The Welfare Effects of Nationalization in a Mixed Duopoly Public Transport Market

ABSTRACT

Recently, many cities have launched new rail transit lines. Once these new rail transit lines start commercial operation, they will play important roles as competitors to conventional bus services. In this paper, the effects of nationalization on equilibria have been studied in a mixed duopoly public transport market, in which one publicly-owned rail transit operator competes with one private bus operator. Two numerical case studies show the nationalization of a rail transit company is always socially desirable both in Bertrand pricing game in the short run and in Nash Non-cooperative game in the long run.

Key words: Mixed duopoly transit market; Nash Non-cooperative competition; Degree of nationalization.

1 INTRODUCTION

Since the 1990s, in parallel with the trend towards deregulation and privatization, the organizations of public transport in many cities have been undergoing a radical transformation. In the bus market, it exhibits a transition from a state-owned monopolistic form to a private competition regime. Under franchise agreements, private bus companies can freely provide local services to residents. Whereas, due to the huge capital requirement and substantial economies of scale, most rail transit systems around the world are typically operated by state-owned enterprises (SOEs). However, since the 2000s, in order to transform SOEs from cost centers to economic entities responsible for certain profit targets, transit authorities have begun to restructure these SOEs into limited liability companies or joint stock companies, which can be termed as "corporatization" or "partial privatization". Consequently, the prevailing organization in urban public transport market presents such a structure that one (semi-) public rail transit operator competes with one or several private bus companies, which suggests the presence of a mixed oligopoly.[1] As an example, Shanghai-one of the largest cities in China, has more than 1000 bus lines, served by more than 10 private bus companies. However, all rail transit lines are only operated by one company—Shengtong Metro Company, which was originally a complete public entity. While, in 2000, it reformed as a semi-public cooperation with 63.65% of its share holding by Shanghai municipality. Mixed oligopolies are also common in developing countries' cities (such as Santiago in Chile, Kuala Lumpur in Malaysia), in which multiple operators with different ownership are vying for passengers. In a mixed oligopoly, the operational strategies chosen by public and private operators may differ due to their respective organisational objectives. As a result of changes in the transport market, pertinent questions emerge: if one rail transit operator chooses to partially maximize consumer surplus rather than to solely maximize its own profit, how will this change affect equilibria and, most importantly, could it really contribute to increasing social welfare as its organisational objective concerns?

The rest of paper is organized as follows. In Section 2, some related literature is briefly reviewed to shed light on the contributions made by this paper. Section 3 provides one Nash non-cooperative game in a mixed duopoly public transport market. In Section 4, to evaluate the effects of nationalization on equilibrium results, two numerical cases for short-run and long-run competitions are conducted by using Suzhou traffic data. Main findings and recommendations for further research are reported in Section 5.

[1] In the field of industrial economics, mixed oligopoly refers to the competition between public firms, which are instructed to maximize their contributions to welfare, and profit-maximization private companies.

2 LITERATURE REVIEW

This paper builds upon two strands of literature. The first strand is the application of non-cooperative game theory in the field of transport analysis. The second strand consists of studies on mixed oligopoly in the field of industrial organization.

Regarding modelling strategic competitions in transport market, the non-cooperative game theory is firstly applied as the Cournot monopoly model. Viewed as a cornerstone, Viton (1981) first started to consider Cournot competition in a purely private duopoly market with service quality and fare as controlled variables. After observing changes in the British bus market, Oldfield and Emmerson (1986) also modelled a Cournot-quality competition between two private bus operators to explain their price setting should follow changes in public transport organization. In the case of one public transport line operated by an arbitrary number of operators, Williams and Abdulaal (1993) employed a Logit model to derive mathematical expressions for equilibrium fares and frequencies. In addition to these pioneering studies on Cournot quantity competition, strategic interactions between transport operators were also modelled as Bertrand oligopoly (Vives, 1999). Braid (1986) adopted a game-theoretical approach to model a Bertrand pricing competition between two congested facilities. Else and James (1995) investigated a private duopoly competition over price and service quality in the rail transit sector. Afterwards, studies on modelling public transport competition with Bertrand-like models have intensified (e.g., De Palma and Leruth, 1989; Wang and Yang, 2005; Wichiensin et al., 2007; Yang and Woo, 2000).

These studies provide considerable insights into inter-modal and/or intra-modal competitions. However, most studies have only focused on the pure oligopoly market in which each operator concentrates on maximizing its own profits. Actually, the coexistence of one public firm and several private firms with distinct objective functions is prevalent both in developing countries and in developed countries. For example, the transit authority of Antwerp recently launched a proposal to build a private tunnel under the Scheldt River to relieve congestion on an existing publicly--owned Kennedy tunnel (Proost et al., 2005). The mixed oligopoly competition can also be observed in the long-distance passenger transport services, in which one state-owned rail company and several private coach companies compete alongside one another. Another real world case is in the metropolis of Chicago, where the local aviation authority decides to privatize Midway airport, while leaving O'Hare airport in public ownership (Noruzoliaee et al., 2015).

Although a larger body of literature has been produced to model strategic interactions within private oligopoly market, only a few studies have been carried out to study the mixed oligopoly competition in transport sector. For the seaport industry, Czerny et al. (2014) explored the effect of privatization on port for a local market. Matsushima and Takauchi (2014) investigated how port privatization affects port charges, profits and welfare elements from an international perspective.

In the context of aviation sector, Matsumura and Matsushima (2012) and Mantin (2012) investigated behavioral patterns of airports for international air transport markets, where one domestic publicly-owned airport competes against one private airport in another country. In addition to these intra-mode competition models, Yang and Zhang (2012) investigated the effect of inter-modal competition between air transport and high speed rail (HSR) in a mixed duopoly framework. Very recently, D'Alfonso et al. (2014) analyzed the impacts of competition between one private airline and one public HSR on the environment and social welfare. In the field of urban transport market, Cantos-Sanchez and Colonques (2006) explored frequency and pricing competition between one private bus and one public rail transit using a quadratic address model. Employing a quadratic utility function, Clark et al. (2009) compared equilibrium results that arise from collusion, applying Cournot and Bertrand competitions for a mixed duopoly bus market[2]. The above studies only focus on full nationalization. In reality, however, the markets are likely to have partially nationalized transport operators, especially in the urban transport market. For example, in China, Hangzhou government holds 51% of the share in Hangzhou Metro Company; the Norwegian Transit Authority holds the majority of shares in 36 bus companies. Despite these prevailing real world cases, the implications of partial privatization for frequency and pricing choices have not been investigated in the transport sector. Our research intends to fill in this gap.

Dating back to the 1960s, Merrill and Schneider (1966) first observed the coexistence of one public firm and several private firms is prevalent in a diverse range of industries, such as telecommunications, energy and banking. They defined this market structure as "mixed oligopoly"(MO). Since then, theoretical analysis of mixed oligopoly has received considerable attentions. Most studies are concerned with whether a public firm should be privatized in the face of competitions from private firms. The evidence suggests that in the context of a quantity-setting oligopoly, nationalization will improve social welfare if there are relatively few private firms, but will lower welfare if there are a high number of private firms (De Fraja and Delbono, 1989, 1990; Anderson et al., 1997; Matsumura, 1998; Ishibashi and Matsumura, 2006). Most of the literature has assumed Cournot competition within a homogeneous market, only little attention has been paid to differentiated Bertrant model. There are a small number of exceptions to this. Cremer et al. (1991) examined the strategic behaviors of firms in choosing location and price in a horizontally differentiated mixed oligopoly market. Another recent example with product differentiation is a study by Fujiwara (2007) that assessed the welfare implications of partial privatization.

[2] An important feature of our study which is distinct from Clark et al. (2009) is that the bus and rail transit operators compete in price and frequency with asymmetric goal functions, rather than competing in quantity or price with identical objective functions.

The review of relevant literature has made clear that all previous studies on MO employed simple linear demand functions, which can easily obtain analytical solutions. But, when exploring price competition among firms, an attraction model with nonlinear form is superior to linear one since it can not only represent reality more closely but also can reflect the nonlinear effects arising in competitive phenomena (Huang et al., 2012). Because of these advantages, we adopt a non-linear demand function (that is, a Logit model) in this paper to obtain more general and practical conclusions. Moreover, most of above studies in mixed oligopoly assumed the aggregate demand for whole market is inelastic with respect to fare or full cost. Actually, this is a very strict assumption. In this paper, a more realistic case will be developed in which total local transport demand is elastic with respect to the combination of fare and frequency.

The contributions of this paper can be summarized as follows: (*i*) given the growing importance of partial nationalization in the transport sector, this paper fills the research gap by analyzing the effects of partially nationalizing a rail transit company that competes with a private bus service. (*ii*) in contrast to most mixed oligopoly studies with linear demand function, we employed an attraction demand function with non-linear form to provide more practical and general insights; (*iii*) Supplemented with a Chinese case study, two time horizons are analyzed for this vertically and horizontally differentiated mixed duopoly competition: a short time horizon in which operators compete only in fares, and a long time horizon in which they compete in frequency along with the price.

3 MODELING FRAMEWORK

3.1 Basic setting of Nash Non-Cooperative game

This study focuses on a mixed duopoly public transport market, in which one semi-public rail transit operator (which is fully or partially concerned with users' benefits) competes with a private bus company (which only consider its own profits). Since each player is trying to "optimize its objective function without prior knowledge of other players' functions" (Evans, 1992), these two players make their decisions simultaneously without any collusion and receive payoffs depending on the service levels they offer. In this setting, one Nash Non-cooperation Game in the context of mixed duopoly has the following features:

1) Players. Nowadays, although multiple private bus companies prevail in some cities, few overlapping operating situations lead us to view them as one virtual operator. Thus, we theoretically assume only one bus operator competes with one (semi-) public rail transit operator on an isolated route during a given period.

2) Strategies. In the field of transport modelling, the strategic interaction occurs either in the price dimension (Bertrand) or in the quantity dimension (Cournot). What type of competition is appropriate not only depends on the strategic variables employed but also largely depends on the production technology and the time horizon (Krepsand Scheinkman, 1983). Cournot competition may be applicable when the capacity is difficult to adjust (e.g. rail, airport) and productions are perceived as perfect substitutes. But, when capacities are flexible (e.g. bus) and products are more differentiated, a Bertrand competition would be more appropriate (Quinet and Vickerman, 2004). In the case of a competition between bus and rail transit, it is not evident which type of competition is more proper based on the ease or difficulty of capacity adjustment. Whereas, bus and rail services are not only horizontally differentiated in terms of their distinct mode features but also vertically differentiated in terms of different service qualities. This implies that the substitution between these two modes is imperfect and the products are differentiated. As a result, we believe a differentiated Bertrand competition may be reasonable for the short run competition.

Besides price and quantity, frequency is another important strategic device. In the short run, since the increasing number of departures could require additional fleets, frequency is difficult to change. Thus, operators only compete over price, assuming frequency is exogenously given. Whereas, in the long run, operators may compete over frequency along side with fare since operators can relocate stops and buying more fleet.

3) Payoff: In the field of mixed oligopoly, the conventional objective function of one public company is to maximize social welfare, which is the sum of consumer surplus, its own profit and the profits of private companies. However, in the context of competition, it is somewhat unreasonable to assume the rail transit company competes against one bus company while also needs to consider its rival's profit. Therefore, we assume, in addition to the rail company's profit, the rail company takes consumers' interests into account when it decides fare and frequency. The profit of the bus company is out of its consideration. This kind of treatment is adopted by much recent literature (Clark et al., 2009; Jørgensen and Santos, 2014). While, as usual, it is reasonable to assume one private bus company is only concerned with maximizing its profit.

3.2 Demand functions and operating cost

In view of the mixed duopoly competition between one public rail transit operator and one private bus operator, this sub-section begins by constructing a demand function for operator *i* (Hereafter, the subscript *r* will be used to denote rail transit and the subscript *b* for bus), which conventionally takes the following form:

$$Q_i = Q^* \left\{ \xi, cc \right\} M_i \qquad i = b, r \tag{1}$$

Where, Q_i is the number of passengers selecting public transport service *i*. $Q^*\{\xi, cc\}$ represents total traffic demand of public transport sector, which is the function of composite costs

(cc) and one positive parameter ξ ($\xi > 0$). M_i denotes the market share of public transport service afforded by operator i.

The total demand for local public transport is supposed to be an exponential function of composite costs (cc)[3]:

$$Q^* = \bar{Q}^* \exp\left[-\xi(cc - \overline{cc})\right] \tag{2}$$

The composite cost (cc) represents the expected disutility of using public transport modes. A bar over a variable denotes its value in a reference case before competition takes place. Thus, $\bar{Q}^*$ is the initial total public transport demand in a benchmark situation and $\overline{cc}$ means the composite cost of reference case. Total demand for local public transport is elastic, because any changes in composite cost will shift the travelling demand between public transport modes and private transport modes (such as private cars, motorbikes, etc.).

To reflect the elastic of total demand change with respect to travellers' composite costs, we further specify the formulation of composite costs as:

$$cc = -\frac{1}{\theta} \ln \sum \exp(-\theta C_i) \tag{3}$$

Where C_i denotes the generalized cost of travelling by public transport mode i. θ is a positive dispersion parameter. In this paper, individual travellers' mode choices are assumed to be based on minimizing their generalized cost per trip (C_i), which equals the sum of the monetary cost (fare) and the travel time weighted by value of time.

$$C_i = P_i + \alpha_1 \rho T_i + \alpha_2 \rho (1/2 f_i) \tag{4}$$

Where P_i is the fare charged by public transport operator i. T_i is the average in-vehicle travelling time using public transport service i, which can be computed based on travelling distance and speed. Normally, during rush hour, the public transport services are much more frequent. Thus, the average waiting time can be roughly estimated from one-half the inverse of frequency (f_i). ρ is the value of time (VOT). α_1 and α_2 are parameters that measure different weights that passengers put on in-vehicle travelling time and waiting time. To focus on the principal aspects, this paper confines itself to the case of one homogeneous passenger group, which indicates all passengers are identical in regard to the value of time. Furthermore, for both bus and rail transit services, since the station location and number of stops have been decided in the planning stage, accessing time costs have not been included in the generalized cost function.

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[3] With this exponential function, the elasticity of total public transport demand will be proportional to composite cost ($\varepsilon_{cc} = \left(\partial Q^* / \partial cc\right)(cc/Q^*) = (-\xi) cc$) .This is a property of public transport demand which has been found in many studies (See Johansen, 2001; Fearnley, et al., 2004).

Turning to the specifications of travelling mode choices, the market share of operator *i* is given by the Binary Logit form:

$$M_i = \frac{\exp(-\theta C_i)}{\sum_i \exp(-\theta C_i)} \tag{5}$$

Inserting Equations (5), (2) and (3) into Equation (1), after some manipulation, the number of passengers that choose public transport service *i* is:

$$Q_i = \bar{Q}^* \left[\sum_i \exp\left(-\theta \bar{C}_i\right) \right]^{-\frac{\xi}{\theta}} \exp\left(-\theta C_i\right) \left[\sum_i \exp\left(-\theta C_i\right) \right]^{\frac{\xi}{\theta}-1} \tag{6}$$

Where $\bar{C}_i$ denotes the generalized cost of mode *i* in reference case. To show the impact of price and frequency on travel demand, we partially differentiate Equation (6) and obtain the following results:

$$\frac{\partial Q_i}{\partial P_i} = -Q_i\left(\xi M_i + \theta M_j\right) < 0 \qquad i, j \in \{b, r\} \qquad i \neq j \tag{7}$$

$$\frac{\partial Q_i}{\partial P_j} = (\theta - \xi) Q_i M_j > 0 \tag{8}$$

$$\frac{\partial Q_i}{\partial f_i} = \frac{\alpha_2 \rho}{2 f_i^2} Q_i (\xi M_i + \theta M_j) > 0 \tag{9}$$

$$\frac{\partial Q_i}{\partial f_j} = (\xi - \theta) \frac{\alpha_2 \rho}{2 f_j^2} Q_i M_j < 0 \tag{10}$$

Various empirical studies proved that $(\theta\text{-}\xi) > 0$ in the urban public transport market. From the above equations, it is clear that a high fare of one public transport service will reduce its own demand and increase the demand of its competitor. Whereas, the increase of frequency will boost its own demand rate and simultaneously reduce the demand of its competitor.

The variable operating costs of public transport service *i* (OC_i) is the sum of marginal passenger cost (c_{i0}) plus the variable cost (c_{i1}) associated with service frequencies:

$$OC_i = c_{i0} Q_i + c_{i1} f_i \tag{11}$$

3.3 Price competition in the short run

In a mixed oligopoly market, the bus operator is viewed as a pure commercially oriented company, which is only concerned with maximizing own profit. The profit of private bus company (π_b) is defined as the fare-box revenue minus the costs associated with bus operation (OC_b):

$$OF_b = \pi_b = P_b Q_b - OC_b \tag{12}$$

As mentioned earlier, the public rail transit operator maximizes a weighted combination of its profit and consumer surplus of whole public transport market. Given the proposed exponential demand function, the explicit form of consumer surplus can be simply expressed as:

$$CS = Q^{*} / \xi \tag{13}$$

This simple expression for consumer surplus was derived by Evans (1992), and has since been used extensively by many studies (Williams and Abdulaal, 1993; Wichiensin et al., 2007). The parameter ξ can be interpreted from two perspectives. Firstly, ξ quantifies the sensitivity of the total local transport demand with respect to composite costs. The second interpretation is that the inverse of ξ is the average perceived benefit experienced by one representative traveler. Accordingly, the rail operator's objective function can be specified as:

$$OF_r = \sigma CS + \pi_r = \sigma (Q^{*} / \xi) + (P_r Q_r - OC_r) \tag{14}$$

The continuous parameter σ, which ranges from 0 and 1, can be referred as the "weight" attached to consumer surplus. It gives a measurement of nationalization degree. σ=0 signifies the rail company is solely concerned with its profits. As σ rises, the weight on consumer surplus becomes heavier in rail transit firm's objective function. When σ=1, the rail transit operator is fully nationalized and aims to maximize the sum of consumer surplus and its own profit.

In this mixed duopoly market, each operator attempt to maximize its distinct objective function with respect to strategic valuables, subject to capacity constraint.

$$\begin{aligned} &\underset{P_i,\, f_i}{Max} \quad OF_i \\ &s.t. \quad Q_i \le K_i\, f_i \end{aligned} \tag{15}$$

K_i is the designed vehicle capacity of public transport services *i*.

In the short run, since the expansion of rolling stocks is strongly limited, the service frequency is assumed to be exogenously given. To this end, competition between two public transport modes is in price only.

By setting the partial derivatives of payoff functions with respect to fare equal to zero, and performing some manipulation, Nash equilibrium prices (P_i^{NE}; the superscript *NE* denotes equilibrium) for operator *i* is given as (Details of the derivation can be referred to Appendix 1):

$$P_i^{NE} = \frac{(1-\sigma)\delta}{\theta M_j + \xi M_i} + c_{i0} \tag{16}$$

The indicator variable δ takes on the value of one for rail transit and 0 for bus.

Next, we further partially differentiate Equation (16) with respect to its rival's price (P_j)

$$\frac{\partial^2 OF_i}{\partial P_i \partial P_j} = \frac{\partial P_i^{NE}}{\partial P_j} = M_j\left(1 - M_j\right)\left(\theta - \xi\right) \tag{17}$$

A closer look at the above mathematical expressions for equilibrium fares gives rise to some interesting insights. Firstly, in the absence of capacity constraints, the equilibrium price is equal to marginal passenger cost of public transport service i (c_{i0}) plus a mark-up, which relates to the market share of public transport services. Secondly, when $\sigma = 1$ (that is, the public rail transit company considers whole consumer surplus), the rail company consequently sets its fare equal to marginal cost (c_{i0}). Finally, since various empirical studies proved that (θ-ξ)>0 in urban public transport market, we have $\partial^2 OF/\partial P_i\,\partial P_j > 0$, which implies one operator's marginal profit increases as its competitor's fare increase. That is, the price set by the bus company decrease (increase) in response to the price reduction (increase) of the rail transit company ($\partial P_i^{NE}/\partial P_j > 0$). This property matches the well-known definition of the strategic complement in oligopolies.

To test the effect of partial nationalization (σ) on equilibrium fares, we invoke the techniques of monotone comparative statics for the following analytical derivations and obtain:

$$\frac{dP_r^{NE}(\sigma)}{d\sigma} = -\frac{\partial^2 OF_r \big/ \partial P_r \partial \sigma}{\partial^2 OF_r \big/ \partial P_r^2} < 0 \tag{18}$$

From Equation (18)， it is clear that with an increase of σ, the equilibrium price falls for rail transit. The explanation could be, an increase in σ provides more incentives to rail transit company to reduce its fare for improving consumer surplus. Furthermore, since the equilibrium fares between bus and rail transit are strategic complements, the bus fare correspondingly falls to compete for passengers. So we can conclude in the mixed duopoly market, as the publicly-owned rail transit operator pays more attention to consumer surplus relative to its own profit, both equilibrium fares of bus and rail transit decreases.

3.3 Frequency-price competition in the long run

In the long run, since the frequency can be easily adjusted, it is reasonably assumed that public transport operators compete in both price and frequency. To derive the reaction functions for service frequency, a similar procedure is made by partially differentiating the payoff function of operator i with respect to its frequency. After some algebraic manipulation, the equilibrium service frequency is given as (Details of the derivation can be referred to Appendix 1):

$$f_i^{NE} = \sqrt{\frac{\alpha_2 \rho}{2c_{i1}} Q_i} \tag{19}$$

Obviously, the Nash Equilibrium frequency also follows the “square root formula". By analogy, the mathematical expression of equilibrium frequency can be further differentiated with respect to the frequency of its competitor:

$$\frac{\partial f_i^{NE}}{\partial f_j} = \sqrt{\frac{\alpha_2 \rho}{2c_{i1}}} \frac{1}{2} Q_i^{-\frac{1}{2}} \frac{\partial Q_i}{\partial f_j} \tag{20}$$

From Equation (10), we have $\partial Q_i / \partial f_j < 0$. Hence, the above equation takes a negative sign, which indicates a marginal increase (decrease) in the frequency of rail transit causes the bus operator to decrease (increase) its service frequency. It is worth noting that the conventional concept of strategic substitution can be appropriately employed to describe the relationship of equilibrium frequencies between bus and rail transit in this mixed oligopoly model[4].

Summarizing the above analysis, we obtain Proposition 1:

Proposition 1: In the mixed duopoly market involving one private bus operator and one public rail transit company, Nash Equilibrium fares are strategic complements. While, Nash Equilibrium frequencies are strategic substitutes.

Next, we investigate the impact of nationalization degree on equilibrium frequency. Given the mathematical specification of equilibrium frequency for rail transit in Equation (20), we partially differentiate it with respect to σ:

$$\frac{\partial f_r^{NE}}{\partial \sigma} = \sqrt{\frac{\alpha_2 \rho}{2c_{i1}}} \frac{1}{2} \left(Q_r^{NE}\right)^{-\frac{1}{2}} \frac{\partial Q_r^{NE}}{\partial P_r^{NE}} \frac{\partial P_r^{NE}}{\partial \sigma} \tag{21}$$

From Equations (7) and (21), we have $\partial Q_r^{NE} / \partial P_r^{NE} < 0$ and $\partial P_r^{NE} / \partial \sigma < 0$, thus we can confirm $\partial f_r^{NE} / \partial \sigma > 0$. This means the equilibrium frequency of rail transit increases when rail operator places more weight on consumer surplus. Since the equilibrium frequencies between bus and rail are strategic substitutes, an increase in σ will decrease the equilibrium frequency for bus (f_b^{NE}).

Proposition 2 summarizes the impact of nationalization degree (σ) on equilibrium fares and service frequencies.

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[4] As pointed out by an anonymous referee, the equilibrium result of Cournot competition is equivalent to the outcome of a two-stage game, where there is a simultaneous capacity choice after which price competition occurs (Kreps and Scheinkman, 1983). In the mixed monopoly, this equivalence found by Kreps and Scheinkman (1983) is still valid, if assumptions of L-shaped marginal cost function and efficient rationing rule hold (A technical proof is available upon request from the authors). However, if two assumptions are violated, the K- S result might not remain valid. Extending the analysis to compare the equilibrium results of Cournot competition with outcomes of two-stage game would be a useful future study, if the demand function can be explicitly converted to inverse demand function.

Proposition 2: In line with the initial objective of increasing consumer surplus, a lower degree of nationalization leads to (*i*) lower equilibrium fares for both rail transit and bus; (*ii*) higher equilibrium frequency for rail transit and lower equilibrium frequency of bus.

Next, we analyze how the change of nationalization degree affects passenger demand, consumer surplus and producer surplus. All results are summarized in the following propositions.

Proposition 3: An increase in the level of nationalization (*i*) increases the total demand of local public transport in the equilibrium ($\partial Q^{NE*} \big/ \partial \sigma > 0$); (*ii*) increases the ridership of rail transit under the equilibrium, but reduces the ridership for bus, $\partial Q_r^{NE} \big/ \partial \sigma > 0$ and $\partial Q_b^{NE} \big/ \partial \sigma < 0$.

Proof: See the Appendix 3

The insights behind Proposition 3 are as follows: When the rail operator attaches a high weight to consumer surplus and a correspondingly low weight to its profit, this welfare concern puts a downward pressure on rail fare and an upward incentive on raising rail frequency. The combination of lower fare and higher frequency makes rail transit services more attractive than before. Thus, its ridership is boosted along with its strengthened market share. However, only concerning its profit, the bus service becomes progressively less attractive, although it also reduces its fares to compete against rail transit.

Proposition 4: Since travellers pay less in the move from a standard duopoly (SD) to a mixed duopoly (MD), the consumer surplus necessarily rises alongside the boost in total public transport demand. At the same time, there is a reduction in producer surplus when we move away from a standard private duopoly (SD) to a mixed duopoly (MD).

Proof: See the Appendix 4

Whilst the effects of nationalization on consumer surplus and producer surplus are straightforward now, we are not certain whether the increase in consumer surplus can compensate for the loss of producer surplus as the rail transit operator puts more weight on consumer surplus. Therefore, the impact of nationalization on social welfare is still ambiguous. So we turn to a numerical study based primarily on a mixed public transport market in a Chinese city.

4 NUMERICAL CASE STUDIES

To shed light on the impacts of partial nationalization on equilibrium outputs under a mixed duopoly market, this section begins with introducing Bertrand pricing game in Case 1, in which service frequency is less flexible in the short run. In Case 2, adding frequency as another strategic device, we illustrate the long-run impact of nationalization on Nash equilibrium solutions. To establish orders of magnitude for key strategic variables and associated welfare elements, Suzhou traffic data are used to gauge the results. Table 1 summarizes all traffic data and parameters used.

Table 1: Traffic Data and Parameters

Description	Measurement	Rail Transit	Bus
Average running speed (V_i)	Km/hour	40	20
In-vehicle travelling time (T_i)	Hour	0.225	0.45
Vehicle capacity (K_{mi})	Passenger/vehicle	960	102
Variable running costs (c_{i1})	CNY/vehicle-km	22	8.9
Marginal passenger cost (c_{io})	CNY/passenger	0.002	0.0015
Fare (P_{i0})	CNY	3	1.5
Frequency (f_i)	Vehicle/hour	12	8
Market share (M_{i0})	Percent (%)	84.46%	15.54%
Patronage of service i (Q_{i0})	Passengers/hour	4,223	777
Total public transport demand ($\overline{Q}^*$)	Passengers/hour	5,000	
Value of time (ρ)	CNY/hour	10.1	
Scale parameter (θ)		1	
Weights of in-vehicle time (α_1)		1	
Weights of waiting time (α_2)		2	
Parameter in total transit demand (ξ)		0.2	

In the following numerical calculations, the traffic data and parameters, such as average fares, general operating characteristics of public transport, and value of time (VOT), were sourced from the *Annual Report of Suzhou Urban Transportation* (2008). Furthermore, concerning operating costs of bus, the information stems from *Suzhou Bus Group Annual Report (2008)*. This report illustrates the variable operating costs of running an extra bus kilometre are 8.9 CNY and the marginal cost per passenger journey 0.0015 CNY. In terms of rail transit, the figures of variable operating costs (22 CNY per kilometre and 0.002 CNY per passenger) are estimated by referring to the accounting report of Suzhou Metro Company. Hong and Zuo (2006) estimated the sensitivity of transit passengers to changes of traveling costs in several Chinese cities. The results indicate that for a 1% reduction in traveling costs, there is a 0.1%-0.4 % increase in public transport patronage. In order to make the following cases more illustrative, operational status is also listed in Table 1. Setting the current situation as a reference case, we can compare how fares, frequencies, market shares, social welfare and its constituent parts change with the degree of nationalization. With 5,000 commuters per morning peak hour, the rail transit line catches a significantly larger market share (84.46%) than the bus (15.54%).

4.1 Case one: the effect of nationalization (σ) on equilibra in the short run

In the short run, public transport operators cannot afford more frequent services due to the additional cost of increasing their fleet. Thus, both operators compete purely on price, assuming service frequencies are exogenously given. In Case one, to assess the impact of the level of nationalization (σ) on equilibrium configurations, the analysis begins with the move from standard private duopoly (SD, $\sigma=0$) to a mixed duopoly (MD, $\sigma>0$) case. As mentioned earlier, the parameter--σ, is continuous. But for the sake of exposition, in Table 2 and 3, we only list numerical equilibrium solutions with σ varying in steps of 0.2.

The first two rows of Table 2 show that, as expected, the equilibrium prices of bus and rail transit decrease when rail operator places more weight on consumer surplus. Thus, for different values of σ, the pair of equilibrium fares are highest in standard private duopoly with 1.219 CNY for bus and 2.643 CNY for rail transit, and lowest in the mixed duopoly situation (σ=1) with only 1.021 CNY for bus and 0.002 CNY for rail transit. Moreover, the synchronous decrease in bus and rail transit fares indicates the prices are strategic complements, which means the prices set by bus company decrease in response to the price reduction of Rail Company. Although both equilibrium fares move in the same direction (downward), the rate of decrease of rail transit fare is much greater. When the nationalization degree exceeds 0.8, rail transit has a competitive advantage over bus in terms of its relatively low fare.

Table 2: Effect of Nationalization (σ) on Equilibrium Solutions in the short run

	σ=0	σ=0.2	σ=0.4	σ=0.6	σ=0.8	σ=1
Bus fare （P_b）	1.219	1.17	1.124	1.081	1.046	1.021
Rail transit Fare （P_r）	2.643	2.326	1.945	1.463	0.828	0.002
Market share of bus (M_b)	21.98%	17.73%	13.36%	9.04%	5.17%	2.39%
Market share of rail transit (M_r)	78.02%	82.27%	86.64%	90.96%	94.83%	97.61%
Total public transport Demand (Q^*)	5458	5754	6145	6701	7546	8850
Passenger demand of bus (Q_b)	1200	1020	821	606	390	211
Passenger demand of rail transit (Q_r)	4258	4734	5324	6095	7156	8639
Producer Surplus (PS)	10053	9499	8573	6869	3623	-2477
Δ Profit of bus (π_b)	1184	875	605	338	91	-101
Δ Profit of rail transit (π_r)	8869	8624	7968	6531	3532	-2376
Consumer Surplus (CS)	27289	28768	30726	33505	37732	44250
Social Welfare (SW)	37742	38267	39299	40374	41355	41773

Note: PS is the producer surplus, which is the sum of bus's profit (π_b) and rail's profit (π_r).
SW is the social welfare, which is the sum of producer surplus (PS) and consumer surplus (CS)

From Figure 1-a, it can be observed that the total demand of local public transport (Q*) and rail transit (Q_r) increase gradually with the increase in σ. On the contrary, since rail transit attracts increasing numbers of travellers from buses due to its relatively low generalized cost, the market share of the bus quickly shrinks from 21.98% to 2.39%. The logic behind the result can be explained as follows. The overall impact of nationalization on patronage is the combination of two effects: the first is increasing total demand for public transport, and the second is the shifting of market share between modes. Regarding rail transit, the more weight the rail transit firm places on consumer surplus, the greater the aggregate demand and rail operator's market share will be. Conversely, since the decreasing market share dominates the increase of total public transport demand, the equilibrium patronage of bus falls with the increasing degree of nationalization. Consequently, a further decrease in both firms' prices triggered by nationalization boosts the total public transport demand. In such a case, the rail transit significantly erodes the market share of bus and the bus operator gradually loses its competitive advantage.

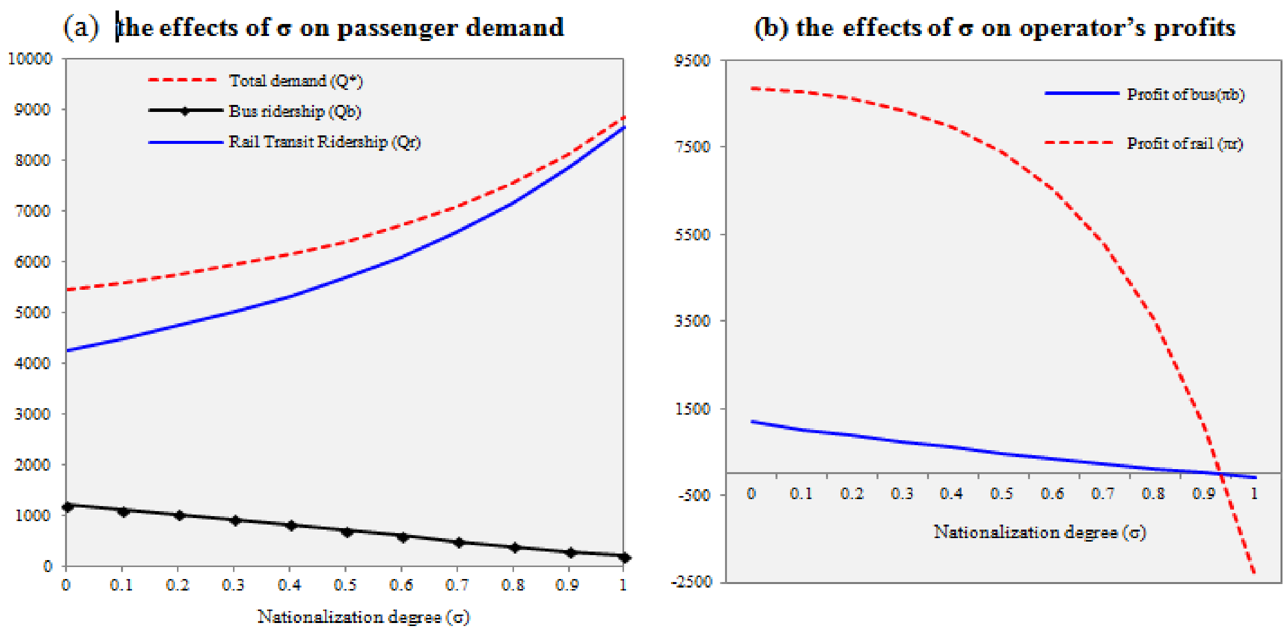

Figure 1: The impacts of partial nationalization on equlibria in short run

As illustrated in Figure 1-b, although the ridership of rail transit rises, the sharp reduction in fares cause the rail transit operator's profits to rapidly tail off. Due to the rail transit operator's relatively high operating cost, the low fare revenue cannot cover its variable operating costs when the degree of nationalization exceeds 0.9. When σ reaches 1, it would be difficult for rail transit operator to remain profitable, the rail transit operator would require subsidies from local government. On the contrary, although the combined effect of fare reduction and shrinking ridership results in a considerable decrease in the profits of the bus, the relatively lower operating cost makes the bus operator profitable when the value of σ does not exceed 0.9. In the extreme case of the rail transit operator fully considering consumer surplus (σ=1), the bus operator cannot break even. As the profit is squeezed, the bus operator might drop out of business.

In the mixed duopoly, lower equilibrium prices and higher total transit demand have positive impacts on consumer surplus, implying consumer surplus rises by a considerable amount. Although the producer surplus falls with the increase in the degree of nationalization, these losses are relatively small compared with the gain in consumer surplus. Consequently, total social welfare increases slightly, this demonstrates the desirability of nationalizing rail transit operators.

The numerical results are consistent with the analytical results obtained in previous section and can be summarized in Remark1:

Remark 1: *An increase in the level of nationalization causes equilibrium prices to fall for both bus and rail transit. Moreover, there is a rise in social welfare when we move away from a standard private duopoly (SD) to a mixed duopoly (MD).*

The above remark states that, in the process of nationalization, as the rail transit operator puts more weights on consumer surplus relative to its profit, the price competition in the mixed duopoly market become fiercer. Since travellers pay less in the move from a standard duopoly (SD) to a mixed duopoly (MD), consumer surplus, thereby boosting total public transport demand. Also, since the increase in consumer surplus can compensate for the loss of producer surplus, partial nationalization is socially preferable in terms of welfare improvement.

4.2 The effect of nationalization (σ) on equilibria in the long run

In the long run, since the operator has the ability to adjust its fleet size to match fierce market competitions, service frequency might be another strategic device. In this numerical case, we turn to Nash price–frequency competition and attempt to demonstrate how the operators respond to the progress of nationalization. The numerical equilibrium solutions for this Nash non-cooperative game are summarized in Table 3.

Table 3. Effect of Nationalization (σ) on Equilibrium Solutions in the long run

	σ=0	σ=0.1	σ=0.3	σ=0.5	σ=0.7	σ=0.9
Bus fare (P_b)	1.253	1.221	1.158	1.097	1.038	0.889
Rail transit Fare (P_r)	2.498	2.373	2.091	1.742	1.279	0.611
Bus frequency (f_b)	13.62	13.02	11.58	9.61	6.35	3.53
Rail Transit frequency (f_r)	14.96	15.39	16.33	17.46	18.9	19.43
Market share of bus (M_b)	25.13%	22.50%	16.93%	10.94%	4.37%	0.75%
Market share of rail transit (M_r)	74.87%	77.50%	83.07%	89.06%	95.63%	99.25%
Total public transport Demand (Q^*)	5857	5987	6395	6709	7320	8327
Passenger demand of bus (Q_b)	1472	1347	1066	734	320	62
Passenger demand of rail transit (Q_r)	4385	4640	5229	5875	7000	8265
Producer Surplus (PS)	8735	8553	7993	6976	5022	965
Δ Profit of bus (π_b)	753	600	306	35	-176	-227
Δ Profit of rail transit (π_r)	7982	7953	7687	6941	5198	1192
Consumer Surplus (CS)	29287	29933	31472	33542	36599	41637
Social Welfare (SW)	38022	38489	39465	40518	4162	42602

As Table 3 shows, the motive of increasing consumer surplus induces rail transit operator to increase its frequencies and reduce fares. Since prices are strategic complements in this price-frequency game, the gradually decreasing rail transit fares provides an incentive to bus operator to defend its market share by responding with a fare reduction. On the contrary, in this case with endogenous frequency, it emerges that service frequencies are strategic substitutes. Thus, responding to the increase in rail transit frequency, the bus operator is forced to reduce its service frequency to prevent further deterioration of profitability (see Figure 2-a).

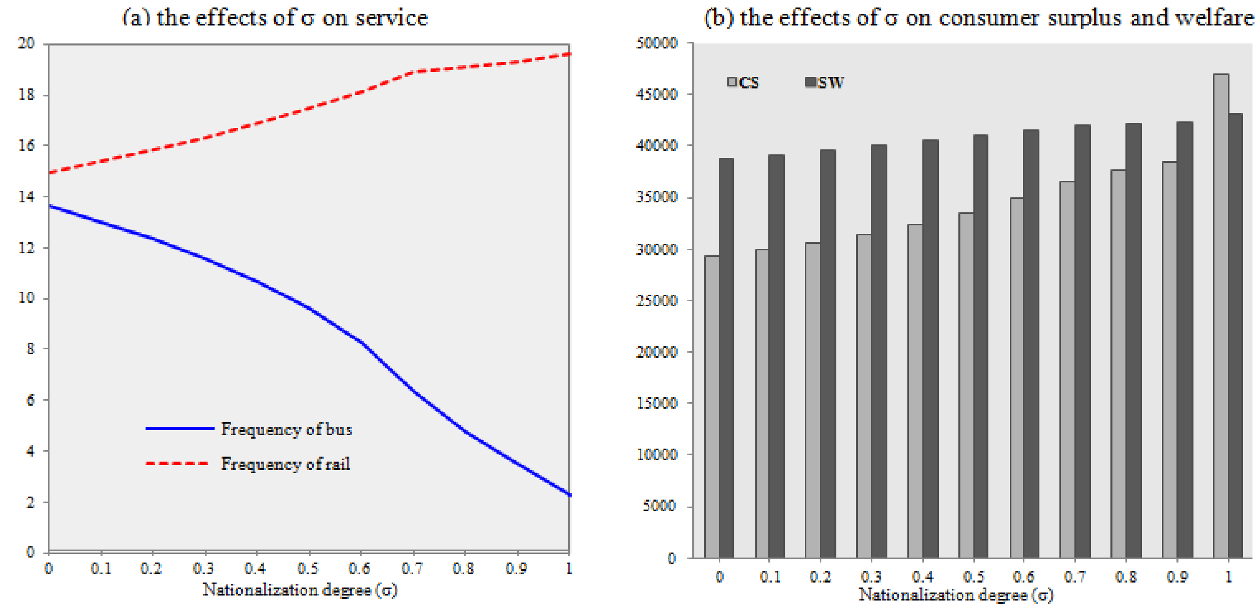

Figure 2. The impacts of partial nationalization on equlibria in long run

In comparison to Case 1, the most likely effect of adding service frequency as another strategic variable is that total public transport ridership has been greatly boosted, because the lower composite costs resulting from additional competitions over frequency increase the attractiveness of public transport. Similarly, when rail operator attaches a high weight to consumer surplus and a correspondingly low weight on profit, its market share will be strengthened. Whereas, conventional bus services become progressively worse off since its service levels are not as attractive as before. The ridership of bus diminishes fast. Then, in the case of full nationalization, a corner solution can be obtained in which the rail operator satisfies all demand. As far as operators' profits are concerned, both operators experience financial losses in the process of nationalization. Once the value of σ exceeds 0.5, the bus loses the commercial feasibility of operating services due to the negative profits. However, the welfare comparisons show that the increase in consumer surplus offsets the losses in operators' profits, which supports the desirability of a mixed oligopoly (see Figure 2-b).

The main finding of Case 2 is summarized in Remark 2.

Remark 2: *In Nash non-cooperative game, with the motive of increasing consumer surplus (i) the rail transit operator intends to reduce fare and increases its service frequency. While, the bus operator strategically choose to reduce both fare and frequency.(ii) the increase in consumer surplus offsets the losses in producer surplus, leading to the increase of social welfare.*

The studies of mixed duopoly in industrial organization suggests that without other regulations, nationalization will heighten social welfare if there are relatively few private firms in the market and will degrade social welfare if there are relatively many private firms, assuming Cournot quantity-setting competition with an inelastic and linear demand function (Anderson et al., 1997; De Fraja and Delbono, 1990). From the above numerical case studies, it can be concluded that full nationalization yields the highest level of social welfare in the public transport market. Thus, the numerical cases provide support for previous findings on Nash non-cooperative game in the context of an elastic and non-linear demand function.

The above numerical case studies employed a large number of parameters and actual data that come from Suzhou area. To test the generalization of these results, a substantial number of sensitivity analyses have been carried out with respect to some important demand and cost parameters. Not surprisingly, we found that the magnitudes of equilibrium configurations are somewhat sensitive (such as fares, service frequencies, demand and welfare elements) to the assumed demand parameters (ξ), to variation in the operating costs (c_{i1}) and to the initial demand rate ($\bar{Q}^*$). However, the qualitative conclusions derived from the numerical case studies are found to be unaffected. All detailed sensitive results are available on request from the authors.

5 CONCLUSIONS

In this paper, the duopolistic interactions between one public rail transit operator and one private bus operator are presented as a Nash non-cooperative game. To investigate the effects of nationalization on equilibrium fares, service frequencies and associated welfare elements, two numerical cases based on Suzhou traffic data are analyzed and the main insights are presented as follows. Firstly, given the less flexible service frequency in the short run, the presence of one publicly-owned rail transit operator is a useful measure for approaching social optima. The more weight the rail operator attached to consumer surplus, resulted in the equilibrium fare being lower, leading to higher consumer surplus and social welfare. Secondly, in Nash non-cooperative game with price and frequency competition, prices are a strategic complement. Due to this, when the rail operator reduces (or increases) its fare level, it forces the bus operator to do the same. Meanwhile, a marginal frequency increase (decrease) in rail transit service causes bus operator to decrease (increase) its frequency, indicating service frequencies are strategic substitutes. Finally, if no additional regulations are implemented, nationalizing one rail transit company is socially desirable both in Bertrand pricing game in the short-run and in Nash non-cooperative game in the long-run. This result is in line with the previous studies' results on mixed oligopolies. That is, when the number of private companies is relatively small, privatizing one public firm degrades social welfare.

The analysis performed in the paper captures the present status of urban public transport market in many Chinese cities where newly introduced state-owned rail transit services compete with extant private bus companies. Although the analysis was based on a simple model, it has potential applicability in the urban public transport market for analyzing the effectiveness of privatization/nationalization. To make the proposed model more application in practice, a new type of modeling framework can be developed in the future by integrating the mixed oligopoly competition model for supply side with the network equilibrium model in the demand side. Then, in addition to mode choices, the travelers' route choices can also be represented by using the complementarity formulations of user equilibrium conditions. This complex two-part equilibrium model can be solved using standard algorithms with some analytical modeling package such as GAMS for example. Additionally, to achieve a more realistic picture, a further extension could be to expand the proposed two-link transport system with only one OD pair to complex network with diversity of topology, more paths and more stakeholders. The third further extension would be to examine the cooperative possibility between bus and rail transit.

Appendix. Formula Derivations

1. Equilibrium price in the short run

Taking the partial derivatives of Equation (15) with respect to fares yields:

(A.1) $$\frac{\partial OF_i}{\partial P_i} = \frac{\sigma}{\xi}\frac{\partial Q^*}{\partial P_i}\delta + Q_i + (P_i - c_{i0})\frac{\partial Q_i}{\partial P_i}$$

Where δ is an indicator variable, which takes on the value of 1 for rail transit and *0* for bus.

Then, plugging Equations (6) and (7) into (A.1), we can obtain the equilibrium fares by setting the corresponding results equal to zero:

(A.2) $$\frac{\partial OF_i}{\partial P_i} = Q_i\left[(1-\sigma)\delta - (P_i - c_{i0})(\xi M_i + \theta M_j)\right] = 0$$

After some manipulation, the above expression reduces to

(A.3) $$P_i^{NE} = \frac{(1-\sigma)\delta}{\theta M_j + \xi M_i} + c_{i0}$$

Next, we examine the effect of nationalization degree (σ) on equilibrium fares. Partially differentiating the first order condition in Equation (A.1) for rail transit with respect to its price yields:

(A.4) $$\frac{\partial OF_r}{\partial P_r} = \frac{\partial Q_r}{\partial P_r}\left\{(1-\sigma) - (P_r - c_r)\left[\theta - (\theta-\xi)M_r\right]\right\} + Q_r\left\{\left[(\theta-\xi)M_r - \theta\right] + (P_r - c_{r0})(\theta-\xi)\frac{\partial M_r}{\partial P_r}\right\}$$

Substituting (7) and $\partial M_r/\partial P_r = \theta M_r(M_r - 1)$ into (24), we can rewrite (A.4) as:

(A.5) $$\frac{\partial^2 OF_r}{\partial P_r^2} = Q_r\left\{(2-\sigma)\left[(\theta-\xi)M_r - \theta\right] - (P_r - c_r)\left[\theta - (\theta-\xi)M_r\right]^2 + (P_r - c_{r0})(\theta^2 - \theta\xi)M_r(M_r - 1)\right\}$$

Because the sign of $\left[(\theta-\xi)M_r-\theta\right]$ and (M_r-1) is negative, and other items in (A.5) are positive, it is easy to prove $\partial^2 OF_r \big/ \partial P_i^2 < 0$.

Taking the first derivative of (A.1) with respect to nationalization degree, we obtain $\partial^2 OF_r \big/ \partial P_r\,\partial\sigma = -\sigma Q_r < 0$, and then we get:

(A.6) $$\frac{dP_r^{NE}(\sigma)}{d\sigma} = -\frac{\partial^2 OF_r \big/ \partial P_r \partial\sigma}{\partial^2 OF_r \big/ \partial P_r^2} < 0$$

2. Equilibrium price and frequency in the short run

Partially differentiating the payoff function of operator *i* with respect to frequency leads to:

(A.7) $$\frac{\partial OF_i}{\partial f_t} = \frac{\sigma}{\xi}\frac{\partial Q^*}{\partial f_i}\delta + \left(P_i - c_{i0}\right)\frac{\partial Q_i}{\partial f_i} - c_{i1}$$

Substituting Equations (7) and (A.3) into (A.7) yields the equilibrium frequency for public transport service *i*:

(A.8) $$f_i^{\,2} = \bar{Q}^*\left[\sum_i \exp(-\theta\bar{C}_i)\right]^{-\frac{\xi}{\theta}} \frac{\alpha_2\rho}{2c_{i1}}\exp(-\theta C_i)\left[\sum_i \exp(-\theta C_i)\right]^{\frac{\xi}{\theta}-1}$$

Recalling the demand function for public transport service *i* in Equation (6), we replace $\bar{Q}^*\left[\sum_i \exp(-\theta\bar{C}_i)\right]^{-(\xi/\theta)} \exp(-\theta C_i)\left[\sum_{i=b,r} \exp(-\theta C_i)\right]^{(\xi/\theta)-1}$ with Q_i and select the positive root:

(A.9) $$f_i^{\,NE} = \sqrt{\frac{\alpha_2\rho}{2c_{i1}}Q_i}$$

3. The effect of nationalization degree on equilibrium demand and welfare

Analogous to the above propositions, we can analytically explore the effects of nationalization degree on equilibrium demand for bus and rail transit:

(A.10) $$\frac{\partial Q_r^{NE}}{\partial\sigma} = \frac{\partial Q_r^{NE}}{\partial f_r^{\,NE}}\frac{\partial f_r^{\,NE}}{\sigma}$$

(A.11) $$\frac{\partial Q_b^{NE}}{\partial\sigma} = \frac{\partial Q_b^{NE}}{\partial f_r^{\,NE}}\frac{\partial f_r^{\,NE}}{\sigma}$$

Recalling Equations (7), (8) and (21), we have $\partial Q_r^{NE} \big/ \partial\sigma > 0$ and $\partial Q_b^{NE} \big/ \partial\sigma < 0$. In terms of total demand for whole public transport, we take the first derivative of Q* with respect to nationalization degree (σ), which yields:

(A.12) $$\frac{\partial Q^{NE*}}{\partial\sigma} = \frac{\partial(Q_r^{NE}+Q_b^{NE})}{\partial\sigma} = \frac{\partial P_r^{NE}}{\partial\sigma}\left(\frac{\partial Q_r^{NE}}{\partial P_r^{NE}} + \frac{\partial Q_b^{NE}}{\partial P_r^{NE}}\right)$$

Substituting Equations (7), (8) and (18) into (A.12) and rearranging items provides the following:

(A.13) $$\frac{\partial Q^{NE*}}{\partial\sigma} = -\xi\frac{\partial P_r^{NE}}{\partial\sigma}Q_r^{NE} > 0$$

4. The effect of nationalization degree on consumer surplus and producer surplus

Recalling the expression of consumer surplus in (13), since $\partial Q^{NE*}/\partial\sigma > 0$, we have $\partial CS^{NE}/\partial\sigma > 0$.

The producer surplus can be decomposed into two parts: the part associated with patronage and the part related to frequency:

(A.14) $$PS = \sum_{i=b,r}\pi_i = \sum_{i=b,r}(P_i - c_{0i})Q_i - \sum_{i=b,r}(c_{1i})f_i$$

Differentiating $\sum_{i=b,r}(P_i - c_{0i})Q_i$ with respect to σ and rearranging the terms gives:

(A.15) $$\frac{\partial\sum_{i=b,r}(P_i^{NE} - c_{i0})Q_i^{NE}}{\partial\sigma} = \frac{\partial P_r^{NE}}{\partial\sigma}Q_r^{NE} + \frac{\partial Q_r^{NE}}{\partial P_r^{NE}}\frac{\partial P_r^{NE}}{\partial\sigma}(P_r^{NE} - c_{r0}) + \frac{\partial P_b^{NE}}{\partial\sigma}Q_b^{NE} + \frac{\partial Q_b^{NE}}{\partial\sigma}(P_b^{NE} - c_{b0})$$

Substituting (7) for $\partial Q_r^{NE}/\partial P_r^{NE}$ and Equation (18) for $\left(P_r^{NE} - c_{r0}\right)$ into (A.15) yields:

(A.16) $$\frac{\partial\sum_{i=b,r}(P_r^{NE} - c_{r0})Q_r^{NE}}{\partial\sigma} = \frac{\partial P_r^{NE}}{\partial\sigma}Q_r^{NE}\sigma + \frac{\partial P_b^{NE}}{\partial\sigma}Q_b^{NE} + \frac{\partial Q_b^{NE}}{\partial\sigma}(P_b^{NE} - c_{b0}) < 0$$

Given $\partial P_r^{NE}/\partial\sigma < 0$, $\partial P_b^{NE}/\partial\sigma < 0$ and $\partial Q_b^{NE}/\partial\sigma < 0$, it can be easily shown that $\partial\sum_{i=b,r}(P_r^{NE} - c_{r0})Q_r^{NE}\Big/\partial\sigma < 0$.

Given the straightforward relationship $M_r^{NE} + M_b^{NE} = 1$ and recalling the expressions for equilibrium frequencies in Equation (20) yields:

(A.17) $$\sum_{i=b,r}(c_{1i})f_i = \sqrt{\frac{\alpha_2\rho c_{r1}}{2}Q^{NE*}M_r^{NE}} + \sqrt{\frac{\alpha_2\rho c_{b1}}{2}Q^{NE*}(1 - M_r^{NE})}$$

Differentiating Equation (A.17) with respect to σ and rearranging the terms yields:

(A.18) $$\frac{\partial\sum_{i=b.r}c_{i1}f_i^{NE}}{\partial\sigma} = \frac{\partial Q^{NE*}}{\partial\sigma}\sqrt{\frac{\alpha_2\rho}{8Q^{NE*}}}\left(\sqrt{c_{r1}M_r^{NE}} + \sqrt{c_{b1}(1 - M_r^{NE})}\right) + \frac{\partial M_r^{NE}}{\partial\sigma}\sqrt{\frac{\alpha_2\rho}{2Q^{NE*}}}\left(\frac{\sqrt{c_{r1}}}{2\sqrt{M_r^{NE}}} - \frac{\sqrt{c_{b1}}}{2\sqrt{(1 - M_r^{NE})}}\right)$$

After observing $\partial Q^{NE*}/\partial\sigma > 0$ and $\partial M_r^{NE}/\partial\sigma > 0$, if the gap in operating costs between rail transit and bus is large enough ($\sqrt{c_{r1}/M_r^{NE}} - \sqrt{c_{b1}/M_b^{NE}} > 0$), then the sign of (A.18) is positive.

Taking both Equations (34) and (36) into account, when the difference in production cost between rail transit and bus is large enough, it is certain that producer surplus decreases with an increase in the degree of nationalization ($\partial\sum\pi_i/\partial\sigma < 0$).

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