Available online at www.sciencedirect.com

ScienceDirect

Procedia - Social and Behavioral Sciences 111 (2014) 800 – 809

Procedia
Social and Behavioral Sciences

EWGT2013 – 16th Meeting of the EURO Working Group on Transportation

# Performance of route suggestions in networks with correlated link congestion

Achille Fonzone[a,*], Jan-Dirk Schmöcker[b]

[a]Transport Research Institute, Edinburgh Napier University, 10 Colinton Road, Edinburgh, EH10 5DT, United Kingdom
[b]Department of urban management, Kyoto University, C1-2-431, Katsura Nishikyo-ku, Kyoto 615-8540, Japan

**Abstract**

We evaluate the performance of route suggestions which can be adopted when no real time information is available. We consider that when the available information is limited, risk-aversion, regret and disappointment may play an important role in decision making. The effect of link travel time correlation on heuristic route choice efficiency is also explored. Monte Carlo simulation is used to study the performance of heuristic decision making in the Chicago network under different levels of congestion. We conclude that finding the shortest path is more difficult and more important – and therefore the value of real time information is higher – in the presence of positive correlation. A simple local search considering frustration proves the best *a priori* strategy in many circumstances.

© 2013 The Authors. Published by Elsevier Ltd.
Selection and/or peer-review under responsibility of Scientific Committee

*Keywords:* Route Choice; Correlated Travel Times; Risk Aversion; Regret; Disappointment.

## 1. Introduction

Collecting and disseminating reliable real time information for route guidance is still not possible in many cases. In consequence the travellers might have only limited information or they might not perceive the given information as accurate. Depending on taste differences travellers might prefer robust route suggestions rather than being advised to take the probabilistically shortest path that might turn out to be not a good option due to

* Corresponding author. Tel.: +44-131-455-2898.
*E-mail address:* a.fonzone@napier.ac.uk

1877-0428 © 2013 The Authors. Published by Elsevier Ltd.
Selection and/or peer-review under responsibility of Scientific Committee
doi:10.1016/j.sbspro.2014.01.114

unforeseen circumstances. However, the most robust or risk-averse option might also not be fully satisfying as travellers might compare their achieved travel time with "what they could have achieved" on the path that, in hindsight, would have been the fastest option. This traveller thinking motivated Fonzone, Schmocker, Ma, and Fukuda (2012) to develop route choice principles considering regret and disappointment as well as risk-aversion. Their approach aims to be simple and of practical route guidance use in the absence of real time information, and avoids the usage of link choice probabilities. They bring forward two heuristics to select paths *a priori*, i.e. when the minimum and the maximum expected travel times of each link are known but the actual state of the network is not.

In this paper we advance that research by considering that the link states, i.e. the congestion levels on different roads, might be correlated. Correlation affects network operations and hence it determines the efficiency of any route choice approach. We present a methodology which allows drawing conclusions on the absolute and relative efficiency of *a priori* suggested paths and on the value of information with and without congestion correlation. We use our methodology to conduct various performance tests on a large transport network.

That link travel times are correlated is well known and has been a research topic in particular since the development of Advanced Traveler Information Systems (ATIS). Our objective is not to find the link correlation that best reflects actual data but to understand the impact of link correlation patterns per se on route choice. Related to our objective is the contribution of Xing and Zhou (2011) who aim to find the most reliable path under consideration of link travel time correlation. They develop an approach based on Lagrangian substitution assuming that mean and variance of link travel times are given. In their paper they show that by considering correlation the difference between the lower bound (shortest path travel time) and upper bound (expected travel time) can be significantly reduced. They do, however, not discuss the impact of correlation on expected link travel times more generally.

The paper is structured as follows. In the following section we firstly review some previous research on route choice. We then provide a more detailed review of the route guidance introduced in Fonzone et al. (2012). In Section 3 we describe our example network and how link state correlation is simulated. Our actual tests are limited to the comparison between consideration of no correlation and one specific correlation pattern. The effect of correlation on network functioning is analysed in Section 4. In Section 5 the performance of the different route choice principles is shown. Finally some conclusions are drawn.

## 2. Route choice under uncertainty

Finding the shortest path through networks has been a central research topic in various disciplines where one deals with networks and uncertain link travel times. One might group the problem depending on whether the link travel times are assumed static or dynamic. The dynamic shortest path problem has been addressed by e.g. Pallottino and Scutellà (1997) as a discrete time problem or e.g. in Ding, Yu, and Qin (2008) as a continuous time problem. In this research we do not include a time dimension explicitly but the mentioned literature provides route choice methods which can be easily implemented to extend our approach.

We assume that the exact link travel times are unknown. The literature for this case might be again grouped into two main areas. Firstly, there are approaches assuming that link travel time distributions are known such as in the above mentioned Xing and Zhou (2011) paper. In this case the expected travel times and probabilistic shortest paths can be obtained as well as the most reliable paths assuming some required reliability definition. Alternative approaches to risk-averse routing have been proposed based on game theory where travellers do not consider probabilities but fear the occurrence of the "worst case" (Cassir and Bell, 2000). This pessimistic approach can also be used to define adaptive route choice strategies as in Bell (2009) where the traveller selects a risk-averse route set and then chooses links from the choice set "dynamically" once a decision point is reached.

Evidence from research on decision making though shows that travellers apply further criteria in addition to "optimism" versus "pessimism". Research in the transport field has recently been interested in "regret". Regret is experienced when realizing that a better outcome could have been achieved if a different decision had been taken. Savage (1954) is generally considered as the first person defining a comprehensive framework for the consideration of regret in decision making. Chorus, Arentze, and Timmermans (2008) and Chorus (2012) include the concept of regret into travel behaviour decision making. The former paper firstly illustrates that regret can explain mode choice decisions and the latter paper shows that regret can also be applied to explain observed traffic equilibriums.

### *2.1. Review of frustration-based route choice utilised in this research*

Fonzone et al. (2012) develop a framework to include frustration deriving from "counterfactual" thinking in route choice. Counterfactual in this case refers to the anticipation of a scenario that eventually does not occur (Coricelli and Rustichini, 2010). This can cause regret with respect to forgone options but also "disappointment" with regards to the performance of the chosen option. They refer to regret and disappointment collectively as "frustration". In line with above mentioned literature originating from game theoretic approaches travellers are assumed to not know (or not trust) travel time distributions. Rather travellers choose their route considering only two possible states for each link: the optimistic free-flow travel time and the pessimistic delayed travel time occurring in case of extreme congestion. The degree to which 'delay is feared' versus 'free flow time is hoped for' is defined as risk-aversion. Two route choice algorithms are proposed to consider frustration, which are named global and local search. Note that in the following the term "shortest path" is used for a path that minimizes a generalized cost including travel time and frustration and not travel time only.

The global search is a standard shortest path search using modified costs defined as follows. Let the risk-averse link cost $r_a$ be defined as in (1) where $k_a^{\varphi}$ denotes the link cost in the optimistic case $\varphi$, $k_a^{\delta}$ the link cost in the pessimistic case $\delta$, and $\alpha$ the level of risk aversion.

$$r_a = (1-\alpha)\cdot k_a^{\varphi} + \alpha \cdot k_a^{\delta} \tag{1}$$

Further let $f_a$ be the anticipated frustration on link $a$ which derives from 1) the disappointment which is experienced if a path using $a$ is selected and the encountered travel time on $a$ is different from that considered in route choice; 2) the regret which is generated if a path using $a$ is selected but paths using competing links from the tail node of $a$ occur to be more convenient. As even in the simple case assuming only two link states the number of network congestion state scenarios becomes very large, Fonzone et al. (2012) specify a frustration function in which travellers evaluate only extreme scenarios: consideration of uncongested versus congested travel time on the immediate downstream link and consideration of best versus worst travel time to reach the destination from the head node of the downstream links under consideration. They further introduce an exponential evaluation of regret with parameter $\beta$. The larger $\beta$, the more important is the consideration of frustration in route choice. Extending the approach of Loomes and Sugden (1982) who propose a utility function that is a sum of the "choiceless" utility of an option and of a regret-rejoice function of the utilities of the considered alternatives, we suggest to use the modified link costs $k'_a$ in (2) in the search for the shortest path.

$$k'_a = r_a + f_a \tag{2}$$

As a fast alternative concept the local search algorithm includes the frustration into the potentials of the head node of the links. In the absence of travel time distributions at each node the optimistic potential is defined as the cost of the shortest path from the node to the destination in free flow conditions. The pessimistic potential is the cost of the shortest path from the node to the destination when all links are in their delayed state. These node potentials are then utilised in the link-to-link route choice as follows. In line with the above notation let $h^{\varphi}_{a^+}$ and $h^{\delta}_{a^+}$ be the optimistic and pessimistic potentials of the head node of link $a$. Let

$$g'_a = k'_a + (1-\alpha)\cdot h^{\varphi}_{a^+} + \alpha \cdot h^{\delta}_{a^+} \tag{3}$$

denote the anticipated cost from the tail of $a$ taking into account frustration. Starting from the origin the path is built up sequentially by adding at each node the link with the least expected $g'_a$. For further details on the frustration function as well as on the two shortest path algorithms we refer the reader to the paper of Fonzone et al. (2012).

In the following we study the performance of these two suggested paths, abbreviated as GloSP and LocSP respectively, plus the optimistic and pessimistic shortest paths, abbreviated as OptSP and PesSP. The latter are shortest paths obtained by considering free flow conditions (OptSP) or the worst case situation where all links of the network are congested. As we determine these routes without knowledge of the actual network situation we refer to these also as "heuristics" or "*a priori* shortest paths" in order to distinguish them from the actual shortest path under a specific network condition. Our four route suggestions hence reflect the situation of a static route guidance device. Finally, as the actual shortest path is obtained by drawing or simulating a network condition from the link travel time distributions, we refer to this path as SimSP.

## 3. Monte Carlo simulation

Since no dataset was available to us that includes information on link travel time joint or marginal distributions and their correlation, we demonstrate the performance of suggested paths using simulated data. Link travel times are commonly assumed lognormal distributed. Calculation of expected values and confidence intervals with large multivariate lognormal distributions is intractable analytically. Therefore we adopt a Monte Carlo approach, in which estimation of probabilities and expectations relies on the analysis of independent simulations of the stochastic phenomenon: $r$ particular realizations of the stochastic variables of concern are randomly extracted replicating their known probability functions. The phenomenon whose probability distribution is unknown (in our case the performance of the suggested paths) is observed under each realization and the unknown probability function is derived by analyzing the results of the $r$ experiments (Asmussen and Glynn, 2007).

### *3.1. Network*

In our simulations we make use of a modified version of the Chicago Sketch Network provided by Bar-Gera (2011). The network we consider is made up of 2096 links and 541 nodes (Fig. 1a). Coefficients are provided by Bar-Gera to obtain the travel time with a classic BPR type volume-delay function (Bureau of Public Roads, 1964). These have been used to generate the link travel time marginal distributions and a semi-positive covariance matrix $\Sigma$ on which the Monte Carlo simulations are based. The definition of the "optimistic" and "pessimistic" expectations will depend on the decision maker's preferences. Their derivation exceeds the scope of this paper. In the following the optimistic travel time is arbitrarily assumed equal to the $5^{th}$ percentile of the travel time distributions (and coinciding with the free flow time of the BPR functions), the pessimistic one to the $90^{th}$. $\Sigma$ has been randomly generated so that the correlation between link travel times decreases with the distance between the links as shown in Fig. 1b. This means that close links tend to have the same level of congestion whereas the delays on distant links are only weakly related.

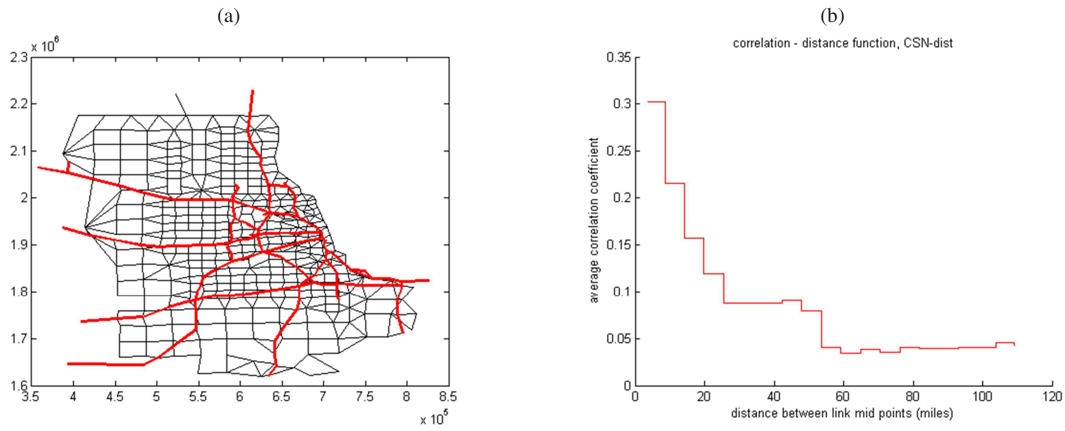


Fig. 1. (a) Network map; (b) Average correlation between link travel times

### *3.2. Simulation*

In this paper a "run" is a particular realisation of link travel times under which the performance of the four route choice heuristics is evaluated. The values of travel times in each run are calculated as follows:

1) Extract randomly $n$ values $z_a$ from the standard normal distribution, where $n$ is the number of links.
2) Calculate $Y = M^TZ$, where $Z = [z_1 \ldots z_n]^T$ and M = C when correlation is considered, M = I when not. C is the Cholesky decomposition of Σ (i.e. the matrix such that $C^TC = \Sigma$. Note that C exists because Σ is semi-positive).
3) Calculate the travel time on link $a$ as $\overline{k}_a = \exp\left(y_a + \mu_a\right)$, where $\mu_a$ is the mean of the (lognormal) distribution of travel time on $a$.

Further, a "trial" is a set of 10,000 runs for the same origin and destination, network characteristics and route choice attitudes. The *a priori* routes are selected on the basis of values of travel time which are characteristic of the travel time distribution functions. Therefore they are common to all the runs in a trial whereas the actual shortest path between origin and destination changes from run to run. The following assumes $\alpha = 0.5$. This means that the traveller considers optimistic as well as pessimistic travel times to the same degree. We further set the frustration parameter $\beta$ to 0.65 following a calibration of regret described in Chorus, Arentze, and Timmermans (2009), though admittedly there regret was not calibrated for route choice.

## 4. Network performance

### *4.1. Link travel times*

We firstly show the impact of correlation on link travel times. For illustration purposes we select three links, namely 803, 784 and 1628. These links are chosen considering the correlation of travel times: Links 784 and 803 have the highest correlation (0.43) in the entire network whereas Link 1628 has been chosen because it is the one with the smallest correlation with Link 803 (0.0002). The correlation between 784 and 1628 is 0.017. Fig. 2

shows the results of one representative trial. On the diagonal the histograms of travel times on single links can be seen (803 in the top left cell, 1628 in the bottom right cell). Off the diagonal are the scatter plots of travel times on pairs of links. Below the x-axes the correlation between link travel times that occurred in the specific trial are reported. Note that the plots with and without correlation have different scales.

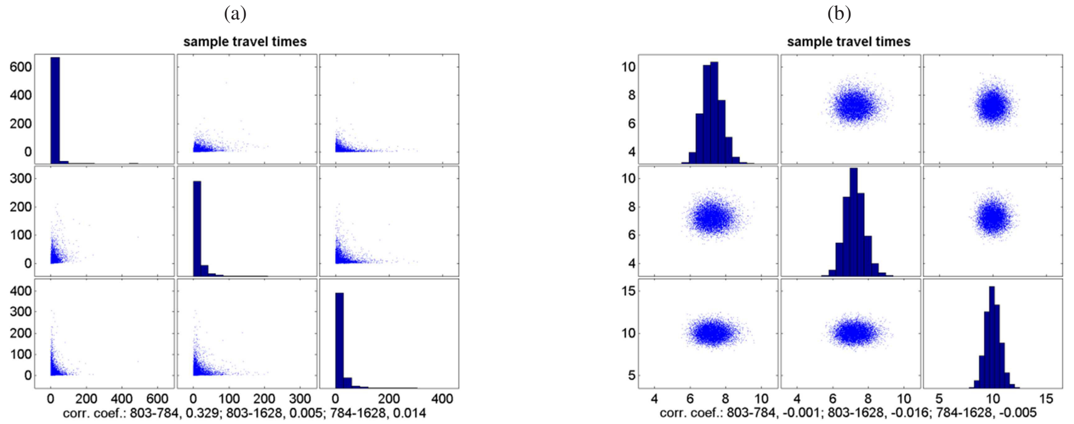


Fig. 2. Distribution (on the diagonals) and correlation of link travel times for links 803, 784 and 1628 (in this order both from top to bottom and from left to right): (a): With Correlation; (b) Without correlation

In the network without correlation no pattern can be observed in the scatter plots, whereas the triangular clouds in the figure concerning the network with correlation shows that link travel times are (to some degree) correlated. The coefficients are low even when congestion correlation is considered; hence data is not aligned along the bisector. When correlation is considered, the average travel times of the three sample links are higher than the expected values calculated by their distribution. The ratios of the average costs (over the 10,000 runs) with and without congestion are 1.64, 1.63 and 1.64 for links 803, 784, and 1628 respectively. The increase of the average costs can be ascribed to the combined effect of the characteristics of the covariance matrix $\Sigma$ and of the shape of the lognormal distribution. All covariances are positive in our simulation which means that in a run the links tend to have the same level of congestion, so that when high congestion occurs in one link it spills over the whole network. This amplifies the influence of the asymmetry of the lognormal distributions on the mean link travel times: In fact the leveling effect of applying C makes both low and high extreme values of the distributions more frequent than in the case without correlation. However since the left tail of the lognormal distribution is much longer than the right one, having more frequent extreme values pushes the mean travel time towards high values. We acknowledge that given the size of the Chicago Sketch Network the assumption on the positivity of all the covariances might not be realistic. However, this is only partly relevant for the objectives of this paper where our focus is to prove the influence of congestion correlation and not to draw conclusions as to the specific network. Nevertheless different structures of correlation should be tested, e.g. situations in which overloading of some areas corresponds to free flow conditions in others and so covariances of links in different areas are negative.

### *4.2. OD travel times*

To study the variability of travel times we divide the test network area into 9 quadrants of equal size. We select an origin in the NW quadrant and extract randomly one destination in each quadrant. Fig. 3 shows the distribution of the travel times between these OD pairs for the same trial considered in Fig. 2. Interestingly although link travel times are 60-70% higher with correlation than without, the (over the 10,000 runs) averaged SimSP travel time between different OD pairs are significantly shorter in the former case. The ratio of the average costs of the SimSPs with and without correlation ranges from 0.79 to 0.94, with a tendency to decrease for increasing values of SimSP cost. In the network without correlation occurrence of congestion is completely random whereas correlation somehow induces an ordered congestion pattern in which some areas are not/less congested. It follows that in the former case it is more difficult to select paths which avoid badly congested links completely. This means that availability of perfect information is more valuable when correlation is present because in that case shorter paths are more likely to exist. This is in line with the findings by Xing and Zhou (2011) who find lower gaps between the upper and lower bound of robust shortest paths in case correlation is considered. The rank of the distances between OD pairs is not altered by the presence of correlation but in one case: i.e. if a destination is further (in terms of shortest path) without consideration of correlation, it is also further with consideration of correlation. As expected more distant destinations have also higher variability of shortest path length. The variability is not influenced by the presence of correlation.

(a) (b)

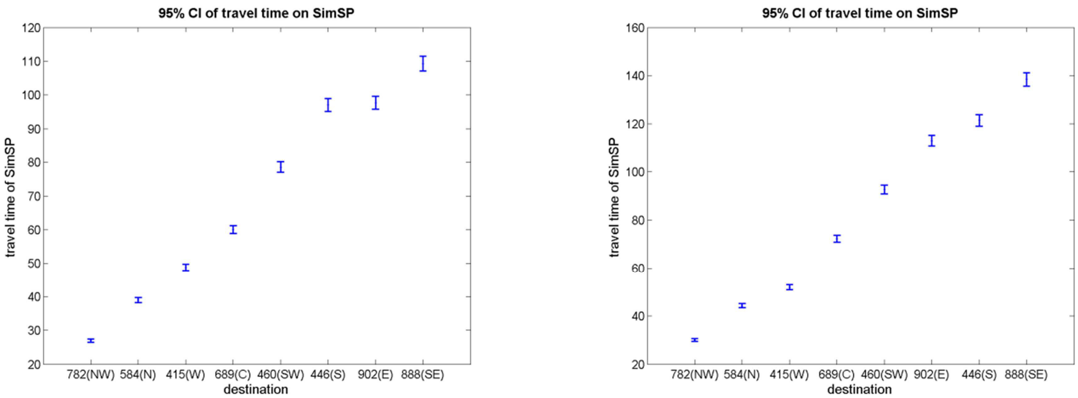


Fig. 3. OD travel times between different OD pairs: (a): With Correlation; (b) Without correlation

## 5. Performance of route suggestions

In the following we compare the performance of the four route choice approaches (LocSP, GloSP, OptSP and PesSP). For this we use two measures: 1) the ratio between the cost of the four suggested paths and that of the actual shortest path (SimSP); 2) the number of runs within a trial that one suggested path turns out to be the best among the four. We refer to these indicators as “inefficiency” and “optimality” respectively.

We study the performance of the *a priori* suggested path by considering the same abovementioned OD pairs. When congestion correlation is present the four heuristics perform worse than without correlation (Fig. 4, note the difference in the y-axis scale between Fig. 4a and 4b). This inefficiency increase can be explained by the combined effect of increasing link travel times (due to the generally higher travel times under the assumption of positive covariance coefficients) and decreasing shortest path costs (related to the congestion pattern induced by the correlation) with respect to the network without correlation. Investigating the performance in more detail we also find that the overlap between suggested links and the links making up the SimSP is smaller in the case with correlation, i.e. route finding is more difficult without ubiquitous and real-time information. LocSP and PesSP perform similarly well. In the non-correlated case the difference between their cost and the cost of the actual shortest path is less than 5% for all OD pairs. In the correlated case the performance is not as good but still in most cases better than for other route suggestions.

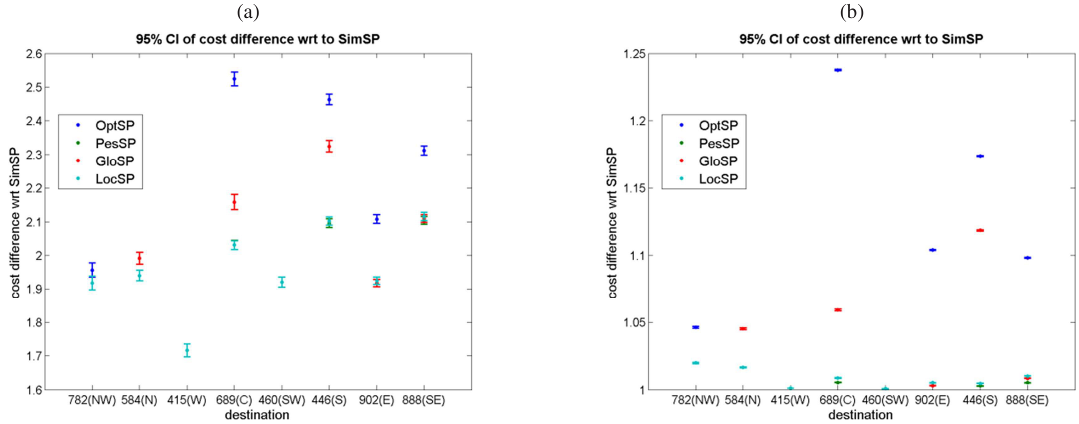


Fig. 4. Inefficiency CI for different OD pairs: (a): With Correlation; (b) Without correlation

Similar to inefficiency, the optimality is significantly dependent on the considered OD pair as illustrated in Fig. 5. Note that when the sum of optimality of the four heuristics is equal to the number of runs (10k) – as in the case of destinations 689(C), 446(S), and 888(SE) – the suggested paths are distinct. Instead when the four suggested paths are identical the sum is 4*10k, as for ODs 415(W) and 460(SW). In the network with congestion correlation, GloSP and PesSP are the best heuristics in terms of optimality, i.e. PesSP has the minimum cost among those of all the *a priori* selected paths in 52.5% of all runs for all OD pairs, GloSP in 54.5% cases. PesSP outperforms the other heuristics in 1 trial (888(SE)), GloSP in 2 (689(C) and 446(S)). When there is no congestion correlation, GloSP proves sub-optimal in most cases. Its optimality is nil in half the trials. Instead, the likelihood of PesSP being the optimal path increases for all OD pairs when there is no congestion correlation. The reduced performance of GloSP might be related to the fact that the frustration of each link is defined in such a way that it contains the information on the potential cost of the whole downstream path. Consider two links *a* and *b* such that *b* follows *a* on a path between a given OD pair. The modified cost of *b* as defined by (2) includes the frustration feared when using the path from the head of *b* to the destination. Similarly the modified cost of *a* depends on the frustration of the path from the head of *a* to the destination, therefore also on the frustration associated with the path from the head of *b* to the destination. Hence the sum of the modified utilities of *a* and *b* double-counts the frustration from *b* to the destination. Because of this issue when using GloSP the fear of mistakes tends to become much more important than the consideration of the actual costs in choosing a path. This attitude looks rewarding when a congestion pattern exists and therefore not many congested links are encountered if the congested areas are avoided, but it is not optimal when congested links are not grouped. Note that LocSP also considers frustration but the "double counting" issue does not arise because the search is local, i.e. at each node only a next link with the minimum modified utility is recommended.

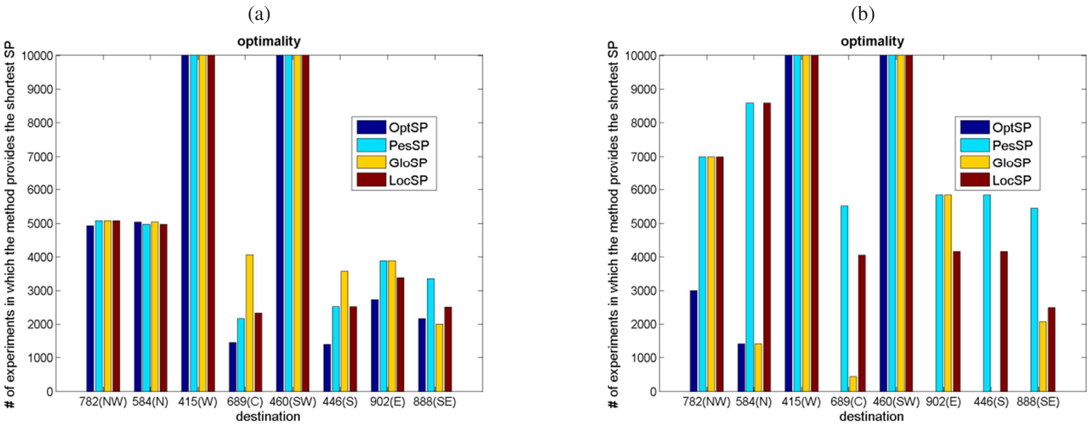


Fig. 5. Optimality CI: (a): With Correlation; (b) Without correlation

Fig. 4 and Fig. 5 illustrate that the best suggested path in terms of optimality is not necessarily the best in terms of expected travel times: for destinations 689(C) and 446(S), adopting GloSP a traveller would reach his target earlier more often than using other *a priori* paths but on average he would spend more time than using PesSP or LocSP.

## 6. Conclusions

This paper demonstrates the effect of travel time correlation on network performance and route choice. Though link correlation has been looked at in the literature we find only limited work analysing the effect of correlation on the performance of route choice principles. We explore the effect of including frustration caused by regret and disappointment in route choice to cope with lack of real time information with and without congestion correlation.

Our case study proves that correlation has relevant effects on network performance and hence on the efficiency of route choice approaches. We illustrate that in case of positive correlation between link travel times it is more difficult and more important to find the actual shortest path. With "difficult" we mean that the *a priori* suggested paths tend to differ more from the actual shortest path. With "important" we mean that *a priori* suggested paths perform significantly worse than the actual shortest path. This suggests that the importance of providing and forecasting actual travel times increases in case link travel time correlation is likely.

We find that in our network the possibility that something goes wrong should be taken into account when congestion is possible but no real time information is available. We test four different *a priori* approaches to route choice and find that path recommendation based on a local search considering risk aversion and frustration performs well under the consideration of correlation. This is a promising result for route guidance applications as this search algorithm is also computationally more efficient than a "global search". Our results show that the route choice heuristic providing the on average shortest path suggestion is not necessarily the best suggestion in the most number of times.

Further analysis on the effects of correlation should be carried out using different networks, different link correlation patterns and/or congestion levels. This work might highlight the conditions under which our simple heuristics can compete with more complex or real time data-hungry approaches. Also the use of *a priori* information, i.e. how the optimistic and pessimistic travel times considered in route choice are defined, the calibration of the parameters describing the attitude towards risk and frustration, as well as different specifications of the cost functions should be explored. Finally, adaptive route choice combining real time information (possibly discounted as discussed in Koutsopoulos and Xu (1993)) with *a priori* shortest paths may provide efficient navigation advice. In this vein, in future development we are aiming to develop a tractable version of the Dong, Vu, and Vo (2011) model on real time route guidance making use of *a priori* shortest paths.

## References

Asmussen, S., & Glynn, P. W. (2007). *Stochastic simulation : algorithms and analysis.* New York: Springer.

Bar-Gera, H. (2011). *Transportation Network Test Problems.* [online] available at http://www.bgu.ac.il/~bargera/tntp/. [accessed 2012].

Bell, M. G. H. (2009). Hyperstar: A multi-path Astar algorithm for risk averse vehicle navigation. *Transportation research. Part B, Methodological, 43*, 97-107.

Bureau of Public Roads. (1964). *Traffic assignment manual for application with a large, high speed computer.* Washington,: U. S. Dept. of Commerce, Bureau of Public Roads, Office of Planning.

Cassir, C., & Bell, M. G. H. (2000). The n+m person game approach to network reliability. In M. G. H. Bell & C. Cassir (Eds.), *Reliability of Transport Networks* (pp. 91-102). Baldock, Hertfordshire, England: Research Studies Press.

Chorus, C. G. (2012). Regret theory-based route choices and traffic equilibria. *Transportmetrica, 8*, 291-305.

Chorus, C. G., Arentze, T. A., & Timmermans, H. J. P. (2008). A random regret-minimization model of travel choice. *Transportation Research Part B-Methodological, 42*, 1-18.

Chorus, C. G., Arentze, T. A., & Timmermans, H. J. P. (2009). Spatial choice: a matter of utility or regret? *Environment and Planning B-Planning & Design, 36*, 538-551.

Coricelli, G., & Rustichini, A. (2010). Counterfactual thinking and emotions: regret and envy learning. *Philosophical Transactions of the Royal Society B-Biological Sciences, 365*, 241-247.

Ding, B., Yu, J. X., & Qin, L. (2008). Finding time-dependent shortest paths over large graphs. In *Proceedings of the 11th international conference on Extending database technology: Advances in database technology*. Nantes, France: ACM.

Dong, W., Vu, H. L., & Vo, Q. B. (2011). Real Time Route Guidance with Correlated Link Cost. *2011 14th International Ieee Conference on Intelligent Transportation Systems (Itsc)*, 1039-1044.

Fonzone, A., Schmocker, J. D., Ma, J. S., & Fukuda, D. (2012). Link-Based Route Choice Considering Risk Aversion, Disappointment, and Regret. *Transportation Research Record*, 119-128.

Koutsopoulos, H., & Xu, H. (1993). An information discounting routing strategy for advanced traveler information systems. *Transportation Research Part C-Emerging Technologies, 1*, 249-264.

Loomes, G., & Sugden, R. (1982). Regret Theory - an Alternative Theory of Rational Choice under Uncertainty. *Economic Journal, 92*, 805-824.

Pallottino, S., & Scutellà, M. G. (1997). *Shortest Path Algorithms in Transportation models: classical and innovative aspects.* University of Pisa.

Xing, T., & Zhou, X. S. (2011). Finding the most reliable path with and without link travel time correlation: A Lagrangian substitution based approach. *Transportation Research Part B-Methodological, 45*, 1660-1679.