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CONVENIENT AND ACCURATE FORMULAS FOR STRESS INTENSITY FACTOR DISTRIBUTION OF SEMI-ELLIPTICAL SURFACE CRACK

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In this paper, the stress intensity factor (SIF) formula F_{ISE} along the crack front of a semi-elliptical surface crack is studied. The exact SIF solution F_{ISE} is used by solving the hypersingular integral equation of the body force method discussed in the previous paper. To obtain the accurate formula, the SIF ratio F_{ISE}/F_{IE} is focused considering the exact solution F_{IE} of an elliptical crack. By applying the least squares method to the ratio F_{ISE}/F_{IE}, accurate and convenient formula is proposed. The proposed formulas may provide the accurate SIF distributions for the aspect ratio a/b=1~4 better than 0.2% accuracy.

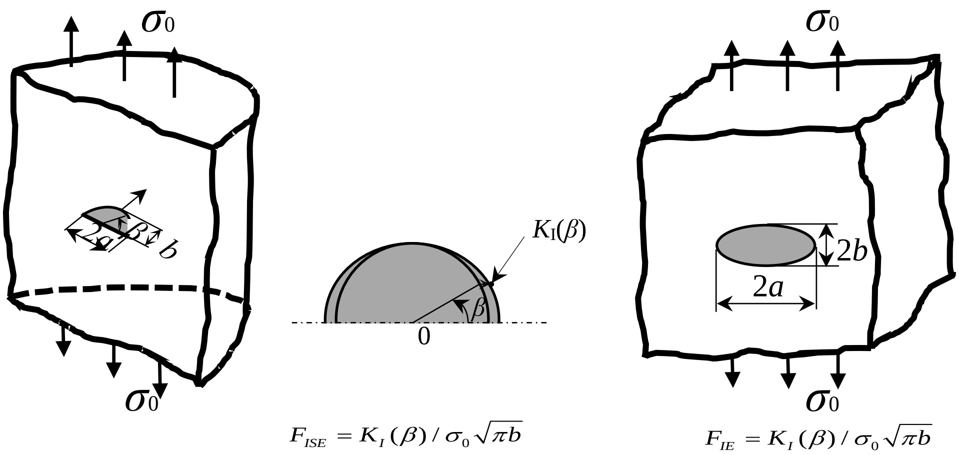
Keywords: Stress Intensity Factor; Semi-elliptical Crack; Elliptical Crack; Approximate Formula.

Notations

The notations used in this paper are summarized below.
ν: Poisson's ratio (= 0.3)
F_{ISE} : Dimensionless stress intensity factor (SIF) of a semi-circular surface crack defined by $F_{ISE}(\beta) = K_I(\beta)/\sigma_0\sqrt{\pi b}$ where $K_I(\beta)$ is the SIF of a semi-circular surface crack
F_{IE} : Dimensionless stress intensity factor of an elliptical cracks defined by $F_{IE}(\beta) = K_I(\beta)/\sigma_0\sqrt{\pi b}$
$F_{ISE}^{\max}$, $F_{ISE}^{\min}$: Maximum and minimum values of dimensionless stress intensity factor for the semi-elliptical surface crack in semi-infinite body
β: Eccentric angle of ellipse (°)
a, b: Major and minor radius of a semi-elliptical crack

1. Introduction

The stress intensity factor (SIF) of a semi-elliptical surface crack lying perpendicular to the surface in Fig. 1 (a) has been used as a fundamental model of actual defects. [1-5] Here, the SIF of an elliptical crack F_{IE} in Fig. 1 (b) is also considered [6] as a reference solution representing an internal defect to clarify the free surface effect. In the previous studies, [7,8] the SIF distributions F_{ISE} were exactly provided for the semi-elliptical crack for a/b=1, 4/3, 2, 4. In this study, the least squares method is applied to the SIF ratio F_{ISE} / F_{IE} Then, a highly accurate calculation formula [9] is proposed for arbitrary *a*/*b*.



(a) Semi-elliptical surface crack in a semi-infinite body (b) Elliptical crack in an infinite body

Fig. 1. Semi-elliptical surface crack and elliptical crack

2. Convenient formula for the SIF distribution of semi-elliptical surface cracks

Eq. (1) denotes the exact solution for the elliptical crack in Fig.1(b). [9] Eq. (2.a) is the formula obtained by the least square method useful for 1°≦β≦20° and 1.0≦*a*/*b*≦4.0. Eq. (2.b) is the obtained formula useful for 10°≦β≦90° and 1.0≦*a*/*b*≦4.0. The F_{ISE} value can be obtained by those formulas with less than 0.2% error for the whole range of *a*/*b* = 1.0~4.0.

Fig.2 illustrates $F_{ISE}(\beta)/F_{IE}(\beta)$ obtained from Eq. (2) and Fig.3 illustrates F_{ISE} (β) from Eq.(1) and Eq. (2). From Fig. 3, it is seen that the maximum value of F_{ISE} appears at $\beta = 3°$ when *a*/*b* = 1.0 ~ 1.27 and at $\beta = 90°$ when *a*/*b* ≥ 1.27. The difference between $F_{ISE}(\beta)$ and $F_{IE}(\beta)$ becomes larger around $\beta = 0$. This is due to the corner point singularity. At the corner point $\beta = 0$, the singular index is different from the singular index at $\beta \neq 0$. Therefore, K_{ISE} behaves in a complicated manner near the corner point $\beta = 0$, and finally $K_{ISE} \to 0$ as $\beta \to 0$. [10,11]

$$K_I = \frac{\sigma}{E(k)}\left(\frac{\pi b}{a}\right)^{1/2}(a^2\sin^2\beta + b^2\sin^2\beta)^{\frac{1}{4}} = \frac{\sigma}{E(k)}\left(\frac{\pi b}{a}\right)^{1/2}\left(\frac{a^2(a/b)^2\tan^2\theta + b^2}{1+(a/b)^2\tan^2\theta}\right)^{1/4}$$

$$F_{IE} = \frac{K_I}{\sigma\sqrt{\pi b}} = \frac{1}{E(k)}(\sin^2\beta + (b/a)^2\cos^2\beta)^{1/4} = \frac{1}{E(k)}\left(\frac{(a/b)^2\tan^2\theta + (b/a)^2}{1+(a/b)^2\tan^2\theta}\right)^{1/4}$$

$$For \quad a \geq b, \quad K = \left(1-\frac{b^2}{a^2}\right)^{1/2}, \quad E(k) = \int_0^{\frac{\pi}{2}}(1-k^2\sin^2\phi)^{1/2}\, d\phi$$

$$For \quad a < b, \quad K_I = \left(1-\frac{a^2}{b^2}\right)^{1/2}, \quad E(k) = \frac{b}{a}E(k_I) \tag{1}$$

When 1°≦β≦20° and 1.0≦a/b≦4.0;

$$
\begin{aligned}
F_{ISE}/F_{IE} &= 1.5713 - 1.1221(b/a) + 1.0408(b/a)^2 - 0.34133(b/a)^3 \\
&+(-0.18718 + 0.56557(b/a) - 0.53103(b/a)^2 + 0.17466(b/a)^3)\beta \\
&+(0.032013 - 0.085978(b/a) + 0.063788(b/a)^2 - 0.015795(b/a)^3)\beta^2 \\
&+(-0.0026331 + 0.0054534(b/a) - 0.0017899(b/a)^2 - 0.00046795(b/a)^3)\beta^3 \\
&+(0.00010321 - 0.00013791(b/a) - 8.9406\times10^{-5}(b/a)^2 + 0.00010013(b/a)^3)\beta^4 \\
&+(-1.5484\times10^{-6} + 8.3237\times10^{-7}(b/a) + 3.9337\times10^{-6}(b/a)^2 - 2.833\times10^{-6}(b/a)^3)\beta^5 \quad (2.a)
\end{aligned}
$$

When 10°≦β≦90° and 1.0≦a/b≦4.0;

$$
\begin{aligned}
F_{ISE}/F_{IE} &= 1.1879 + 0.21553(b/a) - 0.3688(b/a)^2 + 0.15787(b/a)^3 \\
&+(-0.00712 - 0.010195(b/a) + 0.018019(b/a)^2 - 0.0081504(b/a)^3)\beta \\
&+(0.00029402 - 1.8046\times10^{-5}(b/a) - 0.00025984(b/a)^2 + 0.00014923(b/a)^3)\beta^2 \\
&+(-5.6483\times10^{-6} + 4.0129\times10^{-6}(b/a) + 5.2159\times10^{-7}(b/a)^2 - 9.9946\times10^{-7}(b/a)^3)\beta^3 \\
&+(5.1738\times10^{-8} - 5.46\times10^{-8}(b/a) + 1.7392\times10^{-8}(b/a)^2 - 6.4006\times10^{-11}(b/a)^3)\beta^4 \\
&+(-1.8135\times10^{-10} + 2.2527\times10^{-10}(b/a) - 9.5656\times10^{-11}(b/a)^2 + 1.1776\times10^{-11}(b/a)^3)\beta^5 \quad (2.b)
\end{aligned}
$$

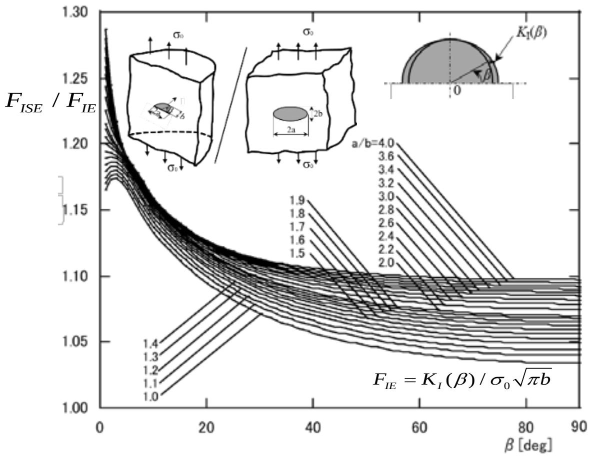


Fig.2. F_{ISE}/F_{IE} from Eq.(2)

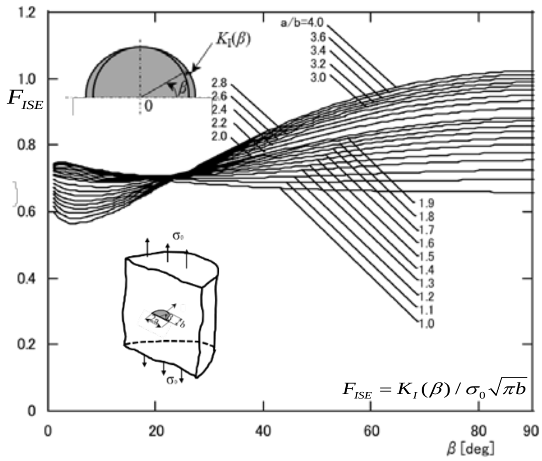


Fig.3. F_{ISE} from Eqs. (1) and (2)

3. Under which aspect ratio *a/b* the SIF becomes nearly constant?

Table 1 shows the maximum and the minimum SIF of F_{IE} (β) . The average SIF value $F_{ISE}^{ave.}$ and standard deviation F_{ISE}^{SD} are also indicated. It is seen that when $a/b \cong 1.2$, F_{ISE} (β) $\cong$ constant. The results suggested that the fatigue crack may propagate under the ratio $a/b = 1.2$.

Table. 1 Stress intensity factor F_{ISE}^{max} , F_{ISE}^{min} , $F_{ISE}^{ave.}$, F_{ISE}^{SD} (ν=0.3)

	a/b							
	1.0	1.18	1.19	1.20	1.21	1.22	1.23	4.0
F_{ISE}^{max}	0.747	0.747	0.747	0.747	0.747	0.746	0.746	1.024
	at $\beta = 3°$	at $\beta = 3°$	at $\beta = 3°$	at $\beta = 3°$	at $\beta = 3°$	at $\beta = 3°$	at $\beta = 3°$	at $\beta = 90°$
F_{ISE}^{min}	0.659	0.699	0.699	0.700	0.701	0.702	0.703	0.564
	at $\beta =$	at $\beta =$	at $\beta =$	at $\beta =$	at $\beta =$	at $\beta =$	at $\beta =$	at $\beta =$
	90°	34°	32°	32°	31°	30°	29°	5°
$F_{ISE}^{ave.}$	0.680	0.711	0.713	0.714	0.716	0.717	0.718	0.837
±SD	±0.026	±0.012	±0.012	±0.012	±0.012	±0.012	±0.012	±0.158

4. Conclusions

In this study, the convenient formula F_{ISE} along the crack front of the semi-elliptical surface crack was proposed. The conclusions can be summarized in the following way.

(1) To obtain the accurate formula, the SIF ratio F_{ISE} / F_{IE} was focused on the basis of the exact solution of an elliptical crack F_{IE}. A convenient SIF formulas was proposed for *a*/*b* = 1.0 ~ 4.0 better than 0.2% accuracy.

(2) It is found that the maximum value of F_{ISE} appears at $\beta = 3°$ when *a*/*b* = 1.0 ~ 1.27 and at $\beta = 90°$ when *a*/*b* $\geq$1.27.

(3) When *a*/*b* $\cong$ 1.2, the SIF F_{ISE} (β) $\cong$ constant. along the crack front. The results suggested that the fatigue crack may propagate under the ratio *a*/*b* = 1.2.

Acknowledgments

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