the number of available vehicles in future periods, as well as the limitations on the number of operating vehicles, it is difficult to calculate the marginal gain from *purchasing* one additional vehicle. Therefore, we consider the fleet's aggregated reliability during a vehicle's average operational life and use an approximation. To do so, we assume that the fleet size is affected only by u_t in period $t+1$; for periods after $t+1$, the fleet size is adjusted by further purchasing decisions $(u_{t+1}, u_{t+2}, \dots)$.

Also, due to the budget limitations, it is reasonable to assume that the HO cannot afford owning and operating a capacity (fleet of vehicles) larger than a certain threshold (i.e., an upper-bound). To determine this threshold, we use a simplified version of the optimization model (8) in which all variables are continuous and all stochastic elements are replaced by their expected values (i.e., $K_{t+1} = \mu$, and $L(x_t) = (1-\gamma)x_t$). We denote the optimal number of vehicles to operate in this simplified problem by $\bar{a}_t$. For details of the deterministic formulation and for a closed-form solution of the deterministic model when demand is constant $(d = 0)$, we refer the reader to Appendix B.

With this information, we can estimate the expected social gain from purchasing additional vehicles in period t, V_t^u, by considering the expected social gain in the next period, assuming that the decision-maker will not operate more than $\bar{a}_t$ vehicles. Therefore, we approximate the social gain of purchasing i vehicles, given the current fleet size x_t, by

$$\begin{aligned} V_t^u(i, x_t) &= \mathbf{E}_{L(x_t)}[V_t^a(\min(L(x_t)+i, \bar{a}_{t+1})) - V_t^a(\min(L(x_t), \bar{a}_{t+1}))] \\ &= \sum_{l=0}^{x_t} P(L(x_t) = l) \left(e^{b_{t+1}[D_{t+1} - \min(l, \bar{a}_{t+1})]} - e^{b_{t+1}[D_{t+1} - \min(l+i, \bar{a}_{t+1})]} \right). \end{aligned} \tag{17}$$

Furthermore, we need to estimate the expected social gain from a *saving* budget for future periods. Estimating the impact of saving on future social gain is complex since the saved budget can be allocated either to operating or to purchasing at any time period in the future. We fit a function to estimate the value of saving, $V^s(s|\theta)$, that increases in the amount of saving. As the deprivation cost function consists of exponential functions, we define $V^s(s|\theta)$ similarly, and propose

$$V^s(s|\theta) = \begin{cases} e^{b'\theta\hat{S}} - e^{b'(\theta\hat{S}-s)} & \text{if } s < \hat{S}, \\ e^{b_m D_m} & \text{otherwise,} \end{cases} \tag{18}$$

where b_m is the convexity factor associated with D_m, $\hat{S} = SB^{max} - r\gamma\mu/c_L - \mu$ is an upper threshold on savings, μ/c_L is an estimation of the fleet size, b' is a coefficient of function V^s which can be determined from the model parameters, and θ is the heuristic's parameter that determines the marginal social gain of saving. The procedure to approximate $V^s(s|\theta)$ and to determine b' is explained in detail in Appendix C.

After estimating V_t^a, V_t^u, and V^s, we have to solve the following allocation model to determine the decisions of the SAO heuristic:

$$\max_{u_t^h, a_t^h} \{V_t^a(a_t^h) + V_t^u(u_t^h, x_t) + V^s\left(SB_t - c_f x_t - c_o a_t^h - p u_t^h|\theta\right)\}, \tag{19}$$

subject to

$$c_f x_t + c_o a_t^h + p u_t^h \leq SB_t, \tag{20}$$

$$a_t^h \leq x_t \quad \text{and} \tag{21}$$

$$a_t^h, u_t^h \in \mathbb{Z}^+. \tag{22}$$

Figure 1 demonstrates the behavior of $V^s(s|\theta)$ under different values of the heuristic parameter θ. Larger values for θ put higher marginal value on savings and lead to a forward-looking behavior, whereas smaller values of θ favor more myopic strategies (i.e., undervaluing future risks). Therefore, as a rule of thumb, larger values of θ that tend to save more of the budget are suitable for situations with high uncertainties and larger variations of missions criticality (i.e., high values for b_t). To find the best parameter value, θ^*, that yields the lowest deprivation costs, we use simulation and the *Quadratic Interpolation method* (Bertsekas 1999). Based on θ^*, we find the heuristic decisions for the number of operating and purchasing vehicles, which we denote by $a_t^h(x_t, SB_t, \theta^*)$ and $u_t^h(x_t, SB_t, \theta^*)$.

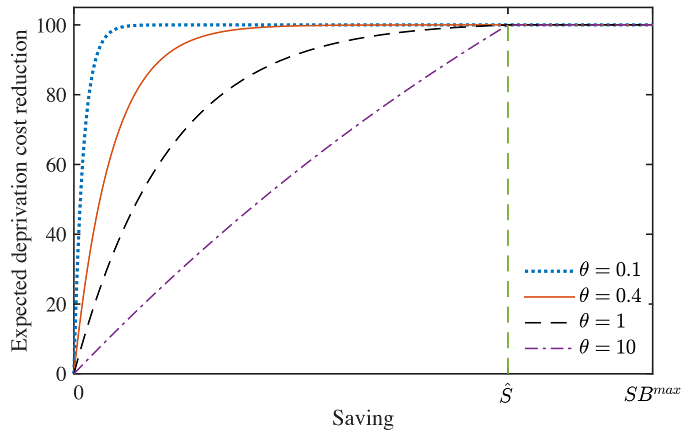


Figure 1 **Estimated social gain from saving ($V^s(s|\theta)$) for different values of θ.**

The following proposition establishes second-order properties of the optimization model (19), which can be used to design an efficient algorithm to solve the problem in a short time in an *online* fashion (i.e., in every time period in which the model is used). The proofs are contained in Appendix D.

Proposition 1 *The maximization function in the optimization model (19) is (non-strictly) concave in a_t^h and in u_t^h and (non-strictly) sub-modular in a_t^h and u_t^h.*

Proposition 1 implies that a_t^h and u_t^h are non-decreasing in the available budget SB_t, which we can use to further restrict the search space of the solution to (19). Finally, the computation of the three functions V_t^a, V_t^u and V^s and the optimal parameter θ^* can be done once in the beginning and stored in memory.

3.3. Sensitivity analysis

In this subsection, we discuss how the decisions of the SAO heuristic depend on the model parameters. Proposition 2 unveils limits on the average fleet size, the operating level and the deprivation costs.

Proposition 2 *For the situation that the average donations are less than the average cost to fulfill completely the demand (i.e., $\mu \le c_L D_m$), and that the limits x^{max} and SB^{max} are sufficiently high, we have the following results:*

1. *It holds that* $\mu = (c_f + \gamma(p - r))\mathbf{E}(x_t) + c_o\mathbf{E}(a_t)$.
2. *The expression* μ/c_L *is a lower limit for the average fleet size,* $\mathbf{E}(x_t)$*, and an upper limit for the average operating level,* $\mathbf{E}(a_t)$.
3. *The average deprivation costs,* $\mathbf{E}(R_t)$*, is bounded below by* $e^{b_m\left(D_m - \frac{\mu}{c_L}\right)} - 1$.

Part 1 of Proposition 2 implies that for a given average budget (μ), an increase in the average fleet size by one vehicle necessitates a reduction in the average operating level of $c_o/(c_f + \gamma(p - r))$ vehicles. As expected, since the average received budget and the residual value of a used vehicle (r) have a positive impact on the total budget, they increase the bound μ/c_L on the fleet size and the operating level. However, it is easy to see that the bound μ/c_L decreases in fixed costs, c_f, operating costs, c_o, purchase price, p, and the probability of vehicle dismissal, γ. Therefore, a well-framed fleet management that maintains vehicles properly and trains drivers frequently seems to decrease not only the costs of fleet management but also, as a consequence, the deprivation costs.

4. Numerical Experiments

In this section, we describe performing numerical experiments to assess the performance (i.e., the optimality gap) of our SAO heuristic and to analyze the sensitivity of the heuristic's decisions to the parameters of the model. To avoid using synthetic data as inputs to our numerical experiments, we use field data to estimate the model parameters empirically. Based on a data set provided by the ICRC, we obtain point estimation for the variables of interest. Our data set contains information on 1,074 Toyota Land Cruisers that the ICRC had owned from 2000 to 2015, in five countries; Iraq, Kenya, Liberia, Syria and Sudan. A main reason for choosing these countries was that, during the past few years, they simultaneously experienced different types of disasters, such as war and political conflict, hunger, and poverty. Consequently, the ICRC has been among the leading organizations that supplied a wide range of humanitarian services to these countries.

Our data set contains information on each vehicle's identification number, date of purchase, purchasing price, number and cost of accidents, maintenance and operating costs, total mileage, mission type, and the delegation (location) in which the vehicle was used. We also know when and how a vehicle has been dismissed (i.e., sold, donated or scrapped) and its residual value. Furthermore, the data set provides us with information on the monthly fleet size of each country over a period of 15 years (mapped in Figure 8, Appendix E).

4.1. Parameter estimation

Similar to Eftekhar et al. (2014), our model is based on vehicle monthly utilization, and we assume a generic average usage value for each variable. To calculate an average operating cost, we took into account each vehicle's total repair and preventive maintenance costs, accident cost, and fuel and driver cost based on the vehicle's cumulative odometer. Then, we divided this cost by vehicle age. The first two cost components were directly found from our data set. However, to calculate the third component, we used an average fuel and driver cost per kilometer in each country of operations that was provided by an ICRC expert. Likewise, we calculated an average fixed cost of keeping an additional vehicle in the fleet, considering driver's training and refreshing cost, management and technical staffing cost, workshop and office cost, and monthly insurance cost. This data was also directly provided by an ICRC expert.

For the point estimations, we applied Bayesian analysis that relies on the assumption that the observed data is fixed while all parameters are random quantities, and provides more robust estimations than frequentist methods (Kruschke et al. 2012). We estimated the posterior mean, standard deviation, and minimum and maximum values of each variable. To estimate the posteriors, we assumed a *non-informative* Uniform prior distribution, and the posteriors were estimated via Markov Chain Monte Carlo (MCMC) sampling. A non-informative prior assigned equal probabilities to all possible states of the parameter space to rectify the subjectivity problem. To increase the accuracy of our simulation results, we used 42,500 MCMC iterations with a warm-up period of 2,500 iterations. The parameter for the seasonal demand variation was obtained from the fleet size information in the data set. For the budget K_t, we used a truncated log-normal distribution (see also Okten and Weisbrod (2000)). The mean of K_t was also obtained from the fleet size information, which gave the more accurate estimate. For the standard deviation of K_t (σ) we used the variation in the budget data. The maximum values for fleet size and budget were chosen to be sufficiently large so that the impact on the results was negligible. We report the results of the cost and demand estimations as well as all parameters used in the numerical experiments in Appendix E.

4.2. Performance assessment of the SAO heuristic

To run our experiments, we used the *Policy Iteration Method* (Puterman 2005) and simulated the system with 50 replications, each of which consisted of 62,000 iterations that included a warm-up period of 2,000 iterations. Due to the size of the problem, it was impossible to obtain an optimal policy for large fleet sizes in an acceptable runtime. Therefore, to assess the optimality gaps, we focused on the data of Sudan and Syria from 2000 to 2005, when both countries had comparably small fleets. We kept the average budget (μ) constant and systematically changed the average level of demand to obtain a ratio of average budget to demand (funding level $\mu/(c_L D_m)$) of 0.5, 0.75 and 1.0. We also changed the factor of mission criticality (ν) from 0.55 to 0.75. To make the scenarios comparable when varying ν, we scaled the total deprivation costs potential to have the same total value under any ν. In all our numerical experiments, we used a three-month interval as one period.

To analyze the performance of the heuristics, we use the *social service level*, (SSL), the ratio of deprivation costs avoided through fleet management to the total potential to avoid deprivation costs, i.e.,

$$SSL = 1 - \frac{\mathbf{E}(R_t)}{e^{b_m D_m} - 1}. \tag{23}$$

Note that $e^{b_m D_m}$ in equation (23) represents the highest possible deprivation costs, when no mission is satisfied in any period. The social service level can attain values between 0% and 100%, with larger values indicating lower deprivation costs.

In Table 1, we compare the optimal, heuristic and benchmark policies, as well as the lower bound (see Proposition 2), and report the deprivation costs, the social service level, (SSL), and the optimality gap. The values are determined with simulation. The half-widths of all 95% confidence intervals are less than 1% of the total performance.

Table 1 Deprivation costs and social service levels of the SAO heuristic and the benchmark policy (data of 2000-2005).

Country	Funding level	Solution	$\nu = 0.55$			$\nu = 0.65$			$\nu = 0.75$		
			dep. c.	SSL	gap	dep. c.	SSL	gap	dep. c.	SSL	gap
Sudan	**0.5**	lower bound	0.50	60.0%	-	2.75	79.0%	-	14.61	94.0%	-
		optimal	0.51	59.1%	-	2.94	77.6%	-	17.41	92.8%	-
		heuristic	0.51	59.0%	0.2%	2.95	77.5%	0.1%	17.77	92.7%	0.2%
		benchmark	0.64	48.2%	18.5%	4.62	64.7%	16.6%	47.85	80.3%	13.5%
	0.75	lower bound	0.22	82.0%	-	0.94	92.8%	-	2.95	98.8%	-
		optimal	0.24	80.8%	-	1.13	91.4%	-	4.53	98.1%	-
		heuristic	0.24	80.6%	0.2%	1.15	91.2%	0.2%	4.74	98.0%	0.1%
		benchmark	0.34	72.9%	9.8%	2.30	82.4%	9.8%	22.68	90.7%	7.6%
	1.00	lower bound	0.00	100.0%	-	0.00	100.0%	-	0.00	100.0%	-
		optimal	0.05	95.6%	-	0.24	98.2%	-	0.81	99.7%	-
		heuristic	0.06	95.3%	0.3%	0.26	98.0%	0.2%	0.91	99.6%	0.0%
		benchmark	0.08	93.8%	1.9%	0.44	96.6%	1.6%	3.41	98.6%	1.1%
Syria	**0.50**	lower bound	0.50	60.0%	-	2.75	79.0%	-	14.61	94.0%	-
		optimal	0.50	59.7%	-	2.83	78.4%	-	15.79	93.5%	-
		heuristic	0.50	59.7%	0.0%	2.83	78.4%	0.1%	16.03	93.4%	0.1%
		benchmark	0.57	54.1%	9.4%	3.85	70.6%	10.0%	35.94	85.2%	8.9%
	0.75	lower bound	0.22	82.0%	-	0.94	92.8%	-	2.95	98.8%	-
		optimal	0.23	81.6%	-	1.01	92.3%	-	3.63	98.5%	-
		heuristic	0.23	81.6%	0.0%	1.03	92.1%	0.1%	3.76	98.4%	0.1%
		benchmark	0.30	76.3%	6.6%	1.86	85.8%	7.0%	16.17	93.3%	5.2%
	1.00	lower bound	0.00	100.0%	-	0.00	100.0%	-	0.00	100.0%	-
		optimal	0.03	97.3%	-	0.13	99.0%	-	0.39	99.8%	-
		heuristic	0.04	97.2%	0.2%	0.15	98.9%	0.1%	0.45	99.8%	0.0%
		benchmark	0.04	96.4%	0.9%	0.23	98.2%	0.8%	1.50	99.4%	0.5%

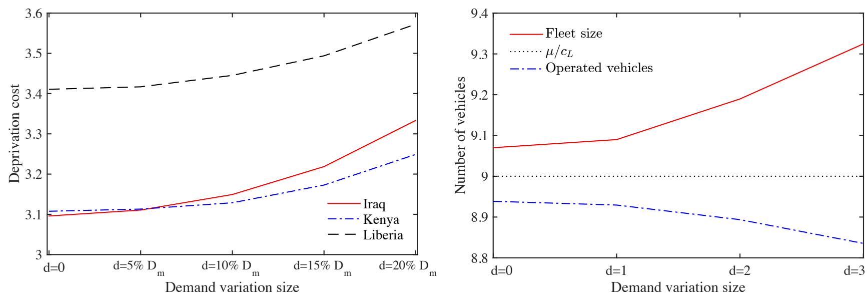
dep. c. = Deprivation costs, gap = Optimality gap.

As shown in Table 1, the optimal policy achieves a performance fairly close to the lower bound. Furthermore, the SAO heuristic demonstrates a performance close to the optimal policy, with an average optimality gap of 0.12%, and it outperforms the benchmark policy (average optimality gap of 7.21%) by a wide margin.

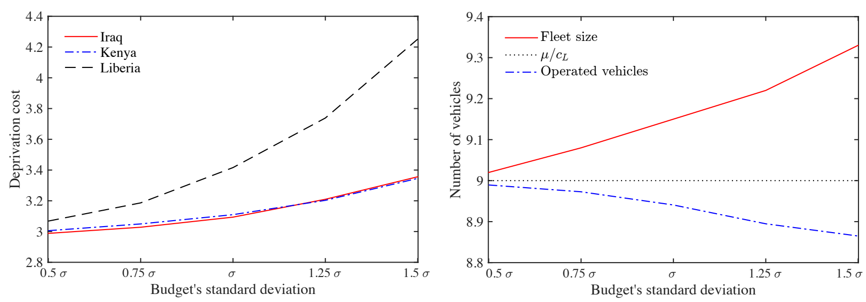
4.3. Sensitivity analyses and discussion

Next, we conducted numerical experiments to conclude managerial insights. Unless otherwise noted, we used a funding level of 75% and a factor for differences in mission criticality of $\nu = 0.75$. Furthermore, for analyses in which we varied the parameters c_L, p, or γ, we also increased the average budget (μ) to keep the level of funding constant.

Impact of uncertainty and variability- Our results indicate that uncertainty in budget and variability in demand have negative consequences on the deprivation costs and on fleet utilization (i.e., the number of vehicles operated compared to the size of the total fleet). These results, graphically shown in Figure 2, are in line with conventional wisdom that uncertainty renders the operations



(a) Impact of demand variation on deprivation costs (b) Impact of demand variation on fleet utilization (data of Syria)



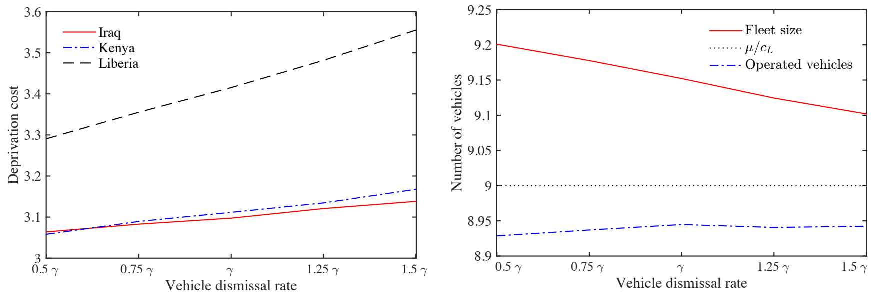
(c) Impact of budget variation on deprivation costs (d) Impact of budget variation on fleet utilization (data of Syria)

Figure 2 Impact of demand and budget variation on deprivation costs and fleet utilization.

problem more complex to manage.

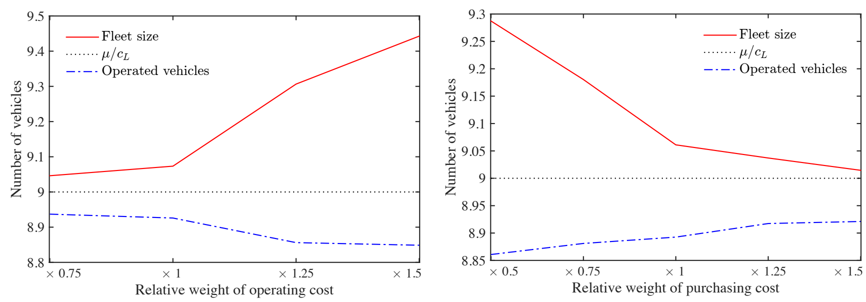
Moreover, an increase in the vehicle dismissal probability, which corresponds to a decrease in the average vehicle life time, increases the deprivation costs (Figure 3a) but reduces the fleet utilization (Figure 3b). In fact, a greater vehicle dismissal probability pushes the delegation to more quickly adjust the fleet size to the seasonality of demand. Yet, ceteris paribus, the impact of the vehicle dismissal probability on the deprivation costs seems to be less severe than the impact of budget uncertainty (cf. Figure 2c).

Impact of cost parameters- The impact of operating cost on the asset management policy is interesting. With an increase in the operating unit cost (c_o) the decision-maker is more conservative and spends resources on the more critical missions. Therefore, as shown in Figure 4a, an increase in operating cost causes a decrease in fleet utilization. On the contrary, when the vehicle purchase price increases, the decision-maker will keep a smaller fleet but will use the available vehicles as often as possible, thus resulting in a higher fleet utilization (Figure 4b).



(a) On deprivation costs (b) On fleet utilization (data of Syria)

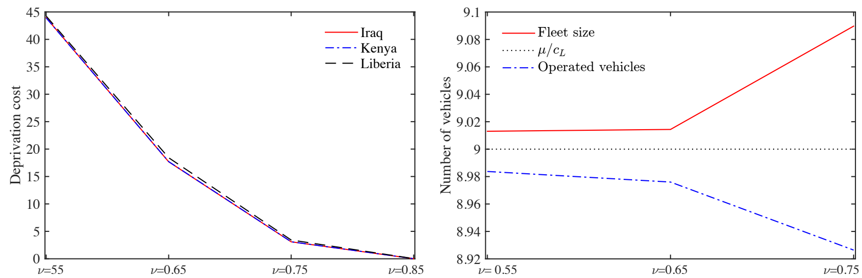
Figure 3 Impact of the rate of vehicle dismissal on service level and fleet utilization.



(a) Impact of operating cost (data of Syria) (b) Impact of purchase price (data of Syria)

Figure 4 Impact of the operating cost and purchasing price on fleet utilization.

Impact of the differences in mission criticality- Figure 5a indicates that the deprivation costs decrease according to the differences of the criticality of the humanitarian missions (ν). This means that in situations where the most critical missions are considerably more important than the least critical missions, the average deprivation costs is less than in situations where the difference between the most and the least critical missions is small. Furthermore, when the differences in mission criticality increase, the number of operating vehicles (i.e., the number of missions fulfilled) and the utilization of the fleet decrease. This happens because the cost of missing an important mission in the future is high, such that a forward-looking delegation avoids covering the less important missions in the current period and preserves the budget for the future; the delegation needs to protect the budget for future *more critical* missions. This observation is confirmed by Figure 6, which indicates that the level of budget savings between periods increases with ν. Our finding is similar to revenue management in the commercial sector, where a firm that does not have enough



(a) On deprivation costs (b) On fleet utilization (data of Syria)

Figure 5 Impact of the differences in mission criticality on deprivation costs and fleet utilization.

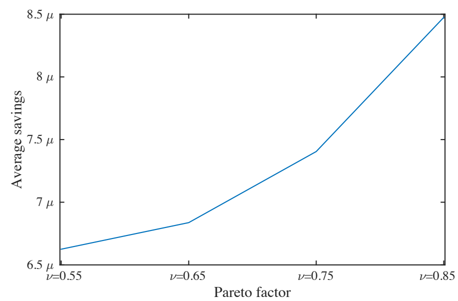


Figure 6 Impact of differences in mission criticality on the savings pattern (data of Iraq).

resources to cover all demands reserves its limited resources for the most profitable demand. Thus, our findings imply that, ceteris paribus, delegations that are engaged in a limited range of programs might maintain a higher fleet utilization over time compared to those delegations engaged in a wide range of services.

5. Model Extensions

In this section, we extend our model to analyze the impact of the flexibility on budget savings as well as the impact of procurement lead time on deprivation costs and fleet utilization.

5.1. Impact of flexibility on budget saving

Until now, we have assumed that financial donations can be fully saved for future periods. This may not always be possible (e.g., for earmarked budgets that have to be used in a given period). We extend our model to allow for partial saving by introducing a new parameter $0 \le \rho \le 1$ that indicates the flexibility of saving; a value of $\rho = 1$ corresponds to the model that we have studied so far in which saving is fully possible, while a value of $\rho = 0$ refers to a situation where budget has to be spent entirely in the period in which it is received.

We can incorporate this new parameter by updating the state equation (11) to

$$SB_{t+1} = \rho S_t + r(x_t - L(x_t)) + K_{t+1}. \tag{24}$$

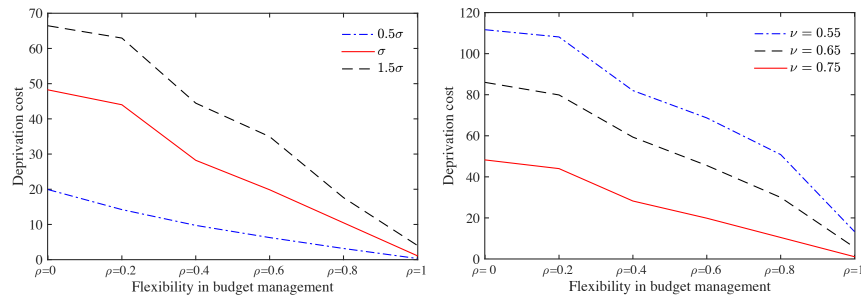
Since the SAO heuristic has been designed for full saving flexibility, we numerically determined the optimal policy to analyze the impact of parameter ρ on the results. Figures 7a-b indicate that the possibility of budget savings can significantly reduce the deprivation costs, regardless of budget variation and differences in mission criticality. The importance of saving seems to be particularly critical when the budget variation is high.

The opportunity of budget savings between periods seems to neutralize the negative effect of budget uncertainty to some degree. This indicates that a non-earmarked budget, which can also be used for later periods, has more value than earmarked budgets. Interestingly, we also observe that the saving option has a positive impact on fleet utilization (Figures 7c-d). This result is similar to that of Besiou et al. (2014) that disclose a negative impact of an earmarked budget on the performance of humanitarian operations.

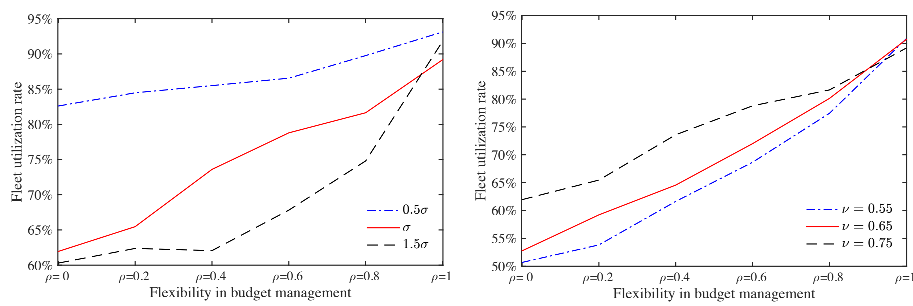
5.2. Impact of procurement lead time

Besiou et al. (2014) compare vehicle procurement models (i.e., centralized, hybrid and decentralized) by analyzing the procurement costs (e.g., vehicle purchasing prices from global versus local markets, lead time costs, etc.). They highlight that vehicle procurement costs in a decentralized model (where each delegation purchases vehicles from the domestic market) are higher than in a centralized model (in which the headquarters purchase vehicles directly from a manufacturer). Nevertheless, their results indicate that a decentralized model provides a higher service level driven by the shorter procurement lead time. In their study, service level is defined as the ratio of available vehicles to the total number of required vehicles. We looked at the same question with a different objective function, where service level is based on deprivation costs.

To analyze the impact of procurement lead time on the deprivation costs, we define two new



(a) On deprivation costs for different budget uncertainties (b) On deprivation costs for different mission criticality values



(c) On fleet utilization for different budget uncertainties (d) On fleet utilization for different mission criticality values

Figure 7 Impact of the possibility for budget saving (data of Sudan).

models, one with a zero procurement lead time and one with two periods of procurement lead time. The case of one period of lead time corresponds to the standard case described in Section 2.

In the scenario with zero lead time, we have to adapt equations (9), (11) and (12) with

$$a_t \leq x_t + u_t, \tag{25}$$

$$SB_{t+1} = \min\{S_t + r(x_t + u_t - L(x_t + u_t)) + K_{t+1}, SB^{max}\}, \tag{26}$$

$$x_{t+1} = \min\{L(x_t + u_t), x^{max}\}, \tag{27}$$

to allow for the immediate availability of u_t vehicles.

In the case of two periods of procurement lead time, we have to add a new state variable w_t to describe the number of vehicles that are purchased in period $t-1$. Hence, we replace equation (12) with

$$x_{t+1} = \min\{L(x_t) + w_t, x^{max}\} \tag{28}$$

and add the additional constraint

$$w_{t+1} = u_t. \tag{29}$$

Models with longer lead times can be defined equivalently through the introduction of additional state variables, but lead to significantly higher computational complexity.

Similar to that of Besiou et al. (2014), our trade-off is based on the assumption that shorter lead times often come at additional procurement costs. Hence, we compare the impact of higher vehicles procurement costs in the case of zero lead time by considering price mark-ups of 0%, 50% and 100% on the base vehicle price. Furthermore, we vary the standard deviation of budget and the rate of vehicle dismissal. Table 2 reports the deprivation costs for each scenario for Syria.

Table 2 Deprivation costs for different scenarios.

			σ			γ	
Lead time	Price markup	$\times 0.5$	$\times 1$	$\times 1.5$	$\times 0.5$	$\times 1$	$\times 1.5$
2	-	3.370	3.755	4.801	3.627	3.755	3.879
1	-	3.254	3.631	4.600	3.556	3.631	3.719
0	0%	3.118	3.502	4.472	3.507	3.502	3.534
0	50%	6.323	6.825	7.987	5.110	6.825	8.701
0	100%	10.453	11.050	12.288	6.990	11.050	15.067

The results indicate that a procurement lead time of one period (which corresponds, in our setting, to three months) has a negative impact on deprivation costs only if the manager is able to procure the same vehicle at the same price and with no lead time, which is nearly impossible in the real world. While decreasing the procurement lead time reduces per se the deprivation costs, this effect is mitigated by the vehicle price markups, which limit the financial resources for current and future operations.

Also, results surprisingly indicate that the advantages of a centralized fleet policy are particularly strong when the degree of uncertainty is high. Finally, we find that the centralized model is more affected by budget variation, i.e., the sensitivity with respect to the budget variation is higher than the sensitivity with respect to the vehicle dismissal rate, whereas the decentralized model seems to be more affected by the vehicle dismissal rate.

6. Conclusion

In this article, we develop a model for purchasing and operating asset capacity in a setting of humanitarian development programs. The objective of our model is to minimize the human suffering due to insufficient humanitarian aid. We extend existing research by considering mission criticality, budget uncertainty and uncertainty in asset replacement. We develop a heuristic based on a portfolio approach, which achieves close-to-optimal results, with an average optimality gap in our numerical experiments of 0.12%, outperforming existing policies by a wide margin. Based on a real data set of the ICRC's vehicle fleets in five different countries from 2000 to 2015, we perform numerical experiments and derive managerial insights.

We find that budget variability increases the deprivation costs and decreases asset utilization. Our results indicate, though, that a way to mitigate the negative effects of system uncertainty (such as budget uncertainty) is to allow the delegations to save a proportion of budget between periods for future operations. Our results also demonstrate that it is not always preferable to operate all assets at capacity in all periods, which renders asset management for humanitarian development programs different from asset management in the commercial sector. Finally, we find that differences in mission criticality, even though they decrease the deprivation costs, lead to a fewer missions served.

We believe that this avenue of research could be further developed by empirical analysis and/or analytical work to better understand deprivation cost in humanitarian development settings. Also, we studied a single-type asset procurement policy, and further research can explore multi-type asset settings. Finally, it seems to be worthwhile to analyze settings in which an HO should trade-off between operational assets and consumable relief items.

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Appendix A: Table of symbols

Model variables and parameters	
a_t	The operating decision in period t
b_m	Convexity factor of the demand function corresponding to D_m
b_t	Convexity factor of the demand function in period t
c_f	Fixed cost of a vehicle per period
c_L	The average per period logistics cost of a vehicle in case of 100% utilization rate
c_o	Operating cost of a vehicle per period
d	Magnitude of demand oscillation
D_m	Average demand
D_t	Demand in period t
γ	The probability of having to dismiss a vehicle in one period
$J_t(x_t, SB_t)$	The average social cost under the optimal policy, with state variables x_t and SB_t, and time period t
K_t	Financial donations received at the beginning of period t
$L(x_t)$	Part of the fleet size in period t that can be used in period $t+1$
μ	The average financial donations received per period
n	Number of periods in one cycle
ν	Pareto parameter identifying the relative importance of the missions
ω_0	Initial phase of the demand function
p	Purchasing cost of a vehicle
r	Residual value of a dismissed vehicle
R_t	The social cost incurred in period t
S_t	Budget remains as savings at the end of period t
SB^{max}	Maximum amount of budget
SB_t	Available budget at the beginning of period t
SSL	Social service level
σ	The standard deviation of financial budget
t	Indicator of a period
u_t	The purchasing decision in period t
x^{max}	Maximum number of vehicles
x_t	Available vehicles at the beginning of period t
Heuristic and benchmark variables and parameters	
a_t^b (u_t^b)	Benchmark operating (purchasing) policy in period t
a_t^h (u_t^h)	The heuristic policy for operating (purchasing) in period t under a given value of θ
$\bar{a}_t$	Optimal number of vehicles to operate in period t of the simplified problem
b'	Coefficient of the gain function V^s
$\hat{S}$	Upper threshold for savings
θ	Heuristic parameter
$V_t^a(i)$	Social gain from operating i vehicles in period t
$V^s(s\|\theta)$	Estimated social gain from a saving s under the heuristic parameter θ
$V_t^u(i, x_t)$	Estimated social gain from purchasing i vehicles in period t, when the fleet size is x_t
$\bar{u}_t$	Optimal purchasing decision in period t of the simplified problem

Appendix B: Optimizing deterministic problem

The deterministic problem of Section 3.2 can be written as follows:

$$\min \qquad \tfrac{1}{n}\textstyle\sum_{t=1}^{n} R_t(\bar{a}_t) \tag{30}$$

subject to

$$\bar{a}_t \le x_t, \tag{31}$$

$$S_t = SB_t - c_f x_t - c_o \bar{a}_t - p\bar{u}_t, \qquad 1 \le t \le n \tag{32}$$

$$SB_{t+1} = \min\{S_t + r\gamma x_t + \mu, SB^{max}\}, \qquad 1 \le t \le n \tag{33}$$

$$x_{t+1} = \min\{(1-\gamma)x_t + \bar{u}_t, x^{max}\}, \qquad 1 \le t \le n \tag{34}$$

$$(x_1, SB_1) = (x_{n+1}, SB_{n+1}), \tag{35}$$

$$\bar{a}_t, \bar{u}_t \ge 0, \qquad 1 \le t \le n \tag{36}$$

where n was defined as the number of seasons and the minimization is taken over the controls $\bar{u}_t$ and $\bar{a}_t$.

For a simple case of constant demand, Proposition 3 expresses the optimal number of vehicles to operate, $\bar{a}_t$, and the optimal number of vehicles to purchase, $\bar{u}_t$, and describes the system in steady state.

Proposition 3 *For a constant demand ($d = 0$),*

- *the optimal number of operating vehicles is equal to the fleet size $\bar{a}_t = x_t$,*
- *the optimal purchasing decision is given by*

$$u_t = \frac{SB_t - c_f x_t - c_o a_t}{p}, \tag{37}$$

- *the steady state fleet size, $\bar{x}$, is given by*

$$\bar{x} = \min\left\{D_m, \frac{\mu}{c_L}\right\}, \tag{38}$$

where, in the steady state, the number of operating and purchasing vehicles are $\bar{a}_t = \bar{x}$ and $\bar{u}_t = \gamma\bar{x}$.

Appendix C: Deriving $V_t^s(s|\theta)$

To design a proper function that estimates the social gain from a certain amount of savings, we note that according to Equation (11), budgets larger than SB^{max} will be lost. Expression $r\gamma x_t + \mu = \mathbf{E}(r(x_t - L(x_t)) + K_{t+1})$ is the expected budget to be received at the end of period t. Therefore, we set the social gain of any value of savings greater than $SB^{max} - r\gamma x_t - \mu$ to zero in our estimation. To simplify, we define the following variable:

$$\hat{S} = SB^T - r\gamma\frac{\mu}{c_L} - \mu, \tag{39}$$

where μ/c_L comes from Proposition 3 and estimates the average fleet size in each period. For any $s < \hat{S}$, the behavior of $V^s(s|\theta)$ should resemble the behavior of $J_t(x_t, SB_t)$ in SB_t. The following Lemma presents an important characteristic of J_t.

Lemma 1 *The average deprivation costs is decreasing in the budget, i.e. $J_t(x_t, SB_t)$ is decreasing in SB_t.*

Since the objective in Equation (19) is to maximize the social gain, which is the inverse of the objective in the main model (minimization of the deprivation costs), we need to propose a function V^s that is increasing in s, so that it provides a similar behavior. Moreover, since our value function is the sum of exponential functions, we also use an exponential function for V^s. We assume $V^s(s|\theta) = -e^{b'(\theta\hat{S}-s)} + b''$, where θ is the heuristic parameter. b' and b'' are functions of θ, where we use the following equations to determine their values:

$$V^s(0|\theta) = 0, \tag{40}$$

$$V^s(s \geq \hat{S}|\theta) = e^{b_m D_m}. \tag{41}$$

Note that $b_m D_m = b_t D_t \ \forall t$, according to Equation (4). Equation (40) implies that the expected social gain from no saving is zero, while Equation (41) expresses that for any $s \geq \hat{S}$, the expected social gain is equal to the social gain from satisfying all the demands in one period. Therefore, $V^s(s)$ can be written as in Equation (18), where b' and b'' are calculated by solving the following equations:

$$e^{b'\hat{S}\theta} - e^{b'\hat{S}(\theta-1)} = e^{b_m D_m}, \tag{42}$$

$$b'' = e^{b'\theta\hat{S}}. \tag{43}$$

Online Appendix

Appendix D: Proofs

Proof of Proposition 1 We first establish concavity of the gain functions that is $V_t^a(a)$ is increasing and concave in a_t, $V_t^u(u_t, x_t)$ is increasing and concave in u_t, and $V^s(s|\theta)$ is increasing and concave in s.

The proof that $V_t^a(a_t)$ is increasing and concave in a_t is straightforward.

Properties of $V_t^u(u|x_t)$: First, we can rewrite $V_t^u(u|x_t)$ as follows:

$$V_t^u(u, x_t) = \mathbf{E}_{L(x_t) \le \bar{a}_{t+1} - u_t}\left(e^{b_{t+1}[D_{t+1} - L(x_t)]} - e^{b_{t+1}[D_{t+1} - L(x_t) - u_t]}\right) +$$
$$\mathbf{E}_{L(x_t)\ |\bar{a}_{t+1} - u_t < L(x_t) \le \bar{a}_{t+1}}\left(e^{b_{t+1}[D_{t+1} - L(x_t)]} - e^{b_{t+1}[D_{t+1} - \bar{a}_{t+1}]}\right).$$

Note that $\mathbf{E}_{L(x_t)\ |L(x_t) > \bar{a}_{t+1}}(e^{b_{t+1}[D_{t+1} - \bar{a}_{t+1}]} - e^{b_{t+1}[D_{t+1} - \bar{a}_{t+1}]}) = 0$.

$V_t^u(u_t, x_t)$ is decreasing in u_t:

For every $x_t \ge 0$ we have:

$$\frac{\partial V_t^u}{\partial u_t} = b_{t+1} e^{b_{t+1}[D_{t+1} - L(x_t) - u_t]} > 0.$$

$V_t^u(u_t, x_t)$ is concave in u_t: For every $x_t \ge 0$ we have:

$$\frac{\partial^2 V_t^u}{\partial u_t^2} = -b_{t+1}^2 e^{b_{t+1}[D_{t+1} - L(x_t) - u_t]} < 0.$$

Properties of $V^s(s|\theta)$: First we show that $b' \ge 0$.

$$\forall s > 0 : V^s(s|\theta) > 0 \implies e^{b'\theta\hat{S}}\left(1 - \frac{1}{e^{b's}}\right) > 0 \implies e^{b's} \ge 1 \implies b' \ge 0. \tag{44}$$

$V^s(s|\theta)$ is increasing in s:

$$\frac{\partial}{\partial s} V^s(s|\theta) = \begin{cases} b' e^{b'(\theta\hat{S} - s)} > 0 & \text{if} \quad s \le \hat{S}, \\ 0 & \text{otherwise.} \end{cases}$$

$V^s(s|\theta)$ is concave in s:

$$\frac{\partial^2}{\partial s^2} V^s(s|\theta) = \begin{cases} -b'^2 e^{b'(\theta\hat{S} - s)} < 0 & \text{if} \quad s \le \hat{S}, \\ 0 & \text{otherwise.} \end{cases}$$

Since s is linear in a_t and u_t, concavity of V^s in s implies the concavity of V^s in a_t and u_t as well. From the above concavity results follows that all three gain functions are concave in u_t and in a_t (not necessarily jointly). Thus, the sum of the functions is also concave.

Recall that submodularity of the saving function $V^s(\cdot)$ in u_t and a_t can be established as $V^s(u_t + 1, a_t) - V^s(u_t, a_t)$ being non-increasing in a_t (we abuse notation and write V^s directly as a function of u_t and a_t).

But this follows directly from the concavity of $V^s(\cdot)$ in s and the fact that $s = SB_t - c_f x_t - c_o a_t - pu_t$, which implies that $V^s((SB_t - c_f x_t - c_o a_t - pu_t) - p|\theta) - V^s(SB_t - c_f x_t - c_o a_t - pu_t|\theta)$ is non-increasing in a_t. Submodularity of the maximization function then follows from the fact that the sum of submodular functions is also submodular and that the two remaining functions $V_t^a(a_t)$ and $V_t^u(u_t, x_t)$ are by definition submodular in both u_t and a_t because they each depend only on a single one of the two parameters.

Proof of Proposition 2 The first part holds, because

$$\mathbf{E}(\text{Cost per period}) = c_f\mathbf{E}(x_t) + c_o\mathbf{E}(a_t) + p\mathbf{E}(u_t) - r\mathbf{E}(L(x_t)) = (c_f + \gamma(p-r)\mathbf{E}(x) + c_o\mathbf{E}(a_t) = \mu, \quad (45)$$

because when in a stable system in which the maximum limits are sufficiently high, the average cost per period corresponds to the average donations per period.

Next, we have to prove that $\mathbf{E}(a_t) \leq \dfrac{\mu}{c_L}$ and $\mathbf{E}(x_t) \geq \dfrac{\mu}{c_L}$.

Since $\mu \leq c_L D_m$, assume $\mathbf{E}(a_t) > \dfrac{\mu}{c_L}$. In this case we also have $\mathbf{E}(x_t) \geq \mathbf{E}(a_t) > \dfrac{\mu}{c_L}$.

$$\mathbf{E}(\text{Cost per period}) = c_f\mathbf{E}(x_t) + c_o\mathbf{E}(a_t) + p\gamma\mathbf{E}(x_t) - r\gamma\mathbf{E}(x_t) > c_f\frac{\mu}{c_L} + c_o\frac{\mu}{c_L} + (p-r)\gamma\frac{\mu}{c_L} = \mu = \mathbf{E}(\text{Budget per period}), \quad (46)$$

which violates the budget constraint. Therefore $\mathbf{E}(a_t) \leq \dfrac{\mu}{c_L}$.

On the other hand, if we assume that $\mathbf{E}(a_t) < \mathbf{E}(x_t) < \dfrac{\mu}{c_L}$, by the same reasoning we get: $\mathbf{E}(\text{Cost per period}) < \mathbf{E}(\text{Budget per period})$. However, it is not an optimal solution, since by increasing $\mathbf{E}(x_t)$ we can also increase $\mathbf{E}(a_t)$ and consequently, decrease the social cost. Therefore in optimality, we have $\mathbf{E}(x_t) \geq \dfrac{\mu}{c_L} \geq \mathbf{E}(a_t)$.

Next, we show that in the optimal solution $\mathbf{E}\left[(a_t - \mathbf{E}(a_t))(b_t - \mathbf{E}(b_t))\right] < 0$. The reward function is $R_t = e^{b_t(D_t - a_t)} - 1 = e^{b_m D_m - b_t a_t} - 1$, since according to Equation (4), we have

$$b_t D_t = b_m D_m \quad \forall\ t \implies \frac{\partial a_t}{\partial b_t} = \frac{-a_t e^{b_m D_m - b_t a_t}}{b_t e^{b_m D_m - b_t a_t}} < 0 \implies \frac{\partial\left(a_t - \mathbf{E}(a_t)\right)}{\partial\left(b_t - \mathbf{E}(b_t)\right)} < 0$$

since $\mathbf{E}(a_t)$ and $\mathbf{E}(b_t)$ are fixed values. From the equation above it is concluded that $\mathbf{E}\left[(a_t - \mathbf{E}(a_t))(b_t - \mathbf{E}(b_t))\right] < 0$. Moreover, according to part 1 of the proposition, $\mathbf{E}(a_t) \leq \dfrac{\mu}{c_L}$. Therefore:

$$\mathbf{E}(b_t a_t) = \mathbf{E}(b_t)\mathbf{E}(a_t) + \mathbf{E}\left[(a_t - \mathbf{E}(a_t))(b_t - \mathbf{E}(b_t))\right] < \mathbf{E}(b_t)\mathbf{E}(a_t) = b_m\mathbf{E}(a_t) \leq b_m\frac{\mu}{c_L} \implies \mathbf{E}(b_t a_t) < b_m\frac{\mu}{c_L}.$$

Therefore:

$$\mathbf{E}(b_m\frac{\mu}{c_L} - b_t a_t) > 0 \implies e^{\mathbf{E}(b_m\frac{\mu}{c_L} - b_t a_t)} > 1 \implies \mathbf{E}\left(e^{b_m\frac{\mu}{c_L} - b_t a_t}\right) > 1 \quad \text{According to Jensen's inequality,}$$

$$\implies e^{b_m\frac{\mu}{c_L}}\mathbf{E}\left(e^{-b_t a_t}\right) > 1 \implies \mathbf{E}\left(e^{-b_t a_t}\right) > e^{-b_m\frac{\mu}{c_L}} \implies e^{b_m D_m}\mathbf{E}\left(e^{-b_t a_t}\right) > e^{b_m D_m}e^{-b_m\frac{\mu}{c_L}}$$

$$\implies \mathbf{E}\left(e^{b_m D_m - b_t a_t}\right) > e^{b_m(D_m - \frac{\mu}{c_L})} \implies \mathbf{E}\left(e^{b_t[D_t - a_t]} - 1\right) > e^{b_m[D_m - \frac{\mu}{c_L}]} - 1 \quad \text{since } b_t D_t = b_m D_m\ \forall t.$$

Proof of Proposition 3 Assume we use the operating policy of $\bar{a}_t = x_t$ and the purchasing policy of $\bar{u}_t = \frac{SB_t - c_f x_t - c_o a_t}{p}$. It is easy to see that under this policy $SB_t = \mu + r\gamma x_{t-1}$ (since $S_t = 0$). Therefore, the state equation describing the fleet size can be written as:

$$x_{t+1} = (1-\gamma)x_t + \bar{u}_t = (1-\gamma)x_t + \frac{\mu + r\gamma x_{t-1} - c_f x_t - c_o \bar{a}_t}{p} \implies x_{t+1} = \frac{\mu}{p} + \left[(1-\gamma) - \frac{c_f + c_o}{p}\right] x_t + \frac{r}{p}\gamma x_{t-1}. \tag{47}$$

First we show that the above sequence converges to a unique number (i.e. $\lim_{t\to\infty} \Delta x_t = 0$).

$$x_{t+2} - x_{t+1} = \left[\frac{\mu}{p} + [(1-\gamma) - \frac{c_f + c_o}{p}]x_{t+1} + \frac{r\gamma}{p} x_t\right] - \left[\frac{\mu}{p} + [(1-\gamma) - \frac{c_f + c_o}{p}]x_t + \frac{r\gamma}{p} x_{t-1}\right] =$$
$$\left[1 - \frac{c_f + c_o + \gamma p}{p}\right] x_{t+1} - \left[1 - \frac{c_f + c_o + \gamma p}{p} - \frac{\gamma r}{p}\right] x_t - \frac{\gamma r}{p} x_{t-1} = \left[1 - \frac{c_f + c_o + \gamma p}{p}\right](x_{t+1} - x_t) + \frac{\gamma r}{p}(x_t - x_{t+1}).$$

Assume $\lim_{t\to\infty} \Delta x_t = v$. According to the above equation we have:

$$v = \left[1 - \frac{c_f + c_o + \gamma p}{p}\right] v + \frac{\gamma r}{p} v \implies \frac{c_f + c_o + \gamma(p - r)}{p} v = 0 \implies v = 0.$$

Therefore, the sequence converges to a single value. In order to find out this value, we assume $\bar{x} = \lim_{t\to\infty} x_t$. Therefore:

$$\bar{x} = \frac{\mu}{p} + \left[(1-\gamma) - \frac{c_f + c_o}{p}\right] \bar{x} + \frac{r}{p}\gamma \bar{x} \implies \frac{c_f + c_o + \gamma p - \gamma r}{p} \bar{x} = \frac{\mu}{p} \implies c_L \bar{x} = c_L \frac{\mu}{c_L} \quad \text{According to Eq. (38),}$$

which concludes that $\bar{x} = \frac{\mu}{c_L}$. Therefore, the Markov chain is absorbing with steady state variables $x = \frac{\mu}{c_L}$ and $SB = \mu + r\gamma \frac{\mu}{c_L}$:

$$\begin{aligned} x_{t+1} &= (1-\gamma)x_t + u = x_t = \frac{\mu}{c_L}, \\ SB_{t+1} &= SB_t - c_o \bar{a}_t - c_f x_t - p\gamma x_t + r\gamma x_t + \mu = \mu + r\gamma \frac{\mu}{c_L} - c_o \frac{\mu}{c_L} - c_f \frac{\mu}{c_L} - p\gamma \frac{\mu}{c_L} + r\gamma \frac{\mu}{c_L} + \mu = SB_t = \mu + r\gamma \frac{\mu}{c_L}. \end{aligned}$$

Therefore, in the steady state we have:

$$\bar{a}_t = \frac{\mu}{c_L} \text{ and } \bar{u}_t = \gamma \frac{\mu}{c_L}.$$

Assuming g as the gain and h as the bias, according to Puterman (2005), we have:

$$\begin{aligned} & g + h(x_t, SB_t) = R(a_t) + h(x_{t+1}, SB_{t+1}) \implies \\ & g + h(\frac{\mu}{c_L}, \mu + r\gamma \frac{\mu}{c_L}) = e^{b(D - \frac{\mu}{c_L})} - 1 + h((1-\gamma)\frac{\mu}{c_L} + \gamma \frac{\mu}{c_L}, \mu + r\gamma \frac{\mu}{c_L} - c_o \frac{\mu}{c_L} - c_f \frac{\mu}{c_L} - p\gamma \frac{\mu}{c_L} + r\gamma \frac{\mu}{c_L} + \mu) \implies \\ & g + h(\frac{\mu}{c_L}, \mu + r\gamma \frac{\mu}{c_L}) = e^{b(D - \frac{\mu}{c_L})} - 1 + h(\frac{\mu}{c_L}, \mu + r\gamma \frac{\mu}{c_L}) \implies g = e^{b(D - \frac{\mu}{c_L})} - 1. \end{aligned}$$

To show that the above policy is optimal, we refer to Puterman (2005), where for a state space S and a finite action space A_s, unichain Markov process, and bounded reward function, the optimality condition in an average reward MDP is expressed as:

$$0 = \min_{a \in A_s} \left\{ r(s,a) - g + \sum_{j \in S} p(j|s,a)h(j) - h(s) \right\}$$

and if (g', h') is any other solution of the average reward optimality equation, then $g = g'$.
Since at the steady state, there is only one absorbing state $(\frac{\mu}{c_L}, \mu + r\gamma\frac{\mu}{c_L})$, which satisfy the above equation, the policy is optimal.

Proof of Lemma 1 To show that $J_t(x_t, SB_t)$ is decreasing in SB_t, we need to show that:

$$\forall x \in \mathbb{Z}^+, SB, \delta \in \mathbb{R}^+ : \; J_t(x, SB + \delta) \leq J_t(x, SB).$$

Assume the above equation holds for period $t+1$ that is for any (x_{t+1}, SB_{t+1}):

$$J_{t+1}(x_{t+1}, SB_{t+1} + \delta) \leq J_{t+1}(x_{t+1}, SB_{t+1}), \tag{48}$$

we have to show that for period t: $J_t(x_t, SB_t + \delta) \leq J_t(x_t, SB_t), \; \forall \; x_t, SB_t$. Let us define $\tilde{J}_t(x_t, SB_t, u, a) = R(x_t, SB_t, u, a) + \mathbf{E}[J_{t+1}(x_{t+1}, SB_{t+1})]$. Assume that $\hat{u}$ and $\hat{a}$ are the optimal solutions of $J_t(x_t, SB_t + \delta)$, and $\breve{u}$ and $\breve{a}$ optimize $J_t(x_t, SB_t)$, we know that:

$$J_t(x_t, SB_t + \delta) = \tilde{J}_t(x_t, SB_t + \delta, \hat{a}, \hat{u}) \leq \tilde{J}_t(x_t, SB_t + \delta, \breve{a}, \breve{u}).$$

The Lemma will be proved if we further show that:

$$\tilde{J}_t(x_t, SB_t + \delta, \breve{a}, \breve{u}) \leq \tilde{J}_t(x_t, SB_t, \breve{a}, \breve{u}) = J_t(x_t, SB_t). \tag{49}$$

Equation (49) can be simplified to:

$$\begin{aligned}
&R_t(\breve{a}) + \mathbf{E}[J_{t+1}(L(x_t) + \breve{u}, SB_t + \delta - c_f x_t - p\breve{u} - c_o\breve{a} + r(x_t - L(x_t)) + K_{t+1})] \\
&\leq R_t(\breve{a}) + \mathbf{E}[J_{t+1}(L(x_t) + \breve{u}, SB_t - c_f x_t - p\breve{u} - c_o\breve{a} + r(x_t - L(x_t)) + K_{t+1})] \\
&\Longrightarrow \mathbf{E}[J_{t+1}(x_{t+1}, SB_{t+1} + \delta)] \leq \mathbf{E}[J_{t+1}(x_{t+1}, SB_{t+1})], \quad \forall \; L(x_t), K_{t+1},
\end{aligned}$$

where $x_{t+1} = L(x_t) + \breve{u}$ and $SB_{t+1} = SB_t - c_f x_t - p\breve{u} - c_o\breve{a} + r(x_t - L(x_t)) + K_{t+1}$. From Equation (48), we know that $\mathbf{E}[J_{t+1}(x_{t+1}, SB_{t+1})]$ is stochastically greater than $\mathbf{E}[J_{t+1}(x_{t+1}, SB_{t+1} + \delta)]$. This ends the proof.

Appendix E: Results of Costs Estimates

Table 3 presents the results of our costs estimations. MCSE is the MCMC standard error and represents the accuracy of our simulation results. Ideally, it must be equal to zero which requires an infinite number of MCMC iterations. Columns 7-8 show the credible interval that provides a range for a parameter such that the probability that the parameter lies in that range is 0.95. For example, the probability that the mean of monthly operating cost for a vehicle in Sudan is between 512.7 and 572 is about 0.95. Column 9 shows the acceptance rate that specifies the proportion of proposed parameter values that was accepted by the algorithm. For example, an acceptance rate of 0.42 (for the purchase price in Sudan) means that 42% out of 40,000 proposal parameter values were accepted by the algorithm. Gelman et al. (1997) show that an optimal acceptance rate is 0.45. Therefore, our results provide near optimal acceptance rate.

Table 3 Cost Estimations.

Country	Variable	Posteriors								Fixed cost (c_f)
						95% credible interval				
		Mean	SD	MCSE	Median	min.	max.	Acc. Rate	Obs.	
Sudan	p	25,645.1	11.28	0.11	25,645.1	25,622.6	25,667	0.42	238	99
	r	13,837.0	5.43	0.05	13,837.0	13,826.2	13,847.5	0.45	205	
	c_o	541.9	15.14	0.15	541.9	512.7	572.0	0.44	205	
Kenya	p	23,950.8	5.63	0.06	23,950.9	23,939.8	23,962.0	0.48	310	255
	r	12,806.1	6.69	0.07	12,806.2	12,792.8	12,819.31	0.43	210	
	c_o	203.9	6.70	0.07	203.9	190.5	216.9	0.42	210	
Liberia	p	25,645.5	4.99	0.05	25,645.6	25,635.7	25,655.3	0.47	157	212
	r	10,937.3	8.42	0.08	10,937.4	10,920.8	10,953.7	0.41	142	
	c_o	394.5	8.41	0.08	394.5	377.8	410.8	0.43	142	
Iraq	p	24,315.7	5.61	0.05	24,315.6	24,304.9	24,326.8	0.40	321	203
	r	8,934.5	6.90	0.07	8,934.4	8,921.0	8,947.9	0.46	211	
	c_o	368.8	6.91	0.07	368.8	355.3	382.2	0.47	211	
Syria	p	23,875.7	10.05	0.10	23,875.7	23,856.1	23,895.6	0.45	49	132
	r	16,007.9	13.09	0.13	16,007.9	15,982.2	16,033.8	0.45	49	
	c_o	525.3	13.06	0.13	525.3	499.7	550.9	0.42	49	

Figure 8 shows the fleet sizes of the five countries studied in the whole period from 2000 to 2015. Table 4 summarizes all parameter values used in the numerical experiments.

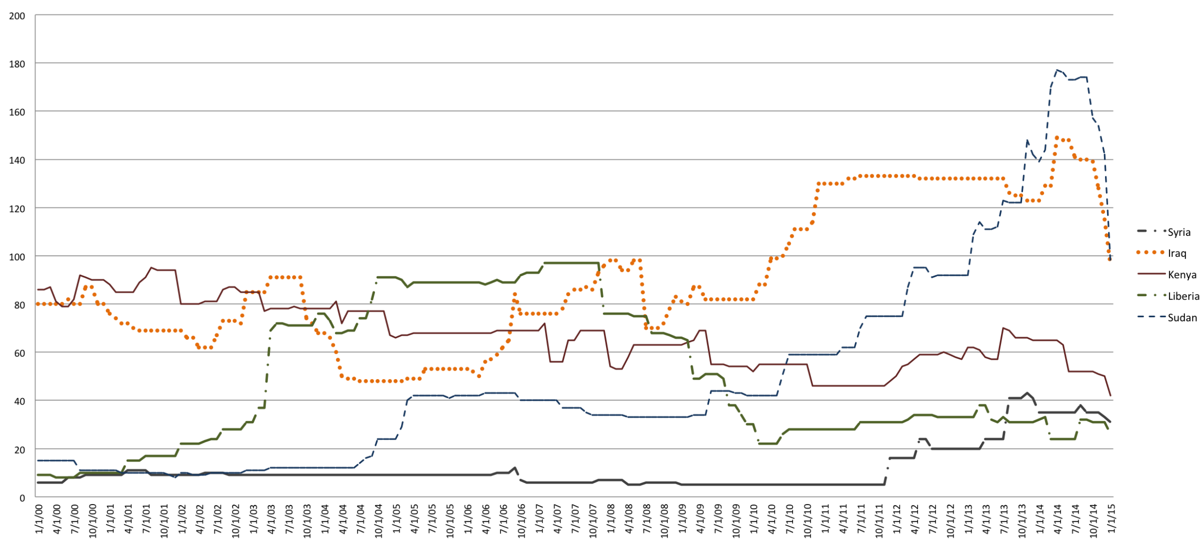

Figure 8 Fleet sizing of the five countries of operations in 2000-2015.

Table 4 Parameters used in the numerical experiments

Country	D_m*	d	c_f	c_o	p	r	γ	μ	σ	b_m**
Syria	9	1	396	1,575.9	23,876	16,008	0.03	19,871.41	17,977.72	0.61
Sudan	12	1	297	1,625.7	25,645	13,837	0.03	27,323.32	37,189.33	0.46
Liberia	49	2	636	1,183.5	25,646	10,937	0.06	132,397.61	108,118.60	0.11
Kenya	67	1	765	611.7	23,951	12,806	0.06	137,040.59	73,276.92	0.08
Iraq	90	2	609	1,106.4	24,316	8,934.5	0.03	195,915.24	106,445.80	0.06

* at a funding level of 100%, ** for $\nu = 0.75$