Applied Composite Materials
https://doi.org/10.1007/s10443-022-10039-y

A Micro–macro Damage Mechanics-based Model for Fatigue Damage and Life Prediction of Fiber-reinforced Composite Laminates

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Received: 7 December 2021 / Accepted: 23 April 2022

Abstract

A multidirectional damage model was proposed to predict fatigue damage evolution and final failure of composite laminates in this paper. A damage characterization model for composite laminates was established to characterize the influence of three main damage modes on the damaged mechanical behavior of composite laminates at micro–macro level. The damage evolution model was also established based on damage mechanics to predict the evolution of the three damage modes and stiffness degradation of composite laminates by means of damage characterization model. Then, a relationship between residual stiffness and residual strength was introduced, from which the residual strength could be obtained according to the predicted residual stiffness. When the residual strength is calculated to decrease to the maximum applied stress of fatigue loading after several cycles, the composite laminate was assumed to fail, and accordingly the fatigue life could be obtained. In order to verify the model, the predicted stiffness degradation and fatigue life of two cross-ply laminates under fatigue loadings with different stress levels were compared to experimental results. The standard derivation of stiffness degradation and average errors of fatigue between prediction results and experimental results were less than 0.1 and 8.26%, respectively, indicating the effectiveness and reliability of proposed model.

Keywords Composite laminates · Fatigue loading · Damage evolution · Fatigue life

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Published online: 17 June 2022

1 Introduction

Nowadays, composite materials are widely utilized for primary bearing structures in aircraft structures due to their excellent mechanical properties and designability. As a result, composite materials are more likely subjected to fatigue loadings, and hence more likely suffer from fatigue damages [1]. As is known, fatigue damages in composite materials could result in significant degradation of stiffness and strength, which greatly influences the bearing capability of composite materials, so the research on fatigue damages in composite materials, including the study on fatigue damage mechanisms, prediction of residual stiffness and fatigue life, has become a very important issue [2].

So far, many researchers have done plenty of studies on fatigue damages in composite materials, including analytical research and experimental research. Analytical models could be sorted into following categories [3, 4]: 1) residual strength model [5, 6], 2) residual stiffness model [7–12], 3) continuum damage mechanics (CDM)-based model [13, 14] and 4) micromechanics-based model [15]. Stojkovic et al. [6] developed a two-parameter analytical model for the prediction of residual strength of composite materials under fatigue loadings. Their model was proposed based on the normalization of the difference between the residual strength and maximal applied load in the constant amplitude cyclic loadings, and a single set of parameter values was used, which could reduce the required experimental effort to determine model parameters. Califano et al. [7–9] proposed a two-parameter residual stiffness degradation model to describe the fatigue behavior of composite materials. In their model, the concept of equivalent residual strength assumption and a modified damage accumulation rule based on the Miner's rule were introduced to study the mechanical behavior of composite materials under spectrum fatigue loadings. Besides, the effects of loading rate on the static and fatigue behavior of composite materials were also investigated by the authors [16]. Aoki et al. [13] proposed a simulation model to evaluate progressive damage in composite laminates by means of CDM. The model distinguished two kinds of damage modes, including intra-laminar damage and inter-laminar damage. They found that when considering both the intra- and inter-laminar fatigue damages, the predicted stiffness value at 300,000 cycles was 96.4% of the initial stiffness while the experimental values were 97.7% and 96.9%, which means that the behavior of the stiffness degradation of composite laminates up to the final failure could be successfully predicted. Llobet et al. [14] extended a mesoscale continuum damage model to predict residual strength and fatigue life. The plasticity under in-plane shear loading was considered in their constitutive model and linear or bilinear functions were adopted for softening laws. A theoretical model based on shear-leg theory was developed by Sorenson et al. [15] to predict fatigue limit of unidirectional fiber composites. They found that higher R-ratio and initial value of the interfacial sliding shear stress could increase the fatigue limit, also smaller fiber volume fraction and inter-facial fracture energy could result in a higher fatigue limit. They also concluded that the effects of residual stresses are insignificant for fiber volume fractions higher than about 50%.

Apart from analytical researches, several experimental studies were also conducted. Adam et al. [17] carried out four-point bending very high cycle fatigue (VHCF) tests on $[\pm 45]_s$ angle-ply laminates to investigate VHCF damage mechanisms. In the tests, transmitted light imaging and thermography were utilized for damage detection, and crack densities and delaminated area fraction were defined as damage variables to characterize the damage evolution process. They found that the basic damage mechanisms are the same for composites under High Cycle Fatigue loading and Very High Cycle Fatigue loading, and the extent of damage development strongly depends on the load level, e.g., delamination damage is found to be less progressive when maximum fatigue loading is smaller than $0.65\varepsilon_c$, where ε_c is the failure strain under static loading. Reis et al. [18] performed fatigue tests on composite laminates made of woven bi-directional layers with different stress ratios at room temperature. In their tests, the loss of laminates stiffness was monitored according to the peak load and displacement data. A series of sequence block tests were also performed to study the fatigue behavior under variable amplitude block loadings. Shen et al. [19] conducted fatigue tests on cross-ply composite laminates to research the fatigue damage initiation and evolution law of composite laminates. The light transmission method was used to observe and record damage state, and the damaged stiffness was also calculated corresponding to the strain–stress data. Krishnan et al. [20] adopted Digital Image Correlation method to monitor the surface strain evolution in composite laminates with open hole, and a progressive fatigue damage model was proposed based on the residual stiffness and strength method. Besides, several experimental studies were also carried out to investigate the damage mechanisms of composite structures, e.g., composites tubes [21] and composite sandwich T-joints [22].

In many analytical models, like the residual strength models, residual stiffness models and some CDM models, the degradation of residual strength and stiffness were described with empirical functions, rather than based on the practical damage mechanisms. In order to predict the degradation of material properties and fatigue life of composite laminate based on the physical damage modes in composite laminates, a micro–macro damage relationship deduced previously was introduced to the fatigue model, which could link the macro phenomenological damage variables to the microscopic practical damage mechanism. Besides, in order to characterize the influence of three specific damage modes on the multidirectional stiffness properties of composite laminates under fatigue loadings, including transverse matrix cracks, local delamination and fiber breakage, three groups of macro damage variables were defined based on the damage mechanics, and the constitutive equations of composite laminates were also deduced in terms of the defined damage variables. Then evolution laws of the three damage modes under fatigue loading were established. In order to predict the initiation and evolution of transverse matrix cracks, the previously presented method based on initial matrix crack initiation life was introduced. The thermodynamic forces relating to local delamination damage variables were introduced as a new damage evolution variable in the model and then a macroscopic phenomenological equation was deduced to describe the evolution of local delamination damage. According to the damage characterization and evolution model, stiffness degradation of composite laminates under fatigue loading could be calculated. To predict the emergence of fiber breakage damage, the relationship between residual stiffness and residual strength was introduced, and then the fatigue life of composite laminates was obtained accordingly.

2 Damage Model for Composite Laminates under Fatigue Loading

2.1 Damage Mechanism of Composite Laminates under Fatigue Loading

Figure 1 shows typical damage modes in composite laminates, especially cross-ply laminates, under fatigue loadings, including matrix cracking, local delamination and fiber breakage [23–25], in which Fig. 1a illustrates the matrix cracking damage [23], Fig. 1b illustrates the local delamination damage [24] and Fig. 1c shows the fiber breakage damage [25]. Under

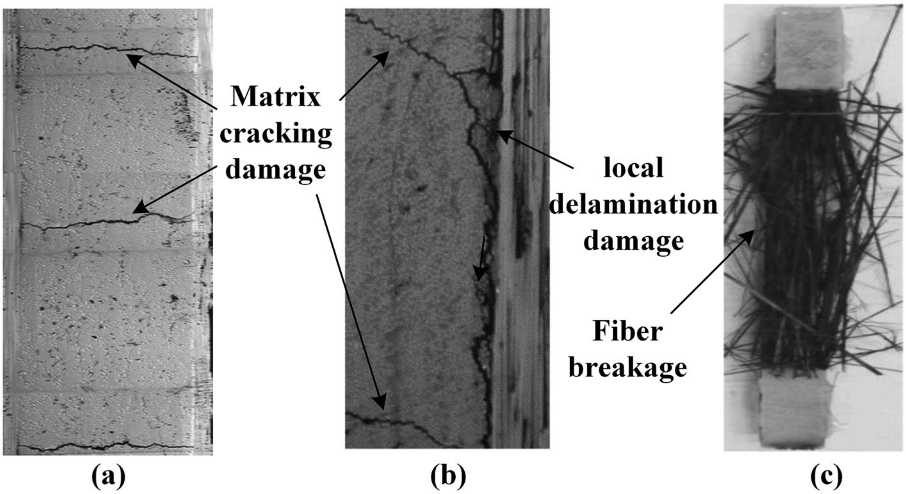

Fig. 1 Typical damage modes in cross-ply composite laminates under fatigue loading. (**a**) matrix cracking damage [23], (**b**) local delamination damage [24], (**c**) fiber breakage damage [25]

cyclic loadings, matrix cracks would first initiate in weaker plies, they are also known as transverse matrix cracks since these cracks are often parallel to fiber direction. With the increase of cycle number, more and more transverse matrix cracks would exist and finally saturate, resulting in the continuous degradation of stiffness, especially the transverse modulus and shear modulus.

As the transverse matrix cracks tend to saturate, local delamination would initiate and develop at the edges of transverse matrix cracks due to the stress concentration effect, leading to the further degradation of stiffness properties. With the emergence of more and more damages in matrix and interface, fibers in composites would bear more and more loading. Although applied stress of fatigue loading could be smaller than fiber strength, the stress concentration effect would increase the local stress in fibers and finally fiber breakage occurs, which causes the catastrophic failure of whole structure. Typical damage evolution in composite materials under fatigue loading is shown in Fig. 2 [26, 27].

2.2 Damage Characterization Model under Fatigue Loading

In order to model the influence of three main damage modes on the mechanical behavior, a damage characterization model for composite laminate was considered. In the previous work [28], the consistency of damage characterization model under quasi-static loading and fatigue loading was discussed and proven valid both under quasi-static and fatigue loading. Hence, the damaged strain energy density of elementary ply in composite laminates under fatigue loading was defined as follow with the assumption of plane-stress state [27],

$$
\begin{aligned}
e_d = \frac{1}{2}\Bigg(&\frac{{\sigma_{11}}^2}{E_1^0\left(1-d^f\right)} + \frac{{\sigma_{22}}^2}{E_2^0\left(1-\left[\sigma_{22}\right]^+ d_{22}^m\right)\left(1-d_{22}^l\right)} \\
&+ \frac{{\tau_{12}}^2}{G_{12}^0\left(1-d_{12}^m\right)\left(1-d_{12}^l\right)} - \frac{2\nu_{12}^0\sigma_{11}\sigma_{22}}{E_1^0\left(1-d^f\right)}\Bigg)
\end{aligned} \tag{1}
$$

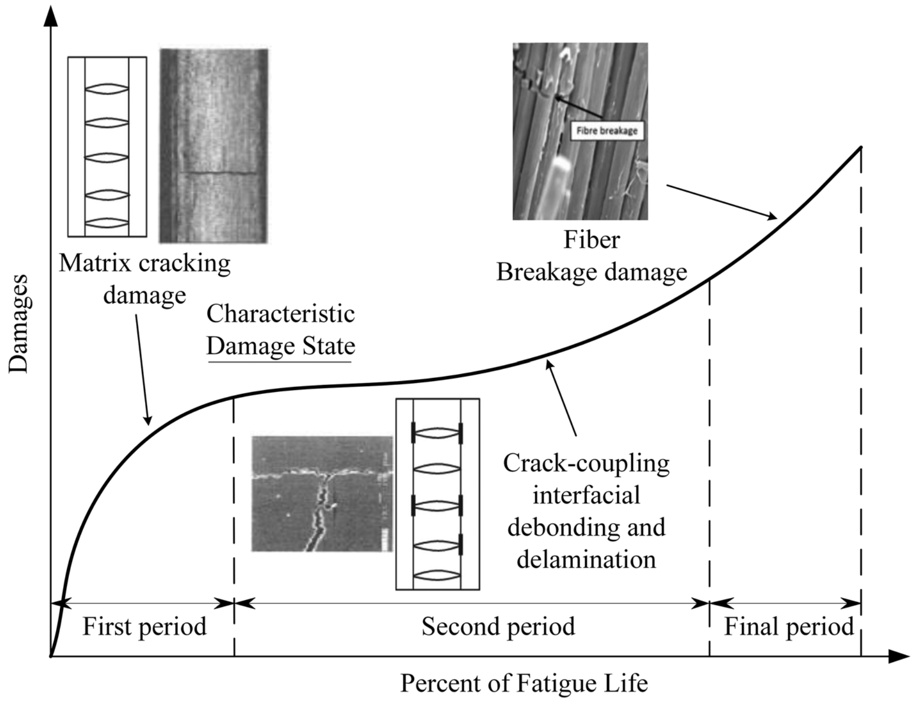

Fig. 2 Damage evolution in composites under fatigue loading [26, 27]

where e_d denotes damaged strain energy density, $E_1{}^0$ denotes original longitudinal modulus of ply, $E_2{}^0$ denotes the original transverse modulus, and $G_{12}{}^0$ is the original shear modulus, $v_{12}{}^0$ denotes the Poisson's ratio. $d_{12}{}^m$ and $d_{22}{}^m$ are macro matrix cracking damage variables, which characterize the influence of matrix cracking damages on the shear modulus and transverse modulus respectively. Similarly, $d_{12}{}^l$ and $d_{22}{}^l$ are macro local delamination damage variables, and d^f denotes fiber breakage damage variable. σ_{11}, σ_{22} and τ_{12} denote the longitudinal stress, transverse stress and shear stress, respectively. The blanket $[\cdot]^+$ represents the Heaviside function. Then the strain–stress relationships of damaged ply could be obtained based on the definition of strain energy release rate,

$$\varepsilon_{ij} = \frac{\partial e_d}{\partial \sigma_{ij}} \tag{2}$$

where ε_{ij} denote strain components. According to Eq. (1), the constitutive equations could be written as [27].

$$\begin{aligned}
\varepsilon_{11} &= \frac{\sigma_{11}}{E_1^0(1-d^f)} - \frac{v_{12}^0}{E_1^0(1-d^f)}\sigma_{22} \\
\varepsilon_{22} &= \frac{\sigma_{22}}{E_2^0\left(1-[\sigma_{22}]^+ d_{22}^m\right)(1-d_{22}^l)} - \frac{v_{12}^0}{E_1^0(1-d^f)}\sigma_{11} \\
\gamma_{12} &= \frac{\tau_{12}}{G_{12}^0(1-d_{12}^m)(1-d_{12}^l)}
\end{aligned} \tag{3}$$

Based on the above equations, the influence of the forementioned three damage modes is modelled, and accordingly, the damaged stiffness properties could be calculated based on Eq. (3). The Heaviside function and macro matrix cracking damage variables characterize the influence of matrix cracking damage on material stiffness properties under different loading conditions, since under tension and shear loading, matrix cracks are always open, resulting in the degradation of stiffness properties, but they are closed under compression loading, which means they won't have influence on the stiffness properties [29]. Also, the macro delamination damage variables and fiber failure damage variable could reflect the influence of local delamination damage and fiber damages on the material mechanical properties under different loading conditions. As a result, the damage characterization model in this paper could be used for different loading boundary conditions. Then, by means of the Classic Laminate Theory (CLT) method, the damaged constitutive equations of entire composite laminates then could be obtained.

It is known that the change of temperature could result in the thermal strain in composite laminates, which could be calculated with following equation [30],

$$\varepsilon^T = \Delta T \alpha \tag{4}$$

where ε^T denotes residual strain, ΔT denotes temperature difference between the loading environment and manufacturing environment, α denotes thermal coefficient of expansion. And then the strain–stress relationship in Eq. (3) could be modified as

$$\begin{aligned} \varepsilon_{11} &= \frac{\sigma_{11}}{E_1^0\left(1-d^f\right)} - \frac{\nu_{12}^0}{E_1^0\left(1-d^f\right)}\sigma_{22} + \alpha_1 \Delta T \\ \varepsilon_{22} &= \frac{\sigma_{22}}{E_2^0\left(1-\left[\sigma_{22}\right]^+ d_{22}^m\right)\left(1-d_{22}^l\right)} - \frac{\nu_{12}^0}{E_1^0\left(1-d^f\right)}\sigma_{11} + \alpha_2 \Delta T \\ \gamma_{12} &= \frac{\tau_{12}}{G_{12}^0\left(1-d_{12}^m\right)\left(1-d_{12}^l\right)} + \alpha_6 \Delta T \end{aligned} \tag{5}$$

2.3 Damage Evolution Model under Fatigue Loadings

In order to model the damage evolution process in composite laminates under fatigue loading, evolution laws of three main damage modes were established, and then stiffness degradation under fatigue loading could be predicted based on the damage characterization model.

2.3.1 Evolution Law of Matrix Cracking

Although the evolution of transverse matrix cracking makes up a small proportion of the whole fatigue life, it could have great influence on the stiffness properties of composite laminates, especially the transverse elastic modulus [31]. When applied maximum stress is smaller than the initial matrix crack initiation stress, matrix cracks will initiate after several cycles. In order to predict the initiation and evolution of transverse matrix cracks in composite laminates in this case, the initial matrix crack initiation life was introduced [28],

$$\lg N_{\mathrm{ini}} = \lg K_0 - 2\lambda \lg \sigma_{\mathrm{ti}}^{90} + 2\lambda \lg \sigma_{\max}^{90} \tag{6}$$

where N_{ini} denotes initial matrix crack initiation life, $\sigma^{90}{}_{\max}$ denotes maximum stress in cracked plies, $\sigma^{90}{}_{\mathrm{ti}}$ denotes the initiation stress of initial matrix crack in cracked plies under quasi static loading, K_0 and λ are material parameters.

According to the above equation, the initial matrix crack initiation life could be determined once the maximum stress in cracked plies is calculated with the CLT method. When the cycle number of fatigue loading reaches the local initial matrix crack initiation life, matrix crack is assumed to initiate. Detailed damage evolution model of matrix cracking in composite laminates under fatigue loading has been described in the previous work [28].

When the applied maximum stress is larger than the initial matrix crack initiation stress, matrix cracks will initiate in the first cycle, which is the same as the quasi-static loading condition. Then, with the increase of cycle number, more and more transverse matrix cracks initiate and finally tend to saturate.

Based on the matrix cracking evolution law, the increase of matrix crack density was predicted, as described in the previous work [28], then the damaged stiffness matrix of cracked plies could be obtained based on the COD theory in micromechanics [27],

$$Q_{\mathrm{d}} = \left[I + \frac{\rho q}{E_1^0} Q_0 U \right]^{-1} Q_0 \tag{7}$$

where Q_{d} denotes the damaged stiffness matrix, ρ denotes the density of transverse crack, I is the unit matrix, q denotes the ratio of the extension length to the total length, which considers the condition that the transverse crack has not run through the width direction of specimen, U denotes the normalized displacement matrix of matrix crack surface. ${E_1}^0$ and Q_0 are the undamaged longitudinal modulus and stiffness matrix respectively. Then, the damaged stiffness matrix of composite laminates could be calculated by means of CLT method, and finally, the damaged elastic modulus of composite laminates could be obtained [28],

$$\begin{aligned} E_1(\rho) &= \frac{Q_{11}^{\mathrm{d}} Q_{22}^{\mathrm{d}} - {Q_{12}^{\mathrm{d}}}^2}{Q_{22}^{\mathrm{d}}} \\ E_2(\rho) &= \frac{Q_{11}^{\mathrm{d}} Q_{22}^{\mathrm{d}} - {Q_{12}^{\mathrm{d}}}^2}{Q_{11}^{\mathrm{d}}} \\ G_{12}(\rho) &= Q_{66}^{\mathrm{d}} \end{aligned} \tag{8}$$

where $E_1(\rho)$, $E_2(\rho)$ and $G_{12}(\rho)$ are damaged longitudinal modulus, damaged transverse modulus and damaged shear modulus respectively.

Besides, according to the definition of macro-level damage variables in Eq. (1) and damaged elastic modulus at micro level in Eq. (8), a macro–micro relationship could be obtained [27],

$$\begin{aligned} d_{11}^m &= 1 - \frac{E_1(\rho)}{E_1^0} = f_{11}(\rho) \\ d_{22}^m &= 1 - \frac{E_{22}(\rho)}{E_2^0} = f_{22}(\rho) \\ d_{12}^m &= 1 - \frac{G_{12}(\rho)}{G_{12}^0} = f_{12}(\rho) \end{aligned} \tag{9}$$

Accordingly, the macro damaged mechanical behavior and stiffness degradation of composite laminates could be modelled based on the practical damage mechanisms at micro level.

2.3.2 Evolution Law of Delamination

As a main damage mode in the second stage of damage evolution process, local delamination often initiates form free edge and tips of matrix cracks at the interface of adjacent plies when transverse matrix cracking tends to saturate [32, 33]. Similar to transverse matrix cracking, local delamination could result in the degradation of stiffness properties, including transverse elastic modulus and in-plane shear modulus [34].

It has been shown that Paris-law equation could well describe the evolution of local delamination in composite materials under fatigue loading [13, 35–37], hence a Paris-like equation is defined based on the damage mechanics at macro level,

$$\frac{\Delta D}{\Delta N} = A(\Delta Y)^{\beta} \tag{10}$$

where A and β are material parameters, D denotes the delamination damage variable which characterizes the delamination damage state in composite materials and is defined as $D=1\text{-}E_1{}^{d}/E_1{}^{0}$, N denotes the cycle number of fatigue loading, ΔY is the range of damage evolution variable. Taking the coupling between transverse and shear stress into consideration, the damage evolution variable Y is defined as follow [27],

$$Y = \sqrt{Y_{12} + kY_{22}} \tag{11}$$

where k is the coupling parameter, Y_{12} and Y_{22} denote the thermodynamic forces relating to damage variables. Based on the damage mechanics theory, they are deduced from the damaged strain energy density as follow [27],

$$\begin{aligned} Y_{22} &= -\frac{\partial e}{\partial d_{22}^{l}} = \frac{1}{2}\frac{{\sigma_{22}}^{2}}{E_2^0\left(1-d_{22}^{m}\right)\left(1-d_{22}^{l}\right)^{2}} \\ Y_{12} &= -\frac{\partial e}{\partial d_{12}^{l}} = \frac{1}{2}\frac{{\tau_{12}}^{2}}{G_{12}^0\left(1-d_{12}^{m}\right)\left(1-d_{12}^{l}\right)^{2}} \end{aligned} \tag{12}$$

The proposed model describes the evolution law of local delamination damage at macro phenomenological level and predicts the stiffness degradation of composite laminate due to local delamination under fatigue loading. It should also be noted that local delaminate damages would initiate before the saturation of transverse cracks under fatigue loadings, especially the loading with small stress levels. This kind of interaction between transverse matrix cracking and local delamination could result in the great difficulty in determining the stress field and the stiffness degradation due to local delamination during this stage, taking these into consideration, it is assumed that local delaminate damages would initiate after the saturation of transverse matrix cracks, and the evolution law of local delamination would begin once the matrix crack density reaches the critical value.

2.3.3 Evolution Law of Fiber Breakage

Under fatigue loading, more and more damage initiates in the matrix and interface with the continuous increase of cycle number, and in consequence the fibers would bear more and more loadings. When the stress in the fibers reaches the local failure strength of fibers, then fiber breakage occurs. Hence, a brittle failure law as shown is often used to predict the initiation of fiber breakage damage [38],

$$\frac{\sigma_f}{S_f} = 1 \tag{13}$$

where σ_f denotes the bearing stress in the fibers and S_f denotes the failure strength of fibers.

In order to utilized the above criterion, the stress in fibers should be determined accurately at micro level. As a result, damage modes including matrix cracks, local delamination and fiber-matrix debonding, and the interaction between different damage modes should all be considered since they all have influence on the stress distribution, making it much difficult to calculate the fiber stress theoretically. Besides, the random distribution of fiber strength due to manufacturing defects, e.g., Weibull distribution, should also be accounted for. To overcome the difficulty of determining bearing stress in fibers at micro level, an equivalent method is adopted in this paper to predict the initiation of fiber breakage damage in composite laminates under fatigue loading at macro level. According to Eq. (13), the increase of bearing stress σ_f due to accumulated damages is equivalent to the decrease of fiber failure strength S_f. Besides, the occurrence of fiber breakage damage is viewed as the reason of structural failure, then the decrease of fiber failure strength S_f is further equivalent to the reduction of failure strength of composite laminate. On the other hand, the accumulated damages would result in the stiffness degradation of composite laminate, so the decrease of composite laminate strength could be linked to its stiffness degradation. The relation between residual strength and residual stiffness of composite laminate is introduced as [39].

$$D_S = \left(D_E\right)^w \tag{14}$$

where w is a material parameter, D_S and D_E denote residual strength damage variable and residual stiffness damage variable respectively which characterize the degradation of residual strength and residual longitudinal modulus, and they are defined as follow [39],

$$\begin{aligned} D_E &= \frac{E_1^0 - E_1^d}{E_1^0 - E_1^{cr}} \\ D_S &= \frac{S_0 - S_r}{S_0 - \sigma_{\max}} \end{aligned} \tag{15}$$

where ${E_1}^0$ and S_0 denote the initial longitudinal elastic modulus and failure strength of composite laminate, ${E_1}^d$ and S_r denote the damaged longitudinal elastic modulus and strength, ${E_1}^{cr}$ is the critical longitudinal elastic modulus when the composite laminate fails under fatigue loading and $\sigma_{\max}$ denotes the maximum longitudinal stress in composite laminate.

Then the residual strength of composite laminate under certain cycle number could be obtained as

$$S_r = S_0 - \left(S_0 - \sigma_{\max}\right)\left(\frac{E_1^0 - E_1^d}{E_1^0 - E_1^{cr}}\right)^w \tag{16}$$

The damaged longitudinal elastic modulus ${E_1}^d$ could be determined based on the evolution law of transverse matrix cracking and local delamination, then the corresponding residual strength could be calculated. When the obtained residual strength is smaller than the applied maximum stress of fatigue loading, the composite laminate was considered to fail finally and the fatigue life could be obtained. It is also assumed that the final failure of composite laminates indicated the occurrence of numerous fiber breakage, and the breakage of single fiber due to the manufacturing defects was neglected, and hence, the occurrence of fiber breakage damages was also predicted accordingly.

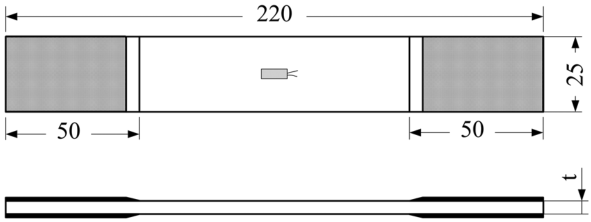

Fig. 3 Specimen configuration of cross-ply laminates

According to the Eq. (16), we could see that when the damaged longitudinal stiffness $E_1{}^{d}$ reaches the critical value $E_1{}^{cr}$, the residual strength of the laminate S_r would decrease to the maximum applied stress σ_{max}, which means that the laminate would fail then. Thus, the definition of fatigue life in this paper based on the occurrence of fiber breakage damages is essentially the same as the definition based on the stiffness degradation limit, which is often utilized in the structural application.

The model could be used to analyze various multidirectional laminates made of unidirectional plies since it is established on the basis of micro–macro damage mechanics and CLT method. The damage characterization model was established on the basis of elementary ply, and the coordinate transformation equations could be adopted to obtain the constitutive equations for any θ-ply in laminates. In the damage evolution model, the initial matrix crack initiation life Eq. (6) could be obtained for cracked plied in multidirectional laminates by means of micro damage mechanics and CLT method. Besides, the evolution law of local delamination and fiber breakage was established based on the macro damage mechanics with phenomenological damage variables, so they are valid for multidirectional laminates.

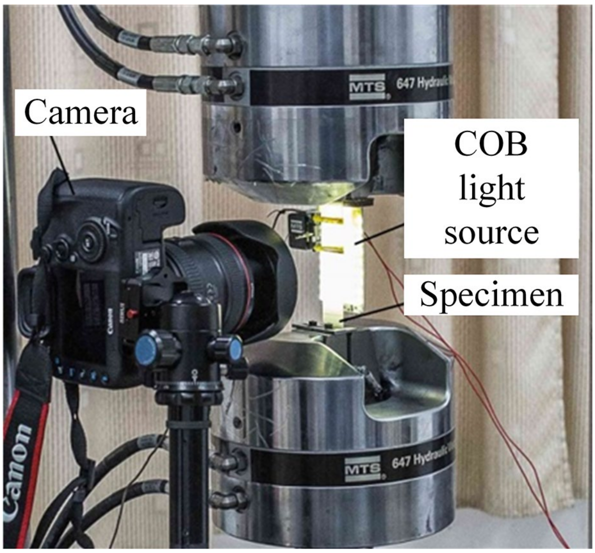

Fig. 4 Illustration of test condition

Table 1 Several basic mechanical properties of E-glass/YPX-3300 [19]

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	ν_{12}
43157	10810	4215	0.306

3 Results and Discussion

3.1 Introduction of Experiments

Experimental results of cross-ply laminates under fatigue loading are used to determine the parameters in the proposed model [19]. The fatigue tests were carried out at a room temperature dry environment with an MTS 370.10 test machine. The loading condition is load-controlled and the loading rate is 8 Hz, which is in accordance with ASTM D3479 test standard. In the test, an MTS extensometer was also used to measure the strain, and two cross-ply composite laminates made from Henderson composite materials co., LTD were used, which consisted of the high strength E-glass fiber and 3300 epoxy resin from Kunshan Yubo Composite Materials Co., Ltd. The composite laminates were manufactured by hand lay-up of pre-preg followed autoclave curing, and the volume fraction of the fibers is 70%, and the experiment condition is shown in Fig. 4.

The lay-up includes $[0/90_4]_s$ and $[0_2/90_4]_s$, and the specimen configuration is also designed based on the ASTM D3479 test standard, as shown in Fig. 3. Experimental results [23] show that in cross-ply laminates, matrix cracks are often formed parallel to the fiber direction in 90º plies, and they could easily initiate and propagate through the whole thickness and width of the cracked plies, and then delamination damages would initiate and develop at the tips of matrix cracks, so these damages in cracked 90º plies and inter-plies could be observed clearly just by looking at the surface in cross-ply laminates of glass-fiber reinforced composite. On the contrary, in multidirectional laminates, matrix cracks in off-axial plies are often complex and have curved patterns due to the inclined principal stress directions. Also, an off-axial ply adjacent to a 90º ply in multidirectional laminates would generate numerous partial cracks, and some of these partial cracks may not form through cracks under fatigue loadings, which would greatly influence the accuracy of damage record results in the experiment. As a result, these cross-ply laminates were considered in the experiment to obtain and record the damage state more clearly and accurately. Besides, some previous experimental researches [32, 40–42] showed that typical damage mechanisms, would exist in composite laminates under fatigue loadings with stress ratio defined as 0.1. Thus, in order to clearly investigate the damage initiation and evolution in composite laminates under fatigue loadings, the stress ration R in our tests was chosen as 0.1 based on previous successful experimental experience, and

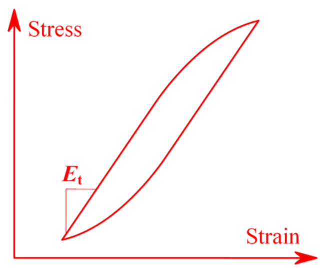

Fig. 5 Illustration of damaged longitudinal modulus under fatigue loading

Table 2 Material parameters in transverse matrix crack evolution

Configuration	K_0	λ
$[0/90_4]_s$	160.63	-4.63
$[0_2/90_4]_s$	72.88	-5.51

six different stress levels were defined corresponding to the tensile failure stress in the test. Among them, the smaller four stress levels were selected based on the initiation stress of the initial matrix crack which was determined in the static tests. The initial matrix crack initiation stress for $[0_2/90_4]_s$ and $[0/90_4]_s$ laminate is 105.07 MPa and 78.59 MPa, respectively, and then the four maximum applied stresses were chosen as 50%, 60%, 70% and 80% of the initiation stress for the four small stress levels. For The larger two stress levels, the maximum applied stresses were selected as 50% and 60% of the tensile failure stress, and the failure stresses are 424.54 MPa and 239.33 MPa for $[0_2/90_4]_s$ and $[0/90_4]_s$ laminate, respectively. The initial matrix crack initiation stress and tensile failure stress were both determined in the static tests [19]. It should be noted that for the purpose of consistence, all the stress levels were expressed corresponding to the tensile failure stress in the figures. Several basic mechanical properties of E-glass/YPX-3300 are shown in Table 1 [19].

The experimental results, including the degradation of longitudinal elastic modulus and evolution of matrix crack density was obtained. In this study, the damaged longitudinal elastic modulus of specimen was defined as the tangent modulus E_t, as shown in Fig. 5, which could be calculated based on the stress–strain curves recorded by the machine and extensometer, and then some cycle numbers and corresponding damaged longitudinal modulus were recorded to obtain the degradation of longitudinal stiffness property under fatigue loading. The normalized longitudinal modulus was defined as dividing the damaged longitudinal modulus of composite laminate by the original value. At the same time, when recording the cycle number, the photo of specimen was also taken to record the damage state, then the matrix crack density was calculated by dividing the numbers of matrix cracks by the gauge length of the specimen, and as a result, the evolution of matrix crack density under fatigue loading could also be obtained. Besides, the large difference between the damage growth rate at the beginning and later period of damage evolution process results in the large difference between the recorded data points distances, the logarithmic

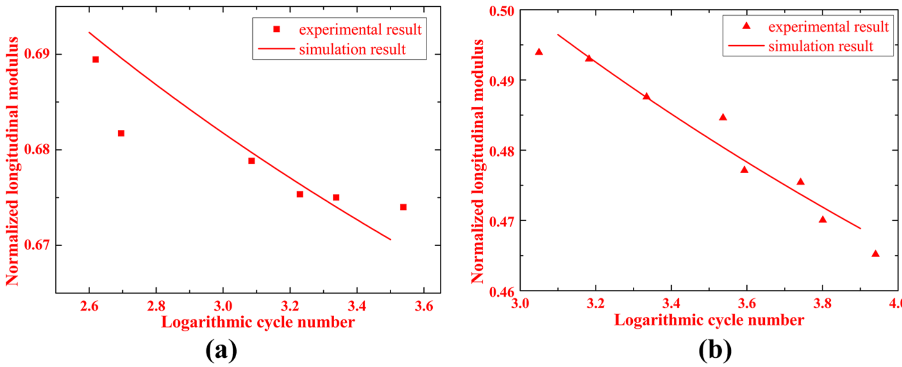

Fig. 6 Comparing the prediction results of normalized longitudinal modulus with experimental results. (**a**) $[0_2/90_4]_s$ laminate, (**b**) $[0/90_4]_s$ laminate

Table 3 R^2 value of local delamination evolution parameters fitting results

Configuration	R^2
$[0/90_4]_s$	0.8573
$[0_2/90_4]_s$	0.9388

value of the fatigue lifetime was used in our fatigue tests for the sake of simplicity and convenience.

At the beginning of fatigue test process, damage growth rate would be rather high, and in order to record the damage evolution process accurately and clearly, the distance between data points was chosen quite small, which is around several hundred cycles. However, at the later period of test process before structure failure, the damage growth rate greatly decreased, hence the distance between data points was chosen quite large, around tens of thousands of cycles or larger. Due to the large difference between the data points distances, the logarithmic value of the fatigue lifetime was used in our manuscript for the sake of simplicity and convenience.

3.2 Determination of Model Parameters

The material parameters in transverse matrix crack evolution law, including K_0 and λ, have been determined previously [28], so they are listed as follow. In the fatigue experiment, the initiation lives of initial matrix crack in cracked plies in laminates under different stress levels were recorded, and then the maximum stress in cracked plies could be determined by means of maximum applied stress and Classic Laminate Theory (CLT). As a result, the initial matrix cracks initiation life curve of composite laminates, which is marked as $S\text{-}N_{\text{ini}}$ curve, could be obtained. Then, the two material parameters could be determined by fitting the $S\text{-}N_{\text{ini}}$ curve with Eq. (6). In Table 2, the material parameters keep two decimals.

The evolution law of local delamination damages is established based on the macroscopic phenomenological method, and for composite laminates with different lay-up, the parameters would be different. So, the material parameters were calibrated by the comparison of the stiffness degradation caused by delamination with experimental results, as shown in Fig. 6, and the material parameters are listed in Table 4. Experimental results [19] show that under

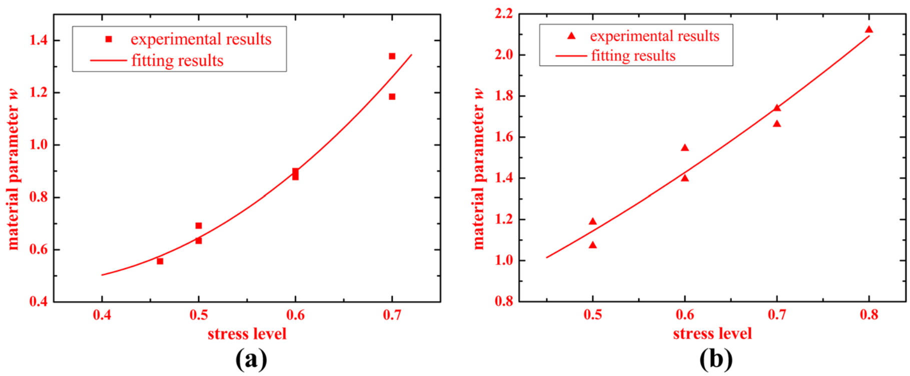

Fig. 7 Fitting results of fiber breakage evolution parameters. (**a**) $[0_2/90_4]_s$ laminate, (**b**) $[0/90_4]_s$ laminate

Table 4 Material parameters in delamination evolution model

Configuration	A	β
$[0/90_4]_s$	0.035	8.1
$[0_2/90_4]_s$	0.021	8.4

fatigue loading with rather low stress levels, transverse matrix cracks in cross-ply laminates would initiate and propagate at a rather low rate, and local delamination damages would initiate much earlier than the saturation of transverse matrix cracks, resulting in the more serious interaction between the transverse matrix cracks and local delamination damages. On the other hand, when suffering fatigue loading with larger stress level, a large quantity of other diffuse damages like micro longitudinal matrix cracks would initiate in the laminates, which was neglected in the proposed model. Taking these reasons into consideration, in order to obtain an accurate result, the 60% stress level corresponding to failure strength was selected. The R^2 values of fitting results for two laminates is shown in Table 3.

It is found that the material parameter w in fiber breakage damage evolution model is related to the composite laminate configuration and stress level, so for different composite laminates, the experimental results of material parameter w under different stress levels are calculated to determine the parameter, as shown in Fig. 7, also the R^2 values of fitting results for two laminates is shown in Table 5.

According to the fitting results, the relationships between the material parameters and stress level are obtained as follow,

$$w = 5.48s^2 - 3.5s + 1.03 \tag{17}$$

$$w = 1.63s^2 + 1.04s + 0.22 \tag{18}$$

where s denotes the stress level corresponding to the failure strength of composite laminate. Equation (17) is valid for $[0_2/90_4]_S$ laminate, and Eq. (18) is valid for $[0/90_4]_S$ laminate.

3.3 Model Verification

In order to verify the proposed model, the prediction results of stiffness degradation curves and *S–N* curves were compared to the experimental results of the two cross-ply laminates under fatigue loadings with six different stress levels. Figure 8 shows the comparison between model prediction results and experimental results of damaged longitudinal modulus of $[0_2/90_4]_s$ cross-ply laminate under fatigue loadings with six stress levels. It should be noted that in the figures, the test result_1 and test result_2 represent the experimental results of two individual specimens, respectively.

The standard deviation values between our prediction results and experimental results of longitudinal modulus of $[0_2/90_4]_s$ laminate under the six stress levels were shown in Table 6.

Table 5 R^2 value of fiber breakage evolution parameters fitting results

Configuration	R^2
$[0/90_4]_s$	0.9612
$[0_2/90_4]_s$	0.9701

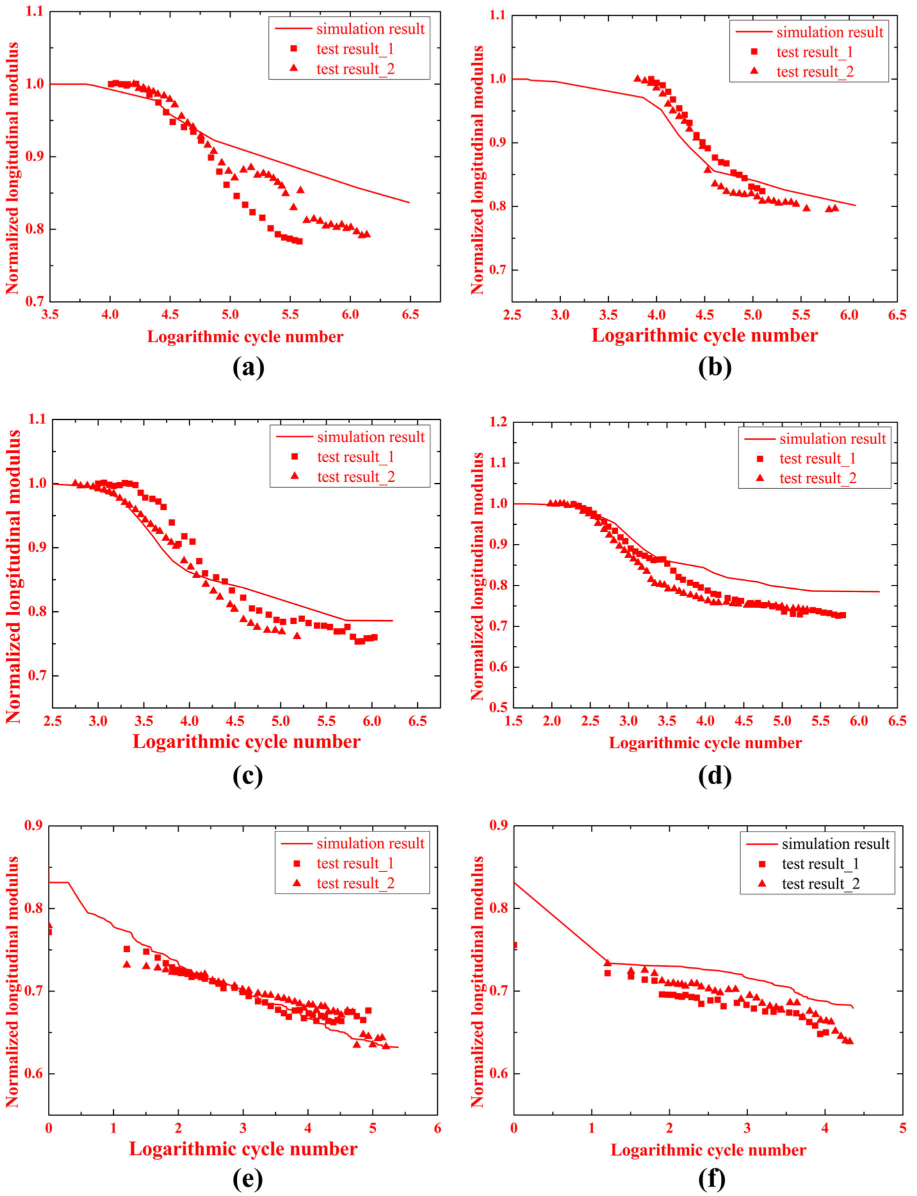

Fig. 8 Comparison between prediction results and experimental results of longitudinal modulus of $[0_2/90_4]_s$ laminate with different stress levels corresponding to tensile failure strength. (**a**) 15%, (**b**) 17%, (**c**) 19%, (**d**) 22%, (**e**) 50%, (**f**) 60%

Figure 9 shows the comparison between model prediction results and experimental results of damaged longitudinal modulus of $[0/90_4]_s$ cross-ply laminate under fatigue loadings with six stress levels.

Table 6 Standard deviation values for stiffness degradation of $[0_2/90_4]_s$ laminate

Stress level	15%	17%	19%	22%	50%	60%
Standard deviation values	0.0483	0.0268	0.0304	0.0474	0.0174	0.0305

The standard deviation values between our prediction results and experimental results of longitudinal modulus of $[0/90_4]_s$ laminate under the six stress levels were shown in Table 7.

The comparison results between model prediction results and experimental results of *S–N* curves of $[0_2/90_4]_s$ and $[0/90_4]_s$ laminate is shown in Fig. 10.

The standard deviation values of logarithmic fatigue life of two cross ply laminates are shown in Table 8.

According to the comparison results in Figs. 8, 9, and 10, and the standard deviation values in Tables 6, 7, and 8, it could be concluded that the tendency of prediction results of damaged longitudinal modulus degradation under six different stress levels and *S–N* curves of two composite laminates are in accordance with experimental results.

3.4 Discussion

In order to evaluate the accuracy of proposed model in the prediction of fatigue life of composite laminates, the average error between obtained prediction results and experimental results of logarithmic fatigue life is calculated and compared to several results available in the literature, as shown in Table 9.

According to the comparison results, the proposed model could predict the fatigue life of composite laminates more accurately, indicating that the proposed model is valid for the prediction of final fatigue life of composite laminates under fatigue loadings.

Based on the standard deviation values in Tables 6 and 7, the prediction results of damaged longitudinal modulus degradation turn out larger than experimental results, especially for fatigue loadings with smaller stress ratios, e.g., for $[0_2/90_4]_s$ laminate under fatigue loadings with 15% stress level in Fig. 8a, and for $[0/90_4]_s$ laminate under fatigue loadings with 23% and 26% stress level in Fig. 9b and c. This is mainly caused by the neglect of micro diffuse damages in the proposed model, such as micro partial cracks, fiber-matrix debonding and so on. These diffuse damages are muck likely to emerge in composite laminates under fatigue loadings with rather low stress level, and they would contribute to the degradation of stiffness properties. Also, the interaction of different damages exists under fatigue loadings, which was also neglected in the proposed model. It was observed in the experiment that local delamination damages would initiate before matrix cracks saturate, and the interaction between local delamination and matrix cracks could result in the further stiffness degradation. As a result, the prediction results turn out greater compared to the experimental results, especially for fatigue loadings with smaller stress levels.

Table 7 Standard deviation values for stiffness degradation of $[0/90_4]_s$ laminate

Stress level	20%	23%	26%	30%	50%	60%
Standard deviation values	0.0823	0.0517	0.0755	0.105	0.0214	0.0408

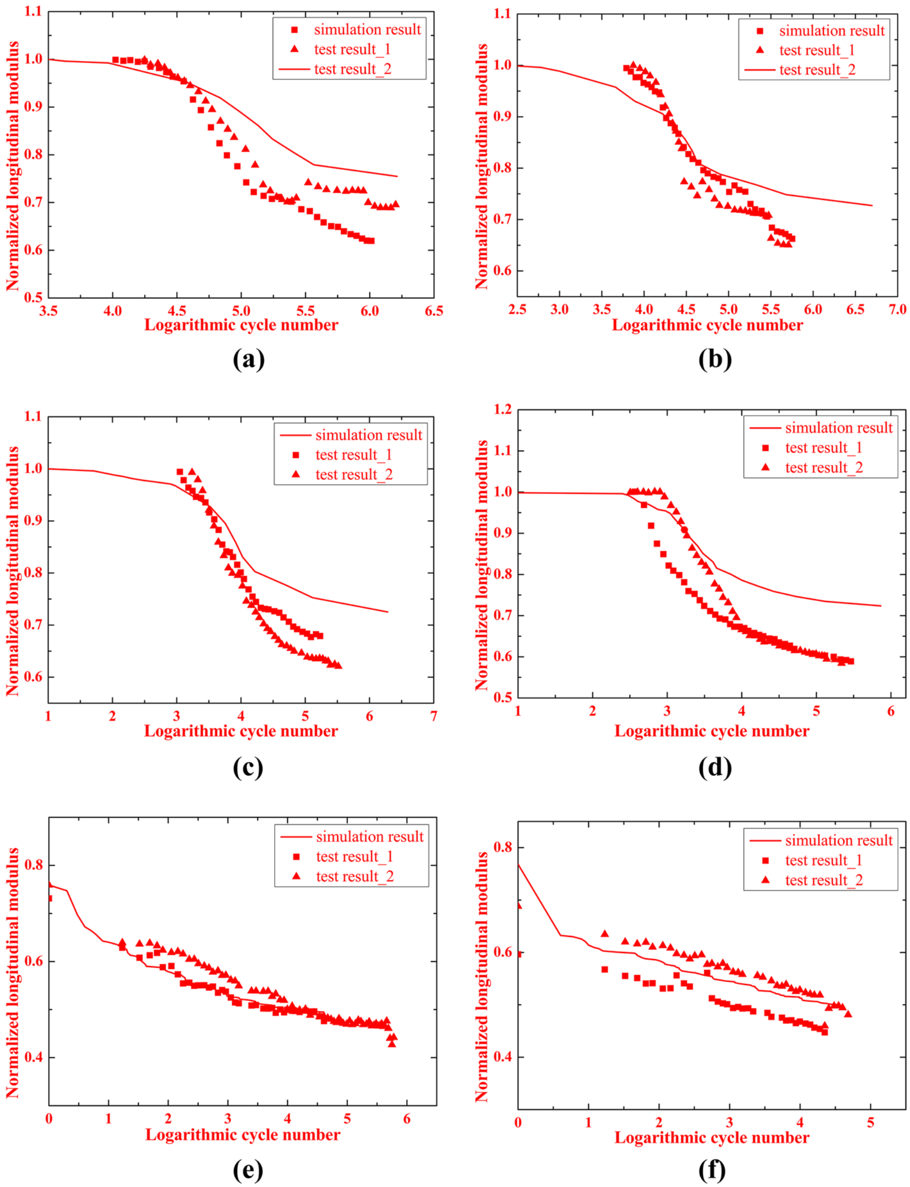

Fig. 9 Comparison between prediction results and experimental results of longitudinal modulus of $[0/90_4]_s$ laminate with different stress levels corresponding to tensile failure strength. (**a**) 20%, (**b**) 23%, (**c**) 26%, (**d**) 30%, (**e**) 50%, (**f**) 60%

The neglect of micro diffuse damages, including micro partial cracks, fiber-matrix debonding as well as diffuse initial defects, e.g., defects induced from manufacture and processing, and interaction between these damages would limit the applicability and accuracy of proposed model in some cases. Experimental results show that local delamination initiated near the existing cracks before transverse cracks density saturated, and the interactions

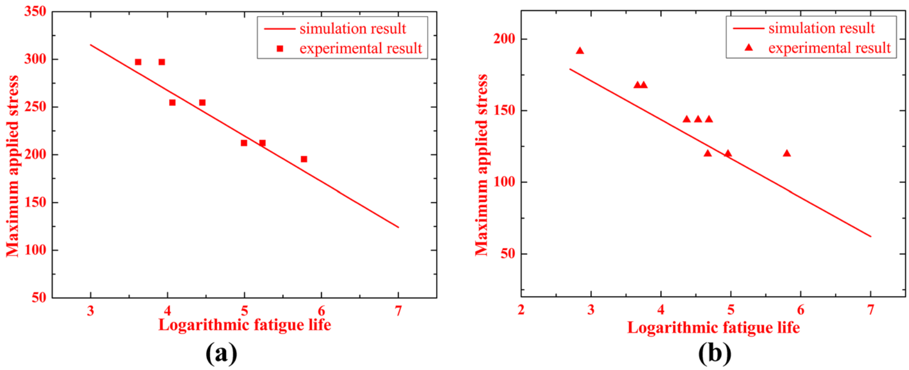

Fig. 10 comparison results between model prediction results and experimental results of *S–N* curves. (**a**) $[0_2/90_4]_s$, (**b**) $[0/90_4]_s$

of these damages could lead to the further degradation of material properties. However, the interactions could make the determination of stresses in cracked laminates generally very difficult to solve analytically, and also the influence of each damage mechanism would be difficult to determine if the interactions are considered.

It should be noted that the proposed model is valid for both glass fiber reinforced laminates and carbon fiber reinforced laminates since their damage mechanisms under fatigue loadings, including main damage modes and damaged mechanical behavior are quite same [45]. However, for woven fabric composites, matrix cracks are always very small and diffused-distributed in woven fabric composites when compared to those in composite laminates due to the structural difference between woven composites and composite laminates at micro level [46], so the damage evolution model for matrix cracking based on Eq. (6) is not suitable for woven fabric composites. On the other hand, the local delamination evolution law and fiber breakage evolution were established based on the Paris-like equation and macro damage mechanics, so they are independent of structural feature, so they are also valid for woven fabric composites. Also, the proposed damage characterization model is valid for woven fabric composites since it was established at macro homogenization level.

For some other different loading conditions, like compression fatigue loadings, the damage mechanisms would be rather different [23], and fibers tend to buckle when suffering compression loadings, which could lead to the formation of kink-band. Finally, catastrophic failure occurs when sufficient fibers break at the kinks in the kink-bands. This kind of damage mechanism was neglected in the proposed damage evolution mode. Thus, the model should be modified for compression loading conditions.

Table 8 Standard deviation values of logarithmic fatigue life of two cross ply laminates

Configuration	$[0_2/90_4]_s$	$[0/90_4]_s$
Standard deviation values	13.357	15.235

Table 9 Comparison between proposed model and previous model on the errors of logarithmic fatigue life

Model	Error
Wu's model [43]	8.36%
Aoki's model [13]	13.74%
Vasiukov's model [44]	9.55%
proposed model	8.26%

4 Conclusion

A fatigue life prediction model for composite laminates was proposed based on damage mechanics in this paper. The model studied three main damage modes in composite laminates under fatigue loadings, including transverse matrix cracking, local delamination and fiber breakage, and could predict the stiffness degradation and fatigue life of composite laminates under fatigue loadings with different stress levels.

In order to characterize the influence of three specific damage modes on the multidirectional stiffness properties of composite laminates under fatigue loadings, three groups of macro damage variables were defined based on the damage mechanics, and the constitutive equations of composite laminates were also deduced in terms of the defined damage variables with CLT method.

A micro–macro damage relationship deduced previously was introduced to the fatigue model to link the macro phenomenological damage variables to the microscopic practical damage mechanism, and then the macro degradation of material properties under fatigue loading could be predicted based on the evolution of micro practical physical damage modes in composite laminates, rather than empirical equations.

In order to describe the evolution of local delamination in composite laminates under fatigue loadings, a new damage evolution variable deduced by means of continuum damage mechanics at macro level in static model was introduced to the fatigue model, which could ensure the consistency between the static model and fatigue model.

The prediction results of two cross-ply laminates under fatigue loading with six stress levels were compared with the experimental results, and the overall tendency of predicted longitudinal stiffness degradation and fatigue life is in accordance with experimental results, indicating that the proposed model is effective and reliable in predicting the stiffness degradation and fatigue life of composite laminates under tensile fatigue loadings with different stress levels.

However, the neglect of micro diffuse damages and the interaction between different damage modes influences the accuracy of the proposed model for fatigue loadings with smaller stress levels. Also, the proposed model is not suitable for woven fabric composites and compression fatigue loading condition due to the quite different damage mechanisms. It is considered that the finite element method could be utilized in the proposed damage characterization model in the future work to characterize the influence of interactions between different damage modes on the stress field and mechanical properties of composite laminates for more accurate results. On the other hand, some advanced damage detecting methods, e.g., optical microscope and X-ray, could be adopted to obtain more detailed and accurate results of micro damages evolution. Besides, some more woven fabric composites fatigue tests and compression fatigue tests should be carried out to study the damage mechanisms, which could be introduced into the damage evolution model for modification.

Data Availability All data generated or analyzed during this study are included in this published article: Shen, H., Yao, W., Qi, W., Zong, J.: Experimental investigation on damage evolution in cross-ply laminates subjected to quasi-static and fatigue loading. Composites Part B: Engineering. 120, (2017). https://doi.org/10.1016/j.compositesb.2017.02.033.

Declarations

Competing interests The authors have no competing interests to declare that are relevant to the content of this article.

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